DTM QUALITY ASSESSMENT

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ABSTRACT

Digital Terrain Models (DTM) are frequently used to make important decisions. In order to judge these decisions DTM quality must be known. The quality of a DTM consists of several components like precision, accuracy, and reliability. This article presents several methods to assess the quality of a DTM, commencing with the data it is based on and proceeding with quality measures for the model itself. These methods are compared to each other and their application is discussed. The outlined techniques are designed for DTM that describe the terrain height as a bivariate function (2.5-dimensional DTM). However, some of them may be extended for application to three-dimensional DTM.

1 INTRODUCTION

The emergence of automatic measurement methods like airborne laserscanning (ALS), image matching and interferometric synthetic aperture radar (InSAR) pushed the spread of digital terrain models (DTM). However, all of these techniques imply the risk of gross errors and ill determined areas. For ALS data, the filtering of off-terrain points is critical (cf. Sithole and Vosselman (2004), and Kraus and Pfeifer (1998)), and may result in large data voids. The same holds for image matching (Bauerhansl *et al.* 2004) and InSAR (Mercer 2004). Image matching may furthermore generate blunders through mismatched points, and the point density decreases in poorly textured areas. DTM derived from InSAR additionally suffer e.g. from phase ambiguities. This paper focuses on the evaluation of DTM from ALS data and photogrammetric imagery, providing data of potentially higher quality.
In general, DTM are deduced from observations of the terrain surface and represent the bare earth at some level of detail. According to Artuso *et al.* (2003) and Elberink *et al.* (2003), the demands on DTM are still growing steadily and the necessity of quality control is evident.
DTM are used in numerous disciplines, ranging from geoinformation to civil engineering. In the course of various applications, DTMs serve as input for decision making, e.g. they are employed for flood hazard analyses. In order to judge these decisions, DTM quality must be quantified using adequate methods and measures. Furthermore, these measures must be communicated to the users. Unfortunately, this is rarely done (Wood and Fisher 1993), and if so, merely global quality measures are provided. However, the spatial variation of DTM quality is of interest for various applications, e.g. for the accuracy estimation of a volume computation based on a DTM. Therefore, this article concentrates on local measures.
However, the simple question for the accuracy of a DTM at some position cannot be answered in general, since the question is defined imprecisely. Accuracy may be measured in height, in horizontal position (e.g. for breaklines), or along the surface normal. Furthermore, accuracies exist for the derivatives of the DTM surface.
Even the standard deviation, as the most common deviation descriptor, has to be differentiated (Mikhail 1971). Standard deviations of the DTM estimated through error propagation of the input data represent interior quality i.e. precision. The redundancy of data gives a statement on the reliability of a DTM, and on the control of observations. The exterior quality i.e. accuracy is computed through comparison with uncorrelated data that did not participate in the generation of the DTM.
In addition to the standard deviation, various other measures exist that answer the following questions:

- "How dense is the data?" Data density represents the amount of discretisation of the terrain surface.
- "To which category belong the nearest data, and what is the distance to them?" In the course of manual measurement processes, data are usually classified, e.g. into lines and points of different quality and meaning. Automatic techniques provide different classes e.g. through provenience from different flight strips.
- "How is the accuracy of the DTM distributed?" Depending on the spatial variation of the accuracy and density of the data, and on the suitability of the interpolation method for a certain relief, DTM quality varies locally and regionally. Depending on the requirements, users may or may not be interested in a homogeneous (and isotropic) distribution of DTM accuracy.
- "What is the accuracy of derived products (Kraus 1994)?" All derived products like lines of equal slope, or borderlines of flood risk areas hold their own accuracy measures.

Before the estimation of model quality, a description of the data quality is necessary, which is illustrated in section 2. Subsequently, diverse measures for the quantification of DTM quality are presented (section 3). Furthermore, empirical formulas are given that allow for the estimation of model accuracy both *a priori* and *a posteriori*.

1.1 Related Work

Numerous publications exist on the topic of global DTM quality that compile all well-established approaches, e.g. by Li (1993)

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and McCullagh (1988).
A sound alternative originating from the field of signal processing is the application of spectral analysis for DTM accuracy estimation, cf. Tempfli (1980) or Frederiksen (1980). The global accuracy is derived from the measurement accuracy and the transfer function of the interpolation method. This function describes the ratio of the amplitudes of the input and output signal i.e. the observed data and the DTM. The method may be used for DTM that were interpolated with a linear estimator (e.g. triangulation, Kriging). However, the computation is time-consuming, especially in case the transfer function is not known beforehand.

2 QUALITY OF THE INPUT DATA

The input data form the basis for the computation of the DTM, consisting of points and / or lines. Automatic measurement techniques like image matching and airborne laserscanning usually output bulk points only, but may be enriched with structure information in a post processing phase (Briese 2004). These input data may hold information on the accuracy of the matching process or the distance determination of the laser range finder. Currently, full-waveform airborne laserscanners offer high potential concerning the quality estimation of ALS points (Wagner *et al.* 2004). In addition to the accuracy in height, there may be given the horizontal accuracy. Supplementary meta data like the date of flight, the flying altitude, or the sensor model further describe the quality of the input data. However, in the simplest case there will only be one single estimation available for the height accuracy of all data. Typically, manual measurements result in different data classes with different qualities: spot heights, breaklines, formlines, coastlines, bulk points, etc. Also, the combination of different data sets yields a classification.
In the following, quality measures are presented which allow to describe the input data. Naturally, they hold a limited expressiveness concerning the quality of the DTM, as the process of interpolation is not regarded.
Three aspects of data quality are investigated. First, the distribution of the data is examined, concerning density, completeness, and type. In the following subsection, the accuracy of the measurements is discussed. While in the first case, only the positioning, or parametrization, respectively, of the measurements is considered, the actual observations are examined in the latter case. Finally, the consistency of the data is analysed in the third subsection, which may reveal discrepancies between different groups within the data.

2.1 Data Distribution

2.1.1 Density Map Data density solely depends on the horizontal positions of the data. A density map may be computed as the number of points per unit area. Therefore, it can be determined easily as a digital image, where the pixel value corresponds to the number of points within the area covered by the pixel. A density map depicts the amount of discretisation of the terrain surface. Regions covered by few or no data become distinguishable. The completeness of the data may be inspected, as density amounts to zero in data voids. Moreover, large data sets can be overviewed better through a density map than by the display of the data themselves. That is, because the higher the data density and the smaller the image scale, the less the data become discriminable. An example for a density map is given in figure 1.
Areas with low data density may be determined automatically by threshold operations. Alternatively, small areas without data may be detected through mathematical morphology (Serra 1982): having binarised the density map, the operator 'Closing' ('Erosion' followed by 'Dilation') is applied. In doing so, the size of the

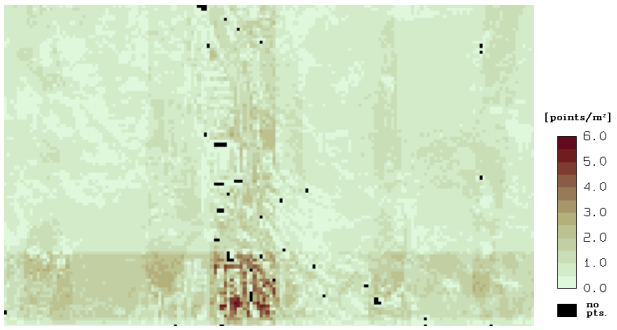

Figure 1: Density map of an ALS data set, computed with a cell size of $100m^2$. Horizontally along the lower side, and upright in the centre, bands of high density are observable. They originate from overlapping ALS strips. Cells that do not contain any data are coloured black.

structure element determines the smallest reported data void.
The density map again can be aggregated to a histogram of densities that allows for a quick inspection of the homogeneity of data distribution, see figure 2.

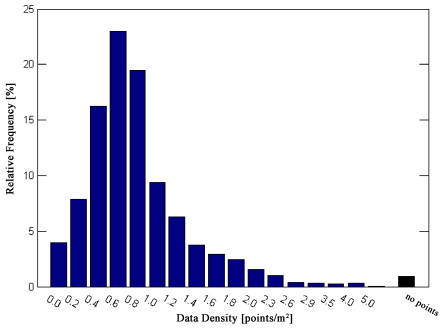

Figure 2: Histogram of the data density pictured in figure 1.

2.1.2 Distance Map A distance map indicates the distance between the centre of each depicted pixel and its nearest data point. It may be computed efficiently using the Chamfer function (Borgefors 1986). The areas that are closest to a certain data point form the Voronoï regions (Okabe *et al.* 2000). Figure 3 (centre) presents the distance map of an MBES (multi beam echo sounding) data set. Outside the convex hull of the data, distances grow to infinity. This map permits the estimation of model reliability, since reliability is the higher, the smaller the distance between the interpolated position and its nearest data point is.
Furthermore, this representation allows for a simple classification into areas of 'interpolation' and 'extrapolation'. Areas where the distance to the nearest data point is larger than a certain threshold may be declared as extrapolation regions. Naturally, the positioning in- or outside the convex hull may be considered, too.
The concept of the distance map can be extended from a pointwise to a regional representation. For instance, the mean of the distances to the n nearest neighbours, or the distance to the nth generation of a Delaunay triangulation may be used. Figure 3 (bottom) shows the mean distances from each pixel centre to its 10 nearest data points.

2.1.3 Data Class Map A data class map shows for each pixel centre the class of the most accurate data inside the area covered by the pixel. The concept may also be used to visualize data classes of different reliability, e.g. manual vs. automatic measurements. However, the quality of interpolated heights can be deduced only to a limited extent, since most interpolation methods employ more than one data point. An example for a data class map of a photogrammetric data set is shown in figure 4.

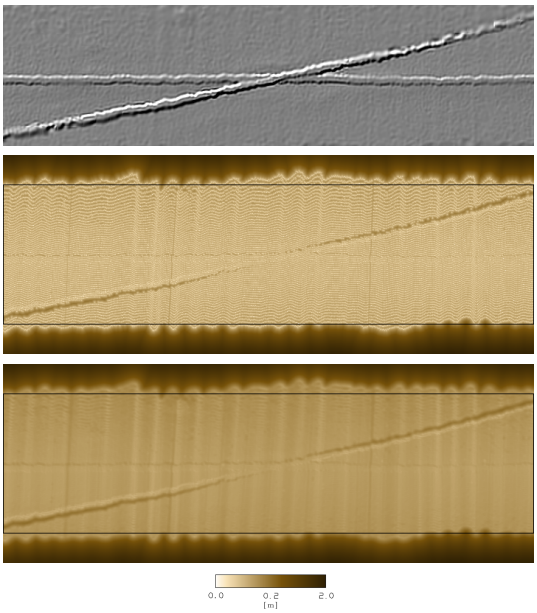

Figure 3: Top: Shaded DSM (digital surface model) of an MBES (multi beam echo sounding) data set featuring two crossing pipelines on the sea bottom. Centre: Distance map of the data. Bottom: Regionalised distance map of the data. For each pixel centre, the mean distance to its 10 nearest data points is visualized. In both distance maps, the area covered by the DSM is indicated with a black rectangle.

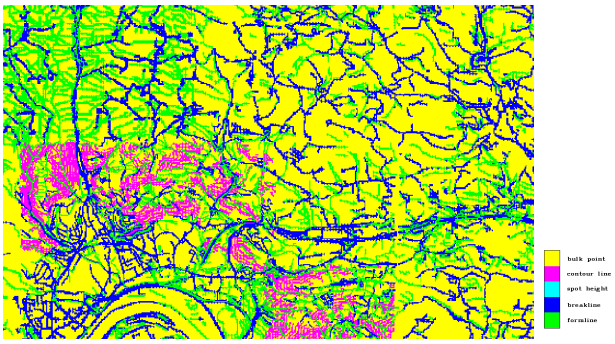

Figure 4: Data class map of a photogrammetric data set consisting of various data types.

2.2 Accuracy of Measurement

The accuracy of the original observations is of crucial importance for the quality of the interpolated DTM. The measurements usually consist of distances and directions, whereof the point coordinates are deduced. The respective computation yields correlations that typically are neglected - frequently, only the accuracy in height is considered. This simplification is admissible, if the horizontal position is determined much better than the elevation, which is the case for e.g. photogrammetric data.
A sigma-z map depicts the distribution of the accuracy of measurement in height. It may be created in case the (automatic) measurement method generated the respective measure, or if the accuracy can be estimated through empirical formulas. This measure holds a limited amount of information about the model quality, since model quality depends on data density, surface complexity, and the interpolation method, too. Similarly, a map of horizontal measurement accuracy may be compiled (sigma-xy). However, these data are rarely available, although the horizontal uncertainty may very well be worse than the one in height (e.g. airborne laserscanner points typically hold the following standard deviations: $\sigma_z \approx 0.15m$, $\sigma_{xy} \approx 0.5m$).

2.3 Consistency

In case a DTM is computed from different, overlapping data sets (e.g. overlapping laserscanner strips), a quality statement can be given within the common areas. Even within a data set different groups of data may be tested for consistency, e.g. photogrammetric profile measurements observed uphill and downhill. The consistency check shows up systematic errors, which are usually not eliminated in the course of DTM generation. Figure 5 depicts differences between overlapping airborne laserscanner strips originating mainly from imprecise sensor orientation (Kager 2004). Figure 6 shows discrepancies as a result of a sensor deficiency.

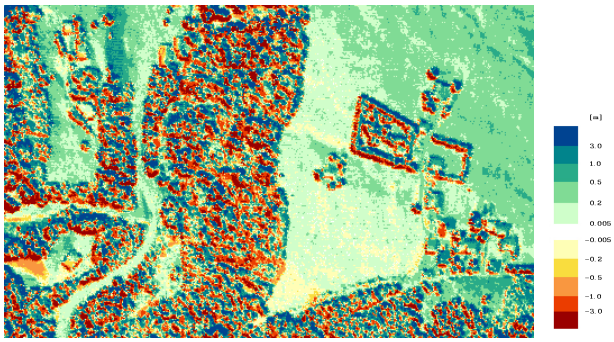

Figure 5: Inconsistencies between different data sets: discrepancies between 2 overlapping airborne laserscanner strips. The differences on roofs and in open areas mainly owe to imprecise sensor orientation. Very large discrepancies are caused by vegetation and occlusion effects (Kager 2004).

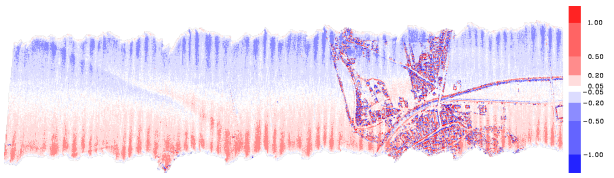

Figure 6: Inconsistencies within a single data set due to a sensor deficiency. The data were captured using a laserscanner with oscillating mirror, which registered the measurements of angles and distances asynchronously (time lag). Because of the mirror's oscillating nature, the systematic difference between points observed on the mirror's way to the left and those on the way to the right could be detected within a single flight strip. The picture illustrates the difference between the surface models derived from the two groups of points.

3 MODEL QUALITY

In addition to a check on the input data quality, the DTM itself may be inspected, whereupon the distinction between the interior (precision) and the exterior quality (accuracy) must be considered. While the first measure describes how well the model fits to the input data, the latter one gives information on its conformity with external data.

3.1 Interior Quality

Besides the quality of the input data and the terrain complexity, model quality also depends on the modelling calculus employed (to a smaller extent, however (Tempfli 1980)). In order to determine the precision, redundancy in the data and its utilization are

necessary. Redundancy is a precondition for the control and reduction of random errors in the modelling process. It may be verified through the sample theorem (cf. Nyquist (1928) and Shannon (1949)). Redundancy is present, if the interval of discretisation is smaller than half the smallest wave length that is contained in the input signal. Practically, this means that the interval of discretisation must be smaller than the radius of the smallest terrain feature to be reconstructed.
An example for an interpolation method that does not take advantage of redundancy is the triangulation of the original data. Thus, this method does not provide accuracy estimations for the model. Furthermore, the data are not controlled i.e. gross errors cannot be detected. These deficiencies may be remedied by smoothing the triangulation, e.g. by using local surface models (Mokhtarian *et al.* 2001). More common methods that employ redundancy for the elimination of random measurement errors are Kriging and methods applying the theory of finite elements. Even by averaging the height of data inside the cells of a grid, redundancy can be taken advantage of, leading to information about the distribution of heights inside each cell.

3.1.1 Error Propagation Using error propagation, the standard deviation in height may be estimated. This estimation can be used to predict the precision of derivatives of the DTM, e.g. slope, curvature, etc. As an alternative for the distinction between extrapolation and interpolation areas described in section 2.1.2, a threshold for the predicted accuracy in height may be applied to classify insufficiently determined areas.
Kriging or linear prediction, respectively (Kraus 1998), constitutes a popular interpolation method that allows for the estimation of prediction errors. However, this precision solely is a function of the distance to the neighbouring data, see figure 7. The magnitude and variation of the error is deduced from the variogram in use, which basically is the same for the whole interpolation area. If the variogram is fitted to the data, it contains the aggregation of characteristics of the actual observations. Concerning the predicted errors, Kriging hence considers the alignment of the local neighbourhood, but disregards the observed elevations. A basic precondition for Kriging is the stationary randomness of the terrain height. Nevertheless, for most landscapes this obviously does not hold true. Thus, either a trend model has to be separated, or a non-stationary variogram has to be used (van den Boogaart 2003). However, there is no definition of the correct trend model; the same holds for the non-stationarity of the variogram. As this free choice affects the predicted errors, they imply some amount of arbitrariness, too.

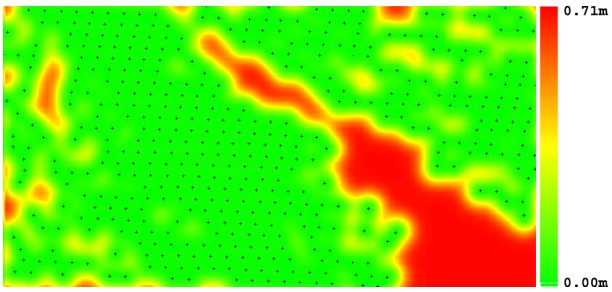

Figure 7: Kriging facilitates the estimation of the standard deviation in height. However, the predicted errors imply some amount of arbitrariness.

3.1.2 Evaluation of Residuals Another measure for the interior accuracy may be generated by computing the residuals i.e. the differences in height between the original points and the interpolated surface. Regarding large data sets and good graphical representation, these residuals should be analysed in a cell structure, too. For each cell, the maximum, root mean square, mean, or median residual may be inspected, see figure 8. The maximum residuals indicate large deviations from the DTM surface. Concerning gross error detection, it has to be considered that the largest residuals do not necessarily need to occur at erroneous observations. Furthermore, the analysis of residuals normalized by their standard deviation a priori has to be preferred in case of observations holding different weights, or accuracies a priori, respectively. The median of the residuals is a robust indicator for systematic errors in height, also their mean value is practical in this context. The RMSE (root mean square error) forms a local measure for the interior quality of the DTM that aggregates the variation of all data contained in each cell.

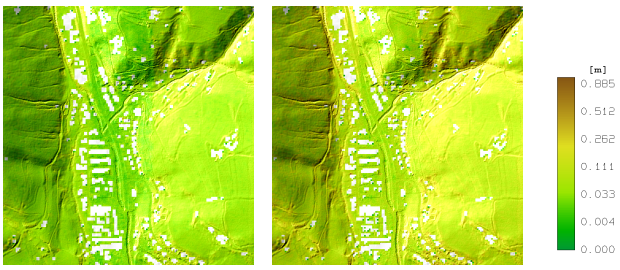

Figure 8: Shaded views of an ALS-DTM, colour coded with a residual map. Left: RMSE (root mean square error). Right: maximum residual per cell.

If the persistent data structure of a DTM is merely an approximation of the interpolated surface, then the terrain is reconstructed in two tiers: a surface is determined by a first (sophisticated) interpolation method (e.g. Kriging, finite elements). This surface is stored in a simplified way, frequently in the form of heights at the points of a regular grid. The actual DTM height is then computed by a second (simple) interpolation method (e.g. bilinear interpolation, adjusting cubic surface patches), using the neighbouring grid point heights. The simplification to the storage format affects DTM quality. Thus, for these two-tiered DTMs, the quality of the surface predicted by the first interpolation method must be distinguished from the quality of the surface deduced from the DTM grid. A significant difference between the residuals of the data to the first surface and the residuals to the surface deduced from the grid is an indication for the grid size being too large. This grid structure may be enhanced with vector data (e.g. spot heights, breaklines, etc.), leading to a hybrid DTM (Kraus 2000b) with a better representation of the terrain relief.

3.2 Exterior Quality

The exterior quality of a DTM can only be determined exactly using external, additional data. These external data must not have been used for DTM generation. In addition, those data are supposed to hold much better accuracy than the input data. The exterior quality describes both the input data and the modelling process. This way, systematic errors of the DTM may be revealed. The residuals between the check points and the DTM may be evaluated like in section 3.1.2.
Occasionally, a small part of the original data is omitted in the DTM creation process. Subsequently, these point heights are tested against the DTM height. However, this form of cross-validation generates information about the interior accuracy that should be interpreted and evaluated using the methods described in section 3.1.
For well-established measurement and modelling techniques there additionally exist empiric models that allow for the estimation of accuracy.

3.2.1 Empirical formulas External data of superior quality usually require additional, costly observations in the field. For that reason, empirical formulas have been defined that allow for the estimation of DTM accuracy in height, basically *a posteriori*. These formulas have been fitted for specific, well-established measurement techniques, setups, filtering and interpolation methods. It is presumed that systematic errors in the data have been minimized using advanced georeferencing (cf. Jacobsen (2004) and Kager (2004)).

For photogrammetric data, the following formula is in use (Kraus 2000a):

$$\sigma_z = \pm \left(0.15‰ h + \frac{0.15}{c} h \tan\alpha \right) \quad (1)$$

Herein, $\tan\alpha$ denotes the terrain slope, $c[mm]$ is the focal length, and h is the flying altitude, which holds the same unit of length as σ_z. Hence, $\frac{h}{c}$ represents the image scale. Knowing the approximate maximum terrain slope, the optimum flying altitude and image scale to meet the requirements for the DTM may be determined *a priori*.

Karel and Kraus (2006) propose the estimation of the accuracy in height of DTMs derived from airborne laserscanning data as:

$$\sigma_z[cm] = \pm \left(\frac{6}{\sqrt{n}} + 30 \tan\alpha \right) \quad (2)$$

Where $n[points/m^2]$ denotes the point density, and $\tan\alpha$ again is the terrain slope. The flying altitude is not contained in the formula, as it does not affect the accuracy in height of ALS points noteworthy. However, data density is related to the flying altitude, depending on the angle step width of the scanner. Therefore, the formula may also be utilized to determine the optimum flying altitude, given the scanner model and the maximum terrain slope.

A more detailed, *a posteriori* alternative for ALS data is described by Kraus *et al.* (2006). It estimates DTM accuracy, based on the original data and the DTM itself. It is applicable regardless of the interpolation method employed for DTM generation. The calculus takes into account the following characteristics:

- the alignment of the data,
- the residuals of the point heights, and
- the DTM curvature.

The DTM standard deviation in height is computed as:

$$\sigma_{DTM} = \pm \sigma_0 \sqrt{q_{a_0}} \quad (3)$$

Herein, σ_0 describes a representative precision of the surrounding original data. In its estimation, the neighbours are weighted according to their distance and the local DTM curvature. q_{a_0} is the cofactor in height of an adjusting plane through the neighbouring original points. Within this adjustment, the same weighting is applied as in the estimation of data precision. In addition, areas of the DTM where the original data is too distant are marked as unusable. Have a look at figure 9 for an example of the estimated accuracy.

4 DISCUSSION AND CONCLUSIONS

Various methods to assess the quality of a DTM are presented above, but the matter of their application, the redundancy among them, and the consequences to take based on them have remained unanswered so far.

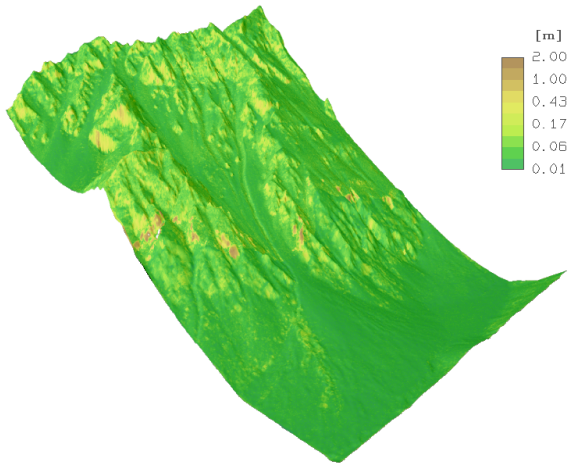

Figure 9: Perspective, shaded view of an ALS-DTM of an alpine landscape. Colour coding with $\hat{\sigma}_{DTM}$, computed with the calculus presented in (Kraus *et al.* 2006).

In the majority of cases, DTM users require simply applicable information on exterior quality. The topic of accuracy is easy to communicate, but frequently the issue of reliability is not. Hence, a combination of a threshold for a minimum amount of reliability and an accuracy measure seems practical.

Areas of the DTM that do not reach the threshold of reliability should be masked appropriately. These regions may either be detected using the method to determine areas of low density (2.1.1), or through the method to distinguish between 'inter'- and 'extrapolation' areas presented in section 2.1.2.

Concerning the accuracy information, the evaluation of residuals to external data of superior quality appears to be convincing, but is too costly for an area-wide evaluation. Moreover, this method merely provides pointwise results. As an alternative, the application of the empirical, *a posteriori* method (3.2.1, (Kraus *et al.* 2006)) is recommended, as it regards all major factors that influence local DTM accuracy. In case a respective routine is not available, the computation of RMSE (3.1.2), or the accuracy estimation through error propagation (3.1.1) constitute viable methods, too. Besides, data class maps (2.1.3) are used in practice.

DTM quality information may be employed by users in several ways, depending on the application. Flood risk can be modelled in hydrological studies, resulting in border lines of a certain inundation scenario. In case such a line comes out to reside in a DTM area masked as not reliable, then e.g. additional surveys have to be carried out, in order to increase reliability.

During the creation of a DTM, deeper insights into data and model quality are needed in order to guarantee acceptable results. Besides a check on the alignment of flight strips and control data, the completeness of data should be inspected by a distance (2.1.2) or density map (2.1.1), or a histogram of densities. If this test fails, the processing has to be stopped until additional data has been gathered. Subsequently, inconsistencies within different groups of data must be explored through difference models (2.3). Eventually, system calibration must further be enhanced. The filtering of off-terrain points should be checked then by inspecting the residuals (3.1.2), whereupon the maximum absolute value is a good indicator of single blunders, and RMSE detects extended failures of the filtering process.

All methods to assess DTM quality presented in this paper have either already been implemented in SCOP++ (SCOP++ 2005), or this is scheduled for the near future. The results of the respective tests are condensed in this article.

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