# 3D quantum gravity?

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MARIE CURIE ACTIONS

with R. Jackiw and N. Johansson: `0805.2610, 0806.4185, 0808.2575, ...`

# Outline

Why 3D?

Which 3D theory?

How to quantize 3D gravity?

What next?

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- Experimental signatures? Data?

## Gravity in lower dimensions

Riemann-tensor $\frac{D^2(D^2-1)}{12}$ components in $D$ dimensions:

- 11D: 1210 (1144 Weyl and 66 Ricci)
- 10D: 825 (770 Weyl and 55 Ricci)
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> - 3D: lowest dimension exhibiting BHs and gravitons
> - Study gravity in 3D!

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## Attempt 1: Einstein–Hilbert

As simple as possible... but not simpler!

Let us start with the simplest attempt. `Einstein-Hilbert` action:

$$I_{\text{EH}} = \frac{1}{16\pi\, G} \int \mathrm{d}^3x\sqrt{-g}\, R$$

Equations of motion:

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**Properties of `Einstein-Hilbert`**

- No gravitons (recall: in $D$ dimensions $D(D-3)/2$ gravitons)
- No BHs
- `Einstein-Hilbert` in 3D is too simple for us!

## Attempt 2: Topologically massive gravity

Deser, Jackiw and Templeton found a way to introduce gravitons!

Let us now add a gravitational Chern–Simons term. TMG action:

$$I_{\text{TMG}} = I_{\text{EH}} + \frac{1}{16\pi\, G} \int \mathrm{d}^3x \sqrt{-g}\, \frac{1}{2\mu}\, \varepsilon^{\lambda\mu\nu}\, \Gamma^\rho{}_{\lambda\sigma} \big(\partial_\mu \Gamma^\sigma{}_{\nu\rho} + \frac{2}{3}\, \Gamma^\sigma{}_{\mu\tau} \Gamma^\tau{}_{\nu\rho}\big)$$

Equations of motion:

$$R_{\mu\nu} + \frac{1}{\mu}\, C_{\mu\nu} = 0$$

with the Cotton tensor defined as

$$C_{\mu\nu} = \frac{1}{2}\, \varepsilon_\mu{}^{\alpha\beta} \nabla_\alpha R_{\beta\nu} + (\mu \leftrightarrow \nu)$$

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**Properties of TMG**

- Gravitons! Reason: third derivatives in Cotton tensor!
- No BHs
- TMG is slightly too simple for us!

# Attempt 3: Einstein–Hilbert–AdS

## Bañados, Teitelboim and Zanelli (and Henneaux) taught us how to get 3D BHs

Add negative cosmological constant to `Einstein-Hilbert` action:

$$I_{\Lambda\mathrm{EH}} = \frac{1}{16\pi\, G} \int \mathrm{d}^3x\sqrt{-g}\,\big(R + \frac{2}{\ell^2}\big)$$

Equations of motion:

$$G_{\mu\nu} = R_{\mu\nu} - \frac{1}{2}\,g_{\mu\nu}R - \frac{1}{\ell^2}\,g_{\mu\nu} = 0$$

Particular solutions: BTZ BH with line-element

$$\mathrm{d}s^2_{\mathrm{BTZ}} = -\frac{(r^2 - r_+^2)(r^2 - r_-^2)}{\ell^2 r^2}\,\mathrm{d}t^2 + \frac{\ell^2 r^2}{(r^2 - r_+^2)(r^2 - r_-^2)}\,\mathrm{d}r^2 + r^2\,\big(\mathrm{d}\phi - \frac{r_+ r_-}{\ell r^2}\,\mathrm{d}t\big)^2$$

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**Properties of `Einstein-Hilbert-AdS`**

- No gravitons
- Rotating BH solutions that asymptote to $\text{AdS}_3$!
- Adding a negative cosmological constant produces BH solutions!

## Cosmological topologically massive gravity

### CTMG is a 3D theory with BHs and gravitons!

We want a 3D theory with gravitons and BHs and therefore take CTMG action

$$I_{\text{CTMG}} = \frac{1}{16\pi\,G}\int \mathrm{d}^3x\sqrt{-g}\,\Big[R + \frac{2}{\ell^2} + \frac{1}{2\mu}\,\varepsilon^{\lambda\mu\nu}\,\Gamma^\rho{}_{\lambda\sigma}\big(\partial_\mu\Gamma^\sigma{}_{\nu\rho} + \frac{2}{3}\,\Gamma^\sigma{}_{\mu\tau}\Gamma^\tau{}_{\nu\rho}\big)\Big]$$

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**Properties of CTMG**

- Gravitons!
- BHs!
- CTMG is just perfect for us. Study this theory!

## Einstein sector of the classical theory

Solutions of Einstein's equations

$$G_{\mu\nu} = 0 \qquad \leftrightarrow \qquad R = -\frac{6}{\ell^2}$$

also have vanishing Cotton tensor

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Line-element of pure AdS:

$$\mathrm{d}s^2_{\mathrm{AdS}} = \bar{g}_{\mu\nu}\,\mathrm{d}x^\mu\,\mathrm{d}x^\nu = \ell^2\big(-\cosh^2\rho\,\mathrm{d}\tau^2 + \sinh^2\rho\,\mathrm{d}\phi^2 + \mathrm{d}\rho^2\big)$$

Isometry group: $SL(2,\mathbb{R})_L \times SL(2,\mathbb{R})_R$
Useful to introduce light-cone coordinates $u = \tau + \phi$, $v = \tau - \phi$

## AdS$_3$-algebra of Killing vectors

### A technical reminder

The $SL(2,\mathbb{R})_L$ generators

$$L_0 = i\partial_u$$

$$L_{\pm 1} = ie^{\pm iu}\left[\frac{\cosh 2\rho}{\sinh 2\rho}\partial_u - \frac{1}{\sinh 2\rho}\partial_v \mp \frac{i}{2}\,\partial_\rho\right]$$

obey the algebra

$$[L_0, L_{\pm 1}] = \mp L_{\pm 1}\,, \qquad [L_1, L_{-1}] = 2L_0$$

and have the quadratic Casimir

$$L^2 = \frac{1}{2}(L_1 L_{-1} + L_{-1} L_1) - L_0^2$$

The $SL(2,\mathbb{R})_R$ generators $\bar{L}_0, \bar{L}_{\pm 1}$ obey same algebra, but with

$$u \leftrightarrow v\,, \qquad L \leftrightarrow \bar{L}$$

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Perhaps most interesting solution:

- Warped AdS (stretched/squashed), see Bengtsson & Sandin

Line-element of space-like warped AdS:

$$\mathrm{d}s^2_{\text{warped AdS}} = \frac{\ell^2}{\nu^2+3}\left(-\cosh^2\rho\,\mathrm{d}\tau^2 + \frac{4\nu^2}{\nu^2+3}\left(\mathrm{d}u + \sinh\rho\,\mathrm{d}\tau\right)^2 + \mathrm{d}\rho^2\right)$$

Sidenote: null-warped AdS in holographic duals of cold atoms:

$$\mathrm{d}s^2_{\text{null warped AdS}} = \ell^2\left(\frac{dy^2 + 2\,\mathrm{d}x^+\,\mathrm{d}x^-}{y^2} \pm \frac{(\mathrm{d}x^-)^2}{y^4}\right)$$

# CTMG as particle mechanics problem

## Stationary and axi-symmetric solutions

Stationarity plus axi-symmetry:

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Reduced action (`Clement`):

$$I_{\mathrm{C}}[\zeta, X^i] \sim \int \mathrm{d}\rho \left[ \frac{\zeta}{2} \dot{X}^i \dot{X}^j \eta_{ij} - \frac{2}{\zeta \ell^2} + \frac{\zeta^2}{2\mu} \epsilon_{ijk} X^i \dot{X}^j \ddot{X}^k \right]$$

Here $\zeta$ is a Lagrange-multiplier and $X^i = (T, X, Y)$ a Lorentzian 3-vector

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It could be rewarding to investigate this mechanical problem systematically and numerically!

## CTMG at the chiral point
### ...abbreviated as CCTMG

Definition: CTMG at the chiral point is CTMG with the tuning

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Calculating the central charges of the dual boundary CFT yields

$$c_L = \frac{3}{2G}\,\big(1 - \frac{1}{\mu\,\ell}\big)\,, \qquad c_R = \frac{3}{2G}\,\big(1 + \frac{1}{\mu\,\ell}\big)$$

Thus, at the chiral point we get

$$c_L = 0\,, \qquad c_R = \frac{3}{G}$$

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Notes:

- Abbreviate "CTMG at the chiral point" as CCTMG
- CCTMG is also known as "chiral gravity"

# Gravitons around $AdS_3$ in CTMG

Linearization around AdS background

$$g_{\mu\nu} = \bar{g}_{\mu\nu} + h_{\mu\nu}$$

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leads to linearized EOM that are third order PDE

$$G^{(1)}_{\mu\nu} + \frac{1}{\mu}\,C^{(1)}_{\mu\nu} = (\mathcal{D}^R\mathcal{D}^L\mathcal{D}^M h)_{\mu\nu} = 0 \tag{1}$$

with three mutually commuting first order operators

$$(\mathcal{D}^{L/R})_\mu{}^\nu = \delta_\mu^\nu \pm \ell\,\varepsilon_\mu{}^{\alpha\nu}\bar{\nabla}_\alpha\,, \qquad (\mathcal{D}^M)_\mu{}^\nu = \delta_\mu^\nu + \frac{1}{\mu}\varepsilon_\mu{}^{\alpha\nu}\bar{\nabla}_\alpha$$

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Three linearly independent solutions to (1):

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At chiral point left ($L$) and massive ($M$) branches coincide!

## Degeneracy at the chiral point

### Will be quite important later!

Li, Song & Strominger found all solutions of linearized EOM.

- Primaries: $L_0, \bar{L}_0$ eigenstates $\psi^{L/R/M}$ with

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$$h_{\mu\nu} = \operatorname{Re} \psi_{\mu\nu}$$

- At chiral point: $L$ and $M$ branches degenerate. Get new solution (DG & Johansson)

$$\psi^{\text{new}}_{\mu\nu} = \lim_{\mu\ell\to 1} \frac{\psi^M_{\mu\nu}(\mu\ell) - \psi^L_{\mu\nu}}{\mu\ell - 1}$$

with property

$$\big(\mathcal{D}^L \psi^{\text{new}}\big)_{\mu\nu} = \big(\mathcal{D}^M \psi^{\text{new}}\big)_{\mu\nu} \neq 0\,, \qquad \big((\mathcal{D}^L)^2 \psi^{\text{new}}\big)_{\mu\nu} = 0$$

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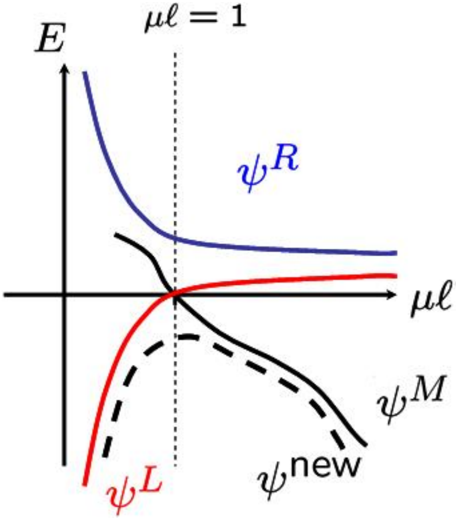
- Either way need a mechanism to eliminate unwanted negative energy objects – either the gravitons or the BHs
- Even at chiral point the problem persists because of the logarithmic mode. See Figure. (Figure: thanks to `N. Johansson`)

Energy for all branches:



# Outline

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Various suggestions to interpret this problem: need cosmic strings, need sum over complex geometries, 3D quantum gravity does not exist by itself

# Li, Song & Strominger attempt

## Is CCTMG dual to a chiral CFT?

Interesting observations:

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**But:**

Disagrees with results by `Carlip, Deser, Waldron & Wise`!

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Such a Jordan form of $L_0, \bar{L}_0$ is defining property of a logarithmic CFT!

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- Either logarithmic or chiral CFT dual (or none)
- Currently unknown which of these alternatives is realized!

## Viability of the logarithmic mode, part 1

### Explicit solution for logarithmic mode (DG & Johansson)

**Is the logarithmic mode really there?**

Collect in the following suggestions how the logarithmic mode could drop out of the physical spectrum and show that none of them is realized.

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Before starting, here is the explicit form of the logarithmic mode:

$$h_{\mu\nu}^{\text{new}} = \frac{\sinh\rho}{\cosh^3\!\rho}\left(c\,\tau - s\ln\cosh\rho\right)\begin{pmatrix} 0 & 0 & 1 \\ 0 & 0 & 1 \\ 1 & 1 & 0 \end{pmatrix}_{\mu\nu} - \tanh^2\!\rho\left(s\,\tau + c\ln\cosh\rho\right)\begin{pmatrix} 1 & 1 & 0 \\ 1 & 1 & 0 \\ 0 & 0 & -a^2 \end{pmatrix}_{\mu\nu} \tag{2}$$

with

$$c = \cos(2u)\,,\qquad s = \sin(2u)\,,\qquad a = \frac{1}{\sinh\rho\,\cosh\rho}$$

# Viability of the logarithmic mode, part 2

## Physical mode with negative energy

**Suggestion 1**

The logarithmic mode is pure gauge?

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$h^{\text{new}}$ does not solve linearized Einstein equations. Thus is not pure gauge.
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$$E^{\mathrm{new}} = -\frac{47}{1152G\,\ell^3}$$

Energy is finite and negative.
Thus logarithmic mode leads to instability but cannot be discarded.

## Viability of the logarithmic mode, part 3

### Boundary conditions beyond Brown–Henneaux

**Suggestion 3**

New mode is not a small perturbation?

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$h^{\mathrm{new}}$ diverges asymptotically like $\rho$, but AdS background diverges asymptotically like $e^{2\rho}$. Thus $h^{\mathrm{new}}$ is really a small perturbation.

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Solution is asymptotically AdS

$$\mathrm{d}s^2 = \mathrm{d}\rho^2 + \left(\gamma_{ij}^{(0)} e^{2\rho/\ell} + \gamma_{ij}^{(1)} \rho + \gamma_{ij}^{(0)} + \gamma_{ij}^{(2)} e^{-2\rho/\ell} + \dots\right) \mathrm{d}x^i \, \mathrm{d}x^j$$

but violates Brown-Henneaux boundary conditions! $(\gamma_{ij}^{(1)}\big|_{\text{BH}} = 0)$
Henneaux et al. showed precedents where this may happen in 3D
New boundary conditions replacing Brown-Henneaux (DG & Johansson)

# Viability of the logarithmic mode, part 4

## Brown–York boundary stress tensor

**Suggestion 5**

New mode leads to ill-defined `Brown-York` boundary stress tensor?

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## Brown–York boundary stress tensor

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New mode leads to ill-defined Brown–York boundary stress tensor? No!

Total action including boundary terms (Kraus & Larsen)

$$I_{\text{total}} = I_{\text{CTMG}} + \frac{1}{8\pi G} \int \mathrm{d}^2x \sqrt{-\gamma} \Big( K - \frac{1}{\ell} \Big)$$

Its first variation leads to Brown–York boundary stress-tensor:

$$\delta I_{\text{total}}\big|_{\text{EOM}} = \frac{1}{32\pi G} \int \mathrm{d}^2x \sqrt{-\gamma^{(0)}}\, T^{ij}\, \delta\gamma_{ij}^{(0)}$$

DG & Johansson: $T_{ij}$ is finite, traceless and chiral:

$$T_{ij} = -\frac{\ell}{16\pi\, G} \begin{pmatrix} 1 & 1 \\ 1 & 1 \end{pmatrix}_{ij}$$

Note: coincides with Brown–York boundary stress-tensor of global $\mathrm{AdS}_3$

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`DG, Jackiw & Johansson`: classical phase space analysis of CCTMG

$$N = \frac{1}{2}\left(2\times D - 2\times N_1 - N_2\right) = \frac{1}{2}\left(2\times 18 - 2\times 14 - 6\right) = 1$$

- $N$: number of physical degrees of freedom (per point)
- $D$: number of canonical pairs in full phase space
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- Conclusion 1: logarithmic mode passed all tests so far
- Conclusion 2: CCTMG is unstable; dual CFT probably logarithmic

# Outline

Why 3D?

Which 3D theory?

How to quantize 3D gravity?

What next?

## Chiral vs. logarithmic

Pivotal open question: does dual CFT exist? is it chiral or logarithmic?

**To Do**

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- Logarithmic: must show consistency of $2^{\text{nd}}$ order perturbations!

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- Might be rewarding exercise for a student

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Two observations:

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`Strominger et al. :`

Suggestive to consider warped AdS as possible groundstate of (C)CTMG

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Thank you for your attention!

# Some recent literature on cosmological topologically massive gravity


- W. Li, W. Song and A. Strominger, JHEP **0804** (2008) 082, 0801.4566.
- D. Grumiller and N. Johansson, JHEP **0807** (2008) 134, 0805.2610.
- S. Carlip, S. Deser, A. Waldron and D. Wise, Phys.Lett. **B666** (2008) 272, 0807.0486
- I. Sachs, JHEP **0809** (2008) 073, 0807.1844.
- G. Giribet, M. Kleban and M. Porrati, JHEP **0810** (2008) 045, 0807.4703.
- S. Carlip, S. Deser, A. Waldron and D. K. Wise, 0803.3998.
- D. Grumiller, R. Jackiw and N. Johansson, 0806.4185.
- S. Carlip, 0807.4152
- A. Strominger, 0808.0506.
- D. Grumiller and N. Johansson, 0808.2575.


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