Gravity in lower dimensions

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MARIE CURIE ACTIONS

Outline

Why lower-dimensional gravity?

Which 2D theory?

Which 3D theory?

How to quantize 3D gravity?

What next?

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Quantum gravity

The Holy Grail of theoretical physics

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- Non-perturbative understanding of quantum gravity?

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- Experimental signatures? Data?

Gravity in lower dimensions

Riemann-tensor $\frac{D^2(D^2-1)}{12}$ components in D dimensions:

- 11D: 1210 (1144 Weyl and 66 Ricci)
- 10D: 825 (770 Weyl and 55 Ricci)
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- 3D: lowest dimension exhibiting BHs and gravitons
- Simplest gravitational theories with BHs and gravitons in 3D

Outline

Attempt 1: Einstein–Hilbert in and near two dimensions

Let us start with the simplest attempt. `Einstein-Hilbert` action in 2 dimensions:

$$I_{\text{EH}} = \frac{1}{16\pi\, G} \int \mathrm{d}^2x \sqrt{|g|}\, R = \frac{1}{2G}\,(1-\gamma)$$

- Action is topological
- No equations of motion
- Formal counting of number of gravitons: -1

Attempt 1: Einstein–Hilbert in and near two dimensions

Let us continue with the next simplest attempt. `Einstein-Hilbert` action in $2{+}\epsilon$ dimensions:

$$I_{\text{EH}}{}^{\epsilon} = \frac{1}{16\pi\, G} \int \mathrm{d}^{2+\epsilon}x \sqrt{|g|}\, R$$

- `Weinberg`: theory is asymptotically safe
- `Mann`: limit $\epsilon \to 0$ should be possible and lead to 2D dilaton gravity
- `DG, Jackiw`: limit $\epsilon \to 0$ yields Liouville gravity

$$\lim_{\epsilon\to 0} I_{EH}{}^{\epsilon} = \frac{1}{16\pi\, G_2} \int \mathrm{d}^2x \sqrt{|g|}\, \left[XR - (\nabla X)^2 + \lambda e^{-2X}\right]$$

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Result of attempt 1:

A specific 2D dilaton gravity model

Attempt 2: Gravity as a gauge theory and the Jackiw-Teitelboim model

`Jackiw, Teitelboim (Bunster)`: (A)dS_2 gauge theory

$$[P_a, P_b] = \Lambda\, \epsilon_{ab} J \qquad [P_a, J] = \epsilon_a{}^b P_b$$

describes constant curvature gravity in 2D. Algorithm:

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$$I = \int X_A F^A = \int \Big[X_a(\mathrm{d}e^a + \epsilon^a{}_b \omega \wedge e^b) + X\, \mathrm{d}\omega + \epsilon_{ab} e^a \wedge e^b\, \Lambda X\Big]$$

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A specific 2D dilaton gravity model

Attempt 3: Dimensional reduction

For example: spherical reduction from D dimensions

Line element adapted to spherical symmetry:

$$\mathrm{d}s^2 = \underbrace{g_{\mu\nu}^{(D)}}_{\text{full metric}} \,\mathrm{d}x^\mu\,\mathrm{d}x^\nu = \underbrace{g_{\alpha\beta}(x^\gamma)}_{2D\text{ metric}}\,\mathrm{d}x^\alpha\,\mathrm{d}x^\beta - \underbrace{\phi^2(x^\alpha)}_{\text{surface area}}\,\mathrm{d}\Omega^2_{S_{D-2}}\,,$$

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Insert into D-dimensional EH action $I_{EH} = \kappa \int \mathrm{d}^D x \sqrt{-g^{(D)}} R^{(D)}$:

$$I_{EH} = \kappa \frac{2\pi^{(D-1)/2}}{\Gamma(\frac{D-1}{2})} \int \mathrm{d}^2 x \sqrt{-g} \, \phi^{D-2} \Big[R + \frac{(D-2)(D-3)}{\phi^2} \big((\nabla\phi)^2 - 1 \big) \Big]$$

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Cosmetic redefinition $X \propto (\lambda\phi)^{D-2}$:

$$I_{EH} = \frac{1}{16\pi \, G_2} \int \mathrm{d}^2 x \sqrt{-g} \Big[X R + \frac{D-3}{(D-2)X} (\nabla X)^2 - \lambda^2 X^{(D-4)/(D-2)} \Big]$$

Result of attempt 3:

A specific class of 2D dilaton gravity models

Attempt 4: Poincare gauge theory and higher power curvature theories

Basic idea: since EH is trivial consider $f(R)$ theories or/and theories with torsion or/and theories with non-metricity

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- Example: `Katanaev-Volovich` model (Poincare gauge theory)

$$I_{\text{KV}} \sim \int \mathrm{d}^2x\sqrt{-g}\,\left[\alpha T^2 + \beta R^2\right]$$

- `Kummer, Schwarz`: bring into first order form:

$$I_{\text{KV}} \sim \int \Big[X_a(\mathrm{d}e^a + \epsilon^a{}_b\omega \wedge e^b) + X\,\mathrm{d}\omega + \epsilon_{ab}e^a \wedge e^b\,(\alpha X^a X_a + \beta X^2)\Big]$$

- Use same algorithm as before to convert into second order action:

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Attempt 5: Strings in two dimensions

Conformal invariance of the σ model

$$I_\sigma \propto \int \mathrm{d}^2\xi \sqrt{|h|}\,\left[g_{\mu\nu} h^{ij}\partial_i x^\mu \partial_j x^\nu + \alpha' \phi \mathcal{R} + \dots\right]$$

requires vanishing of β-functions

$$\beta^\phi \propto -4b^2 - 4(\nabla\phi)^2 + 4\Box\phi + R + \dots$$
$$\beta^g_{\mu\nu} \propto R_{\mu\nu} + 2\nabla_\mu\nabla_\nu\phi + \dots$$

Conditions $\beta^\phi = \beta^g_{\mu\nu} = 0$ follow from target space action

$$I_{\text{target}} = \frac{1}{16\pi\, G_2} \int \mathrm{d}^2x\sqrt{-g}\Big[XR + \frac{1}{X}\,(\nabla X)^2 - 4b^2\Big]$$

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Synthesis of all attempts: Dilaton gravity in two dimensions

Second order action:

$$I = \frac{1}{16\pi\, G_2} \int_{\mathcal{M}} \mathrm{d}^2x \sqrt{|g|} \left[XR - U(X)(\nabla X)^2 - V(X) \right] - \frac{1}{8\pi\, G_2} \int_{\partial\mathcal{M}} \mathrm{d}x \sqrt{|\gamma|} \; [XK - S(X)] + I^{(m)}$$

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- Interesting option: couple 2D dilaton gravity to matter

Recent example: AdS_2 holography

Two dimensions supposed to be the simplest dimension with geometry, and yet...

- extremal black holes universally include AdS_2 factor
- funnily, AdS_3 holography more straightforward
- study charged Jackiw–Teitelboim model as example

$$I_{\mathrm{JT}} = \frac{\alpha}{2\pi} \int \mathrm{d}^2x\sqrt{-g}\,\left[e^{-2\phi}\,\left(R + \frac{8}{L^2}\right) - \frac{L^2}{4}\,F^2\right]$$

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- Coupling constant α usually is positive

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- study charged Jackiw–Teitelboim model as example

$$I_{\text{JT}} = \frac{\alpha}{2\pi} \int \mathrm{d}^2x \sqrt{-g} \left[e^{-2\phi} \left(R + \frac{8}{L^2} \right) - \frac{L^2}{4} F^2 \right]$$

- Metric g has signature $-,+$ and Ricci-scalar $R < 0$ for AdS
- Maxwell field strength $F_{\mu\nu} = 2E\,\varepsilon_{\mu\nu}$ dual to electric field E
- Dilaton ϕ has no kinetic term and no coupling to gauge field
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- Coupling constant α usually is positive
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- δg EOM: complicated for non-constant dilaton...

$$\nabla_\mu \nabla_\nu e^{-2\phi} - g_{\mu\nu} \nabla^2 e^{-2\phi} + \frac{4}{L^2} e^{-2\phi} g_{\mu\nu} + \frac{L^2}{2} F_\mu{}^\lambda F_{\nu\lambda} - \frac{L^2}{8} g_{\mu\nu} F^2 = 0$$

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Some surprising results

Hartman, Strominger = HS   Castro, DG, Larsen, McNees = CGLM

- Holographic renormalization leads to boundary mass term (CGLM)

$$I \sim \int \mathrm{d}x \sqrt{|\gamma|}\, m A^2$$

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- Boundary stress tensor transforms anomalously (HS)

$$(\delta_\xi + \delta_\lambda)\, T_{tt} = 2T_{tt}\partial_t\xi + \xi\partial_t T_{tt} - \frac{c}{24\pi} L\partial_t^3\xi$$

  where $\delta_\xi + \delta_\lambda$ is combination of diffeo- and gauge trafos that preserve the boundary conditions (similarly: $\delta_\lambda J_t = -\frac{k}{4\pi} L\partial_t\lambda$)

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- Positive central charge only for negative coupling constant α (`CGLM`)

$$\alpha < 0$$

Outline

Attempt 1: Einstein–Hilbert

As simple as possible... but not simpler!

Let us start with the simplest attempt. `Einstein-Hilbert` action:

$$I_{\text{EH}} = \frac{1}{16\pi\, G} \int \mathrm{d}^3x \sqrt{-g}\, R$$

Equations of motion:

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Ricci-flat and therefore Riemann-flat – locally trivial!

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Properties of `Einstein-Hilbert`

- No gravitons (recall: in D dimensions $D(D-3)/2$ gravitons)
- No BHs
- `Einstein-Hilbert` in 3D is too simple for us!

Attempt 2: Topologically massive gravity

Deser, Jackiw and Templeton found a way to introduce gravitons!

Let us now add a gravitational Chern–Simons term. TMG action:

$$I_{\text{TMG}} = I_{\text{EH}} + \frac{1}{16\pi\, G} \int \mathrm{d}^3x\sqrt{-g}\,\frac{1}{2\mu}\,\varepsilon^{\lambda\mu\nu}\,\Gamma^{\rho}{}_{\lambda\sigma}\big(\partial_\mu\Gamma^{\sigma}{}_{\nu\rho} + \frac{2}{3}\,\Gamma^{\sigma}{}_{\mu\tau}\Gamma^{\tau}{}_{\nu\rho}\big)$$

Equations of motion:

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Properties of TMG

- Gravitons! Reason: third derivatives in Cotton tensor!
- No BHs
- TMG is slightly too simple for us!

Attempt 3: Einstein–Hilbert–AdS

Bañados, Teitelboim and Zanelli (and Henneaux) taught us how to get 3D BHs

Add negative cosmological constant to `Einstein-Hilbert` action:

$$I_{\Lambda\mathrm{EH}} = \frac{1}{16\pi\, G} \int \mathrm{d}^3x \sqrt{-g}\,\big(R + \frac{2}{\ell^2}\big)$$

Equations of motion:

$$G_{\mu\nu} = R_{\mu\nu} - \frac{1}{2}\, g_{\mu\nu} R - \frac{1}{\ell^2}\, g_{\mu\nu} = 0$$

Particular solutions: BTZ BH with line-element

$$\mathrm{d}s^2_{\mathrm{BTZ}} = -\frac{(r^2 - r_+^2)(r^2 - r_-^2)}{\ell^2 r^2}\, \mathrm{d}t^2 + \frac{\ell^2 r^2}{(r^2 - r_+^2)(r^2 - r_-^2)}\, \mathrm{d}r^2 + r^2\,\big(\mathrm{d}\phi - \frac{r_+ r_-}{\ell r^2}\, \mathrm{d}t\big)^2$$

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Properties of `Einstein-Hilbert-AdS`

- No gravitons
- Rotating BH solutions that asymptote to AdS_3!
- Adding a negative cosmological constant produces BH solutions!

Cosmological topologically massive gravity

CTMG is a 3D theory with BHs and gravitons!

We want a 3D theory with gravitons and BHs and therefore take CTMG action

$$I_{\rm CTMG} = \frac{1}{16\pi\, G}\int \mathrm{d}^3x\sqrt{-g}\,\Big[R + \frac{2}{\ell^2} + \frac{1}{2\mu}\,\varepsilon^{\lambda\mu\nu}\,\Gamma^\rho{}_{\lambda\sigma}\big(\partial_\mu\Gamma^\sigma{}_{\nu\rho} + \frac{2}{3}\,\Gamma^\sigma{}_{\mu\tau}\Gamma^\tau{}_{\nu\rho}\big)\Big]$$

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Properties of CTMG

- Gravitons!
- BHs!
- CTMG is just perfect for us. Study this theory!

Einstein sector of the classical theory

Solutions of Einstein's equations

$$G_{\mu\nu} = 0 \qquad \leftrightarrow \qquad R = -\frac{6}{\ell^2}$$

also have vanishing Cotton tensor

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Line-element of pure AdS:

$$\mathrm{d}s^2_{\mathrm{AdS}} = \bar{g}_{\mu\nu}\,\mathrm{d}x^\mu\,\mathrm{d}x^\nu = \ell^2\big(-\cosh^2\rho\,\mathrm{d}\tau^2 + \sinh^2\rho\,\mathrm{d}\phi^2 + \mathrm{d}\rho^2\big)$$

Isometry group: $SL(2,\mathbb{R})_L \times SL(2,\mathbb{R})_R$
Useful to introduce light-cone coordinates $u = \tau + \phi$, $v = \tau - \phi$

AdS$_3$-algebra of Killing vectors

A technical reminder

The $SL(2,\mathbb{R})_L$ generators

$$L_0 = i\partial_u$$

$$L_{\pm 1} = ie^{\pm iu}\Big[\frac{\cosh 2\rho}{\sinh 2\rho}\partial_u - \frac{1}{\sinh 2\rho}\partial_v \mp \frac{i}{2}\,\partial_\rho\Big]$$

obey the algebra

$$[L_0, L_{\pm 1}] = \mp L_{\pm 1}\,, \qquad [L_1, L_{-1}] = 2L_0$$

and have the quadratic Casimir

$$L^2 = \frac{1}{2}(L_1 L_{-1} + L_{-1} L_1) - L_0^2$$

The $SL(2,\mathbb{R})_R$ generators $\bar{L}_0, \bar{L}_{\pm 1}$ obey same algebra, but with

$$u \leftrightarrow v\,, \qquad L \leftrightarrow \bar{L}$$

Cotton sector of the classical theory

Solutions of CTMG with

$$G_{\mu\nu} \neq 0$$

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Perhaps most interesting solution:

- Warped AdS (stretched/squashed), see `Bengtsson & Sandin`

Line-element of space-like warped AdS:

$$\mathrm{d}s^2_{\text{warped AdS}} = \frac{\ell^2}{\nu^2+3}\left(-\cosh^2\rho\,\mathrm{d}\tau^2 + \frac{4\nu^2}{\nu^2+3}\left(\mathrm{d}u + \sinh\rho\,\mathrm{d}\tau\right)^2 + \mathrm{d}\rho^2\right)$$

Sidenote: null-warped AdS in holographic duals of cold atoms:

$$\mathrm{d}s^2_{\text{null warped AdS}} = \ell^2\left(\frac{dy^2 + 2\,\mathrm{d}x^+\,\mathrm{d}x^-}{y^2} \pm \frac{(\mathrm{d}x^-)^2}{y^4}\right)$$

CTMG as particle mechanics problem
Stationary and axi-symmetric solutions

Stationarity plus axi-symmetry:

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Reduced action (`Clement`):

$$I_{\mathrm{C}}[\zeta, X^i] \sim \int \mathrm{d}\rho \left[\frac{\zeta}{2} \dot{X}^i \dot{X}^j \eta_{ij} - \frac{2}{\zeta \ell^2} + \frac{\zeta^2}{2\mu} \epsilon_{ijk} X^i \dot{X}^j \ddot{X}^k \right]$$

Here ζ is a Lagrange-multiplier and $X^i = (T, X, Y)$ a Lorentzian 3-vector

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It could be rewarding to investigate this mechanical problem systematically and numerically!

CTMG at the chiral point
...abbreviated as CCTMG

Definition: CTMG at the chiral point is CTMG with the tuning

$$\mu \, \ell = 1$$

between the cosmological constant and the Chern–Simons coupling.

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$$c_L = \frac{3}{2G}\,\big(1 - \frac{1}{\mu\,\ell}\big)\,, \qquad c_R = \frac{3}{2G}\,\big(1 + \frac{1}{\mu\,\ell}\big)$$

Thus, at the chiral point we get

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Notes:

- Abbreviate "CTMG at the chiral point" as CCTMG
- CCTMG is also known as "chiral gravity"

Gravitons around AdS_3 in CTMG

Linearization around AdS background

$$g_{\mu\nu} = \bar{g}_{\mu\nu} + h_{\mu\nu}$$

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leads to linearized EOM that are third order PDE

$$G^{(1)}_{\mu\nu} + \frac{1}{\mu}\,C^{(1)}_{\mu\nu} = (\mathcal{D}^R\mathcal{D}^L\mathcal{D}^M h)_{\mu\nu} = 0 \tag{1}$$

with three mutually commuting first order operators

$$(\mathcal{D}^{L/R})_\mu{}^\nu = \delta^\nu_\mu \pm \ell\,\varepsilon_\mu{}^{\alpha\nu}\bar{\nabla}_\alpha\,, \qquad (\mathcal{D}^M)_\mu{}^\nu = \delta^\nu_\mu + \frac{1}{\mu}\varepsilon_\mu{}^{\alpha\nu}\bar{\nabla}_\alpha$$

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Three linearly independent solutions to (1):

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At chiral point left (L) and massive (M) branches coincide!

Degeneracy at the chiral point

Will be quite important later!

Li, Song & Strominger found all solutions of linearized EOM.

- Primaries: $L_0, \bar{L}_0$ eigenstates $\psi^{L/R/M}$ with

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$$h_{\mu\nu} = \mathrm{Re}\, \psi_{\mu\nu}$$

- At chiral point: L and M branches degenerate. Get new solution (DG & Johansson)

$$\psi^{\text{new}}_{\mu\nu} = \lim_{\mu\ell \to 1} \frac{\psi^M_{\mu\nu}(\mu\ell) - \psi^L_{\mu\nu}}{\mu\ell - 1}$$

with property

$$\left(\mathcal{D}^L \psi^{\text{new}}\right)_{\mu\nu} = \left(\mathcal{D}^M \psi^{\text{new}}\right)_{\mu\nu} \neq 0\,, \qquad \left((\mathcal{D}^L)^2 \psi^{\text{new}}\right)_{\mu\nu} = 0$$

Sign oder nicht sign?

That is the question. Choosing between Skylla and Charybdis.

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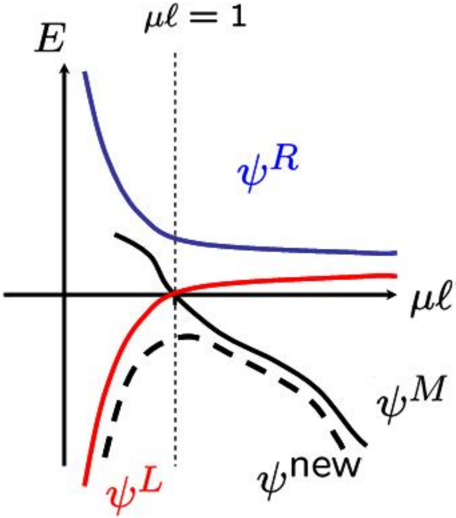
- Either way need a mechanism to eliminate unwanted negative energy objects – either the gravitons or the BHs
- Even at chiral point the problem persists because of the logarithmic mode. See Figure. (Figure: thanks to `N. Johansson`)

Energy for all branches:



Outline

Witten's attempt

Different approach (without gravitons!):

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Various suggestions to interpret this problem: need cosmic strings, need sum over complex geometries, 3D quantum gravity does not exist by itself

Li, Song & Strominger attempt

Is CCTMG dual to a chiral CFT?

Interesting observations:

1. If left-moving sector is trivial, $Z_L = 1$, then problem of holomorphic factorization
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But:

Disagrees with results by `Carlip, Deser, Waldron & Wise`!

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Note: called "logarithmic CFT" because some correlators take the form

$$\langle \psi^{\text{new}}(z) \psi^{\text{new}}(0) \rangle \sim \ln z + \dots$$

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- Currently unknown which of these alternatives is realized!

Viability of the logarithmic mode, part 1

Explicit solution for logarithmic mode (DG & Johansson)

Is the logarithmic mode really there?

Collect in the following suggestions how the logarithmic mode could drop out of the physical spectrum and show that none of them is realized.

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Before starting, here is the explicit form of the logarithmic mode:

$$
\begin{aligned}
h_{\mu\nu}^{\text{new}} = & \frac{\sinh\rho}{\cosh^3\rho}\left(c\,\tau - s\ln\cosh\rho\right)\begin{pmatrix} 0 & 0 & 1 \\ 0 & 0 & 1 \\ 1 & 1 & 0 \end{pmatrix}_{\mu\nu} \\
& - \tanh^2\rho\left(s\,\tau + c\ln\cosh\rho\right)\begin{pmatrix} 1 & 1 & 0 \\ 1 & 1 & 0 \\ 0 & 0 & -a^2 \end{pmatrix}_{\mu\nu}
\end{aligned}
\tag{2}
$$

with

$$
c = \cos(2u)\,, \qquad s = \sin(2u)\,, \qquad a = \frac{1}{\sinh\rho\,\cosh\rho}
$$

Viability of the logarithmic mode, part 2

Physical mode with negative energy

Suggestion 1

The logarithmic mode is pure gauge?

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The logarithmic mode is pure gauge? No!

h^{new} does not solve linearized Einstein equations. Thus is not pure gauge.
Note: confirmed by Sachs who considered logarithmic quasi-normal modes

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Logarithmic mode has infinite energy and thus must be discarded?

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Suggestion 2

Logarithmic mode has infinite energy and thus must be discarded? No!

$$E^{\text{new}} = -\frac{47}{1152G\,\ell^3}$$

Energy is finite and negative.
Thus logarithmic mode leads to instability but cannot be discarded.

Viability of the logarithmic mode, part 3

Boundary conditions beyond Brown–Henneaux

Suggestion 3

New mode is not a small perturbation?

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New mode is not a small perturbation? It is!

h^{new} diverges asymptotically like ρ, but AdS background diverges asymptotically like $e^{2\rho}$. Thus h^{new} is really a small perturbation.

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Solution is asymptotically AdS

$$\mathrm{d}s^2 = \mathrm{d}\rho^2 + \left(\gamma_{ij}^{(0)} e^{2\rho/\ell} + \gamma_{ij}^{(1)} \rho + \gamma_{ij}^{(0)} + \gamma_{ij}^{(2)} e^{-2\rho/\ell} + \dots\right) \mathrm{d}x^i \, \mathrm{d}x^j$$

but violates Brown-Henneaux boundary conditions! ($\gamma_{ij}^{(1)}\big|_{\text{BH}} = 0$)
Henneaux et al. showed precedents where this may happen in 3D
New boundary conditions replacing Brown-Henneaux (DG & Johansson)

Viability of the logarithmic mode, part 4

Brown–York boundary stress tensor

Suggestion 5

New mode leads to ill-defined `Brown-York` boundary stress tensor?

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Total action including boundary terms (Kraus & Larsen)

$$I_{\text{total}} = I_{\text{CTMG}} + \frac{1}{8\pi G} \int \mathrm{d}^2x\sqrt{-\gamma}\left(K - \frac{1}{\ell}\right)$$

Its first variation leads to Brown-York boundary stress-tensor:

$$\delta I_{\text{total}}\big|_{\text{EOM}} = \frac{1}{32\pi G} \int \mathrm{d}^2x\sqrt{-\gamma^{(0)}}\, T^{ij}\, \delta\gamma_{ij}^{(0)}$$

DG & Johansson: T_{ij} is finite, traceless and chiral:

$$T_{ij} = -\frac{\ell}{16\pi\, G} \begin{pmatrix} 1 & 1 \\ 1 & 1 \end{pmatrix}_{ij}$$

Note: coincides with Brown-York boundary stress-tensor of global AdS_3

Viability of the logarithmic mode, part 5

Artifact of linearization?

Suggestion 6

Maybe some non-linear "magic" kills the new mode?

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`DG, Jackiw & Johansson`: classical phase space analysis of CCTMG

$$N = \frac{1}{2}\left(2 \times D - 2 \times N_1 - N_2\right) = \frac{1}{2}\left(2 \times 18 - 2 \times 14 - 6\right) = 1$$

- N: number of physical degrees of freedom (per point)
- D: number of canonical pairs in full phase space
- $N_{1(2)}$: number of linearly independent first (second) class constraints

confirmed in more general calculation by `Carlip`

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- Conclusion 1: logarithmic mode passed all tests so far
- Conclusion 2: CCTMG is unstable; dual CFT probably logarithmic

Outline

Chiral vs. logarithmic

Pivotal open question: does dual CFT exist? is it chiral or logarithmic?

To Do

- Chiral route: must show consistency of truncation!
- Logarithmic: must show consistency of $2^{\rm nd}$ order perturbations!

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$$\bar{L}_{-1}\psi^{\text{new}}_{\mu\nu} = Y_{\mu\nu} = X_{\mu\nu} + \mathcal{L}_\xi \bar{g}_{\mu\nu}$$

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- straightforward but somewhat lengthy calculation
- expand metric around AdS background up to second order:

$$g_{\mu\nu} = \bar{g}_{\mu\nu} + h^{\text{new}}_{\mu\nu} + h^{(2)}_{\mu\nu}$$

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- Might be rewarding exercise for a student

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Two observations:

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`Strominger et al. :`

Suggestive to consider warped AdS as possible groundstate of (C)CTMG

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Thank you for your attention!

Some literature

- D. Grumiller, W. Kummer, and D. Vassilevich, "Dilaton gravity in two dimensions," *Phys. Rept.* **369** (2002) 327–429, hep-th/0204253.
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