

# Spatial Error Concealment Using Optimized Directional Decision and Extrapolation

Yan Zhao\*, Qin Chen\*, Hexin Chen\*, Markus Rupp†

\*School of Communication Engineering, Jilin University, Changchun, China  
Email: {zhao, chx}@email.jlu.edu.cn

†Institute of Communications and Radio-Frequency Engineering, Vienna University of Technology, Vienna, Austria  
Email: mrupp@nt.tuwien.ac.at

**Keywords:** Spatial, error concealment, Sobel, Hessian, directional extrapolation

## Abstract

In this paper, we present a new spatial error concealment algorithm with optimized directional decision and extrapolation. First, we employ the Sobel operator and a Hessian matrix to determine the exact direction in the cases of existing both step edges and line edges in the image. Then, with the accurately estimated direction, lost MBs are recovered by directional extrapolation pixel-by-pixel. From experimental results, the proposed spatial error concealment method provides both subjective and objective quality improvement of concealment especially for images with more lines.

## 1 Introduction

With the development of third-generation (3G) networks, mobile multimedia services such as video streaming and video telephone become reality at present. However, the network still does not provide enough bandwidth to guarantee error-free transmission of the information packet. We still need to conceal the corrupted regions in the received images or videos due to channel errors.

Spatial error concealment techniques recover the lost pixels using the correlation between adjacent pixels in an image. They have been widely studied by researchers. The method of project-onto-convex-sets (POCS) estimates the edge information by spatial and spectral domain [10]. First, a constraint-satisfying iterative process is applied. Then, directional filtering is used for recovery of the missing MB. A content-adaptive spatial error concealment method for video communication is proposed in [8]. In our previous work, a spatial error concealment using directional extrapolation, which addresses concealment of missing image blocks on a pixel-by-pixel basis, was proposed [12]. In order to recover edge information inside the corrupted regions, the edge feature in neighbouring MBs is required to estimate in the spatial domain. Many researches measure the edge feature by using a Sobel operator [1-10,12,13]. However, the Sobel operator is not a very efficient tool for detecting line edges and their direction since it is a first-order operator. Thus, the detection of edges and their direction is not accurate for corrupted regions including line edges.

In this work, we propose a new solution to spatial error concealment, which can measure both step and line edges by using a Sobel operator combined with a Hessian matrix. The proposed spatial error concealment algorithm consists of two parts: edge direction estimation and pixel extrapolation.

The rest of this paper is organized as follows. The edge direction estimation process is presented in Section 2. Section 3 describes an extrapolation algorithm to conceal missing image pixels. Section 4 gives the experimental results. Section 5 presents the conclusions.

## 2 Edge direction estimation

### 2.1 Hessian matrix

For line edge, the normal direction of the edge,  $(n_x, n_y)$  corresponds to the eigenvector of the maximum absolute eigenvalue of its Hessian matrix. Therefore, the normal direction of the edge  $(n_x, n_y)$  and its second order derivative can be obtained by calculating the maximum magnitude of the eigenvalue and its corresponding eigenvector of the Hessian matrix.

The Hessian matrix of an image is given by:

$$\mathbf{H}(x, y) = \begin{bmatrix} g_{xx} & g_{xy} \\ g_{xy} & g_{yy} \end{bmatrix} \quad (1)$$

where partial derivatives  $g_x$ ,  $g_y$ ,  $g_{xx}$ ,  $g_{xy}$ ,  $g_{yy}$  are obtained by convoluting the image and the Gauss convolution kernels, which are as follows.

$$\begin{cases} h_x(x, y) = h'(x)h(y) \\ h_y(x, y) = h(x)h'(y) \\ h_{xx}(x, y) = h''(x)h(y) \\ h_{xy}(x, y) = h'(x)h'(y) \\ h_{yy}(x, y) = h(x)h''(y) \end{cases} \quad (2)$$

For an image, the first order derivative at the central point of the line edge is zero. That is the point for which the first order directional derivative of the edge direction is zero and the second order directional derivative takes the maximum

absolute value. This is considered the central point of the line edge. The point with the first order derivative zero, lies in the current pixel if  $|tn_x \leq 0.5|$  and  $|tn_y \leq 0.5|$ , where

$$t = -\frac{n_x g_x + n_y g_y}{n_x^2 g_{xx} + n_x n_y g_{xy} + n_y^2 g_{yy}} .$$

The current pixel is

determined as the center of the line edge if the point, which the first order derivative is zero, lies in the pixel and the second order derivative of  $(n_x, n_y)$  is larger than a given threshold.

We use the mask of size 3×3 to determine edge and its direction. The mask is shown in Figure 1.

$z_1$	$z_2$	$z_3$
$z_4$	$z_5$	$z_6$
$z_7$	$z_8$	$z_9$

Figure 1: The 3×3 mask

That is to say, the image mentioned above is actually a 3×3 subimage or mask in our implementation.

Let  $\lambda$  is the maximum absolute eigenvalue,  $\theta$  is the angle of the line edge direction, then we have:

$$\tan \theta = -\frac{\lambda - g_{yy}}{g_{xy}} \quad (3)$$

From  $\|(n_x, n_y)\| = 1$ , we can get:

$$n_x = \frac{-\tan \theta}{\sqrt{\tan^2 \theta + 1}} \quad (4)$$

$$n_y = \frac{1}{\sqrt{\tan^2 \theta + 1}} \quad (5)$$

We determine the line edge exists in the mask if  $|tn_x \leq 0.5|$ ,  $|tn_y \leq 0.5|$  and the maximum absolute eigenvalue of Hessian matrix, which is also the second order derivative of  $(n_x, n_y)$ , is larger than a given threshold  $\eta$ .

### 2.2 Sobel operator

For step edge detection, the 3×3 Sobel is used. This is achieved by simply calculating

$$\tan \theta = \frac{G_y}{G_x} \quad (6)$$

where  $\theta$  is the angle of the edge direction.

$$G_y = (z_7 + 2z_8 + z_9) - (z_1 + 2z_2 + z_3)$$

$$G_x = (z_3 + 2z_6 + z_9) - (z_1 + 2z_4 + z_7)$$

Obviously,  $\frac{G_y}{G_x} = \frac{g_y}{g_x}$ . The magnitude of this edge direction, denoted  $df$ , where

$$df = [G_x^2 + G_y^2]^{1/2} \quad (7)$$

### 2.3 Directional determination

For every mask, we first determine if there is a line edge in it by using the Hessian matrix. If yes, we calculate the direction, that is  $\tan \theta$ , and the magnitude of the line edge direction is given an available maximum value. Then, the step edge is detected based on the Sobel operator if there is no line edge existing in the mask. If the step edge with some direction exists in the mask, we calculate further the magnitude. Otherwise, the magnitude of this direction is set zero.

## 3 Pixel extrapolation

The pixels in the missing block are recovered pixel-by-pixel in our algorithm. Assume the pixel to be recovered is  $x$ , ten edge directions which are likely to traverse  $x$  are shown in Figure 2.

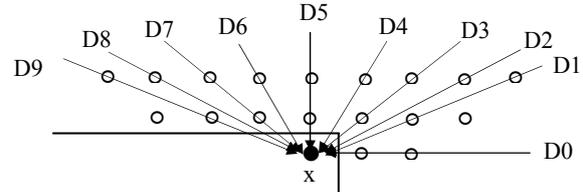


Figure 2: Ten edge directions which have chance to traverse the to-be-recovered pixel

We use ten masks with the size of 3×3, each for one direction, to determine if there is an edge with one of the ten directions traversing  $x$ . Pixels in the mask are denoted by  $z_1$  to  $z_9$  as shown in Figure 1. The nearest pixel to  $x$  on any direction is denoted  $x_1$  and the second nearest pixel is denoted  $x_2$ . Example positions of  $x_1$  and  $x_2$  on directions D3 and D9 are shown in Figure 3. The 3×3 masks for directions D0, D1, D3,

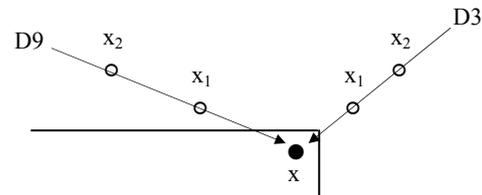


Figure 3: Example positions of  $x_1$  and  $x_2$  on directions D3 and D9

D5, D7 and D9 are centered at  $x_2$ , that is, the pixel  $z_5$  in the 3×3 mask for these directions is  $x_2$ . The 3×3 masks for directions D2, D4, D6 and D8 are centered at  $x_1$ , that is, the

pixel  $z_5$  in the  $3 \times 3$  mask for directions D2, D4, D6 and D8 is  $x_1$ . For each direction, we first determine if the line or step edge with this direction exists in its corresponding mask. Then, the  $\tan \theta$  of this direction is calculated if there is a line or step edge in the mask.

For the example of direction  $D_1$ , if  $\tan \theta$  lies in the range between  $1/4$  and  $7/12$ , it indicates that the edge with direction  $D_1$  exists in the mask. The relationship between ranges of  $\tan \theta$  and ten directions are illustrated in Table 1.

The direction with the maximum magnitude is considered as the edge direction which traverses the pixel to be recovered. If the magnitudes of ten directions are all zero, it indicates that there is not any edge traversing the pixel to be recovered. In other words, the pixel  $x$  is supposed to be in a flat region. If there is an edge direction traversing the pixel  $x$ , it will be recovered through extrapolation of two pixels  $x_1$  and  $x_2$  along the determined direction as:

$$x = \frac{2}{3}x_1 + \frac{1}{3}x_2 \quad (8)$$

If pixel  $x$  is in a flat region, it will be recovered based on weighted pixel averaging [11].

Range of $\tan \theta$	Direction
$-1/4 \leq \tan \theta < 1/4$	D0
$1/4 \leq \tan \theta < 7/12$	D1
$7/12 \leq \tan \theta < 5/6$	D2
$5/6 \leq \tan \theta < 3/2$	D3
$3/2 \leq \tan \theta < 9/2$	D4
$\tan \theta \geq 9/2$ or $\tan \theta < -9/2$	D5
$-9/2 \leq \tan \theta < -3/2$	D6
$-3/2 \leq \tan \theta < -5/6$	D7
$-5/6 \leq \tan \theta < -7/12$	D8
$-7/12 \leq \tan \theta < -1/4$	D9

Table 1: Relationship between the ranges of  $\tan \theta$  and ten directions

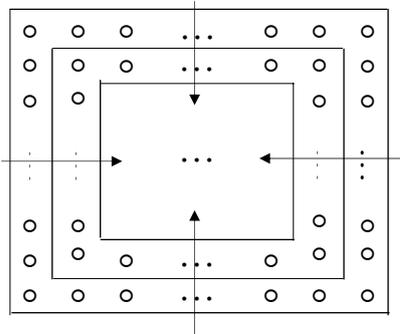


Figure 4: The order of pixels to be recovered

The order of pixels to be recovered in the missing block is from the outer layer to the inner layer, as shown in Figure 4.

In each layer, the pixels of the upper and down rows are recovered first. Then, the pixels of the left and right columns are recovered. In the upper and down rows, pixels are recovered from left to right. The recovery order of pixels in the left column is from up to down, whereas, the recovery order of pixels in the right column is from down to up. The previously recovered pixels can be used for direction decision and extrapolation of the following to-be-recovered pixels.

## 4 Experimental results

Two images are used to evaluate our proposed algorithm. One image is Lena with the size of  $512 \times 512$ . Another image is a typical image as it appears in the stock market, which is a StockCurve with the size of  $512 \times 384$ . We test our algorithm with block size  $8 \times 8$ . The threshold  $\eta$  is set to 500 in our experiments. We use the traditional PSNR as the objective measure of the recovered image.

Table 2 presents the PSNR comparison between the proposed algorithm and the previous work.

Image	Previous [12] dB	Proposed dB
Lena	33.22	33.24
StockCurve	23.81	25.05

Table 2: Performance comparison in PSNR (dB)

Table 2 shows that the PSNR performance for the Lena image over the previous method is about 0.02dB, while for the StockCurve image, which has more lines in the image, has about 1.2 dB gains over the previous method. Figure 5 shows the comparison of the corresponding reconstructed Lena images recovered by the previous work [12] and the proposed method: (a) is the original image, (b) is the corrupted image, (c) is the reconstructed image using algorithm in [12] and (d) is the reconstructed image given by the proposed technique. Figure 6 shows the comparison of the corresponding reconstructed StockCurve images recovered by the previous work [12] and the proposed method: (a) is the original image, (b) is the corrupted image, (c) is the reconstructed image using algorithm in [12] and (d) is the reconstructed image given by the proposed method. It can be observed from Figure 5 and Figure 6 that the proposed algorithm can also recover the corrupted block with better subjective quality, especially for images with more lines.

## 5 Conclusions

This paper presents a new spatial error concealment algorithm. Many spatial error concealment methods detect edge directions by using the Sobel operators only. However, for line edges, the Sobel operator is not effective. We propose to use both a Hessian matrix together with a Sobel operator to implement optimized direction decision both for line and step edges. From experimental results, the proposed spatial error concealment algorithm can provide both objective and subjective improvement of the previous work, especially for images with more lines.



Figure 5: (a) Original Lena image. (b) Corrupted image. (c) Reconstructed image by [12], PSNR = 33.22dB. (d) Reconstructed image by proposed method, PSNR = 33.24dB.

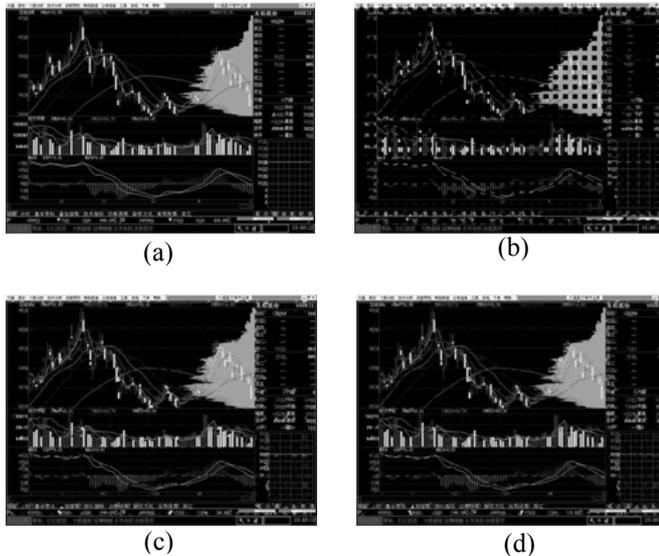


Figure 6: (a) Original StockCurve image. (b) Corrupted image. (c) Reconstructed image by [12], PSNR = 23.81dB. (d) Reconstructed image by proposed method, PSNR = 25.05dB.

## Acknowledgements

This work was supported by the project of Science and Technology Development Plan of Jilin Province under Grant 20070528 and the project of Ministry of Science and Technology, under Grant 2005DFA10300.

The authors would like to thank the support of EURASIA-PACIFIC UNINET scholarship.

## References

- [1] Jihua Cao, Jichang Guo, Fengting Li. "A spatial domain error concealment algorithm with texture recovery for MPEG2", Canadian Conference on Electrical and Computer Engineering, Volume 1, pp. 265-268, (2004).
- [2] M.-H. Jo, H.-N. Kim and W.-J. Song, "Hybrid error concealments based on block content", IET Image Processing, Volume 1, pp. 141-148, (2007).
- [3] W. Kwok and H. Sun, "Multi-directional interpolation for spatial error concealment", IEEE Trans. Consum. Electron., Volume 39, pp. 455-460, (1993).
- [4] Wei-Ying Kung, Chang-Su Kim and C.-C. Jay Kuo, "Spatial and Temporal Error Concealment Techniques for Video Transmission Over Noisy Channels", IEEE Trans. on Circuits and System for Video Technology, Volume 16, pp. 789-802, (2006).
- [5] Wonki Kim, Jasung Koo and Jechang Jeong. "Fine Directional Interpolation for Spatial Error Concealment", IEEE Transactions on Consumer Electronics, Volume 52, pp.1050-1056, (2006).
- [6] O. Nemethova, A. Al Moghrabi, M. Rupp. "An Adaptive Error Concealment Mechanism for H.264/AVC Encoded Low-Resolution Video Streaming", Proceedings of 14th European Signal Processing Conference (EUSIPCO), Florence, Italy, 4-8. Sept. (2006).
- [7] O. Nemethova, A. Al Moghrabi, M. Rupp. "Flexible Error Concealment for H.264 Based on Directional Interpolation", Proceedings of the 2005 International Conference on Wireless Networks Communications and Mobile Computing, WirelessCom05, Maui, Hawaii, Volume 2, pp. 1255-1260, June 13-16, (2005).
- [8] Z. Ronfu, Z. Yuanhua, and H. Xiaodong, "Content-adaptive spatial error concealment for video communication", IEEE Trans. Consum. Electron., Volume 50, pp. 335-341, (2004).
- [9] J.W. Suh and Y. S. Ho. "Error concealment based on directional interpolation", IEEE Trans. Consum. Electron., Volume 43, pp. 295-302, (1997).
- [10] H. Sun and W. Kwok, "Concealment of damaged block transform coded images using projections onto convex sets," IEEE Trans. Image Processing, Volume 4, pp. 470-477, (1995).
- [11] Ye-Kui Wang, M. M. Hannuksela, Viktor Varsa, Ari Hourunranta and Moncef Gabbouj. "The error concealment feature in the H.26L test model," Proc. of ICIP 2002, Volume II, pp. 729-732, (2002).
- [12] Y. Zhao, H. Chen, X. Chi, J. S. Jin, "Spatial Error Concealment Using Directional Extrapolation", Proceedings of Digital Image Computing: Techniques and Applications, pp. 278-283, (2005).
- [13] Y. Zhao, D. Tian, M.M.Hannukasela, M. Gabbouj, "Spatial Error concealment Based on Directional Decision and Intra Prediction", IEEE International Symposium on Circuits and Systems, Volume 3, pp.2899-2902, (2005).