Practical Successive Interference Cancellation in the Binary-Input Gaussian Multiple-Access Channel

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***Abstract*—In the multiple access channel, successive interference cancellation (SIC) can be used to achieve the boundary points of the capacity region. In this paper, we investigate the practical potential of SIC by employing a set of practical moderate-blocksize Low-Density Parity-Check codes in the Multiple Access additive white Gaussian noise channel; in particular we consider binary modulation. The theoretically achievable points in the capacity region are compared with the practically achievable operating points at a low bit-error rate. It is shown that SIC makes available a higher transmission rate compared with simple separate detection that treats the signal of the other user as noise. This statement is also true for the use of binary modulation and a practical efficient implementation with channel codes of moderate blocksize.**

I. Introduction

In the multiple access channel (MAC), all users share the common channel resources to transmit information to a destination. Time-Division Multiple Access (TDMA) and Frequency Division Multiple Access (FDMA) systems are based on the orthogonal division of the resources "time" and "frequency", with only one user occupying a frequency at a time. Contrary to that in CDMA each user is assigned a distinct signature sequence that allows for orthogonal signal transmission with more than one user occupying a frequency at the same time [1]; the price to pay is the increased bandwidth due to the "signature sequence" that has to be transmitted to send one information bit. Still, theoretical analysis shows that CDMA is a more efficient multiple access method [2] than TDMA and FDMA, mainly due to its greater flexibility.

In principle, the CDMA idea can also be used for channel-coded sequences without any explicit "spreading" code and in this case it is known from Information Theory [3] that Successive Interference Cancellation (SIC) is a technique that can achieve the upper limits of the capacity region of the Gaussian Multiple Access Channel. Signal superposition together with SIC theoretically guarantee that one of the users can transmit at the capacity-approaching rate, as in the single-user AWGN channel, while the other user can still realise reliable transmission at a non-zero rate that is determined by the Gaussian MAC capacity region: essentially, the second user's signal is treated as "noise" by the first user, i.e., the first user suffers from a reduced rate due to interference. When the first user's rate is sufficiently low, the signal can be reliably decoded at the receiving end, and the re-encoded signal can be subtracted (cancelled) from the total received signal, leaving only the Gaussian receiver noise. This means the receiver "sees" a Gaussian channel without any interference for the second user.

Although, at a first glance, this theoretical concept seems also attractive for a practical application there are some potential drawbacks. First, the theory assumes that Gaussian signal alphabets are used, which is not possible in practice. Hence, we go to the opposite extreme and investigate the performance of SIC schemes when a binary modulation signal alphabet is used. This is motivated by the idea to have very simple transmitters such as wireless network nodes communicating to a receiving "base station" where we allow for some higher complexity to perform more advanced signal processing such as successive interference cancellation. Second, as it is impossible to transmit strictly reliable in practice, the first user's transmit signal can not always be reconstructed correctly at the receiver, which means the subtraction of the erroneous signal would lead to propagation of the first user's decoding errors into the decoding process for the second user. Hence, it is unclear what the performance benefits of SIC will be in practice: this is exactly the topic of the presented work in which we investigate soft multiple user demodulators in which the interference cancellation in carried out in a "soft" sense using bit-reliability information.

In our work we use low-density parity-check (LDPC) codes which are very powerful error correcting codes. Well-designed and with very large blocksize these codes can achieve very low bit-error-rates (BER) [4]. In practical applications, however, delay constraints usually put an upper limit on the coding blocksize. Therefore, we use practical moderate-blocksize LDPC codes and investigate their performance in the Gaussian MAC with soft successive interference cancellation.

II. System description and Channel Capacities

A. System Model (Two Users)

In a Multiple Access Channel (MAC), also called uplink multiuser channel, all users send signals to one receiver. The discrete-time model for the two-user AWGN MAC is shown in Figure 1. The receive signal is given by

$$y[i] = x_a[i] + x_b[i] + n[i], \quad (1)$$

with i the time index, $n[i] \sim N(0, \sigma_w^2)$ the i.i.d. Gaussian receiver noise with power spectral density (PSD) $N_0/2$, the variance $\sigma_w^2 = N_0 \cdot B$ with B the base-band system bandwidth. For sake of simplicity we assume real baseband signals; an extension to complex baseband signals is straightforward. The received user signals are $x_a[i]$ and $x_b[i]$. We integrate any power scaling and any fixed path-loss into the average power constraints such that $\mathrm{E}\{(x_{a,b}[i])^2\} = P_{a,b}$.

B. Capacity of the single-user discrete-time AWGN Channel

The capacity for the ideal additive white Gaussian noise (AWGN) channel with average input power constraint P is achieved with a Gaussian distribution of the input symbols. The capacity equals

$$C = \frac{1}{2}\log_2(1 + \frac{P}{\sigma_w^2}) \text{ bits per transmission.} \quad (2)$$

For the band-limited AWGN channel with bandwidth B, the capacity of the channel can be rewritten as

$$C = B\log_2(1 + \frac{P}{N_0 B}) \text{ bits per second.} \quad (3)$$

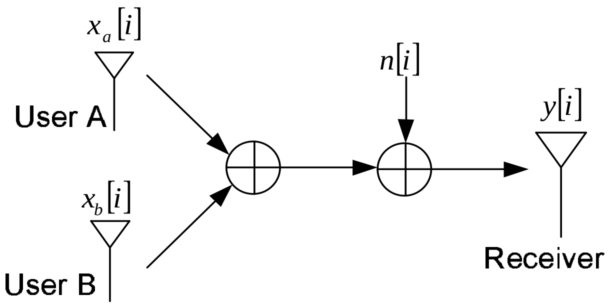

Fig. 1. Multiple access channel.

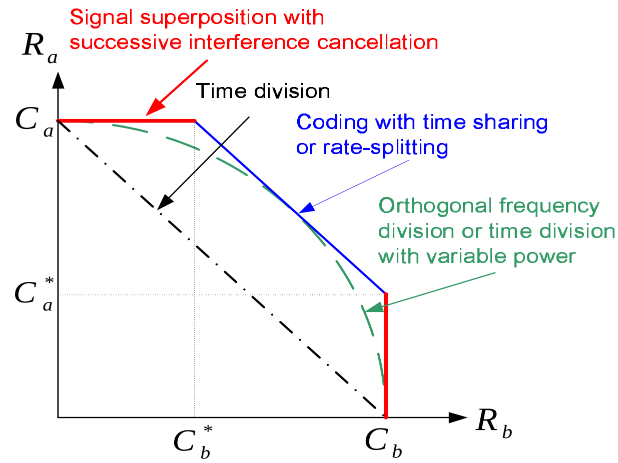

Fig. 2. Capacity region of the two-user multiple access channel [5].

The channel capacity for the binary-input discrete-time AWGN channel with equiprobable input bits $\xi \in \{0,1\}$ and the channel output signal y is

$$C = \sum_{\xi \in \{0,1\}} \int_{-\infty}^{\infty} \frac{p(y|\xi)}{2} \log_2 \frac{2 \cdot p(y|\xi)}{p(y|\xi=1) + p(y|\xi=0)} dy \tag{4}$$

with C in bits per transmission (channel use). For binary phase shift keying modulation (with coherent detection) we map the input bits $\xi \in \{0,1\}$ to the modulation signal "constellation" $\{+1,-1\}$. However, as we have integrated any power scaling and path loss into the power constraints P_a, P_b we have for the received modulation signals

$$x_{a,b}(\xi) = \sqrt{P_{a,b}} \cdot (1 - 2 \cdot \xi) \tag{5}$$

Hence, the probability density function (pdf) of the Gaussian noise equals

$$p(y|\xi) \doteq \frac{1}{\sqrt{2\pi\sigma_w}} \exp(-\frac{1}{2\sigma_w^2}(y - x_{a,b}(\xi))^2) \ . \tag{6}$$

C. Capacity Region for the two-user AWGN MAC

The two-user capacity region is given by all rate vectors (R_a, R_b) satisfying the following constrains [3]:

$$\begin{aligned} R_a &\leq B \log_2 \left(1 + \frac{P_a}{N_0 B}\right) \text{ bits / second} \\ R_b &\leq B \log_2 \left(1 + \frac{P_b}{N_0 B}\right) \text{ bits / second} \\ R_a + R_b &\leq B \log_2 \left(1 + \frac{P_a + P_b}{N_0 B}\right) \text{ bits / second} \end{aligned} \tag{7}$$

The interpretation for the capacity region is straightforward: The first two lines of (7) say that the transmission rates for the individual users can not exceed the Gaussian capacity limits in the single-user AWGN channel. The last line of (7) puts an upper limit on the sum rate of the users: this sum can not exceed the rate limit that we obtain on a Gaussian channel with the sum of the powers of both users.

The capacity region for the two-user Gaussian MAC is shown in Figure 2 [5], where C_k and C_k^* are given by

$$C_k = B \log_2 \left(1 + \frac{P_k}{N_0 B}\right), \ k = a, b, \tag{8}$$

$$C_a^* = B \log_2 \left(1 + \frac{P_a}{N_0 B + P_b}\right), \tag{9}$$

$$C_b^* = B \log_2 \left(1 + \frac{P_b}{N_0 B + P_a}\right). \tag{10}$$

The points $(R_b, R_a) = (0, C_a)$ and $(C_b, 0)$ are achieved when one transmitter sends its data at its maximum rate in the single-user AWGN channel while the other user is silent. The rate pairs on the straight line between $(0, C_a)$ and $(C_b, 0)$ are achieved by a time-division strategy between two transmitters operating at their maximum rates with fixed power. The rate pairs on the line $(0, C_a) - (C_b^*, C_a)$ and the rate pairs on the line $(C_b, C_a^*) - (C_b, 0)$ are achieved by using signal superposition together with SIC. The rate pairs on the straight line connecting (C_b^*, C_a) and (C_b, C_a^*) are achieved by time-sharing between the two points or a rate-splitting technique [6]. For the latter a user splits its data stream into multiple substreams and encodes these substreams as if they originated from different virtual users.

For comparison, Figure 2 also contains the rate region resulting from orthogonal signalling (TDMA, FDMA) without SIC: the rate region achieved by those schemes touches the boundary of the capacity region in exactly one point, which corresponds to the rate at which we obtain the maximum sum rate. In all other points orthogonal signalling is strictly suboptimal. The problem is particularly significant, when the users have strongly different receive power: in this case the weaker user will hardly get any rate at all in the capacity-achieving point [7, pp. 232–234].

III. Application of Low-Density Parity-Check Codes in Successive Interference Cancellation

A. Codes

LDPC codes were invented by Gallager [8], [9] in the early 1960s and largely ignored until they were re-discovered [10] almost 40 years later. LDPC codes are linear block codes with a particular structure for the parity check matrix **H** in which the fraction of nonzero entries is small. This allows graph-based decoders (e.g. Sum Product Algorithm SPA [11]) to be applied efficiently with remarkable performance. A key feature of the Sum Product Decoding algorithm is that soft reliability information from the channel output can be exploited for decoding.

We don't deal with details of LDPC codes and the SPA decoder as such in this paper (full details can be found, e.g., in [11]). We rather provide a scheme with which log-likelihood ratios (L-values) [12] can be obtained for the SIC scheme that we apply in the MAC: L-values are the key to a successful application of soft-in put channel decoding by the sum-product algorithm.

B. Signal Superposition and Successive Interference Cancellation

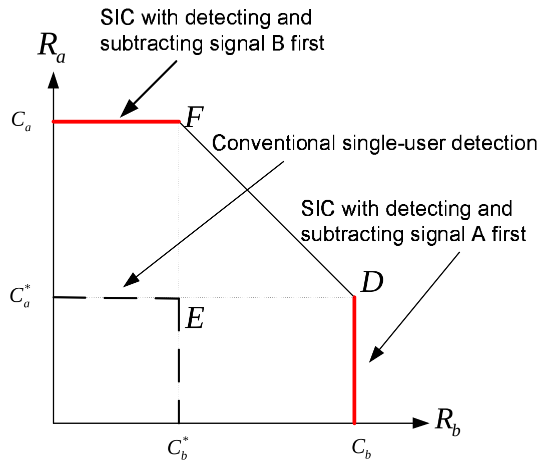

Fig. 3. Capacity boundary for SIC and conventional single-user detection in two-user multiple access channel.

Successive Interference Cancellation (SIC) is a multiuser detection technique which can be used to achieve the boundary of the capacity region of a Multiple-Access Channel [7]. In contrast to conventional single-user detection (e.g., [2]), in which other users' signals are treated as noise when decoding one user's signal, SIC is based on subtracting (cancelling) the already detected signals from the received signal before the detection of the next signal. The more disparate the users' powers are the higher the potential gain from SIC.

The point D in Figure 3 indicates that user B could transmit at capacity-approaching rate in the point-to-point transmission model, while user A can still transmit at the maximun rate of C_a^* without errors. If signal A is the first one to be decoded, signal B is treated as Gaussian noise to user A in the first signal detection stage.

In Figure 3, the line $C_b - D$ is achieved by signal superposition together with SIC when signal A is first detected and subtracted. If the order of the signal detection is reversed and signal B is first detected, by SIC the capacity boundary $C_a - F$ could be achieved. The capacity boundary encompassed by C_a^*, E and C_b^* is for signal superposition without SIC which is the conventional single-user detection method. The theoretical capacity region shows that signal superposition with SIC is superior to the conventional single-user detection method.

In practice, however, reliable communication with channel codes of limited block size is impossible in a strict sense. Therefore, instead of "cancelling by signal subtraction" (which would cause error propagation in case of incorrect decoding), we resort to a "soft" *a posteriori* probability (APP) demodulator, which can deal with reliability information for bits rather than with hard decisions only. This concept (in the given framework) is known from e.g. [13], [14]. In what follows we consider the special case of coherently detected binary phase-shift keying modulation (BPSK) and we state a formulation of a multi-user detector that lends itself to an efficient implementation by L-value algebra. We also compare the simulation results with the unconstrained Gaussian MAC capacity region.

C. Multiuser APP-Demodulator for BPSK Modulation

The basic idea is to extract the *a posteriori* probability for the transmitted bits $B_k[i]$ of all users $k = 1, 2, ..., K$ to be "one" or "zero" given the received channel value y (see Fig. 1 for the two-user case). The channel model for the general K-user case is given by

$$y[i] = \sum_{k=1}^{K} x_k[i] + n[i] \ , \tag{11}$$

where

$$x_k(b_k[i]) = \sqrt{P_k}(1 - 2b_k[i]) \tag{12}$$

and $b_k[i] \in \{0, 1\}$ is the bit sent by user k at time i; P_k is the receive power for the signal transmitted by user k. We now calculate the APP L-value [12] of the bit $b_k[i]$:

$$L(B_k[i]|y) = \log \frac{\Pr(B_k[i] = 0|y)}{\Pr(B_k[i] = 1|y)} \tag{13}$$

where $B_k[i]$ indicates the random variable of the bit and $b_k[i] \in \{0, 1\}$ its realisation. As we consider only one time-instant in the detector, we omit the time index i in what follows as long as there is no risk of confusion.

We expand

$$\Pr(B_k = 0|y) = \sum_{\forall\{b_1,...,b_K\}:b_k=0}^{K} \Pr(B_1 = b_1, ..., B_K = b_K|y) \tag{14}$$

and use the Bayes-rule which immediately leads to

$$L(B_k|y) = \log \frac{\sum\limits_{\forall \mathbf{b}:b_k=0} p(y|\mathbf{b}) \cdot \Pr(\mathbf{b})}{\sum\limits_{\forall \mathbf{b}:b_k=1} p(y|\mathbf{b}) \cdot \Pr(\mathbf{b})} \tag{15}$$

with the bit vector realisations $\mathbf{b} \doteq \{b_1, b_2, ..., b_K\}$ (we use the same definition for the bit vector random "variable" $\mathbf{B}$). The notation $\forall \mathbf{b}: b_k = 0$ denotes all possible bit combinations of the other users, with the bit value of "zero" for the user k under consideration. For later use we further define $\mathbf{b}\backslash b_k \doteq \{b_1, b_2, ..., b_{k-1}, b_{k+1}, ..., b_K\}$. Moreover, we assume that the users' code bits are independent (rather realistic assumption for independent users), so that

$$\Pr(\mathbf{b}) = \prod_{k=1}^{K} \Pr(b_k) \ . \tag{16}$$

(above, we use the abbreviation $\Pr(B_k = b_k) = \Pr(b_k)$).

If we assume a Gaussian channel, we can write the probability density function (PDF) of the channel as follows:

$$p(y|\mathbf{b}) = \frac{1}{\sqrt{2\pi}\sigma_w} \exp\left(-\frac{1}{2\sigma_w^2}\left(y - \sum_{k=1}^{K} x_k(b_k)\right)^2\right) \tag{17}$$

Up to this stage, the equation are also valid for non-binary symbols, but now we start to exploit the special BPSK modulation scheme: we reformulate the exponent of the PDF in (17): with $x_0 \doteq -y$ and the abbreviation $x_k \doteq x_k(b_k), k = 1, 2, ...$ we obtain

$$\begin{aligned}\left(y - \sum_{k=1}^{K} x_k(b_k)\right)^2 &= \left(\sum_{k=0}^{K} x_k\right)^2 \\ &= \sum_{k=0}^{K} x_k^2 + 2\sum_{k=0}^{K-1} x_k \sum_{l=k+1}^{K} x_l \\ &= y^2 + \sum_{k=1}^{K} x_k^2 - 2y \cdot \sum_{k=1}^{K} x_k + 2\sum_{k=1}^{K-1} x_k \sum_{l=k+1}^{K} x_l \end{aligned} \tag{18}$$

As $x_k = \sqrt{P_k}(1-2b_k)$, with $b_k \in \{0,1\}$, we have $x_k^2 = P_k$, which does *not* depend on the choice of the bit b_k. Therefore, we can re-write (15) as follows:

$$L(B_k|y) = \log \frac{\Pr(B_k=0) \sum_{\forall \mathbf{b}: b_k=0} p(y|\mathbf{b}) \cdot \Pr(\mathbf{b}\backslash b_k)}{\Pr(B_k=1) \sum_{\forall \mathbf{b}: b_k=1} p(y|\mathbf{b}) \cdot \Pr(\mathbf{b}\backslash b_k)}$$

$$= L(B_k) + \log \frac{\sum_{\forall \mathbf{b}: b_k=0} e^{f(y,b_1,...,b_K)/\sigma_w^2} \cdot \Pr(\mathbf{b}\backslash b_k)}{\sum_{\forall \mathbf{b}: b_k=1} e^{f(y,b_1,...,b_K)/\sigma_w^2} \cdot \Pr(\mathbf{b}\backslash b_k)} \quad (19)$$

with

$$f(y, b_1, ..., b_K) = y \cdot \sum_{\kappa=1}^{K} x_\kappa - \sum_{\xi=1}^{K-1} x_\xi \sum_{l=\xi+1}^{K} x_l \quad (20)$$

Note that, as above, $x_\kappa = \sqrt{P_\kappa}(1 - 2 \cdot b_\kappa)$ for BPSK modulation. We now separate out the bit b_k in (20), i.e., we isolate $x_k = x_k(b_k)$:

$$f(y, b_1, ..., b_K) = x_k \Big(y - \sum_{\substack{\xi=1 \\ \xi \neq k}}^{K} x_\xi\Big) + y \sum_{\substack{l=1 \\ l \neq k}}^{K} x_l - \sum_{\substack{\xi=1 \\ \xi \neq k}}^{K-1} x_\xi \sum_{\substack{l=\xi+1 \\ l \neq k}}^{K} x_l$$

$$= x_k \cdot y + \Big(y - x_k\Big) \sum_{\substack{\xi=1 \\ \xi \neq k}}^{K} x_\xi - \sum_{\substack{\xi=1 \\ \xi \neq k}}^{K-1} x_\xi \sum_{\substack{l=\xi+1 \\ l \neq k}}^{K} x_l \quad (21)$$

When we use (21) in (19) with $x_k(b_k) = \sqrt{P_k}(1-2b_k)$ we find

$$L(B_k|y) = L(B_k) + \log \frac{\exp(y \cdot x_k(0)/\sigma_w^2)}{\exp(y \cdot x_k(1)/\sigma_w^2)} + L_E(B_k)$$

$$= L(B_k) + 2\frac{\sqrt{P_k}}{\sigma_w^2} \cdot y + L_E(B_k) \quad (22)$$

where

$$L_E(B_k) = \log \frac{\sum_{\forall \mathbf{b}\backslash b_k} e^{g_0(\mathbf{b}\backslash b_k)/\sigma_w^2} \cdot \Pr(\mathbf{b}\backslash b_k)}{\sum_{\forall \mathbf{b}\backslash b_k} e^{g_1(\mathbf{b}\backslash b_k)/\sigma_w^2} \cdot \Pr(\mathbf{b}\backslash b_k)} \quad (23)$$

with

$$g_\rho(\mathbf{b}\backslash b_k) = \Big(y - x_k(\rho)\Big) \sum_{\substack{\xi=1 \\ \xi \neq k}}^{K} x_\xi - \sum_{\substack{\xi=1 \\ \xi \neq k}}^{K-1} x_\xi \sum_{\substack{l=\xi+1 \\ l \neq k}}^{K} x_l \;, \quad (24)$$

with $\rho \in \{0,1\}$ and $x_k = \sqrt{P_k}(1-2b_k)$. As long as we are using binary modulation, (23) can be evaluated for, e.g., up to $K = 10$ users without serious complexity problems. If more users are in the system and the received powers P_i for their signals are significantly different, we can approximate the sums in (24) by only considering a subset of the "strongest" users.

We can further develop (23) by using (16):

$$\Pr(\mathbf{b}\backslash b_k) = \prod_{\substack{l=1 \\ l \neq k}}^{K} \Pr(b_l) \; . \quad (25)$$

We now express the probabilities $\Pr(b_l)$ of the bits of user l by its corresponding "a-priori" L-value

$$L(B_l) \doteq \log \frac{\Pr(B_l = 0)}{\Pr(B_l = 1)} \; . \quad (26)$$

By inversion of (26) we obtain

$$\Pr(B_l = b) = \frac{e^{-L(B_l)\cdot b}}{1 + e^{-L(B_l)}}, \quad b \in \{0,1\} \; . \quad (27)$$

We use (25) to find

$$\Pr(\mathbf{b}\backslash b_k) = A \cdot \exp\left(-\sum_{\substack{l=1 \\ l \neq k}}^{K} L(B_l) \cdot b_l\right) \quad (28)$$

with the constant A that will cancel out in (23). We combine (28) and (24) conveniently for use in (23) and obtain

$$h_\rho(\mathbf{b}\backslash b_k) = \frac{1}{\sigma_w^2} g_\rho(\mathbf{b}\backslash b_k) - \sum_{\substack{l=1 \\ l \neq k}}^{K} L(B_l) \cdot b_l$$

$$= \frac{1}{\sigma_w^2}\left(\Big(y - x_k(\rho)\Big) \sum_{\substack{\xi=1 \\ \xi \neq k}}^{K} x_\xi - \sum_{\substack{\xi=1 \\ \xi \neq k}}^{K-1} x_\xi \sum_{\substack{l=\xi+1 \\ l \neq k}}^{K} x_l\right) - \sum_{\substack{l=1 \\ l \neq k}}^{K} L(B_l) \cdot b_l \quad (29)$$

with

$$L_E(B_k) = \log \frac{\sum_{\forall \mathbf{b}\backslash b_k} \exp(h_0(\mathbf{b}\backslash b_k))}{\sum_{\forall \mathbf{b}\backslash b_k} \exp(h_1(\mathbf{b}\backslash b_k))} \quad (30)$$

As only the factor $x_k(\rho)$ is different in the numerator and denominator of (30), most of $h_\rho(\mathbf{b}\backslash b_k)$ needs only to be computed once. Moreover, the probability-weighting by the L-values for the bits of other users appears as a simple sum of L-values (rightmost term in (30), which are scaled by the bit-values $b_l \in \{0,1\}$: hence, only the "one"-bits in the bit-vector sum-"index" $\mathbf{b}\backslash b_k$ need to be considered. The double-sum in (29) is most conveniently computed by starting with $\xi = K-1$: the result of the inner sum can then be recursively used to compute the next result. As long as the channel-SNRs don't change and with a limited number of users in the system, all sums in (29), apart from the last term, can in principle be pre-computed and stored as they don't depend on y or on the L-values of the other users' bits. Moreover, we can ignore terms in the sums that contribute very little to the total result (which stem typically from users with very low channel SNR); this helps to cope with a larger number of users in the system. In the simulations below we have used (22), (29) and (30) for the special case of $K = 2$ users.

IV. Simulation Results and Analysis

The LDPC codes used here are randomly generated regular LDPC codes[1] of blocksize 1200 with column weight 3. For the theoretical capacity calculation, (2) is for Gaussian input symbols which could not be implemented in the real world, while the capacity equation (4) for binary input symbols could be used as a benchmark for BPSK modulation. In the following simulations, this set of LDPC codes is used in the two-user MAC with BPSK modulation in order to compare the achievable rates by signal superposition and SIC with the capacity region. The theoretical signal-to-noise ratio (SNR) to achieve error free transmission and actual SNR for those middle blocksize LDPC codes to achieve BER around 10^{-5} in the single-user AWGN

[1]The regular random LDPC matrices were constructed using the online software available at http://www.cs.toronto.edu/∼radford/ldpc.software.html This program generates random regular LDPC codes of any specified rate and block length and is capable of expurging four cycles in the LDPC constraint graph.

TABLE I
THEORETICAL SNR AND ACTUAL SNR FOR THE 1200 BLOCKSIZE LDPC CODES BY BPSK MODULATION.

Code rate	0.4	0.5	0.7	0.8	0.9
Theoretical SNR (E_s/N_0 in dB)	-4.2182	-2.8233	-0.277	1.0704	2.7393
Actual SNR (E_s/N_0 in dB)	-1.9791	-0.8103	1.4510	3.0103	4.2424

TABLE II
SIMULATION RESULTS FOR $P_a = 1.5 * P_b$

Point	SNR & BER	Signal A	Signal B
(0.9, 0.4)	SNR (dB)	6.0033	4.2424
	BER	0	3.33×10^{-5}
(0.9, 0.5)	SNR (dB)	6.0033	4.2424
	BER	0.0295	0.034
(0.4, 0.9)	SNR (dB)	6.3849	4.6240
	BER	0	1.91×10^{-5}

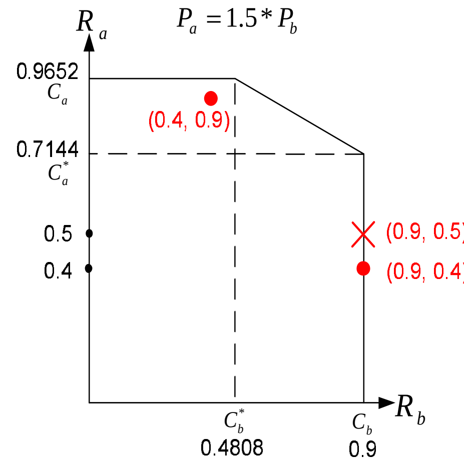

Fig. 4. Practical achievable point by using middle blocksize LDPC codes. $P_a = 1.5 * P_b$

channel are listed in Table I. The SNR is represented by E_s/N_0 which is the ratio of energy per transmitted code symbol and the one-sided power spectral density N_0 of the Gaussian channel noise.

In the multi-user framework below, the SNR next refers to a user's signal power P_k over the background Gaussian noise variance σ_w^2 in the channel, and the background Gaussian noise variance σ_w^2 is conveniently set to "1" (0dB) for the simulations.

A. First scenario: $P_a = 1.5 * P_b$

In this scenario, the received power of signal A is 1.5 times that of the received power of signal B. The parameters used to set up the simulation are SNR 6.00dB for signal A and SNR 4.24dB for signal B. The theoretical SNR for 0.9 information bits per channel use should be 2.74dB shown in Table I, but for this middle blocksize LDPC code SNR of 4.24dB (about 1.5dB more than the theoretical value) is needed to achieve BER below 10^{-4}. According to the SNRs for the signal A and signal B, the binary input theoretical capacity values C_a and C_b could be obtained by (4). C_a^* is obtained by (4) according to the assumption in the capacity calculation that the channel noise and the signal B both represent Gaussian interference to signal A. This Gaussian interference assumption will be analysed later. C_b^* is obtained in the same way by assuming signal A and channel noise are both "Gaussian interference" to B. The theoretical binary-input capacity calculation shows that under this scenario, as the rate of signal B (C_b) is fixed, the rate of signal A could achieve as far as C_a^*. In a similar way, as the rate of signal A is fixed at C_a, the rate of signal B could be increased to as far as C_b^* with error free transmission. By signal superposition and soft SIC, the first signal is detected as described in Section III-C and the channel code is decoded by SPA algorithm [11]. If the first user's signal is successfully decoded (indicated by CRC check), the re-encoded signal of the first user is directly subtracted from the total received signal. The "left" signal is for detecting and decoding the second user's signal. If the first user's signal is not totally successfully decoded (failure in CRC check), soft SIC described in Section III-C is needed for the second user's detection. The simulation result is shown in table II and the theoretical capacity value and the practical achievable region for this scenario is shown in Figure 4.

1) point (0.9, 0.4): The simulation result shows that by using signal superposition and SIC, error free transmission (or very low BER for user A) could be achieved when the signal A with stronger received power transmits at the rate of 0.4 and the signal B with weaker received power transmits at the rate of 0.9. The stronger signal A is detected first.

2) point (0.9, 0.5): According to the theoretical capacity calculation, (0.9, 0.5) should be within the achievable region. But the results show that this rate pair is not achievable. The equivalent SNR for the first decoded user A in the multiple access channel is -0.09dB under the assumption that the background Gaussian channel noise (variance 0dB) and the signal B are Gaussian interference to the signal A. But for the middle blocksize rate 0.5 LDPC code, SNR of -0.81dB is enough for error free transmission in single-user AWGN channel. It should be noticed that to the first signal A, the noise with power $(\sigma_w^2 + P_b)$ is the superposition of channel noise satisfying Gaussian distribution and the signal B satisfying the Bernoulli distribution (which takes value -1 with probability p and +1 with probability $1-p$ in the BPSK modulation). The pdf of this combined noise with power $(\sigma_w^2 + P_b)$ is

$$g(x) = p \cdot f(x+a) + (1-p) \cdot f(x-a), \tag{31}$$

in which $f(x) = \frac{1}{\sqrt{2\pi}\sigma_w} e^{-\frac{1}{2\sigma_w^2}(x)^2}$ and a represents the symbol power after BPSK. From (31) we could see that it is no longer Gaussian distributed noise to the first user. So the decoding of the first signal is not as good as in the single-user AWGN channel even though higher SNR is provided.

3) point (0.4, 0.9): According to the setup SNR in this scenario, the maximum code rate for user A by the theoretical calculation should be 0.9652. Here the code rate 0.9 is used instead in order to analyse the impact by reversing the order of detection. The decoding order for the point (0.4, 0.9) is to detect signal B with weaker power first, then after operating SIC the data for signal A with stronger signal is decoded. BER below 10^{-4} is also achievable for this point but the corresponding SNR needs to be (6.38dB, 4.62dB) which is higher than the SNR needed for point (0.9, 0.4). It means that if the decoding order is reversed to decode the weaker signal first, more signal power is needed to achieve the error free transmission in the MAC. Here, the power of one user is only 1.5 times of the other. If the two users have quite disparate power received at the destination, the decoding order will have an obvious influence on the total signal power to meet given target transmission rates [7].

B. Second scenario: $P_a = 2.0 * P_b$

In this scenario, the received power of signal A is twice of the received power of signal B. The parameters used to set up the simulation are listed in Table III. By detecting the stronger signal A first and operating soft SIC to obtain the second signal B when the first signal is not correctly decoded, point (0.8, 0.4) and point (0.8, 0.5) are achievable while the point (0.8, 0.7) could not be reached. The simulation results are shown in Table III and Figure 5.

TABLE III
SIMULATION RESULTS FOR $P_a = 2.0 * P_b$

Point	SNR & BER	Signal A	Signal B
(0.8, 0.4)	SNR (dB)	6.02	3.01
	BER	0	6.5×10^{-6}
(0.8, 0.5)	SNR (dB)	6.02	3.01
	BER	3.41×10^{-6}	1.13×10^{-5}
(0.8, 0.7)	SNR (dB)	6.02	3.01
	BER	0.082	0.128

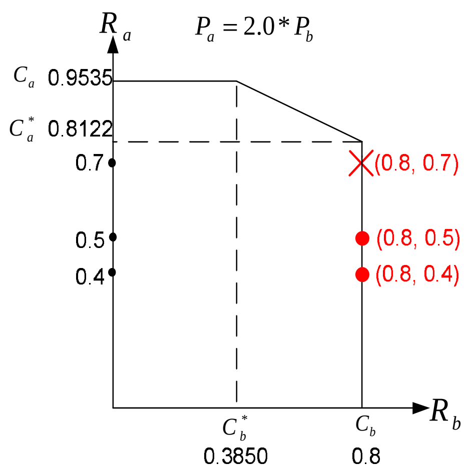

Fig. 5. Practical achievable point by using middle blocksize LDPC codes. $P_a = 2.0 * P_b$.

V. CONCLUSION

Signal superposition together with SIC is the technique which achieves the boundary of the capacity region for the Gaussian Multiple Access Channel. A set of practical LDPC codes with moderate blocksize are used to investigate the practical performance of interference cancellation. In order to match the BER performance under the BPSK modulation, the binary-input channel capacity equation is used to obtain the theoretical values as the benchmark for the simulations. In practice SIC provides a higher transmission rate for the second detected user compared with single-user detection method. The order of the signal detection and subtraction also has an impact on the required signal power to achieve the desired transmission rate. If the previously detected signal is just subtracted, error propagation will occur when the signal was not decoded correctly. For that reason we used a "soft" interference cancellation scheme that takes into account the reliabilities with which bit decisions after channel decoding can be taken. The soft interference canceller was developed into a new form that lends itself for an efficient implementation that will also allow for iterative interference cancellation; this will be a topic of further work.

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REFERENCES

[1] John G. Proakis, *Digital Communications*, The McGraw-Hill Companies,Inc., 2001.

[2] D. Koulakiotis and A.H. Aghvami, "Data detection techniques for ds/cdma mobile systems: a review," *Personal Communications, IEEE [see also IEEE Wireless Communications]*, vol. 7, no. 3, pp. 24–34, June 2000.

[3] Joy A. Thomas Thomas M. Cover, *Elements of Information Theory*, John Wiley & Sons, Inc., second edition, 2006.

[4] T.J. Richardson and R.L. Urbanke, "The capacity of low-density parity-check codes under message-passing decoding," *Information Theory, IEEE Transactions on*, vol. 47, no. 2, pp. 599–618, Feb 2001.

[5] Andrea Goldsmith, *Wireless Communications*, Cambridge University Press, 2005.

[6] A.J. Grant, B. Rimoldi, R.L. Urbanke, and P.A. Whiting, "Rate-splitting multiple access for discrete memoryless channels," *Information Theory, IEEE Transactions on*, vol. 47, no. 3, pp. 873–890, March 2001.

[7] David Tse and Pramod Viswanath, *Fundamentals of Wireless Communication*, Cambridge University Press, 2005.

[8] R.G. Gallager, *Low Density Parity Check Code*, Cambridge:MA:MIT Press, 1963.

[9] R. Gallager, "Low-density parity-check codes," *Information Theory, IEEE Transactions on*, vol. 8, no. 1, pp. 21–28, Jan 1962.

[10] D.J.C. MacKay and R.M. Neal, "Near shannon limit performance of low density parity check codes," in *Electronics Letters*, 29 Aug. 1996, vol. 32, p. 1645.

[11] S. Lin and Daniel J. Costello, *Error Control Coding*, Pearson Prentice Hall, 2004.

[12] J. Hagenauer, E. Offer, and L. Papke, "Iterative decoding of binary block and convolutional codes," *Information Theory, IEEE Transactions on*, vol. 42, no. 2, pp. 429–445, March 1996.

[13] G. Caire, R. Müller, and T. Tanaka, "Iterative multiuser joint decoding: Optimal power allocation and low-complexity implementation," *IEEE Transactions on Information Theory*, vol. 50, no. 9, pp. 1950–1973, Sep. 2004.

[14] E.G. Larsson and B.R. Vojcic, "Cooperative transmit diversity based on superposition modulation," *Communications Letters, IEEE*, vol. 9, no. 9, pp. 778–780, Sep 2005.