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Field measurement of soil moisture dynamics and numerical simulation using the kinematic wave approximation

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Abstract

We investigate in this paper the effect of plane and profile curvatures on the soil moisture distribution and related fluxes obtained from field experiments. Today, there is a need also to confirm the theoretical approaches of the kinematic wave approximation with the real field measurements. Within a research project dedicated to the model-based description of runoff formation processes, field measurements of soil moisture dynamics and numerical simulation, using one simply formulated kinematic wave model, are combined. It is shown that the measurement of the soil moisture movement downwards within a hillslope can be interpreted by the kinematic wave model and vice versa how the model can be validated in its physical explanations by the field measurements. It is revealed that the soil moisture fluxes can be described by kinematic waves crossing the soil mantle of the hillslope. Based on these analyses it is shown that the two topographic characteristics plane and profile curvatures of the hillslope control the subsurface and saturation flow. The obtained results are related to the previous theoretical analysis of Fan and Bras [Fan Y, Bras RL. Analytical solutions to hillslope subsurface storm flow and saturation overland flow. Water Resour Res 1998;34(4):921–7], Troch et al. [Troch PA, Paniconi C, van Loon EE, Bijkerk B, Hilberts A. Behaviour of a hillslope-storage Boussinesq model for subsurface storm flow and saturation overland flow. International Workshop on Catchment, Wageningen 2000; Poster] and Troch et al. [Troch PA, van Loon C, Hilberts A. Analytical solutions to a hillslope-storage kinematic wave equation for subsurface flow. Adv Water Resour 2002;25:637–49] which have shown considerable progress in the modelling using the kinematic wave approximation.

Keywords: Field measurement; Kinematic wave approximation; Soil moisture; Subsurface flow; Hillslope; Plane and profile curvature

1. Introduction

Kinematic wave approximations are used by several authors to describe the subsurface flow and saturated excess overland flow in the hillslopes with a soil layer overlaying a relatively impermeable bedrock [2,4,13,14]. Computations of runoff, groundwater level and soil moisture content under utilisation of the kinematic wave approach were performed in several rainfall-runoff models [1]. Unfortunately, the scale problem limits often the ability of these models to describe the hillslope processes in detail or to validate them with field studies. It seems necessary to validate the concept of the kinematic wave theory. Computer simulations and process studies were combined in this study in order to describe the lateral water fluxes in the hillslopes and to estimate the model parameters from soil characteristics. With regard to the measurement of a smaller scale (single hillslope) was chosen.

From a theoretical point of view the geomorphology of the hillslope is considered as the most significant control over storm flow and saturation [9,12]. For this

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reason, partitioning a catchment into converging and diverging hillsides seems to be appropriate [7,15]. This idea was also used in modelling subsurface flow by [6]. He introduced the geometric pattern function. But these studies, except for the field work done by [3], were often restricted to geometric abstractions of the topography such as a plane (constant slope) or a perfect cone (constant convergence). Recently [5], solved a similar problem with infiltration as formulated by [11]. In many cases these solutions were applied to overland flow only under conditions of simplified geometrical characteristics of surfaces.

The literature shows that the recent progress is still theoretic and more analytic. For example, [4] give a detailed overview of the previous kinematic wave studies and present analytical solution using the second-order polynomial slope profile curvature. The research work of [10] supplemented the work of [4] by formulating the analytical solution considering the third-order polynomial slope profile curvature. New and more general significant solution by considering the *n*-order polynomial slope profile curvature were investigated in [14].

We investigate in this paper the experimental watershed of 2.6 km^2 which used to provide the input data for the application of the kinematic wave model. The hillslope characteristics needed are geomorphological characteristics such as slope, width variation, profile variation of the hillslopes etc. One hillslope was instrumented in order to observe the soil moisture dynamics such as soil moisture movement and groundwater level evolution. Based on the geomorphological structure, the subsurface storm flow and the overland flow were described essentially from the field measurement.

2. Partition method of the catchment to detailed hillsides

The experimental watershed is situated in the Northwest Germany in a humid mountain-climate region with an annual average temperature of 8 °C and an mean annual precipitation of 1158 mm. The soil types are: brown soils, pseudogley soils, stagnogley soils, colluvic soils and gley soils. The two first types are derived from periglacial cover-beds which are affected by solifluction. The nature of the hillslopes is characterized in its major cases by gentle slopes (10°–20°) with converging and diverging curvatures. The land use is mostly pasture with some few dispersed patches of forest.

The watershed was subdivided into drainage areas and hillsides as it is shown in Fig. 1. These hillsides can be classified into three generalized types: divergent, convergent and uniform. Fig. 1 shows: (a) catchment boundary and channel network, (b) internal ridge lines, (c) channel head drainage areas, and (d) finer discretisation into hillslopes. The instrumented hillslope (11) is shown in the finer discretisation of Fig. 1(d). All these subdivisions were obtained from application of the Geographical Information System (GIS).

If we assume that the subsurface and overland flow are follow the slope gradient, we obtained that the hillsides have a tendency to concentrate the runoff to the stream. These sides have a width larger at their ridges than at their bases (Fig. 1c). They are described as convergent hillslopes with decreasing plan curvature. Hillsides with a width larger at the stream than at the ridge, are called divergent hillslopes. The other type have a constant plan curvature and are named uniform hillslopes. This difference in geometry imposed by the topography may influence the runoff processes (concentration or distribution of the runoff) and plays a significant role for the appearance of saturated surfaces [4].

Fig. 2 gives the studied hillslope profile with different soil layers and horizons. As shown, the soil mantle is composed of graduation varying from the silt at the soil surface to the silty clay. Soil moisture conditions were registered at four locations (stations) specified in Fig. 2. Each station is equipped with several tensiometers installed at different depths 20, 50, 80, 120, 150 and 200 cm. The measurements were registered automatically every 10 min.

3. Geomorphological effect on the runoff formation

Considering a catchment with permeable soil where subsurface storm flow is important. Fig. 3 illustrates schematically the three types of hillsides: divergent, convergent, and uniform. Only the permeable soil mantle (soil cover) is shown, which overlies an impermeable bedrock surface. The three-dimensional soil cover, on

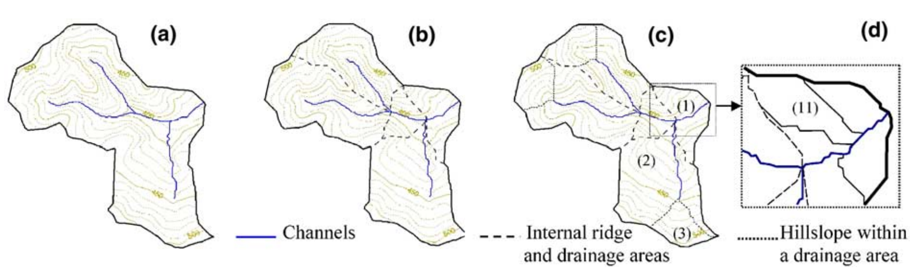


Fig. 1. Schematic illustration of partitioning of the watershed into hillslopes.

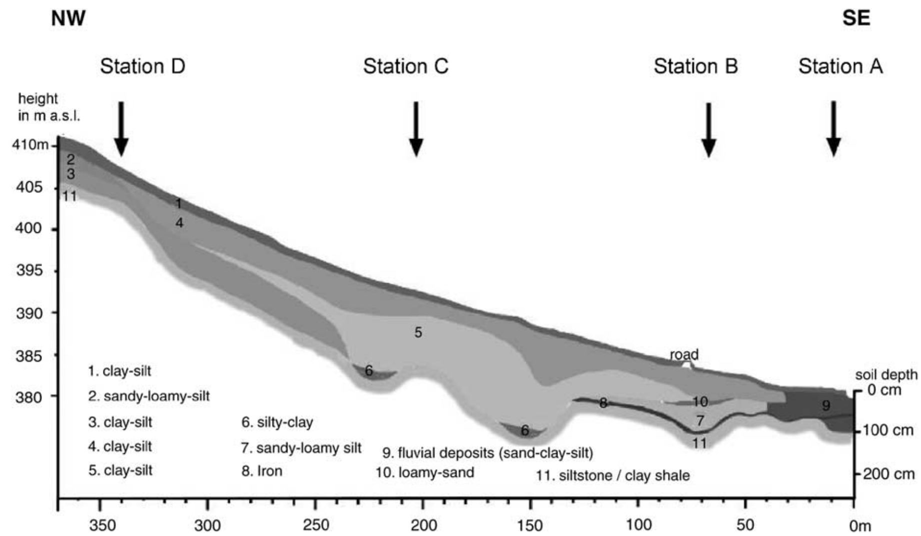


Fig. 2. Soil profile of the analysed hillslope.

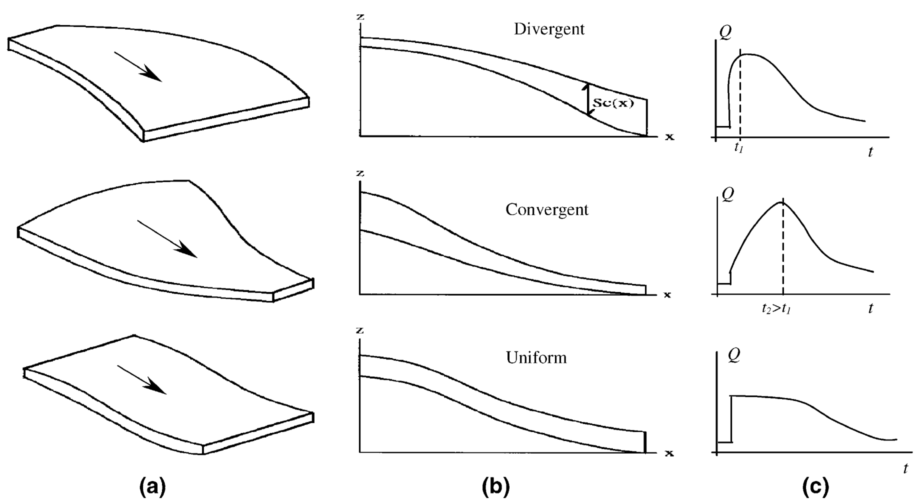


Fig. 3. Plan curvatures types of [4] with modification.

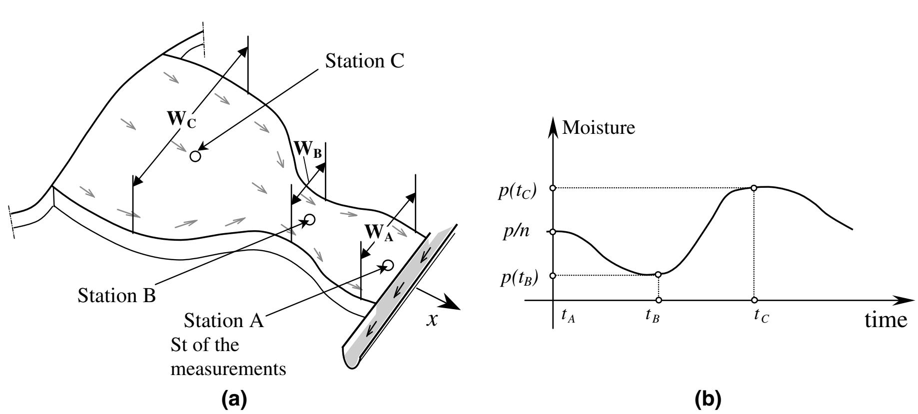
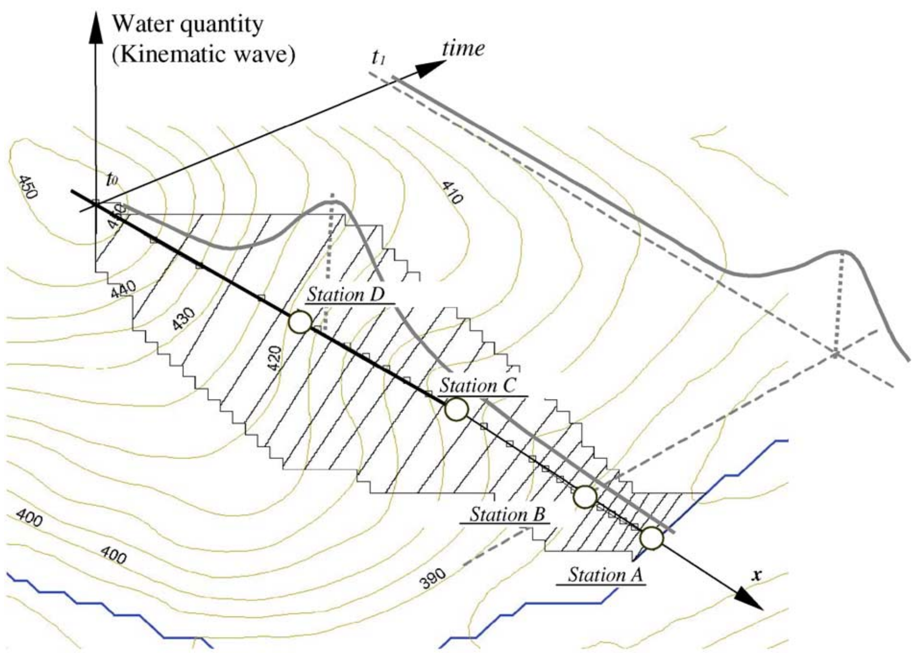


Fig. 4. Qualitative illustration of hillslope soil moisture dynamics.

column (a), can be described by a two-dimensional cross-section made of the drainable pore space on column (b) of Fig. 3 (column (a) and (b) from [4]). Column (c) shows the variation in the resulting temporal runoff at the bottom of the hillslope which dependents on the hillslope geomorphological characteristics.

In the following, an example is given to explain the physical process of the kinematic wave crossing the hillslope after a rainfall event. After one impulse of precipitation rainfall P [mm], one can measure the soil moisture changes using tensiometers at different points of the hillslope. Two different tensiometer reactions can be differentiated: the immediate reaction and the long-term reaction. The difference between these two reactions are related to different processes. In the following, we assume a three-dimensional element of the soil cover with an effective porosity n. Theoretically, at a given point the increase of the amount of the water stored in the soil is P/n when the total rainfall P is infiltrate. This reaction is defined as immediate reaction. For example a rainfall of 20 mm gives an instantaneously increase in the height of soil water of 67 mm in a soil cover with an effective porosity of $n = 0.3$. The long-term reaction starts some time later by the moving down, towards the stream, of the water quantity accumulated in the largest part of the hillslope. When the pressure impulse is crossing the tensiometers (as it is shown schematically in Fig. 4b), one can register an increase or a decrease in the soil moisture content accordingly to the width above (larger or smaller). This is, what can be described here as a kinematic wave, which depends strongly on the hillslope width situated above.



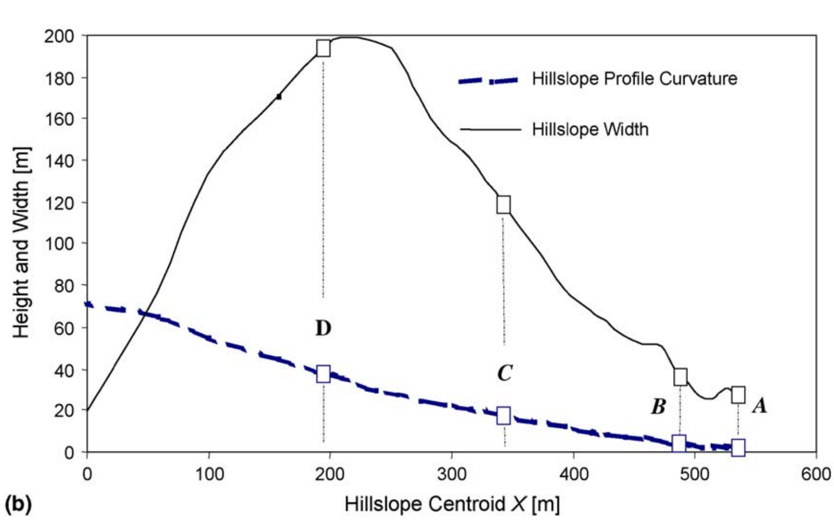


Fig. 5. (a) Locations of measurement stations along the hillslope and its width, (b) hillslope profile curvature and width variation.

The concept of the modelling is explained in Fig. 4. We are dealing here with a homogenous straight profile of the hillslope with a constant soil cover thickness. At initial time t_A, just after the rainfall impulse, the immediate tensiometer reaction at point A can be measured. Here the pressure increases to p_A which is equal to P/n in the area A. In the area B the pressure is p_B and in the area C the pressure is p_C. After time t_B the pressure at point A is reduced as the moisture flux from the part of the hillslope with smaller width (area B) reaches the point A. At time t_C the part of the hillslope around point C drains through A. Here the pressure increases as the runoff is concentrated by the smaller width of the hillslope in the transect A. As it is shown in Fig. 4, the pressure at point A can be therefore characterized by the relationship $p(t_B) = (w_B/w_A) \cdot P/n$ and/or $p(t_C) = (w_C/w_A) \cdot P/n$. Vice versa one can estimate the slope width of the hillslope at C from the tensiometer measurements using the following relationship:

$$w_C = ((pt_C)/(pt_A))w_A \tag{1}$$

4. Interpretation of the hillslope measurements

In Fig. 5(a), the experimental hillslope designated with (11) in Fig. 1(d), is used to illustrate schematically the mechanism of a kinematic wave passing from the hillslope to the flood plain. This quasi-convergent hillslope is instrumented with tensiometers installed in different depths, at four stations along the water course. Station A at the flood plain near the stream, station B, C and D are located at different distances to the flood plain. Fig. 5(b) shows the shape of the correspondent profile curvature and the width function. After a precipitation event the most water quantity belongs to the part of the hillslope with the largest width and forms the peak of the moisture wave. This water quantity flows down towards the river. The tensiometers installed in the station B and A will measure the passing of this wave and its delay. The peak of the wave can be related to the hillslope width and its delay to the length of the hillslope. Within a uniform hillslope with straight profile, this wave formation should not exist because the width function is constant.

In Fig. 6, some field measurements using time domain reflectometry (TDR) along the hillslope surface are shown and carried out in order to give information on the first 15 cm depth. From the presented contours the soil moisture converge towards the station C and more to the station B and after that to station A. As the soil moisture path follows the soil gradient, it has therefore a tendency to concentrate the moisture following the hillslope plan curvature and allows the saturation excess overland flow to appear in the floodplain beside the stream. During rainy periods the area between the stations B and A is not able to transmit the total water amount coming down from the area situated above as interflow. Here return flow is formed.

Piezometers were also used to monitor the groundwater in the flood plain and in a transect parallel to the river, Fig. 7(a). The groundwater surface was registered by pressure at 10 gauges. During a precipitation event in autumn of year 2000 (≈5 mm), the rising of the groundwater surface was measured as it is shown in Fig. 7(b). The higher rise in groundwater results from

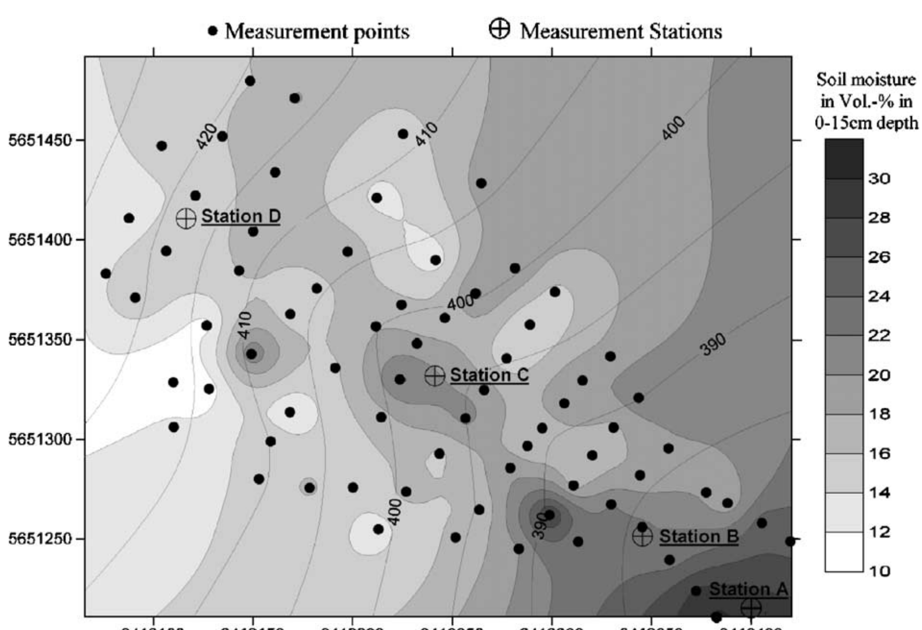


Fig. 6. Soil moisture distribution in the analysed hillslope 24.08.2001.

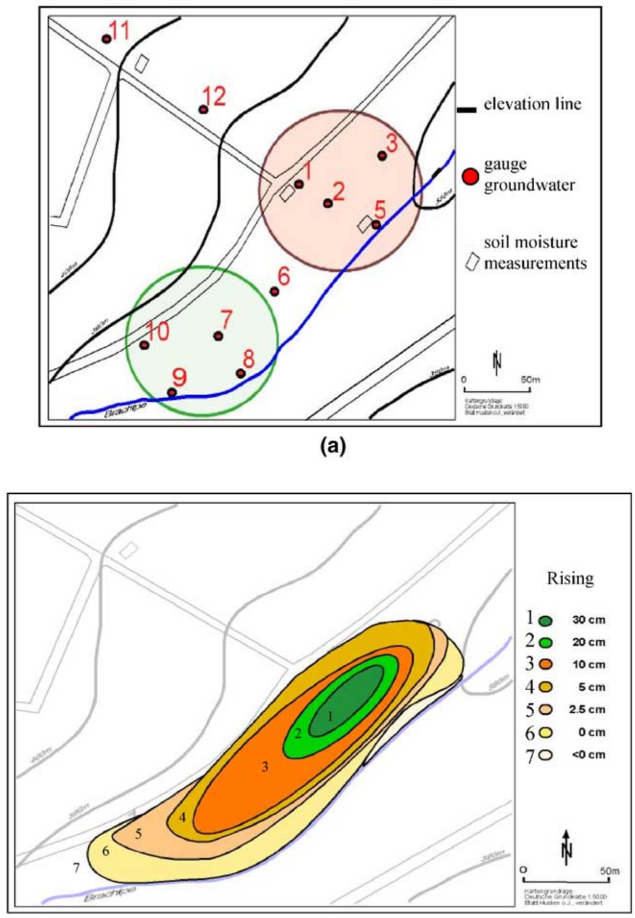


Fig. 7. Groundwater monitoring in the hillslope. (a) Spatial distribution of the GW-piezometers, (b) GW-iso-line sketch showing the groundwater rising during a rainfall event.

the converging plan curvature of the studied hillslope (gauges: 1, 2, 3 and 5) and the lower rise is located in front of the diverging plan curvature of the neighbouring hillslope (gauges: 7, 8, 9 and 10, see also Fig. 1(d)). Fig. 7(b) shows that the same process observed in Fig. 6.

In Fig. 8, the results of tensiometer measurements are shown and presented as suction values. The water potential curves at stations A and B are described respectively in Fig. 8 by the curves (2) and (4). The variation of the groundwater surface is shown in the same stations respectively by the curves (3) and (5). The water level in the river is presented by the curve (1). As the groundwater level is derived from the measured water potential, we can therefore focus our comments on the groundwater evolution and on the water level in the river. A similarity between the water level variation in the stream and the groundwater level variation in the floodplain (station A) becomes evident. The distance of station A to the bank is short (about 2 m apart). Station B is more distant which is nearly 40 m away from the river. We noticed here a "long-term" reaction of the groundwater rising wave after each precipitation event. For example in summer 2000, the precipitation event of 11.07.2000 has given a "long-term" groundwater rise of 50–60 cm registered with a delay of about one day. Fig. 9 shows a similar situation just after the event of 01 October 2000.

A more detailed analysis can be given from Fig. 10. Here the event of 22–23 January 2001 with 36 mm precipitation has produced a rise of the groundwater about 90 cm which forms a wave passing the station B with about 30 hours delay, (Fig. 10). This event make the station A in the floodplain a complete saturated area. As

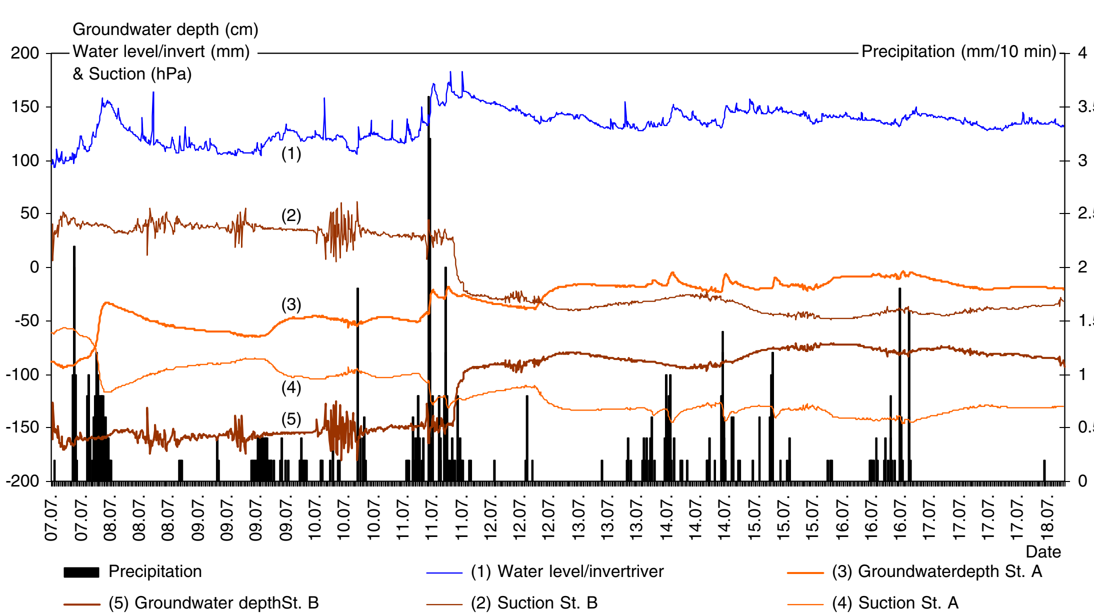


Fig. 8. Field measurement of soil water changes 07–18 July 2000.

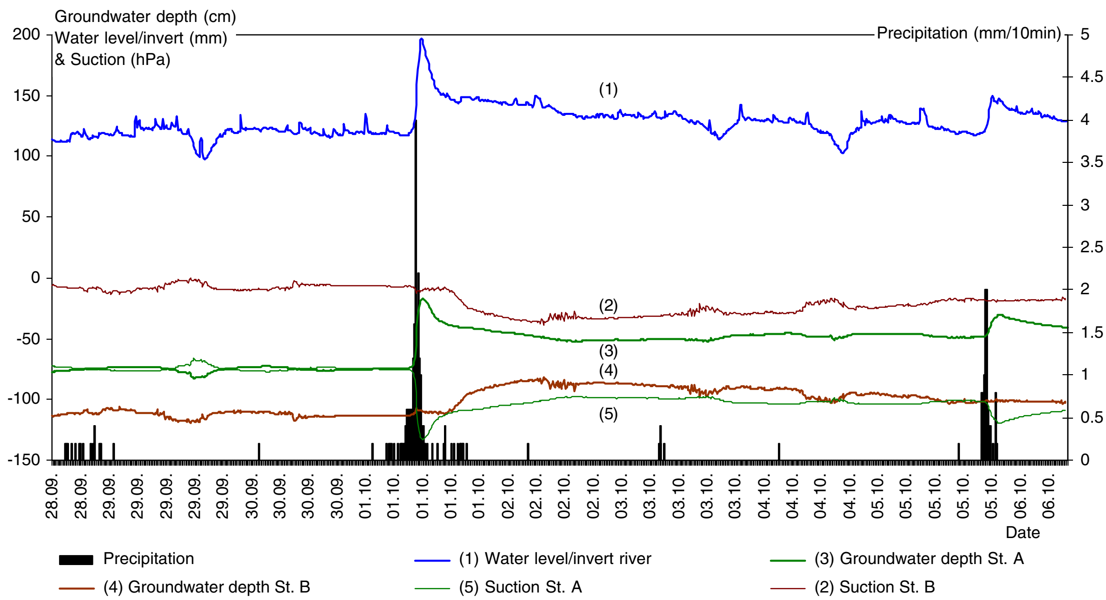


Fig. 9. Field measurement of soil water changes 28 September 2000 to 06 October 2000.

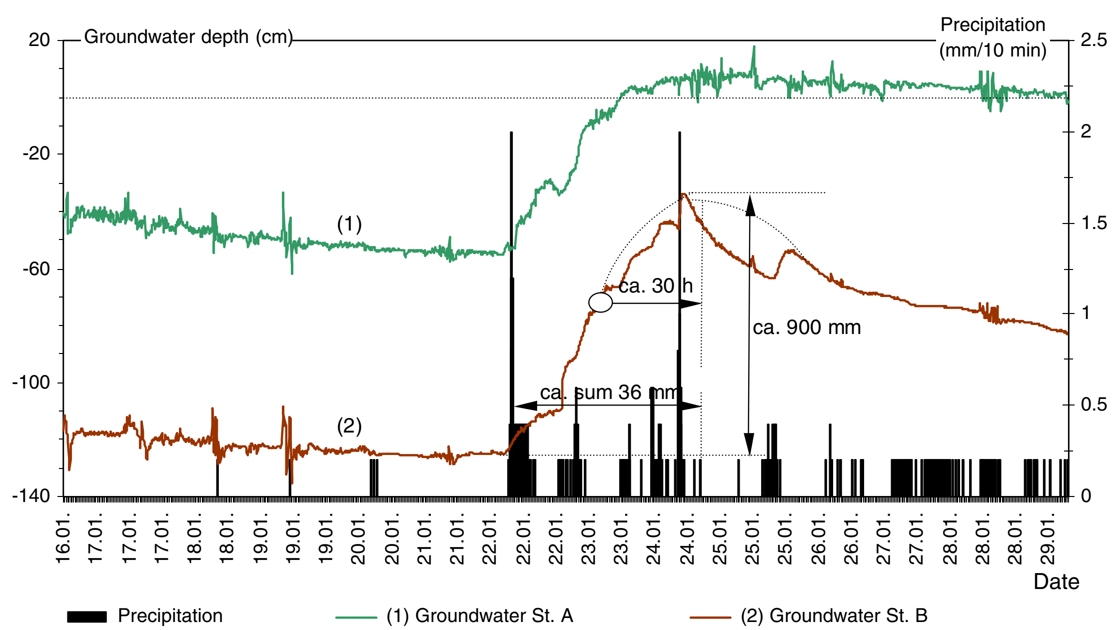


Fig. 10. Field measurement of groundwater level at two stations (A and B) during a rainfall event (16–29 January 2001).

the groundwater rising wave is limited by the soil surface, the surface runoff is consequently formed by return flow. If the observed reaction is immediate, this precipitation of 36 mm (sum of events) would produce groundwater rising wave of only 12 cm (soil porosity of $n = 0.3$) or only 9 cm (if the porosity would be 0.4). Here the groundwater rise was 90 cm as it is shown in Fig. 10. In this case, the soil porosity should be 0.04. So, this reaction cannot be an immediate reaction. These results confirms the hypothesis of a kinematic wave crossing the hillslope which was mentioned before as a long-term reaction. For the constant slope (linear profile) of this hillslope the width maximum of the hillslope situated upstream from station B can be estimated from the groundwater wave of 90 cm. For this purpose the width factor relationship given by Eq. (1) can be used. For a porosity of 0.3 the width factor is 7.5, and respectively equal 10 in case of porosity of 0.4. That means, the maximal width must be 7.5 (or 10) times the width at station B. The topographic analysis shows for the hillslope one factor of about 8 (Fig. 5(b)) which is reasonably approved. The effective drainable porosity of the hillslope therefore seems to be in the range between 0.3 and 0.4. The delay itself is much more difficult to interpret since it is depends on more unknown parameters like lateral permeabilities.

5. Modelling

A mathematical hillslope-based formulation of subsurface storm flow and saturation overland flow described by a modified kinematic wave with a non-linear approximation is used here. A hillslope is defined as the area between two streamlines originating from two ends of a channel link or from the channel head and ending at the ridgelines of the hillslope. The three-dimensional soil cover of a hillslope is reduced to a one-dimensional profile, along which the soil moisture and discharge are modelled. The equation of continuity, non-linear kinematic form of the modified Darcy's law, and the overland flow equation, lead to non-linear wave equations which can be solved using the finite differences method. Under the known kinematic wave assumptions [2,4,13], we have:

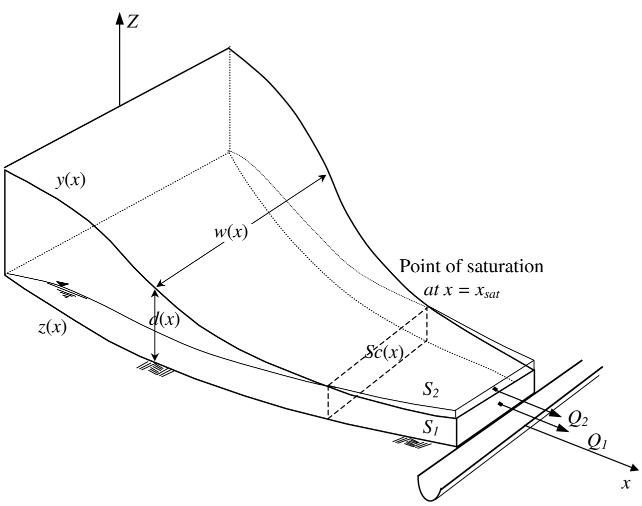


Fig. 11. Scheme of a three-dimensional hillslope.

$$\frac{\partial(S_1+S_2)}{\partial t}+\frac{\partial(Q_1+Q_2)}{\partial x}=i(t)w(x) \tag{2}$$

$$Q_1=-\frac{kS_1}{n}\left(\frac{\partial}{\partial x}(S_1/nw)+\tan\theta\right)\cos\theta \tag{3}$$

$$Q_2=-Cy'(x)S_2 \tag{4}$$

Fig. 11 schematically represents the conditions for subsurface, return flow and overland flow formulation.

Here $S_1(x,t)$ is the soil moisture [m^2] expressed as an area which can be compared with the cross-section area of the soil cover of the hillslope, $S_2(x,t)$ is cross-section area of the surface runoff resulting from return flow after the accumulation of the water fluxes near the base [m^2]. $Q_1(x,t)$ is subsurface flow (a volume). θ is the local bedrock-gradient [–], and $\tan\theta = z'(x)$ is bedrock gradient along the centroid profile (dividing the area into two equally sides), and $y'(x)$ is the terrain gradient. $Q_2(x,t)$ is the surface flow (volume) resulting from return flow [m^3/s], $i(t)$ is the effective rainfall rate [m/s], k is the effective lateral conductivity for saturated conditions [m/s], C is the overland flow conductance [m/s]. When the storage $S(x,t)$ exceeds the storage capacity $S_C(x)=w(x)d(x)n$ return flow is formed which results in overland runoff $Q_2\cdot S_C(x)$ denotes the thickness of the pore space as the soil moisture capacity, where $w(x)$ is the slope width [m], $d(x)$ is the soil depth [m],

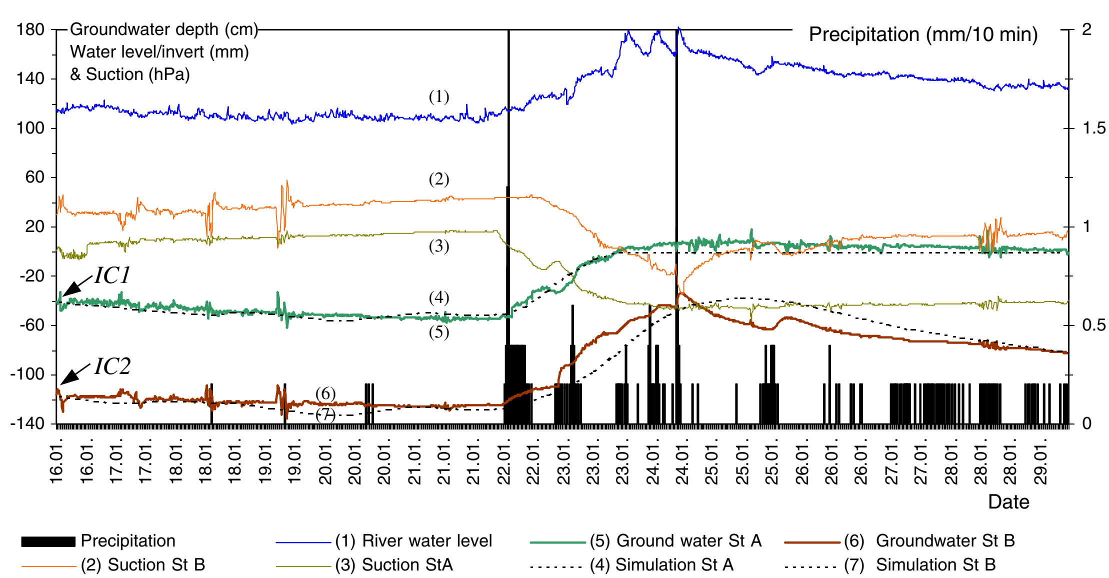


Fig. 12. Comparison of field measurement with simulations 16–29 January 2001.

and n is the effective drainable porosity. $S_C(x)$ decreases downwards in the convergent case. The mathematical model presented here, includes a non-linearity which makes it difficult to solve it analytically. With simplifications, it becomes possible to achieve solutions. These simplifications consist mostly of a simplified form of Darcy's law in Eq. (3), which is also used by [4,14] to develop their new analytical solutions.

The simulations were based on a discretisation of Eqs. (2)–(4) using Lax-method of finite differences [8]. The measured soil moisture values (points *IC1* and *IC2*), at the beginning in Fig. 12, are used as initial condition for the calculations. The simulation was calibrated to reproduce the experimental curves by varying the saturated permeability k and the effective porosity n of the soil cover. The result shown in Fig. 12 is obtained with the parameters: $n = 3.3$ and $k = 9.0\text{E}{-4}$ m/s. In physical term, it is very questionable if these values are reasonable or not for silt to silty clay effected by the macro porosity due to vegetation (see Fig. 2).

6. Conclusion

In this paper we have investigated the effect of the hillslope plane and profile curvatures on the soil moisture distribution and related fluxes using the kinematic wave approximation. Within a research project dedicated to the model-based description of runoff formation processes, field measurement of soil moisture dynamics and numerical simulation are combined. It is desired in this study to highlight the originality of the results of the real field measurements, simplicity of the kinematic wave approximation like it is formulated here and validation of the results by combination experiment-modelling.

The studied watershed was subdivided into drainage areas and hillsides obtained from application of the GIS. These hillsides can be classified into three generalized types: divergent, convergent and uniform hillslopes. One convergent hillslope was instrumented in order to observe the soil moisture dynamics such as soil moisture movement and groundwater level evolution. Soil moisture conditions were registered at four locations (stations). Each station is equipped with several tensiometers installed at different depths. After a precipitation event the most water quantity belongs to the part of the hillslope with the largest width and forms the peak of the moisture wave. The tensiometers installed in the stations measure the passing of this wave and its delay. The peak of the wave can be related to the hillslope width and its delay to the length of the hillslope.

Time domain reflectometry (TDR) were also used along the hillslope surface in order to give information on the first 15 cm soil depth. It was observed that the soil moisture path follows the soil gradient, it has therefore tendency to concentrate the moisture following the hillslope plan curvature and allows the saturation excess overland flow to appear in floodplain beside the stream.

Piezometers were also used to monitor the groundwater in the flood plain and in a transect parallel to the river. It was observed by precipitation event that the higher rise in groundwater (wave peak) results from the converging studied hillslope however the lower rise was observed in the diverging neighbouring hillslope. As the groundwater rising wave in the converging hillslope is limited by the soil surface, the surface runoff is consequently formed by return flow. These results confirms the hypothesis of a kinematic wave crossing the hillslope which was mentioned in this paper as a long-term reaction.

The observed soil moisture movement downwards within a hillslope can be interpreted as a kinematic wave which has a peak proportional to the largest hillslope width. However the delay of the peak is difficult to interpret, as it is related to the soil type and the geomorphological condition. A mathematical application of kinematic wave approach taking the geomorphological factors into account (plan and profile curvatures) can be performed successfully by finite differences method to solve the coupled continuity equation and the modified Darcy equation. May be these results and these conclusions are not unexpected, however it is nice in our opinion to see by field experiment the basic principles and assumptions related to kinematic wave approximation tested in the real field and confirmed. The proposed modelling of the kinematic wave approximation, like it is formulated here, seems to approximate adequately the phenomena, to describe well the process measured and to predict relatively good the results for the proposed analysed converged hillslope.

This field experiments should be extended to divergent and uniform hillslopes. Moreover, it will be interesting also to monitor this behaviour when the profile curvature changes. The concave, the convex and the straight forms of the hillslope may have a significant effect as it was shown by [14] in their original analytical solution.

Acknowledgments

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