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Three-Dimensional Cone FIR Filters Design using the McClellan Transform

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Abstract- **This paper discusses the design of three-dimensional (3-D) FIR cone-shaped filters using the McClellan transform. The new approach proposes a closed form analytical solution for the transform coefficients in design of this type of filers. A simultaneous determination of the 1-D cut-off frequency of the prototype filter is also included in the method. Further, some discussion is addressed to the design of a scaled transform. The performance of the generated transformation function is presented, followed by several 3-D magnitude responses obtained with the proposed method. The newly derived expressions are very attractive for real-time adaptive filter design.**

I. Introduction

Recently, multidimensional (M-D) data processing has been widely applied in different areas (geophysics, biomedical engineering, etc.). A very important class of M-D filters is the FIR class. Several methods exist in the literature for design of this type of filters. One of the most popular design techniques is based on the McClellan transformation [1] of a linear phase 1-D FIR filter prototype. This transformation method proposes an efficient and flexible technique for designing M-D filters.

A few different approaches suitable for design of 3-D FIR filters have been also presented. Some of them are obtained as a generalization of existing techniques for 2-D filters. Design of 3-D cone-shaped filters using McClellan transform is given in [2]. In order to construct transformation function, Runze [2] used a second order Lagrange interpolation polynomial with some limitations on choosing slope of the cone. Further contribution on design of 3-D cone filters is done in work [3] applying numerical optimization and 3-D discrete Fourier transform (DFT) to obtain 3-D transfer function. Some of the authors consider design of 3-D cone filters (e.g. [4]) as a special example of their methods derived for M-D symmetric FIR filters. 3-D spatio-temporal cone filter banks have been constructed [5,6], as well.

Three- and higher-dimensional FIR filters are designed in [7], using least squares (LS) method and closed-form solution. The 3-D octagonally symmetric filter is designed to demonstrate applicability of this approach. Ogle and Smith [8] proposed new McClellan transform method for design of M-D FIR filters where the coefficients of transform function are obtained using LS optimization along to a series of contour points. They presented a design example with 3-D ellipsoidal magnitude response. M-D multiplierless hyperspherically symmetric FIR filters are derived by Charalambous [9] generalizing McClellan transform method for 2-D filters. A new 3-D filtering approach, based on the transform method and its application in biomedical engineering (for brain potential mapping) are shown in [10].

The purpose of this paper is to present new analytical method for design of 3-D FIR cone-shaped filters using the McClellan transform. We wish to obtain scaled transform function and to determine simultaneously the value of optimal 1-D cut-off frequency. Description of the new approach will be shown in Section II of the work. Section III deals with the scaling problem of transformation function. Some simulation results and conclusions are presented in the last two Sections.

II. Description of the Method

A. *The McClellan transform*

We start our investigations with the first order McClellan transform in three frequency variables:

$$\begin{aligned} F_3(\omega_1,\omega_2,\omega_3) &= t_{000} + t_{001}\cos(\omega_3) + t_{010}\cos(\omega_2) + \\ &+ t_{011}\cos(\omega_2)\cos(\omega_3) + t_{100}\cos(\omega_1) + t_{101}\cos(\omega_1)\cos(\omega_3) + \\ &+ t_{110}\cos(\omega_1)\cos(\omega_2) + t_{111}\cos(\omega_1)\cos(\omega_2)\cos(\omega_3) \end{aligned} \tag{1}$$

which is one choice from the original McClellan transform from (I,J,K)-order:

$$F_{OM}(\omega_1,\omega_2,\omega_3) = \sum_{i=0}^{I}\sum_{j=0}^{J}\sum_{k=0}^{K} t_{ijk}\cos(i\omega_1)\cos(j\omega_2)\cos(k\omega_3).$$

Let we consider a 1-D zero-phase FIR filter of odd length $2N+1$ with a frequency response:

$$H(\omega) = \sum_{n=0}^{N} a(n)T_n[\cos(\omega)],$$

where $T_n[\cdot]$ is the n-th order Chebyshev polynomial [11] and the coefficients $a(n)$ can be expressed in terms of the impulse response samples of the filter. Applying McClellan transform, the 3-D frequency response is obtained using the substitution $\cos(\omega)=F_3(\omega_1,\omega_2,\omega_3)$ as follow:

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$$H(\omega_1,\omega_2,\omega_3) = H(\omega)\Big|_{\cos(\omega)=F_3(\omega_1,\omega_2,\omega_3)} =$$
$$= \sum_{n=0}^{N} a(n) T_n | F_3(\omega_1,\omega_2,\omega_3) |,$$

where ω is the 1-D frequency variable and $(\omega_1,\omega_2,\omega_3)$ is a point in the 3-D space. We denote below 1-D cut-off frequency of the prototype filter with ω_c. In order to obtain a scaled transform function the following condition should be satisfied:

$$-1 \le F_3(\omega_1,\omega_2,\omega_3) \le 1, \quad \forall \omega_1,\omega_2,\omega_3 \in [-\pi,\pi]. \qquad (2)$$

The 3-D surfaces created by McClellan transformation will be called isopotential surfaces. It is known [1,11], that transform function $F_3(\omega_1,\omega_2,\omega_3)$ defines the shapes of isopotential surfaces of designed 3-D filters. The values of the magnitude along these surfaces will be determined by 1-D filter response.

B. Closed-form relations for transform parameters and 1-D cut-off frequency

We consider below design problem of a special type of 3-D FIR filter with a conical type of magnitude response. The ideal double cone filter oriented in ω_3 direction is given in Fig.1. The following equation holds on the surface of the cone:

$$\omega_1^2 + \omega_2^2 - \frac{\omega_3^2}{\gamma^2} = 0, \qquad (3)$$

where γ is the slope of the cone and determines the angle between the cone-surface and (ω_1,ω_2)-plane. In other words, the angle of inclination of the cone θ could be written as:

$$\theta = \arctan(\gamma).$$

Further, we impose the following four conditions on the transform function:

$$\begin{aligned} F_3(0,0,\pi) &= \cos(0) = 1 \\ F_3(\pi,\pi,\pi) &= \cos(\pi) = -1 \\ F_3(\pi,0,0) &= \cos(\pi) = -1 \\ F_3(0,\pi,0) &= \cos(\pi) = -1 \end{aligned} \qquad (4)$$

These conditions are necessary to have a true mapping from 1-D lowpass to 3-D lowpass cone filter. With the first condition

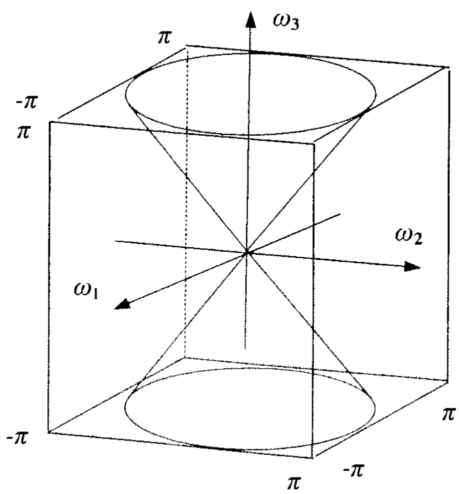

Fig. 1. Ideal frequency response of a cone filter (with γ=1).

we require mapping of 1-D point ω=0 into the point $(0,0,\pi)$ from passband of the cone. Analogously, the π-point (1-D plane) is transformed into the 3-D stopband point (π,π,π). As the points $(\pi,0,0)$ and $(0,\pi,0)$ are outside of the cone, we assume validity of the last two conditions of (4).

The above conditions allow us to reduce the number of transform parameters from eight to four. Using (4), we get the following system of equations:

$$\left| \begin{aligned} t_{000} - t_{001} + t_{010} - t_{011} + t_{100} - t_{101} + t_{110} - t_{111} &= 1 \\ t_{000} + t_{001} + t_{010} + t_{011} - t_{100} - t_{101} - t_{110} - t_{111} &= -1 \\ t_{000} + t_{001} - t_{010} - t_{011} + t_{100} + t_{101} - t_{110} - t_{111} &= -1 \\ t_{000} - t_{001} - t_{010} + t_{011} - t_{100} + t_{101} + t_{110} - t_{111} &= -1 \end{aligned} \right.$$

and four of the parameters are expressed as a function of the rest of them:

$$\begin{aligned} t_{000} &= t_{111} - 0.5 \qquad & t_{001} &= t_{110} - 0.5 \\ t_{010} &= t_{101} + 0.5 \qquad & t_{011} &= t_{100} - 0.5 \end{aligned} \qquad (5)$$

Taking into account (1) and (5), we rewrite our transform function as:

$$\begin{aligned} F_3(\omega_1,\omega_2,\omega_3) = t_{111}&(1+\cos(\omega_1)\cos(\omega_2)\cos(\omega_3)) + \\ &+ t_{110}(\cos(\omega_3) + \cos(\omega_1)\cos(\omega_2)) + \\ &+ t_{101}(\cos(\omega_2) + \cos(\omega_1)\cos(\omega_3)) + \\ &+ t_{100}(\cos(\omega_1) + \cos(\omega_2)\cos(\omega_3)) + \\ &+ 0.5\cos(\omega_2)(1-\cos(\omega_3)) - 0.5(1+\cos(\omega_3)). \end{aligned} \qquad (6)$$

We formulate our approximation problem as follows: under given angle $\theta \in (0,\pi/2)$, to determine the coefficients t_{100}, t_{101}, t_{110}, and t_{111} of transform function (6) and 1-D cut-off frequency ω_c, such that the resulting isopotential surface corresponding to ω_c (called the optimal cut-off isopotential surface), approximates the ideal cone surface. The idea of this approach could be considered as an extension of 2-D fan-filters design [12] via McClellan transform.

The above problem tends to be highly nonlinear and we are going to solve it in approximate sense. To this end, let we define a deviation function on the cut-off isopotential surface (for $\omega=\omega_c$):

$$E(\omega_1,\omega_2,\omega_3,\omega) = F_3(\omega_1,\omega_2,\omega_3) - \cos(\omega) \qquad (7)$$

and consider only the small values of cosine arguments:

$$\cos(x) \approx 1 - \frac{x^2}{2}. \qquad (8)$$

Other authors have also used the same approximation for $\cos(x)$ in designing 2-D fan filters. Using (6), (7), and (8), we have derived the following relation from the equation $E(\omega_1,\omega_2,\omega_3,\omega)=0$:

$$\omega_1^2 + \omega_2^2 - \frac{\omega_3^2}{r/(1-r)} = \frac{2}{r}\big(2r - 1 - \cos(\omega)\big) + \frac{2R}{r}, \qquad (9)$$

where: $\quad r = t_{100}+t_{101}+t_{110}+t_{111}$, $r \neq 0, 1$

and
$$R = \frac{\omega_1^2\omega_2^2}{4}(t_{111}+t_{110}) + \frac{\omega_2^2\omega_3^2}{4}(t_{111}+t_{100}-0.5) + \frac{\omega_1^2\omega_3^2}{4}(t_{111}+t_{101}) - \frac{\omega_1^2\omega_2^2\omega_3^2}{8}t_{111}.$$

We would like the cut-off isopotential surface to approximate as closely as possible to the cone surface (3). Assuming this equivalence between (3) and (9), we obtain:

$$2r-1=\cos(\omega_c), \qquad \gamma^2=r/(1-r).$$

As a result, the following closed-form relations for the transform parameters and 1-D cut-off frequency are derived:

$$\left|\begin{array}{l} t_{000}=-t_{010}=-t_{100}=\dfrac{\cos(2\theta)-2}{4} \\ t_{001}=-\dfrac{\cos(2\theta)+2}{4} \\ t_{011}=t_{101}=t_{110}=-t_{111}=-\dfrac{\cos(2\theta)}{4} \end{array}\right. , \quad \omega_c=\pi-2\theta. \tag{10}$$

Using (10), the transformation function $F_3(\omega_1,\omega_2,\omega_3)$ cold be written as a function of θ:

$$\begin{aligned} F_3(\omega_1,\omega_2,\omega_3) &= \frac{\cos(2\theta)}{4}\,\mathrm{X} \\ &\mathrm{X}\begin{pmatrix} 1-\cos(\omega_1)-\cos(\omega_2)-\cos(\omega_3)-\cos(\omega_1)\cos(\omega_2)- \\ -\cos(\omega_1)\cos(\omega_3)-\cos(\omega_2)\cos(\omega_3)+\cos(\omega_1)\cos(\omega_2)\cos(\omega_3) \end{pmatrix}+ \\ &+0.5\left(\cos(\omega_1)+\cos(\omega_2)-\cos(\omega_3)-1\right) \end{aligned} \tag{11}$$

The equation of isopotential surfaces can be obtained by solving $E(\omega_1,\omega_2,\omega_3,\omega)=0$ for the frequency ω_3 as a function of ω_1 and ω_2. This equation determined for the cut-off isopotential surface and applying new expression (11) is:

$$\omega_3=I(\omega_1,\omega_2)=\arccos\left(\frac{4\cos(\omega_c)-\cos(2\theta)(1-A_1-A_2)-2(A_1-1)}{\cos(2\theta)(A_2-A_1-1)-2}\right),$$

where: $\quad A_1=\cos(\omega_1)+\cos(\omega_2)$

$\qquad\quad A_2=\cos(\omega_1)\cos(\omega_2)$

Applying $\cos(\omega_c)=-\cos(2\theta)$, we get:

$$\omega_3=I(\omega_1,\omega_2)=\arccos\left(\frac{2(A_1-1)-\cos(2\theta)(A_1+A_2-5)}{\cos(2\theta)(A_1-A_2+1)+2}\right). \tag{12}$$

III. SCALED TRANSFORM FUNCTION

In this Section we investigate whether our transform function is properly scaled (see the condition (2)). Our analysis for the function (11) has shown that:

$$F_{3\max}=1 \qquad F_{3\min}=-2+\cos(2\theta).$$

Therefore $|F_{3\min}|>1$ and we need a special scaling procedure. In order to obtain a scaled function we have applied method proposed by Mersereau *et al.* [13]. The surfaces defined by $F_3(\omega_1,\omega_2,\omega_3)=constant$ will not be changed by using scaled function:

$$F_3'(\omega_1,\omega_2,\omega_3)=c_1F_3(\omega_1,\omega_2,\omega_3)-c_2, \tag{13}$$

where:

$$c_1=\frac{2}{F_{3\max}-F_{3\min}}=\frac{2}{3-\cos(2\theta)}, \qquad c_2=c_1F_{3\max}-1=\frac{\cos(2\theta)-1}{3-\cos(2\theta)}.$$

It was shown, that the shapes of the surfaces obtained with (13) will be the same as those obtained with non-scaled function. There will be only change of a 1-D frequency associated with each surface. The new 1-D cut-off frequency corresponding to the scaled function has to be calculated as:

$$\omega_c'=\cos^{-1}(c_1\cos(\omega_c)-c_2)$$

Using relation (13), the scaled transform is:

$$\begin{aligned} F_3'(\omega_1,\omega_2,\omega_3) &= t_{111}'(1+\cos(\omega_1)\cos(\omega_2)\cos(\omega_3))+ \\ &+t_{110}'(\cos(\omega_3)+\cos(\omega_1)\cos(\omega_2))+ \\ &+t_{101}'(\cos(\omega_2)+\cos(\omega_1)\cos(\omega_3))+t_{100}'(\cos(\omega_1)+\cos(\omega_2)\cos(\omega_3))+ \\ &+\left(3-\cos(2\theta)\right)^{-1}(\cos(\omega_2)-\cos(\omega_3)-\cos(\omega_2)\cos(\omega_3)-\cos(2\theta)) \end{aligned} \tag{14}$$

with the following new relations for transform coefficients and corresponding 1-D cut-off frequency ω_c':

$$\left|\begin{array}{l} t_{111}'=-t_{110}'=-t_{101}'=\dfrac{\cos(2\theta)}{2(3-\cos(2\theta))} \\ t_{100}'=\dfrac{2-\cos(2\theta)}{2(3-\cos(2\theta))} \end{array}\right. , \quad \omega_c'=\cos^{-1}\left(\frac{1-3\cos(2\theta)}{3-\cos(2\theta)}\right). \tag{15}$$

It was proved, that the equation of cut-off isopotential surface (for $\omega=\omega_c'$) with the scaled transform function $F_3'(\omega_1,\omega_2,\omega_3)$ is the same as this one of non-scaled function (see eq. (12)). A graphical view of a few cut-off isopotential surfaces plotted for different angles θ is given in Fig. 2. The results show that our transform function approximates better cone-shape for smaller frequencies (in accordance with applied approximation (8)) and for angles θ grater then $\pi/4$ (or 45 degrees).

Our investigations show also that $F_3'(0,0,0)=\cos(\omega_c')$ and this expression may be explained by the fact that the cut-off isopotential surface of the cone passes through the origin of $(\omega_1,\omega_2,\omega_3)$-plane.

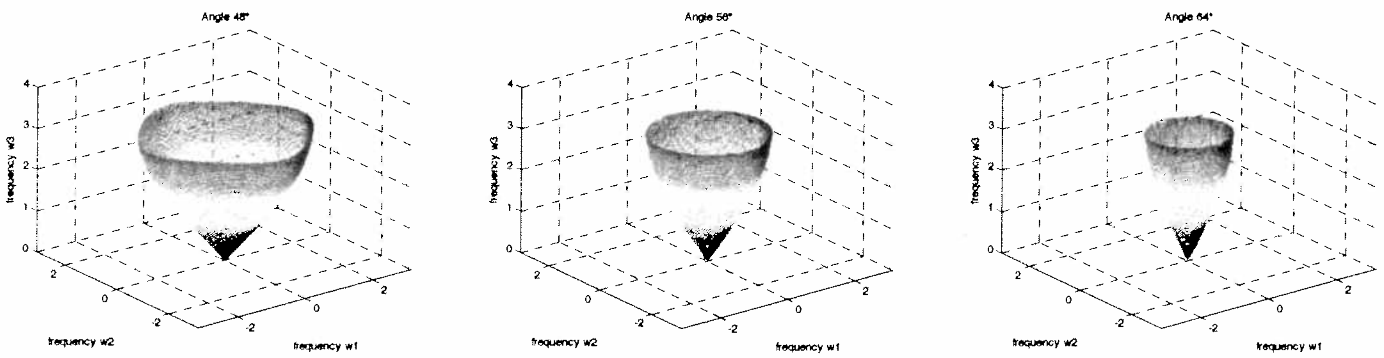

Fig. 2. The cut-off isopotential surfaces for three different angles θ

IV. Design Examples and Magnitude Responses

Several numerical examples have been performed to evaluate the effectiveness of the proposed approach. The results are obtained with Matlab simulation using the derived expressions.

A few slices of 3-D magnitude response plotted for a specific value of frequency ω_3 (for $\omega_3=\omega_c'$) and different angles θ are given in Fig. 3. The McClellan-Parks algorithm is used for design of 1-D FIR filters prototypes (for these examples length of the prototypes is $2N+1=35$ and transition band is 0.1π).

The magnitude responses of designed 3-D cone filter plotted for $\omega_3=0.05\pi$ and $\omega_3=\omega_c'=0.203\pi$ and corresponding contour plots in (ω_1,ω_2)-plane (angle $\theta=65°$) are shown in Fig. 4. The 1-D equiripple FIR filter with length 21 and transition band 0.15π has been used as a prototype. As we expected, the magnitude responses of designed 3-D cone lowpass filters are equiripple (as 1-D equiripple prototypes determine the values of the magnitude along the isopotential surfaces).

V. Summary

This paper proposes several new analytical relations for transform parameters in design of 3-D FIR cone filters as a function of angle of inclination of the cone. The scaling problem of transformation function and choice of a suitable 1-D cut-off frequency is also under discussion.

The proposed solution is in closed-form and it does not require any iterative procedures or optimization techniques. Method could be applied for angle of the cone filter greater than 45 degrees. The simulation results obtained for both cut-off isopotential surfaces and 3-D magnitude responses prove the correctness of the new expressions. This approach could be extended for design of other type of 3-D FIR filters (e.g. with elliptical or spherical magnitude responses). It is also possible, some efficient implementation structures based on the new transform function to be obtained.

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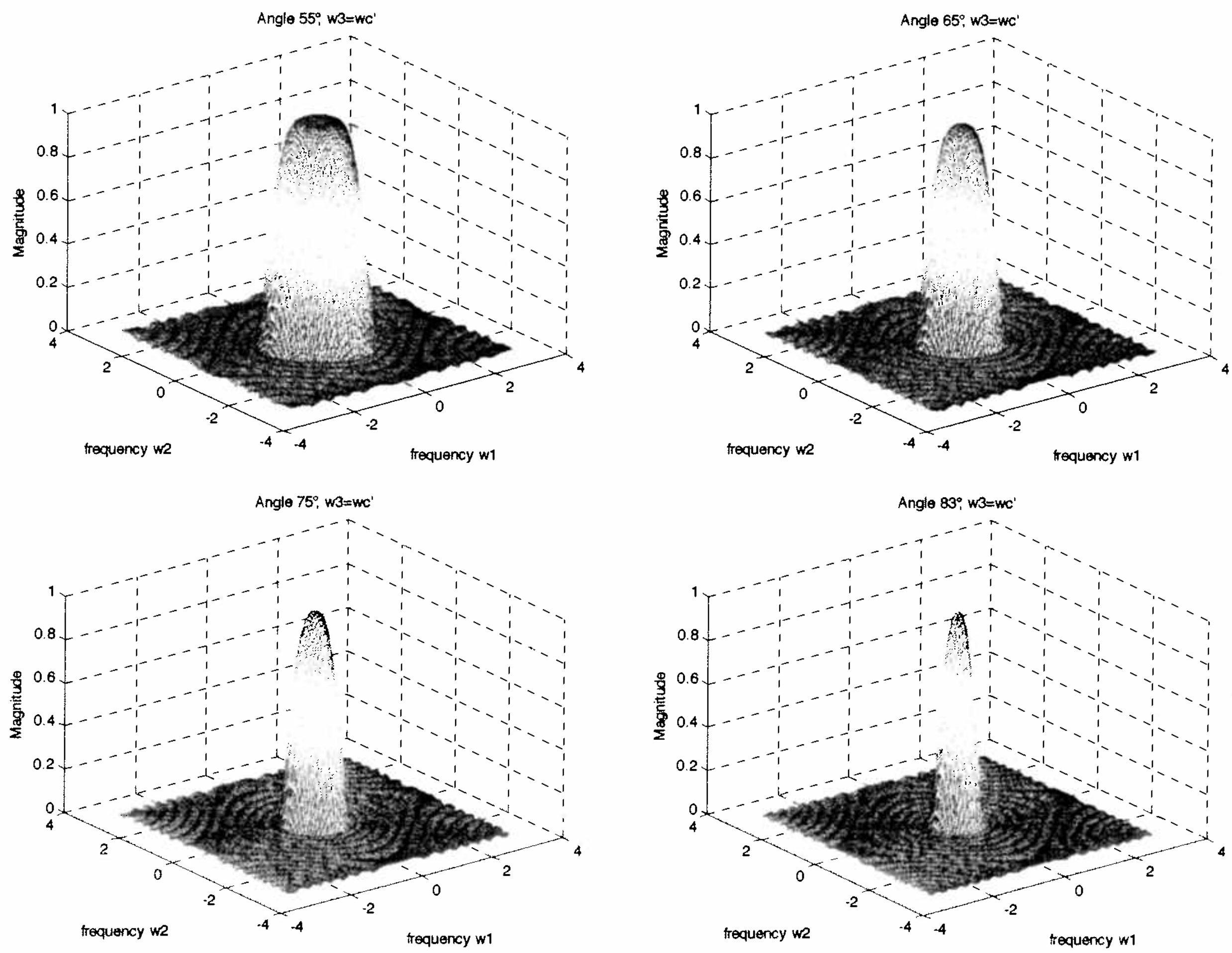

Fig. 3. Magnitude responses of 3-D filters obtained for $\omega_3=\omega_c'$ and different angles of the cone θ.

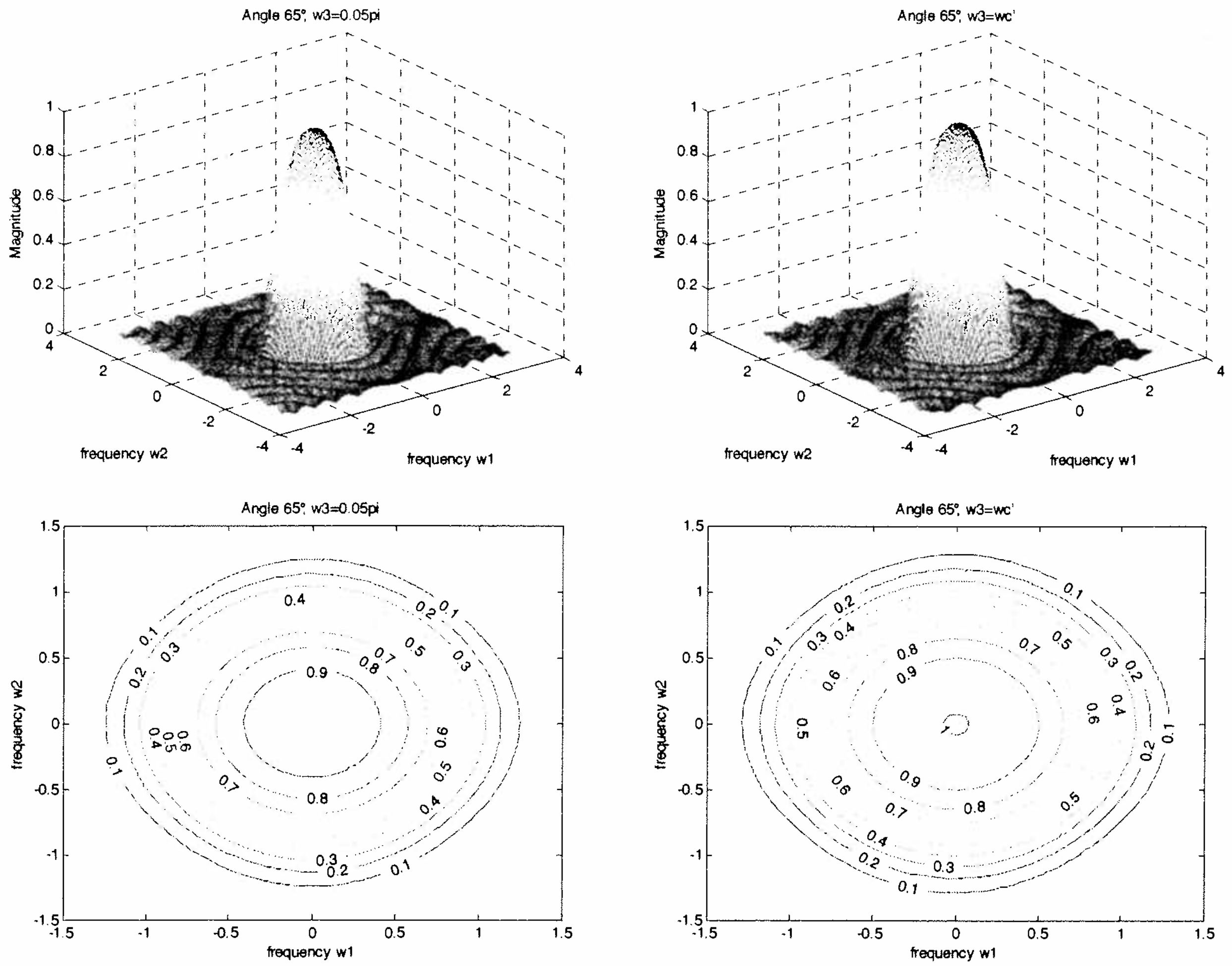

Fig. 4. Magnitude response and contour plots for θ=65°.

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