A Shared-Relay Cooperative Diversity Scheme Based on Joint Channel and Network Coding in the Multiple Access Channel

Xiaoyan Xu, Mark F. Flanagan, Norbert Goertz
Institute for Digital Communications, Joint Research Institute for Signal & Image Processing
The University of Edinburgh, Scotland, UK
Email: X.Xu@ed.ac.uk, mark.flanagan@ieee.org, Norbert.Goertz@ieee.org

***Abstract*—In this paper we propose a cooperative diversity scheme for the scenario of two sources sharing a single relay. The scheme uses algebraic code superposition relaying in the multiple access fading channel to create spatial diversity under the constraint of limited communications resources. We also describe in detail a novel computationally efficient message passing algorithm at the destination's decoder which extracts the substantial spatial diversity contained in the code superposition and signal superposition. The decoder is based on a sliding window structure where certain *a posteriori* LLRs are retained as *a priori* LLRs for the next decoding. We show that despite the simplicity of the proposed scheme, diversity gains are efficiently leveraged by the simple combination of channel coding at the sources and network coding at the relay.**

I. Introduction

While wireless channels suffer from fading, at the same time the broadcast nature of wireless channels provides the possibility of a third party other than the destination "overhearing" the information that the source transmits. Thus apart from the original transmission channel, the same information could be transmitted to the destination through another independently fading channel. This generated spatial diversity can effectively combat the deleterious effect of fading [1]. In recent years, there has been increasing interest in applying the idea of algebraic code superposition, also called "network coding" [2]–[6] to the cooperative communications scenario. The network coding approach provides an efficient way to generate spatial diversity under the constraint of limited resources. One challenge is the problem of decoder design which should be able to cope with the complicated decoding situation at the destination [2]–[6].

In [5]–[7], the model of a typical network coding unit is considered in which the packets from the two sources are linearly combined at the relay. In our work we also focus on this cooperative transmission model for the situation where it is impractical for one mobile user to "capture" the other user's signal during its uplink transmission in the cellular network. Moreover, relay-based cooperative processing provides greater security than direct user cooperation in which user information must be shared.

In [4], a code superposition scheme employing low-density generator matrix (LDGM) codes is proposed to reduce the decoding complexity at the destination. But in order to do the graph-based decoding, the systematic bits must be retained without superposition which means that the potential superposition diversity is lower than that obtainable from fully superposed codewords. In [6], a combined LDPC code construction scheme including two channel code components and one network code component is produced by random parity-check matrix generation under certain constraints. The network codes are actually the parity checks for two channel codewords; this necessitates more complicated relay operations than simple superposition.

In this work we propose a cooperative coding scheme which is different from the previous work of [2]–[6] where superposed codewords experience a channel orthogonal to that of the original transmission, and also different from the previous work of [8] where simple codeword retransmission is employed in the multiple access Gaussian relay channel. Our scheme allows continuous transmission of superposed codewords by the relay and at the same time targets the challenge of coping with the interference introduced by the multiple access channel, thus making efficient use of communication resources to leverage spatial diversity gains. We also detail a novel efficient decoding algorithm based on message passing on the destination node's factor graph for the purpose of exploiting the spatial diversity contained in the algebraically superposed codewords together with the signal superposition introduced by the multiple access channel. The algorithm attains a separation of the three soft-input soft-output (SISO) decoder modules corresponding to each received signal stream; for convolutional codes, this separation affords a complexity advantage over decoding of the "nested code" [2], [3]; for LDPC codes, it affords a more efficient Tanner graph schedule than fully parallel decoding [6].

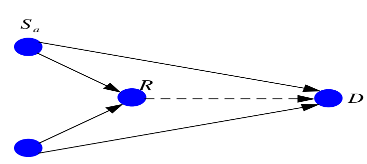

Fig. 1. Four-node communications network. Sources S_a and S_b share a common relay R as well as having direct links to the destination D.

Time Slot	0		1		…	t		t+1		…	L-1		L	
Half Slot	0	1	2	3	…	2t	2t+1	2t+2	2t+3	…	2L-2	2L-1	2L	2L+1
Source Transmits	$\mathbf{c}_0 = \mathbf{a}_0$	$\mathbf{c}_1 = \mathbf{b}_0$	$\mathbf{c}_2 = \mathbf{a}_1$	$\mathbf{c}_3 = \mathbf{b}_1$	…	$\mathbf{c}_{2t} = \mathbf{a}_t$	$\mathbf{c}_{2t+1} = \mathbf{b}_t$	$\mathbf{c}_{2t+2} = \mathbf{a}_{t+1}$	$\mathbf{c}_{2t+3} = \mathbf{b}_{t+1}$	…	$\mathbf{c}_{2L-2} = \mathbf{a}_{L-1}$	$\mathbf{c}_{2L-1} = \mathbf{b}_{L-1}$		
Relay Transmits		$\mathbf{d}_1 = \pi(\mathbf{a}_0)$	$\mathbf{d}_2 = \pi(\mathbf{b}_0) \oplus \mathbf{a}_0$	$\mathbf{d}_3 = \pi(\mathbf{a}_1) \oplus \mathbf{b}_0$	…	$\mathbf{d}_{2t} = \pi(\mathbf{b}_{t-1}) \oplus \mathbf{a}_{t-1}$	$\mathbf{d}_{2t+1} = \pi(\mathbf{a}_t) \oplus \mathbf{b}_{t-1}$	$\mathbf{d}_{2t+2} = \pi(\mathbf{b}_t) \oplus \mathbf{a}_t$	$\mathbf{d}_{2t+3} = \pi(\mathbf{a}_{t+1}) \oplus \mathbf{b}_t$	…	$\mathbf{d}_{2L-2} = \pi(\mathbf{b}_{L-2}) \oplus \mathbf{a}_{L-2}$	$\mathbf{d}_{2L-1} = \pi(\mathbf{a}_{L-1}) \oplus \mathbf{b}_{L-2}$	$\mathbf{d}_{2L} = \pi(\mathbf{b}_{L-1}) \oplus \mathbf{a}_{L-1}$	$\mathbf{d}_{2L+1} = \mathbf{b}_{L-1}$
Destination Receives	$\mathbf{e}_0$	$\mathbf{e}_1$	$\mathbf{e}_2$	$\mathbf{e}_3$	…	$\mathbf{e}_{2t}$	$\mathbf{e}_{2t+1}$	$\mathbf{e}_{2t+2}$	$\mathbf{e}_{2t+3}$	…	$\mathbf{e}_{2L-2}$	$\mathbf{e}_{2L-1}$	$\mathbf{e}_{2L}$	$\mathbf{e}_{2L+1}$
Destination Decodes			$\mathbf{a}_0$	$\mathbf{b}_0$	…	$\mathbf{a}_{t-1}$	$\mathbf{b}_{t-1}$	$\mathbf{a}_t$	$\mathbf{b}_t$	…	$\mathbf{a}_{L-2}$	$\mathbf{b}_{L-2}$	$\mathbf{a}_{L-1}$	$\mathbf{b}_{L-1}$

Fig. 2. Transmission schedule of proposed cooperative coding scheme.

Time Slot	0		1		…	t		t+1		…	L-1		
Half Slot	0	1	2	3	…	2t	2t+1	2t+2	2t+3	…	2L-2	2L-1	2L
Source Transmits	$\mathbf{c}_0 = \mathbf{a}_0$	$\mathbf{c}_1 = \mathbf{b}_0$	$\mathbf{c}_2 = \mathbf{a}_1$	$\mathbf{c}_3 = \mathbf{b}_1$	…	$\mathbf{c}_{2t} = \mathbf{a}_t$	$\mathbf{c}_{2t+1} = \mathbf{b}_t$	$\mathbf{c}_{2t+2} = \mathbf{a}_{t+1}$	$\mathbf{c}_{2t+3} = \mathbf{b}_{t+1}$	…	$\mathbf{c}_{2L-2} = \mathbf{a}_{L-1}$	$\mathbf{c}_{2L-1} = \mathbf{b}_{L-1}$	
Relay Transmits		$\mathbf{d}_1 = \pi(\mathbf{a}_0)$	$\mathbf{d}_2 = \pi(\mathbf{b}_0)$	$\mathbf{d}_3 = \pi(\mathbf{a}_1)$	…	$\mathbf{d}_{2t} = \pi(\mathbf{b}_{t-1})$	$\mathbf{d}_{2t+1} = \pi(\mathbf{a}_t)$	$\mathbf{d}_{2t+2} = \pi(\mathbf{b}_t)$	$\mathbf{d}_{2t+3} = \pi(\mathbf{a}_{t+1})$	…	$\mathbf{d}_{2L-2} = \pi(\mathbf{b}_{L-2})$	$\mathbf{d}_{2L-1} = \pi(\mathbf{a}_{L-1})$	$\mathbf{d}_{2L} = \pi(\mathbf{b}_{L-1})$
Destination Receives	$\mathbf{e}_0$	$\mathbf{e}_1$	$\mathbf{e}_2$	$\mathbf{e}_3$	…	$\mathbf{e}_{2t}$	$\mathbf{e}_{2t+1}$	$\mathbf{e}_{2t+2}$	$\mathbf{e}_{2t+3}$	…	$\mathbf{e}_{2L-2}$	$\mathbf{e}_{2L-1}$	$\mathbf{e}_{2L}$
Destination Decodes		$\mathbf{a}_0$	$\mathbf{b}_0$	$\mathbf{a}_1$	…	$\mathbf{b}_{t-1}$	$\mathbf{a}_t$	$\mathbf{b}_t$	$\mathbf{a}_{t+1}$	…	$\mathbf{b}_{L-2}$	$\mathbf{a}_{L-1}$	$\mathbf{b}_{L-1}$

Fig. 3. Transmission schedule of consecutive relaying scheme in the multiple access channel.

II. Proposed Cooperative Coding Scheme

We consider the four-node communications network depicted in Figure 1, with two sources S_a and S_b, one relay R, and one destination D common to the two sources. The communication period is divided into $L+1$ *time slots* $t = 0, 1, \cdots, L$; each time slot $t \in \{0, 1, \cdots, L\}$ is further subdivided into 2 *half slots* ($2t$, $2t+1$). Source S_a has L messages to transmit, which it encodes into L n-bit codewords $\{\mathbf{a}_t : t = 0, 1, \cdots, L-1\}$. The code used at source S_a is $\mathcal{C}_a$ and is defined by the $m_a \times n$ parity-check matrix $\mathbf{H}_a = (H_a(j,i))$. Similarly, source S_b has L messages to transmit, which it encodes into L n-bit codewords $\{\mathbf{b}_t : t = 0, 1, \cdots, L-1\}$. The code used at source S_b is $\mathcal{C}_b$ and is defined by the $m_b \times n$ parity-check matrix $\mathbf{H}_b = (H_b(j,i))$. Thus, the codes $\mathcal{C}_a$ and $\mathcal{C}_b$ have the same length but not necessarily the same rate. In general, the codes used at the two sources can be LDPC or convolutional; in this paper we concentrate on LDPC codes. S_a and S_b broadcast their modulated codewords to the relay and destination nodes using TDMA and there is no cooperation between the two sources. For each $t \in \{0, 1, \cdots, 2L-1\}$, let $\mathbf{c}_t$ denote the codeword broadcast by the source in half slot t; thus $\mathbf{c}_{2t} = \mathbf{a}_t$ and $\mathbf{c}_{2t+1} = \mathbf{b}_t$ for $t \in \{0, 1, \cdots, L-1\}$.

The relay decodes and then re-encodes each codeword received from the source (the cooperative scheme is based on the scenario where the source is quite close to the relay). The relay also has a buffer in which it stores the codewords it obtained in the previous two half slots. At each half slot ($t = 2, 3, \cdots, 2L$), the relay interleaves the codeword obtained in half slot $t-1$ and superposes it (XOR operation) with the codeword obtained in half slot $t-2$; it then transmits the resulting codeword to the destination in the fading multiple access channel (MAC) whose channel resources are also shared with the source transmission. Special cases arise at half slots 1 and $2L+1$ in which only a single codeword is stored at the relay and no XOR operation is performed. Let $\mathbf{d}_t$ denote the codeword transmitted from the relay to the destination in half slot $t \in \{1, 2, \cdots, 2L+1\}$; thus $\mathbf{d}_t = \pi(\mathbf{c}_{t-1}) \oplus \mathbf{c}_{t-2}$, for $t = 2, 3, \cdots 2L$. Let $\mathbf{e}_t$ denote the signal stream received by the destination in half slot $t \in \{0, 1, \cdots, 2L+1\}$. The use of the MAC allows for no extra expense in terms of channel resources for the proposed cooperative transmission scheme as compared to the non-cooperative scheme. As we shall see, the spatial diversity gain generated by the superposition relaying outweighs the signal interference degradation inherent in using the MAC. To simplify the analysis, we assume that directed antennas are employed at the relay, so that interference between the source-relay link and the relay-destination link may be neglected.[1] For each $t = 0, 1, \cdots L-1$, source S_a's codeword $\mathbf{a}_t$ is decoded at the end of half slot $2t+2$ and source S_b's codeword $\mathbf{b}_t$ is decoded at the end of half slot $2t+3$. The transmission schedule for this cooperative coding scheme is illustrated in Figure 2. It can be seen that spatial diversity for each message is contained in three separate transmissions spanning three half slots.

As will be seen in Section IV, the interleaver π provides the "interleaver gain" for decoding at the destination. The interleaver is not in general necessary in the case of LDPC coding; however it may be used to avoid a large multiplicity of 8-cycles in the Tanner graph for the case where $\mathbf{H}_a = \mathbf{H}_b$.

[1] To avoid full-duplex at the relay in a practical communication system, a second relay R' which employs a simple amplify-and-forward (AF) scheme could be used between R and D in the current cell frequency $f1$. The transmission from R to R' could employ the neighboring cell frequency $f2$, and thus the uplink MAC still maintain $f1$ for this cell. In the counterpart neighboring cell with frequency $f2$, $f1$ is used in the R to R' link. In the overall communications system, there will be no extra frequency band occupied and only negligible inter-cell interference. The decoding procedure at the destination node is then identical to that presented in this paper.

As one reference system, the transmission schedule for consecutive relaying in the MAC is depicted in Figure 3; here the relay simply re-transmits the (interleaved) previously received codeword rather than a codeword superposition. It is easily seen that in this scheme, spatial diversity for each message is contained in two separate transmissions spanning two half slots. A simulation-based comparison of the two schemes described in this section under a transmit power constraint will be given in Section V.

III. Soft Demodulator for BPSK Modulation in the Multiple Access Channel

In the following we define $\mathcal{I} = \{1, 2, \cdots, n\}$, and $x^{(i)}$ denotes the i-th bit of codeword $\mathbf{x}$ while $e^{(i)}$ denotes the i-th received channel value of signal stream $\mathbf{e}$. Considering a single half slot $t \in \{1, \cdots, 2L-1\}$, the channel model is given by

$$e_t^{(i)} = \phi_t^{(i)} \cdot \alpha(c_t^{(i)}) + \psi_t^{(i)} \cdot \beta(d_t^{(i)}) + n_t^{(i)}, \quad i \in \mathcal{I}\,, \tag{1}$$

where $\phi_t^{(i)}$ and $\psi_t^{(i)}$ represent the fading processes on the source-destination and relay-destination links, respectively, and $n_t^{(i)}$ is complex AWGN with variance σ^2 per dimension. Also

$$\begin{aligned} \alpha(c_t^{(i)}) &= \sqrt{P_S}(1-2c_t^{(i)}) && (2) \\ \beta(d_t^{(i)}) &= \sqrt{P_R}(1-2d_t^{(i)}) && (3) \end{aligned}$$

holds for BPSK modulation, with P_S and P_R representing the receive power for the symbols transmitted by the source and relay respectively.

The function of the soft demodulator is to take as input extrinsic LLRs on the transmitted bits (LLRs in the absence of channel information); without loss of generality we consider the bit $c_t^{(i)}$,

$$L^E(c_t^{(i)}) = \ln\left(\frac{\Pr\left(c_t^{(i)}=0\right)}{\Pr\left(c_t^{(i)}=1\right)}\right) \tag{4}$$

and compute the new extrinsic LLRs (incremental LLRs expressing new information derived from the channel)

$$L^O(c_t^{(i)}) = L(c_t^{(i)}) - L^E(c_t^{(i)}) \tag{5}$$

where the *a posteriori* LLRs $L(c_t^{(i)})$ (incorporating channel information) are given by

$$\begin{aligned} L(c_t^{(i)}) &= \ln\left(\frac{\Pr\left(c_t^{(i)}=0|e_t^{(i)}\right)}{\Pr\left(c_t^{(i)}=1|e_t^{(i)}\right)}\right) \\ &= \ln\left(\frac{\sum_{\{c_t^{(i)},d_t^{(i)}\}:c_t^{(i)}=0} \Pr\left(c_t^{(i)},d_t^{(i)}|e_t^{(i)}\right)}{\sum_{\{c_t^{(i)},d_t^{(i)}\}:c_t^{(i)}=1} \Pr\left(c_t^{(i)},d_t^{(i)}|e_t^{(i)}\right)}\right) \\ &= \ln\left(\frac{\sum_{\{c_t^{(i)},d_t^{(i)}\}:c_t^{(i)}=0} p\left(e_t^{(i)}|c_t^{(i)},d_t^{(i)}\right)\cdot\Pr(c_t^{(i)},d_t^{(i)})}{\sum_{\{c_t^{(i)},d_t^{(i)}\}:c_t^{(i)}=1} p\left(e_t^{(i)}|c_t^{(i)},d_t^{(i)}\right)\cdot\Pr(c_t^{(i)},d_t^{(i)})}\right) \end{aligned} \tag{6}$$

We assume that the users' code bits are independent (a realistic assumption for independent users), so that

$$\Pr(c_t^{(i)}, d_t^{(i)}) = \Pr(c_t^{(i)}) \cdot \Pr(d_t^{(i)}) \tag{7}$$

Also, the probability density function (PDF) of the receive channel value conditioned on the transmitted bits may be written as

$$\begin{aligned} p(e_t^{(i)}|c_t^{(i)}, d_t^{(i)}) &= \frac{1}{2\pi\sigma^2}\cdot \\ &\exp\left(-\frac{1}{2\sigma^2}\left|e_t^{(i)} - \phi_t^{(i)}\cdot\alpha(c_t^{(i)}) - \psi_t^{(i)}\cdot\beta(d_t^{(i)})\right|^2\right) \end{aligned} \tag{8}$$

Therefore, (5) may be re-written as (9), for which we use the shorthand $L^O(c_t^{(i)}) = f_1(e_t^{(i)}, L^E(d_t^{(i)}))$. In the same way, the new extrinsic LLR of the code bit $d_t^{(i)}$ may be obtained from (10), for which we use the shorthand $L^O(d_t^{(i)}) = f_2(e_t^{(i)}, L^E(c_t^{(i)}))$.

IV. Decoding Algorithm at Destination Node

Without loss of generality, we consider the decoding of codeword $\mathbf{a}_t$ at the end of half slot $2t+2$, for $t \in \{0, 1, \cdots, L-2\}$. The two codewords contained in the received signal stream $\mathbf{e}_{2t+2}$ are:

$$\begin{aligned} \mathbf{c}_{2t+2} &= \mathbf{a}_{t+1} \\ \mathbf{d}_{2t+2} &= \pi(\mathbf{c}_{2t+1}) \oplus \mathbf{c}_{2t} \\ &= \pi(\mathbf{b}_t) \oplus \mathbf{a}_t \end{aligned}$$

We assume that *a priori* LLRs on $\mathbf{c}_{2t+1}$ and $\mathbf{c}_{2t}$, denoted $\{L_1(c_{2t+1}^{(i)})\}$ and $\{L_2(c_{2t}^{(i)})\}$ respectively, are available from the previous decoding. In addition to decoding of $\mathbf{a}_t$, the decoder will produce *a posteriori* LLRs $\{L_1(c_{2t+2}^{(i)})\}$ and (updated) *a posteriori* LLRs $\{L_2(c_{2t+1}^{(i)})\}$; these will be used as *a priori* LLRs in the next decoding.

Next, we provide a concise description of the factor graph based decoding algorithm [9] at the destination decoder. The factor graph for the decoding is illustrated in Figure 4, where circles depict variable nodes and squares depict factor nodes. Extrinsic information is exchanged between three soft-input soft-output (SISO) decoder modules for the constituent codes (two codes $\mathcal{C}_a$ and one code $\mathcal{C}_b$), via the factor nodes $\{F_i\}$ which correspond to the network coding operation at the relay and the factor nodes $\{G_i\}$ which correspond to the soft demodulator for the MAC as given by (9) and (10). For simplicity, the graph is illustrated for the case $n = 3$, and where $\mathcal{C}_a$ and $\mathcal{C}_b$ are (trivial) LDPC codes. For convolutional constituent codes, the SISO modules execute BCJR algorithms. Note that in the convolutional case, the separation of the two (e.g. M-state) decoder SISO modules gives a complexity advantage over schemes which use a larger (e.g. M^2-state) decoder to decode the "nested" code generated at the relay (see e.g. [2], [3]). In the LDPC case this separation of SISO modules effects a more efficient message-passing schedule than does fully parallel decoding on the Tanner graph of the nested code (see e.g. [6]).

$$L^O(c_t^{(i)}) = \ln\left(\frac{\exp\left(-\frac{1}{2\sigma^2}\left|e_t^{(i)} - \sqrt{P_S}\phi_t^{(i)} - \sqrt{P_R}\psi_t^{(i)}\right|^2 + L^E(d_t^{(i)})\right) + \exp\left(-\frac{1}{2\sigma^2}\left|e_t^{(i)} - \sqrt{P_S}\phi_t^{(i)} + \sqrt{P_R}\psi_t^{(i)}\right|^2\right)}{\exp\left(-\frac{1}{2\sigma^2}\left|e_t^{(i)} + \sqrt{P_S}\phi_t^{(i)} - \sqrt{P_R}\psi_t^{(i)}\right|^2 + L^E(d_t^{(i)})\right) + \exp\left(-\frac{1}{2\sigma^2}\left|e_t^{(i)} + \sqrt{P_S}\phi_t^{(i)} + \sqrt{P_R}\psi_t^{(i)}\right|^2\right)}\right) \tag{9}$$

$$L^O(d_t^{(i)}) = \ln\left(\frac{\exp\left(-\frac{1}{2\sigma^2}\left|e_t^{(i)} - \sqrt{P_R}\psi_t^{(i)} - \sqrt{P_S}\phi_t^{(i)}\right|^2 + L^E(c_t^{(i)})\right) + \exp\left(-\frac{1}{2\sigma^2}\left|e_t^{(i)} - \sqrt{P_R}\psi_t^{(i)} + \sqrt{P_S}\phi_t^{(i)}\right|^2\right)}{\exp\left(-\frac{1}{2\sigma^2}\left|e_t^{(i)} + \sqrt{P_R}\psi_t^{(i)} - \sqrt{P_S}\phi_t^{(i)}\right|^2 + L^E(c_t^{(i)})\right) + \exp\left(-\frac{1}{2\sigma^2}\left|e_t^{(i)} + \sqrt{P_R}\psi_t^{(i)} + \sqrt{P_S}\phi_t^{(i)}\right|^2\right)}\right) \tag{10}$$

Next we introduce some notational conventions pertaining to the following algorithm description. In all cases, the letter λ is used to denote LLRs corresponding to messages passed on the factor graph (i.e. extrinsic LLRs). The interleaving "π" is interpreted as

$$\mathbf{x} = \pi(\mathbf{y}) \quad \Longleftrightarrow \quad x^{(i)} = y^{(\pi(i))} \ \ \forall i \in \mathcal{I}$$

Some index sets are defined as follows. $\mathcal{J}_a = \{1, 2, \cdots m_a\}$; $\mathcal{J}_b = \{1, 2, \cdots m_b\}$; $\mathcal{N}_a(i) = \{j \in \mathcal{J}_a : H_a(j,i) = 1\}$; $\mathcal{N}_b(i) = \{j \in \mathcal{J}_b : H_b(j,i) = 1\}$; $\mathcal{M}_a(j) = \{i \in \mathcal{I} : H_a(j,i) = 1\}$; $\mathcal{M}_b(j) = \{i \in \mathcal{I} : H_b(j,i) = 1\}$. Also, $\boxplus$ denotes the (commutative and associative) "box-plus" operation [10], i.e.

$$\boxplus_{s\in\mathcal{S}}\ \lambda_s = \log\left(\frac{1 + \prod_{s\in\mathcal{S}}\tanh(\lambda_s/2)}{1 - \prod_{s\in\mathcal{S}}\tanh(\lambda_s/2)}\right)$$

N denotes the maximum number of decoding iterations.

Factor Graph Based Decoding Algorithm at Destination Node – Decoding of Codeword $\mathbf{a}_t$

Initialization:

- For $i \in \mathcal{I}$,

$$\lambda_{c_1}^{(i)} = L_2(c_{2t}^{(i)}) \tag{11}$$
$$\lambda_{c_2}^{(i)} = L_1(c_{2t+1}^{(i)}) \tag{12}$$
$$\lambda_8^{(i)} = 0 \tag{13}$$
$$\lambda_{15}^{(i)} = 0 \tag{14}$$

- For $i \in \mathcal{I}$, $j \in \mathcal{N}_a(i)$

$$\lambda_2^{(i,j)} = 0 \tag{15}$$

- For $i \in \mathcal{I}$, $j \in \mathcal{N}_b(i)$

$$\lambda_4^{(i,j)} = 0 \tag{16}$$

- For $i \in \mathcal{I}$, $j \in \mathcal{N}_a(i)$

$$\lambda_6^{(i,j)} = 0 \tag{17}$$

Main Loop: For $k = 1$ to N do

- For $i \in \mathcal{I}$,

$$\lambda_{14}^{(i)} = \lambda_{c_1}^{(i)} + \lambda_8^{(i)} \tag{18}$$

- Network coding constraints: for $i \in \mathcal{I}$,

$$\lambda_{10}^{(\pi(i))} = \lambda_{14}^{(i)} \boxplus \lambda_{15}^{(i)} \tag{19}$$

- For $i \in \mathcal{I}$,

$$\lambda_9^{(i)} = \lambda_{c_2}^{(i)} + \lambda_{10}^{(i)} \tag{20}$$

- SISO decoder $\mathcal{C}_b$: for $i \in \mathcal{I}$, $j \in \mathcal{N}_b(i)$

$$\lambda_3^{(i,j)} = \lambda_9^{(i)} + \sum_{l\in\mathcal{N}_b(i)\setminus\{j\}} \lambda_4^{(i,l)} \tag{21}$$
$$\lambda_4^{(i,j)} = \boxplus_{l\in\mathcal{M}_b(j)\setminus\{i\}}\ \lambda_3^{(l,j)} \tag{22}$$

For $i \in \mathcal{I}$,

$$\lambda_{12}^{(i)} = \sum_{j\in\mathcal{N}_b(i)} \lambda_4^{(i,j)} \tag{23}$$

- Obtain the *a posteriori* LLR (prior for decoding in next half slot)

$$L_2(c_{2t+1}^{(i)}) = \lambda_9^{(i)} + \lambda_{12}^{(i)} \tag{24}$$

- For $i \in \mathcal{I}$,

$$\lambda_{11}^{(i)} = \lambda_{c_2}^{(i)} + \lambda_{12}^{(i)} \tag{25}$$
$$\lambda_{16}^{(i)} = \lambda_{14}^{(i)} \boxplus \lambda_{11}^{(\pi(i))} \tag{26}$$

- Soft demodulation: for $i \in \mathcal{I}$,

$$\lambda_{17}^{(i)} = f_1(e_{2t+2}^{(i)}, \lambda_{16}^{(i)}) \tag{27}$$

- SISO decoder $\mathcal{C}_a$: for $i \in \mathcal{I}$, $j \in \mathcal{N}_a(i)$

$$\lambda_5^{(i,j)} = \lambda_{17}^{(i)} + \sum_{l\in\mathcal{N}_a(i)\setminus\{j\}} \lambda_6^{(i,l)} \tag{28}$$
$$\lambda_6^{(i,j)} = \boxplus_{l\in\mathcal{M}_a(j)\setminus\{i\}}\ \lambda_5^{(l,j)} \tag{29}$$

For $i \in \mathcal{I}$,

$$\lambda_{18}^{(i)} = \sum_{j\in\mathcal{N}_a(i)} \lambda_6^{(i,j)} \tag{30}$$

- Obtain the *a posteriori* LLR (prior for decoding in next half slot)

$$L_1(c_{2t+2}^{(i)}) = \lambda_{17}^{(i)} + \lambda_{18}^{(i)} \tag{31}$$

- Soft demodulation: for $i \in \mathcal{I}$,

$$\lambda_{15}^{(i)} = f_2(e_{2t+2}^{(i)}, \lambda_{18}^{(i)}) \tag{32}$$
$$\lambda_{13}^{(i)} = \lambda_{15}^{(i)} \boxplus \lambda_{11}^{(\pi(i))} \tag{33}$$

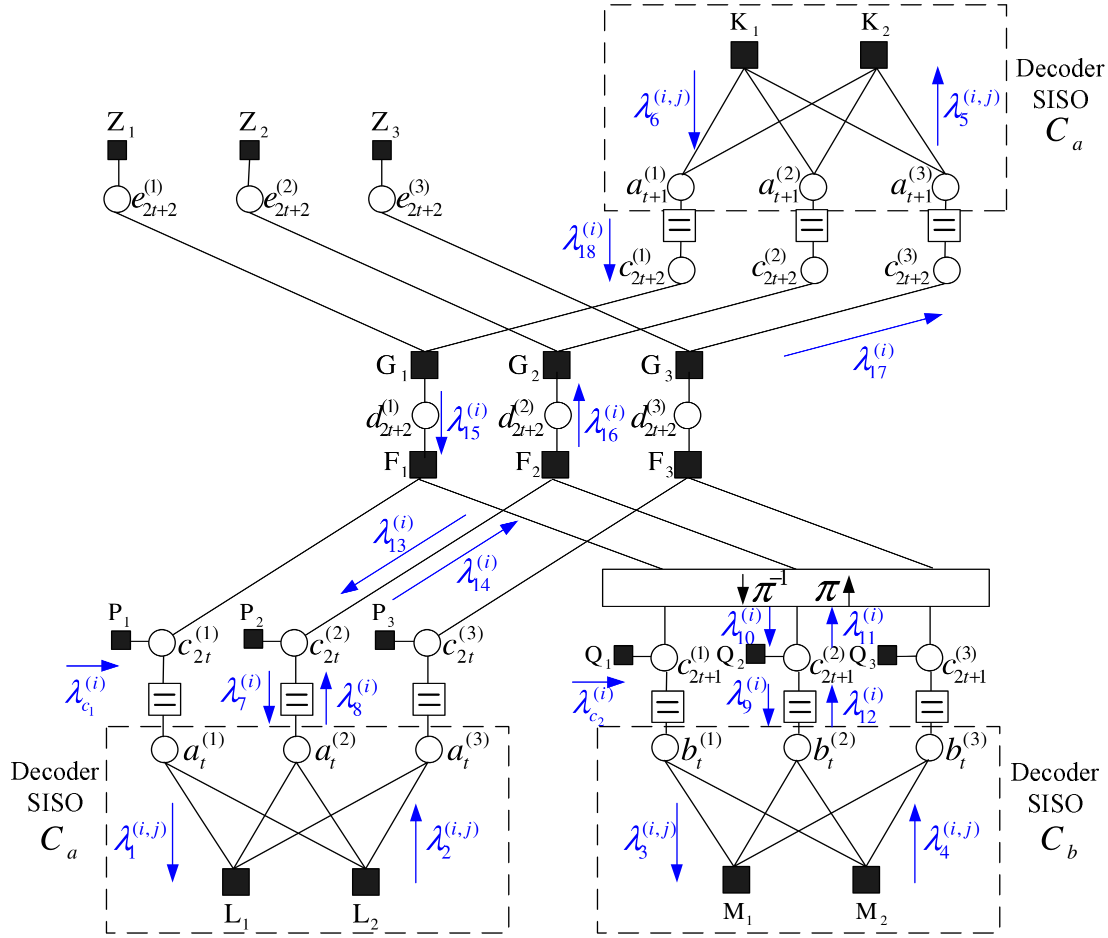

Fig. 4. Factor graph corresponding to the destination's decoding of codeword $\mathbf{a}_t$. The three decoder SISO modules exchange extrinsic information via the factor nodes $\{F_i\}$ which correspond to the network coding operation at the relay and the factor nodes $\{G_i\}$ which correspond to the soft demodulator for the MAC. For ease of presentation, the factor graph is illustrated for the case $n = 3$ and trivial codes $\mathcal{C}_a$, $\mathcal{C}_b$.

- For $i \in \mathcal{I}$,

$$\lambda_7^{(i)} = \lambda_{c_1}^{(i)} + \lambda_{13}^{(i)} \tag{34}$$

- SISO decoder $\mathcal{C}_a$: for $i \in \mathcal{I}$, $j \in \mathcal{N}_a(i)$

$$\lambda_1^{(i,j)} = \lambda_7^{(i)} + \sum_{l \in \mathcal{N}_a(i) \setminus \{j\}} \lambda_2^{(i,l)} \tag{35}$$

$$\lambda_2^{(i,j)} = \boxplus_{l \in \mathcal{M}_a(j) \setminus \{i\}} \lambda_1^{(l,j)} \tag{36}$$

For $i \in \mathcal{I}$,

$$\lambda_8^{(i)} = \sum_{j \in \mathcal{N}_a(i)} \lambda_2^{(i,j)} \tag{37}$$

- Calculate *a posteriori* LLRs for codeword $\mathbf{a}_t$:

$$L(a_t^{(i)}) = \lambda_7^{(i)} + \lambda_8^{(i)} \tag{38}$$

- Make decisions on the code bits

$$\hat{a}_t^{(i)} = \begin{cases} 0 & \text{if } L(a_t^{(i)}) \geq 0 \\ 1 & \text{if } L(a_t^{(i)}) < 0 \end{cases}$$

If $\hat{\mathbf{a}}_t \mathbf{H}_a^T = \mathbf{0}$ then *break;*

Endfor

It may be seen that the decoding of $\mathbf{a}_t$ spans three transmission frames (half slots) $2t \to 2t+1 \to 2t+2$, during which the decoding follows the three-step evolution $L_1(c_{2t}^{(i)}) \to L_2(c_{2t}^{(i)}) \to \hat{a}_t^{(i)}$.

The preceding presentation was for the general case of decoding (half slots $t = 2, 3, \cdots, 2L-1\}$). Special cases are handled in a straightforward manner as follows. In half slot $t = 0$, a single LDPC decoding of $\mathbf{a}_0$ based on LLRs derived from the receive stream $\mathbf{e}_0$ produces $\{L_1(c_0^{(i)})\}$. In half slot $t = 1$, decoding proceeds as in the general case except that $\lambda_{14}^{(i)}$ is set to $+\infty$ for all $i \in \mathcal{I}$; $\{L_1(c_1^{(i)})\}$ and $\{L_2(c_0^{(i)})\}$ are produced (note that this decoder has the same structure as the decoder for the reference scheme of consecutive relaying in the fading MAC as shown in Figure 3). In half slot $t = 2L$, the decoder structure used is that for superposition decoding as described in [2], [3]; also only $\{L_2(c_{2L-1}^{(i)})\}$ are produced. In time slot $t = 2L+1$, a single LDPC decoding of $\mathbf{b}_{L-1}$ is performed based on the sum of the LLRs derived from the receive stream $\mathbf{e}_{2L+1}$ and the LLRs $\{L_2(c_{2L-1}^{(i)})\}$ obtained from the previous decoding.

V. Simulation Results

In this section, we provide a comparison of the proposed cooperative scheme with two reference schemes. The first is consecutive relaying in the fading MAC as shown in Figure 3. The second is a simple TDMA transmission scheme without cooperation or relaying. Fair comparison of the three cooperative schemes is based on the constraint that in simulations, each scheme uses the same codes $\mathcal{C}_a$ and $\mathcal{C}_b$, and the same total energy E for transmission of the $2L$ source messages.

The LDPC codes used for the simulations are randomly generated rate $1/2$ regular LDPC codes of block length $n = 1200$ with column weight 3 and no 4-cycles in the Tanner graph. In simulations we choose $\mathbf{H}_a = \mathbf{H}_b$, and assume BPSK modulation for all three systems. A random interleaver π is used to avoid 8-cycle multiplicity in the Tanner graph. We consider a quasi-static Rayleigh fading channel, for which the fading coefficients are constant within each half slot (one codeword) and change independently from one half slot to the next. We assume equal signal-to-noise ratio (SNR) on the two source-destination links and the relay-destination link, and we assume that the destination has perfect knowledge of the channel fading coefficients and noise variances. As for the two source-relay links, which play a key role in the performance of the system since poor link quality may lead to catastrophic error propagation at the destination decoder, the simulation setup is such that the outage probabilities of the source-relay links are both around 10^{-5} – this is a suitable value for the operation of cooperative schemes. The overall performance of both cooperative schemes will degrade as the outage probability of the source-relay links increases; results under varying outage probability on the source-relay links are omitted due to the space limitations.

Simulated performance results for bit error rate (BER) and frame (codeword) error rate (FER) are shown in Figures 5 and 6 respectively. The curve corresponding to the proposed cooperative scheme exhibits the steepest slope (due to increased diversity gain) of the three, outperforming the others in the SNR region of interest. The scheme attains approximately an order of magnitude decrease in both BER and FER over consecutive relaying in the fading MAC at an E_b/N_0 of 10 dB.

VI. Acknowledgment

This work was supported by the Engineering and Physical Sciences Research Council (EPSRC), grant number EP/D 002184/1, and by the Scottish Funding Council for the Joint Research Institute with Heriot-Watt University which is a part of the Edinburgh Research Partnership.

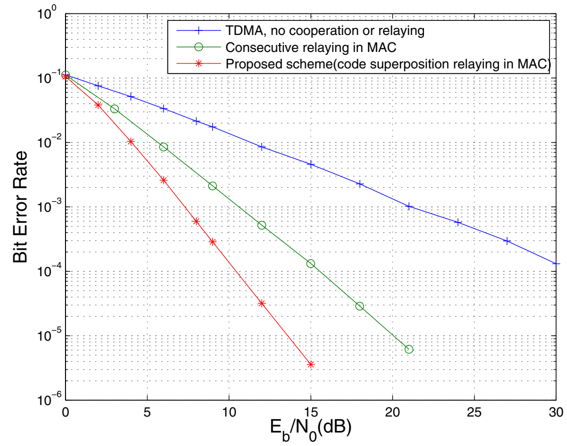

Fig. 5. Comparative BER performance for the three communication schemes.

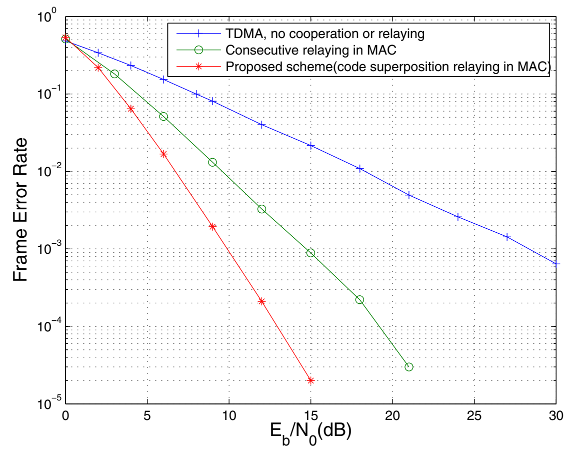

Fig. 6. Comparative FER performance for the three communication schemes.

References

[1] A. Nosratinia, T. E. Hunter, and A. Hedayat, "Cooperative communication in wireless networks," *IEEE Communications Magazine*, pp. 74–80, Octorber 2004.

[2] L. Xiao, T. Fuja, J. Kliewer, and D. Costello, "Cooperative diversity based on code superposition," *IEEE International Symposium on Information Theory (ISIT)*, 2006.

[3] L. Xiao, T. E. Fuja, J. Kliewer, and D. J. Costello, "A network coding approach to cooperative diversity," *IEEE Trans. Information Theory*, vol. 53, no. 10, pp. 3714–3722, Oct. 2007.

[4] L. Xiao, T. Fuja, J. Kliewer, and D. Costello, "Algebraic superposition of LDGM codes for cooperative diversity," *IEEE International Symposium on Information Theory (ISIT)*, 2007.

[5] C. Hausl and P. Dupraz, "Joint network-channel coding for the multiple-access relay channel," in *Sensor and Ad Hoc Communications and Networks, 2006. SECON '06. 2006 3rd Annual IEEE Communications Society on*, vol. 3, 28-28 Sept. 2006, pp. 817–822.

[6] C. Hausl, F. Schreckenbach, I. Oikonomidis, and G. Bauch, "Iterative network and channel decoding on a tanner graph," *43rd Annual Allerton Conference on Communication, Control, and Computing, Monticello, USA*, 2005.

[7] S. Zhang, Y. Zhu, S.-C. Liew, and K. B. Letaief, "Joint design of network coding and channel decoding for wireless networks," in *Wireless Communications and Networking Conference, 2007.WCNC 2007. IEEE*, 11-15 March 2007, pp. 779–784.

[8] M. A. Khojastepour, N. Ahmed, and B. Aazhang, "Code design for the relay channel and factor graph decoding," *Thirty-Eighth Asilomar Conference on Signal, Systems and Computers*, vol. 2, pp. 2000–2004, 2004.

[9] F. R. Kschischang, B. J. Frey, and H. A. Loeliger, "Factor graphs and the sum-product algorithm," *IEEE Trans. Information Theory*, vol. 47, no. 2, pp. 498–519, Feb. 2001.

[10] J. Hagenauer, E. Offer, and L. Papke, "Iterative decoding of binary block and convolutional codes," *Information Theory, IEEE Transactions on*, vol. 42, no. 2, pp. 429–445, March 1996.