Gravity in lower dimensions

Daniel Grumiller

Institute for Theoretical Physics
Vienna University of Technology

Uppsala University, December 2008

MARIE CURIE ACTIONS

Outline

Why lower-dimensional gravity?

Which 2D theory?

Which 3D theory?

How to quantize 3D gravity?

What next?

Outline

Why lower-dimensional gravity?

Which 2D theory?

Which 3D theory?

How to quantize 3D gravity?

What next?

Quantum gravity
The Holy Grail of theoretical physics

There is a lot we do know about quantum gravity already

Quantum gravity
The Holy Grail of theoretical physics

There is a lot we do know about quantum gravity already

- It should exist in some form

Quantum gravity

The Holy Grail of theoretical physics

There is a lot we do know about quantum gravity already

- It should exist in some form
- String theory: (perturbative) theory of quantum gravity

There is a lot we do know about quantum gravity already

- It should exist in some form
- String theory: (perturbative) theory of quantum gravity
- Microscopic understanding of extremal BH entropy

Quantum gravity

The Holy Grail of theoretical physics

There is a lot we do know about quantum gravity already

- It should exist in some form
- String theory: (perturbative) theory of quantum gravity
- Microscopic understanding of extremal BH entropy
- Conceptual insight — information loss problem resolved

Quantum gravity
The Holy Grail of theoretical physics

There is a lot we do know about quantum gravity already

- It should exist in some form
- String theory: (perturbative) theory of quantum gravity
- Microscopic understanding of extremal BH entropy
- Conceptual insight — information loss problem resolved

There is a lot we still do not know about quantum gravity

Quantum gravity
The Holy Grail of theoretical physics

There is a lot we do know about quantum gravity already

- It should exist in some form
- String theory: (perturbative) theory of quantum gravity
- Microscopic understanding of extremal BH entropy
- Conceptual insight — information loss problem resolved

There is a lot we still do not know about quantum gravity

- Reasonable alternatives to string theory?

Quantum gravity

The Holy Grail of theoretical physics

There is a lot we do know about quantum gravity already

- It should exist in some form
- String theory: (perturbative) theory of quantum gravity
- Microscopic understanding of extremal BH entropy
- Conceptual insight — information loss problem resolved

There is a lot we still do not know about quantum gravity

- Reasonable alternatives to string theory?
- Non-perturbative understanding of quantum gravity?

Quantum gravity

The Holy Grail of theoretical physics

There is a lot we do know about quantum gravity already

- It should exist in some form
- String theory: (perturbative) theory of quantum gravity
- Microscopic understanding of extremal BH entropy
- Conceptual insight — information loss problem resolved

There is a lot we still do not know about quantum gravity

- Reasonable alternatives to string theory?
- Non-perturbative understanding of quantum gravity?
- Microscopic understanding of non-extremal BH entropy?

There is a lot we do know about quantum gravity already

- It should exist in some form
- String theory: (perturbative) theory of quantum gravity
- Microscopic understanding of extremal BH entropy
- Conceptual insight — information loss problem resolved

There is a lot we still do not know about quantum gravity

- Reasonable alternatives to string theory?
- Non-perturbative understanding of quantum gravity?
- Microscopic understanding of non-extremal BH entropy?
- Experimental signatures? Data?

Gravity in lower dimensions

Riemann-tensor $\frac{D^2(D^2-1)}{12}$ components in D dimensions:

- 11D: 1210 (1144 Weyl and 66 Ricci)
- 10D: 825 (770 Weyl and 55 Ricci)
- 5D: 50 (35 Weyl and 15 Ricci)
- 4D: 20 (10 Weyl and 10 Ricci)

Gravity in lower dimensions

Riemann-tensor $\frac{D^2(D^2-1)}{12}$ components in D dimensions:

- 11D: 1210 (1144 Weyl and 66 Ricci)
- 10D: 825 (770 Weyl and 55 Ricci)
- 5D: 50 (35 Weyl and 15 Ricci)
- 4D: 20 (10 Weyl and 10 Ricci)
- 3D: 6 (Ricci)
- 2D: 1 (Ricci scalar)

Gravity in lower dimensions

Riemann-tensor $\frac{D^2(D^2-1)}{12}$ components in D dimensions:

- 11D: 1210 (1144 Weyl and 66 Ricci)
- 10D: 825 (770 Weyl and 55 Ricci)
- 5D: 50 (35 Weyl and 15 Ricci)
- 4D: 20 (10 Weyl and 10 Ricci)
- 3D: 6 (Ricci)
- 2D: 1 (Ricci scalar)

> - 2D: lowest dimension exhibiting black holes (BHs)
> - Simplest gravitational theories with BHs in 2D

Gravity in lower dimensions

Riemann-tensor $\frac{D^2(D^2-1)}{12}$ components in D dimensions:

- 11D: 1210 (1144 Weyl and 66 Ricci)
- 10D: 825 (770 Weyl and 55 Ricci)
- 5D: 50 (35 Weyl and 15 Ricci)
- 4D: 20 (10 Weyl and 10 Ricci)
- 3D: 6 (Ricci)
- 2D: 1 (Ricci scalar)

> - 2D: lowest dimension exhibiting black holes (BHs)
> - Simplest gravitational theories with BHs in 2D

> - 3D: lowest dimension exhibiting BHs and gravitons
> - Simplest gravitational theories with BHs and gravitons in 3D

Outline

Attempt 1: Einstein–Hilbert in and near two dimensions

Let us start with the simplest attempt. `Einstein-Hilbert` action in 2 dimensions:

$$I_{\mathrm{EH}} = \frac{1}{16\pi\, G} \int \mathrm{d}^2 x \sqrt{|g|}\, R = \frac{1}{2G}\,(1-\gamma)$$

- Action is topological
- No equations of motion
- Formal counting of number of gravitons: -1

Attempt 1: Einstein–Hilbert in and near two dimensions

Let us continue with the next simplest attempt. `Einstein-Hilbert` action in $2+\epsilon$ dimensions:

$$I_{\mathrm{EH}}{}^{\epsilon} = \frac{1}{16\pi\,G}\int \mathrm{d}^{2+\epsilon}x\sqrt{|g|}\,R$$

- `Weinberg`: theory is asymptotically safe
- `Mann`: limit $\epsilon \longrightarrow 0$ should be possible and lead to 2D dilaton gravity
- `DG, Jackiw`: limit $\epsilon \longrightarrow 0$ yields Liouville gravity

$$\lim_{\epsilon\longrightarrow 0} I_{EH}{}^{\epsilon} = \frac{1}{16\pi\,G_2}\int \mathrm{d}^2x\sqrt{|g|}\,\left[XR - (\nabla X)^2 + \lambda e^{-2X}\right]$$

Attempt 1: Einstein–Hilbert in and near two dimensions

Let us continue with the next simplest attempt. Einstein-Hilbert action in $2+\epsilon$ dimensions:

$$I_{\text{EH}}{}^{\epsilon} = \frac{1}{16\pi\, G} \int \mathrm{d}^{2+\epsilon}x \sqrt{|g|}\, R$$

- Weinberg: theory is asymptotically safe
- Mann: limit $\epsilon \to 0$ should be possible and lead to 2D dilaton gravity
- DG, Jackiw: limit $\epsilon \to 0$ yields Liouville gravity

$$\lim_{\epsilon\to 0} I_{EH}{}^{\epsilon} = \frac{1}{16\pi\, G_2} \int \mathrm{d}^2x \sqrt{|g|}\, \left[XR - (\nabla X)^2 + \lambda e^{-2X}\right]$$

Result of attempt 1:

A specific 2D dilaton gravity model

Attempt 2: Gravity as a gauge theory and the Jackiw-Teitelboim model

`Jackiw, Teitelboim (Bunster)`: (A)dS$_2$ gauge theory

$$[P_a, P_b] = \Lambda\,\epsilon_{ab} J \qquad [P_a, J] = \epsilon_a{}^b P_b$$

describes constant curvature gravity in 2D. Algorithm:

Attempt 2: Gravity as a gauge theory and the Jackiw-Teitelboim model

`Jackiw, Teitelboim (Bunster)`: (A)dS_2 gauge theory

$$[P_a, P_b] = \Lambda\,\epsilon_{ab} J \qquad [P_a, J] = \epsilon_a{}^b P_b$$

describes constant curvature gravity in 2D. Algorithm:

- Start with $SO(1,2)$ connection $A = e^a P_a + \omega J$

Attempt 2: Gravity as a gauge theory and the Jackiw-Teitelboim model

`Jackiw, Teitelboim (Bunster)`: (A)dS$_2$ gauge theory

$$[P_a, P_b] = \Lambda \, \epsilon_{ab} J \qquad [P_a, J] = \epsilon_a{}^b P_b$$

describes constant curvature gravity in 2D. Algorithm:

- Start with $SO(1,2)$ connection $A = e^a P_a + \omega J$
- Take field strength $F = \mathrm{d}A + \frac{1}{2}[A, A]$ and coadjoint scalar X

Attempt 2: Gravity as a gauge theory and the Jackiw-Teitelboim model

`Jackiw, Teitelboim (Bunster)`: (A)dS$_2$ gauge theory

$$[P_a, P_b] = \Lambda\, \epsilon_{ab} J \qquad [P_a, J] = \epsilon_a{}^b P_b$$

describes constant curvature gravity in 2D. Algorithm:

- Start with $SO(1,2)$ connection $A = e^a P_a + \omega J$
- Take field strength $F = \mathrm{d}A + \frac{1}{2}[A, A]$ and coadjoint scalar X
- Construct non-abelian BF theory

$$I = \int X_A F^A = \int \Big[X_a(\mathrm{d}e^a + \epsilon^a{}_b \omega \wedge e^b) + X\,\mathrm{d}\omega + \epsilon_{ab} e^a \wedge e^b\, \Lambda X\Big]$$

Attempt 2: Gravity as a gauge theory and the Jackiw-Teitelboim model

`Jackiw, Teitelboim (Bunster)`: (A)dS$_2$ gauge theory

$$[P_a, P_b] = \Lambda\,\epsilon_{ab} J \qquad [P_a, J] = \epsilon_a{}^b P_b$$

describes constant curvature gravity in 2D. Algorithm:

- Start with $SO(1,2)$ connection $A = e^a P_a + \omega J$
- Take field strength $F = \mathrm{d}A + \frac{1}{2}[A, A]$ and coadjoint scalar X
- Construct non-abelian BF theory

$$I = \int X_A F^A = \int \Big[X_a(\mathrm{d}e^a + \epsilon^a{}_b \omega \wedge e^b) + X\,\mathrm{d}\omega + \epsilon_{ab} e^a \wedge e^b\,\Lambda X\Big]$$

- Eliminate X_a (Torsion constraint) and ω (Levi-Civita connection)

Attempt 2: Gravity as a gauge theory and the Jackiw-Teitelboim model

`Jackiw, Teitelboim (Bunster)`: (A)dS_2 gauge theory

$$[P_a, P_b] = \Lambda\,\epsilon_{ab} J \qquad [P_a, J] = \epsilon_a{}^b P_b$$

describes constant curvature gravity in 2D. Algorithm:

- Start with $SO(1,2)$ connection $A = e^a P_a + \omega J$
- Take field strength $F = \mathrm{d}A + \frac{1}{2}[A, A]$ and coadjoint scalar X
- Construct non-abelian BF theory

$$I = \int X_A F^A = \int \Big[X_a(\mathrm{d}e^a + \epsilon^a{}_b \omega \wedge e^b) + X\,\mathrm{d}\omega + \epsilon_{ab} e^a \wedge e^b\,\Lambda X\Big]$$

- Eliminate X_a (Torsion constraint) and ω (Levi-Civita connection)
- Obtain the second order action

$$I = \frac{1}{16\pi\, G_2} \int \mathrm{d}^2x \sqrt{-g}\, X\,[R - \Lambda]$$

Attempt 2: Gravity as a gauge theory and the Jackiw-Teitelboim model

`Jackiw, Teitelboim (Bunster)`: (A)dS_2 gauge theory

$$[P_a, P_b] = \Lambda\, \epsilon_{ab} J \qquad [P_a, J] = \epsilon_a{}^b P_b$$

describes constant curvature gravity in 2D. Algorithm:

- Start with $SO(1,2)$ connection $A = e^a P_a + \omega J$
- Take field strength $F = \mathrm{d}A + \frac{1}{2}[A, A]$ and coadjoint scalar X
- Construct non-abelian BF theory

$$I = \int X_A F^A = \int \Big[X_a(\mathrm{d}e^a + \epsilon^a{}_b \omega \wedge e^b) + X\,\mathrm{d}\omega + \epsilon_{ab} e^a \wedge e^b\, \Lambda X\Big]$$

- Eliminate X_a (Torsion constraint) and ω (Levi-Civita connection)
- Obtain the second order action

$$I = \frac{1}{16\pi\, G_2} \int \mathrm{d}^2 x \sqrt{-g}\, X\, [R - \Lambda]$$

Result of attempt 2:

A specific 2D dilaton gravity model

Attempt 3: Dimensional reduction

For example: spherical reduction from D dimensions

Line element adapted to spherical symmetry:

$$\mathrm{d}s^2 = \underbrace{g_{\mu\nu}^{(D)}}_{\text{full metric}} \,\mathrm{d}x^\mu\,\mathrm{d}x^\nu = \underbrace{g_{\alpha\beta}(x^\gamma)}_{2D\text{ metric}}\,\mathrm{d}x^\alpha\,\mathrm{d}x^\beta - \underbrace{\phi^2(x^\alpha)}_{\text{surface area}}\,\mathrm{d}\Omega^2_{S_{D-2}}\,,$$

Attempt 3: Dimensional reduction

For example: spherical reduction from D dimensions

Line element adapted to spherical symmetry:

$$\mathrm{d}s^2 = \underbrace{g_{\mu\nu}^{(D)}}_{\text{full metric}} \mathrm{d}x^\mu \, \mathrm{d}x^\nu = \underbrace{g_{\alpha\beta}(x^\gamma)}_{2D \text{ metric}} \mathrm{d}x^\alpha \, \mathrm{d}x^\beta - \underbrace{\phi^2(x^\alpha)}_{\text{surface area}} \mathrm{d}\Omega^2_{S_{D-2}} \,,$$

Insert into D-dimensional EH action $I_{EH} = \kappa \int \mathrm{d}^D x \sqrt{-g^{(D)}} R^{(D)}$:

$$I_{EH} = \kappa \frac{2\pi^{(D-1)/2}}{\Gamma(\frac{D-1}{2})} \int \mathrm{d}^2 x \sqrt{-g} \, \phi^{D-2} \Big[R + \frac{(D-2)(D-3)}{\phi^2} \left((\nabla\phi)^2 - 1 \right) \Big]$$

Attempt 3: Dimensional reduction

For example: spherical reduction from D dimensions

Line element adapted to spherical symmetry:

$$\mathrm{d}s^2 = \underbrace{g_{\mu\nu}^{(D)}}_{\text{full metric}} \mathrm{d}x^\mu \, \mathrm{d}x^\nu = \underbrace{g_{\alpha\beta}(x^\gamma)}_{2D \text{ metric}} \mathrm{d}x^\alpha \, \mathrm{d}x^\beta - \underbrace{\phi^2(x^\alpha)}_{\text{surface area}} \mathrm{d}\Omega^2_{S_{D-2}} \,,$$

Insert into D-dimensional EH action $I_{EH} = \kappa \int \mathrm{d}^D x \sqrt{-g^{(D)}} R^{(D)}$:

$$I_{EH} = \kappa \frac{2\pi^{(D-1)/2}}{\Gamma(\frac{D-1}{2})} \int \mathrm{d}^2 x \sqrt{-g} \, \phi^{D-2} \Big[R + \frac{(D-2)(D-3)}{\phi^2} \big((\nabla\phi)^2 - 1 \big) \Big]$$

Cosmetic redefinition $X \propto (\lambda\phi)^{D-2}$:

$$I_{EH} = \frac{1}{16\pi \, G_2} \int \mathrm{d}^2 x \sqrt{-g} \Big[XR + \frac{D-3}{(D-2)X} (\nabla X)^2 - \lambda^2 X^{(D-4)/(D-2)} \Big]$$

Result of attempt 3:

A specific class of 2D dilaton gravity models

Attempt 4: Poincare gauge theory and higher power curvature theories

Basic idea: since EH is trivial consider $f(R)$ theories or/and theories with torsion or/and theories with non-metricity

Attempt 4: Poincare gauge theory and higher power curvature theories

Basic idea: since EH is trivial consider $f(R)$ theories or/and theories with torsion or/and theories with non-metricity

- Example: `Katanaev-Volovich` model (Poincare gauge theory)

$$I_{\mathrm{KV}} \sim \int \mathrm{d}^2x\sqrt{-g}\,\left[\alpha T^2 + \beta R^2\right]$$

- `Kummer, Schwarz`: bring into first order form:

$$I_{\mathrm{KV}} \sim \int \Big[X_a(\mathrm{d}e^a + \epsilon^a{}_b\omega\wedge e^b) + X\,\mathrm{d}\omega + \epsilon_{ab}e^a\wedge e^b\,(\alpha X^a X_a + \beta X^2)\Big]$$

- Use same algorithm as before to convert into second order action:

$$I_{\mathrm{KV}} = \frac{1}{16\pi\,G_2}\int \mathrm{d}^2x\sqrt{-g}\Big[XR + \alpha(\nabla X)^2 + \beta X^2\Big]$$

Attempt 4: Poincare gauge theory and higher power curvature theories

Basic idea: since EH is trivial consider $f(R)$ theories or/and theories with torsion or/and theories with non-metricity

- Example: `Katanaev-Volovich` model (Poincare gauge theory)

$$I_{\rm KV} \sim \int \mathrm{d}^2x\sqrt{-g}\,\left[\alpha T^2 + \beta R^2\right]$$

- `Kummer, Schwarz`: bring into first order form:

$$I_{\rm KV} \sim \int \left[X_a(\mathrm{d}e^a + \epsilon^a{}_b\omega\wedge e^b) + X\,\mathrm{d}\omega + \epsilon_{ab}e^a\wedge e^b\,(\alpha X^a X_a + \beta X^2)\right]$$

- Use same algorithm as before to convert into second order action:

$$I_{\rm KV} = \frac{1}{16\pi\,G_2}\int \mathrm{d}^2x\sqrt{-g}\left[XR + \alpha(\nabla X)^2 + \beta X^2\right]$$

Result of attempt 4:

A specific 2D dilaton gravity model

Attempt 5: Strings in two dimensions

Conformal invariance of the σ model

$$I_\sigma \propto \int \mathrm{d}^2\xi \sqrt{|h|}\,\left[g_{\mu\nu}h^{ij}\partial_i x^\mu \partial_j x^\nu + \alpha' \phi \mathcal{R} + \dots\right]$$

requires vanishing of β-functions

$$\beta^\phi \propto -4b^2 - 4(\nabla\phi)^2 + 4\Box\phi + R + \dots$$

$$\beta^g_{\mu\nu} \propto R_{\mu\nu} + 2\nabla_\mu\nabla_\nu\phi + \dots$$

Conditions $\beta^\phi = \beta^g_{\mu\nu} = 0$ follow from target space action

$$I_{\text{target}} = \frac{1}{16\pi\,G_2}\int \mathrm{d}^2x\sqrt{-g}\Big[XR + \frac{1}{X}\,(\nabla X)^2 - 4b^2\Big]$$

where $X = e^{-2\phi}$

Attempt 5: Strings in two dimensions

Conformal invariance of the σ model

$$I_\sigma \propto \int \mathrm{d}^2\xi \sqrt{|h|}\,\left[g_{\mu\nu}h^{ij}\partial_i x^\mu \partial_j x^\nu + \alpha' \phi \mathcal{R} + \dots\right]$$

requires vanishing of β-functions

$$\beta^\phi \propto -4b^2 - 4(\nabla\phi)^2 + 4\Box\phi + R + \dots$$
$$\beta^g_{\mu\nu} \propto R_{\mu\nu} + 2\nabla_\mu\nabla_\nu\phi + \dots$$

Conditions $\beta^\phi = \beta^g_{\mu\nu} = 0$ follow from target space action

$$I_{\text{target}} = \frac{1}{16\pi G_2}\int \mathrm{d}^2x\sqrt{-g}\Big[XR + \frac{1}{X}(\nabla X)^2 - 4b^2\Big]$$

where $X = e^{-2\phi}$

Result of attempt 5:

A specific 2D dilaton gravity model

Synthesis of all attempts: Dilaton gravity in two dimensions

Second order action:

$$I = \frac{1}{16\pi\, G_2} \int_{\mathcal{M}} \mathrm{d}^2 x \sqrt{|g|} \left[XR - U(X)(\nabla X)^2 - V(X) \right] - \frac{1}{8\pi\, G_2} \int_{\partial\mathcal{M}} \mathrm{d}x \sqrt{|\gamma|} \; \left[XK - S(X) \right] + I^{(m)}$$

Synthesis of all attempts: Dilaton gravity in two dimensions

Second order action:

$$
\begin{aligned}
I = & \frac{1}{16\pi\, G_2} \int_{\mathcal{M}} \mathrm{d}^2x \sqrt{|g|}\, \big[XR - U(X)(\nabla X)^2 - V(X)\big] \\
& - \frac{1}{8\pi\, G_2} \int_{\partial\mathcal{M}} \mathrm{d}x \sqrt{|\gamma|}\;\; [XK - S(X)] + I^{(m)}
\end{aligned}
$$

- Dilaton X defined by its coupling to curvature R

Synthesis of all attempts: Dilaton gravity in two dimensions

Second order action:

$$I = \frac{1}{16\pi\, G_2} \int_{\mathcal{M}} \mathrm{d}^2 x \sqrt{|g|}\, \big[XR - U(X)(\nabla X)^2 - V(X)\big]$$
$$- \frac{1}{8\pi\, G_2} \int_{\partial\mathcal{M}} \mathrm{d}x \sqrt{|\gamma|}\ \ [XK - S(X)] + I^{(m)}$$

- Dilaton X defined by its coupling to curvature R
- Kinetic term $(\nabla X)^2$ contains coupling function $U(X)$

Synthesis of all attempts: Dilaton gravity in two dimensions

Second order action:

$$I = \frac{1}{16\pi\, G_2} \int_{\mathcal{M}} \mathrm{d}^2x \sqrt{|g|}\, \big[XR - U(X)(\nabla X)^2 - V(X)\big]$$
$$- \frac{1}{8\pi\, G_2} \int_{\partial\mathcal{M}} \mathrm{d}x \sqrt{|\gamma|}\ \ [XK - S(X)] + I^{(m)}$$

- Dilaton X defined by its coupling to curvature R
- Kinetic term $(\nabla X)^2$ contains coupling function $U(X)$
- Self-interaction potential $V(X)$ leads to non-trivial geometries

Synthesis of all attempts: Dilaton gravity in two dimensions

Second order action:

$$I = \frac{1}{16\pi\, G_2} \int_{\mathcal{M}} \mathrm{d}^2x \sqrt{|g|}\, \left[X R - U(X)(\nabla X)^2 - V(X) \right] - \frac{1}{8\pi\, G_2} \int_{\partial\mathcal{M}} \mathrm{d}x \sqrt{|\gamma|}\;\; [XK - S(X)] + I^{(m)}$$

- Dilaton X defined by its coupling to curvature R
- Kinetic term $(\nabla X)^2$ contains coupling function $U(X)$
- Self-interaction potential $V(X)$ leads to non-trivial geometries
- Gibbons–Hawking–York boundary term guarantees Dirichlet boundary problem for metric

Synthesis of all attempts: Dilaton gravity in two dimensions

Second order action:

$$I = \frac{1}{16\pi\, G_2} \int_{\mathcal{M}} \mathrm{d}^2x\sqrt{|g|}\,\left[XR - U(X)(\nabla X)^2 - V(X)\right]$$
$$- \frac{1}{8\pi\, G_2} \int_{\partial\mathcal{M}} \mathrm{d}x\sqrt{|\gamma|}\;\left[XK - S(X)\right] + I^{(m)}$$

- Dilaton X defined by its coupling to curvature R
- Kinetic term $(\nabla X)^2$ contains coupling function $U(X)$
- Self-interaction potential $V(X)$ leads to non-trivial geometries
- Gibbons–Hawking–York boundary term guarantees Dirichlet boundary problem for metric
- Hamilton–Jacobi counterterm contains superpotential $S(X)$

$$S(X)^2 = e^{-\int^X U(y)\,\mathrm{d}y} \int^X V(y) e^{\int^y U(z)\,\mathrm{d}z}\,\mathrm{d}y$$

  and guarantees well-defined variational principle $\delta I = 0$

Synthesis of all attempts: Dilaton gravity in two dimensions

Second order action:

$$\begin{aligned} I &= \frac{1}{16\pi\, G_2}\int_{\mathcal{M}} \mathrm{d}^2x\sqrt{|g|}\,\big[XR - U(X)(\nabla X)^2 - V(X)\big] \\ &\quad - \frac{1}{8\pi\, G_2}\int_{\partial\mathcal{M}} \mathrm{d}x\sqrt{|\gamma|}\;\;[XK - S(X)] + I^{(m)} \end{aligned}$$

- Dilaton X defined by its coupling to curvature R
- Kinetic term $(\nabla X)^2$ contains coupling function $U(X)$
- Self-interaction potential $V(X)$ leads to non-trivial geometries
- Gibbons–Hawking–York boundary term guarantees Dirichlet boundary problem for metric
- Hamilton–Jacobi counterterm contains superpotential $S(X)$

  $$S(X)^2 = e^{-\int^X U(y)\,\mathrm{d}y}\int^X V(y)e^{\int^y U(z)\,\mathrm{d}z}\,\mathrm{d}y$$

  and guarantees well-defined variational principle $\delta I = 0$
- Interesting option: couple 2D dilaton gravity to matter

Recent example: AdS_2 holography

Two dimensions supposed to be the simplest dimension with geometry, and yet...

- extremal black holes universally include AdS_2 factor
- funnily, AdS_3 holography more straightforward
- study charged Jackiw–Teitelboim model as example

$$I_{\mathrm{JT}} = \frac{\alpha}{2\pi} \int \mathrm{d}^2x\sqrt{-g}\,\left[e^{-2\phi}\,\left(R + \frac{8}{L^2}\right) - \frac{L^2}{4}\,F^2\right]$$

Recent example: AdS_2 holography

Two dimensions supposed to be the simplest dimension with geometry, and yet...

- extremal black holes universally include AdS_2 factor
- funnily, AdS_3 holography more straightforward
- study charged Jackiw–Teitelboim model as example

$$I_{\text{JT}} = \frac{\alpha}{2\pi} \int \mathrm{d}^2x \sqrt{-g} \left[e^{-2\phi} \left(R + \frac{8}{L^2} \right) - \frac{L^2}{4} F^2 \right]$$

- Metric g has signature $-,+$ and Ricci-scalar $R < 0$ for AdS

Recent example: AdS_2 holography

Two dimensions supposed to be the simplest dimension with geometry, and yet...

- extremal black holes universally include AdS_2 factor
- funnily, AdS_3 holography more straightforward
- study charged Jackiw–Teitelboim model as example

$$I_{\text{JT}} = \frac{\alpha}{2\pi} \int \mathrm{d}^2 x \sqrt{-g} \left[e^{-2\phi} \left(R + \frac{8}{L^2} \right) - \frac{L^2}{4} F^2 \right]$$

- Metric g has signature $-, +$ and Ricci-scalar $R < 0$ for AdS
- Maxwell field strength $F_{\mu\nu} = 2E\,\varepsilon_{\mu\nu}$ dual to electric field E

Recent example: AdS_2 holography

Two dimensions supposed to be the simplest dimension with geometry, and yet...

- extremal black holes universally include AdS_2 factor
- funnily, AdS_3 holography more straightforward
- study charged Jackiw–Teitelboim model as example

$$I_{\mathrm{JT}} = \frac{\alpha}{2\pi} \int \mathrm{d}^2x\sqrt{-g}\,\left[e^{-2\phi}\,\left(R+\frac{8}{L^2}\right) - \frac{L^2}{4}\,F^2\right]$$

- Metric g has signature $-,+$ and Ricci-scalar $R < 0$ for AdS
- Maxwell field strength $F_{\mu\nu} = 2E\,\varepsilon_{\mu\nu}$ dual to electric field E
- Dilaton ϕ has no kinetic term and no coupling to gauge field

Recent example: AdS_2 holography

Two dimensions supposed to be the simplest dimension with geometry, and yet...

- extremal black holes universally include AdS_2 factor
- funnily, AdS_3 holography more straightforward
- study charged Jackiw–Teitelboim model as example

$$I_{\mathrm{JT}} = \frac{\alpha}{2\pi} \int \mathrm{d}^2x \sqrt{-g} \left[e^{-2\phi} \left(R + \frac{8}{L^2} \right) - \frac{L^2}{4} F^2 \right]$$

- Metric g has signature $-, +$ and Ricci-scalar $R < 0$ for AdS
- Maxwell field strength $F_{\mu\nu} = 2E\,\varepsilon_{\mu\nu}$ dual to electric field E
- Dilaton ϕ has no kinetic term and no coupling to gauge field
- Cosmological constant $\Lambda = -\frac{8}{L^2}$ parameterized by AdS radius L

Recent example: AdS_2 holography

Two dimensions supposed to be the simplest dimension with geometry, and yet...

- extremal black holes universally include AdS_2 factor
- funnily, AdS_3 holography more straightforward
- study charged Jackiw–Teitelboim model as example

$$I_{\mathrm{JT}} = \frac{\alpha}{2\pi} \int \mathrm{d}^2x\sqrt{-g}\,\Big[e^{-2\phi}\,\Big(R + \frac{8}{L^2}\Big) - \frac{L^2}{4}\,F^2\Big]$$

- Metric g has signature $-,+$ and Ricci-scalar $R < 0$ for AdS
- Maxwell field strength $F_{\mu\nu} = 2E\,\varepsilon_{\mu\nu}$ dual to electric field E
- Dilaton ϕ has no kinetic term and no coupling to gauge field
- Cosmological constant $\Lambda = -\frac{8}{L^2}$ parameterized by AdS radius L
- Coupling constant α usually is positive

Recent example: AdS_2 holography

Two dimensions supposed to be the simplest dimension with geometry, and yet...

- extremal black holes universally include AdS_2 factor
- funnily, AdS_3 holography more straightforward
- study charged Jackiw–Teitelboim model as example

$$I_{\text{JT}} = \frac{\alpha}{2\pi} \int \mathrm{d}^2x \sqrt{-g} \left[e^{-2\phi} \left(R + \frac{8}{L^2} \right) - \frac{L^2}{4} F^2 \right]$$

- Metric g has signature $-, +$ and Ricci-scalar $R < 0$ for AdS
- Maxwell field strength $F_{\mu\nu} = 2E\,\varepsilon_{\mu\nu}$ dual to electric field E
- Dilaton ϕ has no kinetic term and no coupling to gauge field
- Cosmological constant $\Lambda = -\frac{8}{L^2}$ parameterized by AdS radius L
- Coupling constant α usually is positive
- $\delta\phi$ EOM: $R = -\frac{8}{L^2} \qquad \Rightarrow \qquad$ AdS_2!

Recent example: AdS_2 holography

Two dimensions supposed to be the simplest dimension with geometry, and yet...

- extremal black holes universally include AdS_2 factor
- funnily, AdS_3 holography more straightforward
- study charged Jackiw–Teitelboim model as example

$$I_{\text{JT}} = \frac{\alpha}{2\pi} \int \mathrm{d}^2x\sqrt{-g}\,\Big[e^{-2\phi}\,\Big(R+\frac{8}{L^2}\Big)-\frac{L^2}{4}\,F^2\Big]$$

- Metric g has signature $-,+$ and Ricci-scalar $R<0$ for AdS
- Maxwell field strength $F_{\mu\nu}=2E\,\varepsilon_{\mu\nu}$ dual to electric field E
- Dilaton ϕ has no kinetic term and no coupling to gauge field
- Cosmological constant $\Lambda=-\frac{8}{L^2}$ parameterized by AdS radius L
- Coupling constant α usually is positive
- $\delta\phi$ EOM: $R=-\frac{8}{L^2}$ $\Rightarrow$ AdS_2!
- δA EOM: $\nabla_\mu F^{\mu\nu}=0$ $\Rightarrow$ $E=\text{constant}$

Recent example: AdS_2 holography

Two dimensions supposed to be the simplest dimension with geometry, and yet...

- extremal black holes universally include AdS_2 factor
- funnily, AdS_3 holography more straightforward
- study charged Jackiw–Teitelboim model as example

$$I_{\mathrm{JT}} = \frac{\alpha}{2\pi} \int \mathrm{d}^2x \sqrt{-g}\,\Big[e^{-2\phi}\,\Big(R + \frac{8}{L^2}\Big) - \frac{L^2}{4}\,F^2\Big]$$

- Metric g has signature $-, +$ and Ricci-scalar $R < 0$ for AdS
- Maxwell field strength $F_{\mu\nu} = 2E\,\varepsilon_{\mu\nu}$ dual to electric field E
- Dilaton ϕ has no kinetic term and no coupling to gauge field
- Cosmological constant $\Lambda = -\frac{8}{L^2}$ parameterized by AdS radius L
- Coupling constant α usually is positive
- $\delta\phi$ EOM: $R = -\frac{8}{L^2}$ $\quad\Rightarrow\quad$ AdS_2!
- δA EOM: $\nabla_\mu F^{\mu\nu} = 0$ $\quad\Rightarrow\quad$ $E = \text{constant}$
- δg EOM: complicated for non-constant dilaton...

$$\nabla_\mu \nabla_\nu e^{-2\phi} - g_{\mu\nu}\,\nabla^2 e^{-2\phi} + \frac{4}{L^2}\,e^{-2\phi}\,g_{\mu\nu} + \frac{L^2}{2}\,F_\mu{}^\lambda\,F_{\nu\lambda} - \frac{L^2}{8}\,g_{\mu\nu}\,F^2 = 0$$

Recent example: AdS_2 holography

Two dimensions supposed to be the simplest dimension with geometry, and yet...

- extremal black holes universally include AdS_2 factor
- funnily, AdS_3 holography more straightforward
- study charged Jackiw–Teitelboim model as example

$$I_{\text{JT}} = \frac{\alpha}{2\pi}\int \mathrm{d}^2x\sqrt{-g}\,\Big[e^{-2\phi}\,\Big(R+\frac{8}{L^2}\Big)-\frac{L^2}{4}\,F^2\Big]$$

- Metric g has signature $-,+$ and Ricci-scalar $R< 0$ for AdS
- Maxwell field strength $F_{\mu\nu}=2E\,\varepsilon_{\mu\nu}$ dual to electric field E
- Dilaton ϕ has no kinetic term and no coupling to gauge field
- Cosmological constant $\Lambda=-\frac{8}{L^2}$ parameterized by AdS radius L
- Coupling constant α usually is positive
- $\delta\phi$ EOM: $R=-\frac{8}{L^2}$ $\Rightarrow$ AdS_2!
- δA EOM: $\nabla_\mu F^{\mu\nu}=0$ $\Rightarrow$ $E=\text{constant}$
- δg EOM: ...but simple for constant dilaton: $e^{-2\phi}=\frac{L^4}{4}\,E^2$

$$\nabla_\mu\nabla_\nu e^{-2\phi}-g_{\mu\nu}\,\nabla^2 e^{-2\phi}+\frac{4}{L^2}\,e^{-2\phi}\,g_{\mu\nu}+\frac{L^2}{2}\,F_\mu{}^\lambda\,F_{\nu\lambda}-\frac{L^2}{8}\,g_{\mu\nu}\,F^2=0$$

Some surprising results

Hartman, Strominger = HS   Castro, DG, Larsen, McNees = CGLM

- Holographic renormalization leads to boundary mass term (CGLM)

$$I \sim \int \mathrm{d}x \sqrt{|\gamma|}\, m A^2$$

Nevertheless, total action gauge invariant

Some surprising results

Hartman, Strominger = HS   Castro, DG, Larsen, McNees = CGLM

- Holographic renormalization leads to boundary mass term (CGLM)

$$I \sim \int \mathrm{d}x \sqrt{|\gamma|}\, m A^2$$

  Nevertheless, total action gauge invariant
- Boundary stress tensor transforms anomalously (HS)

$$(\delta_\xi + \delta_\lambda)\, T_{tt} = 2T_{tt}\partial_t\xi + \xi\partial_t T_{tt} - \frac{c}{24\pi} L \partial_t^3 \xi$$

  where $\delta_\xi + \delta_\lambda$ is combination of diffeo- and gauge trafos that preserve the boundary conditions (similarly: $\delta_\lambda J_t = -\frac{k}{4\pi} L \partial_t \lambda$)

Some surprising results

Hartman, Strominger = HS   Castro, DG, Larsen, McNees = CGLM

- Holographic renormalization leads to boundary mass term (CGLM)

$$I \sim \int \mathrm{d}x \sqrt{|\gamma|}\, m A^2$$

  Nevertheless, total action gauge invariant

- Boundary stress tensor transforms anomalously (HS)

$$(\delta_\xi + \delta_\lambda)\, T_{tt} = 2T_{tt}\partial_t \xi + \xi \partial_t T_{tt} - \frac{c}{24\pi} L \partial_t^3 \xi$$

  where $\delta_\xi + \delta_\lambda$ is combination of diffeo- and gauge trafos that preserve the boundary conditions (similarly: $\delta_\lambda J_t = -\frac{k}{4\pi} L \partial_t \lambda$)

- Anomalous transformation above leads to central charge (HS, CGLM)

$$c = -24\alpha e^{-2\phi} = \frac{3}{G_2} = \frac{3}{2} k E^2 L^2$$

Some surprising results

Hartman, Strominger = HS   Castro, DG, Larsen, McNees = CGLM

- Holographic renormalization leads to boundary mass term (CGLM)

$$I \sim \int \mathrm{d}x \sqrt{|\gamma|}\, mA^2$$

  Nevertheless, total action gauge invariant
- Boundary stress tensor transforms anomalously (HS)

$$(\delta_\xi + \delta_\lambda)\, T_{tt} = 2T_{tt}\partial_t\xi + \xi\partial_t T_{tt} - \frac{c}{24\pi} L\partial_t^3\xi$$

  where $\delta_\xi + \delta_\lambda$ is combination of diffeo- and gauge trafos that preserve the boundary conditions (similarly: $\delta_\lambda J_t = -\frac{k}{4\pi} L\partial_t\lambda$)
- Anomalous transformation above leads to central charge (HS, CGLM)

$$c = -24\alpha e^{-2\phi} = \frac{3}{G_2} = \frac{3}{2} kE^2 L^2$$

- Positive central charge only for negative coupling constant α (CGLM)

$$\alpha < 0$$

Outline

Attempt 1: Einstein–Hilbert

As simple as possible... but not simpler!

Let us start with the simplest attempt. `Einstein-Hilbert` action:

$$I_{\text{EH}} = \frac{1}{16\pi\, G} \int \mathrm{d}^3x \sqrt{-g}\, R$$

Equations of motion:

$$R_{\mu\nu} = 0$$

Ricci-flat and therefore Riemann-flat – locally trivial!

Attempt 1: Einstein–Hilbert

As simple as possible... but not simpler!

Let us start with the simplest attempt. `Einstein-Hilbert` action:

$$I_{\text{EH}} = \frac{1}{16\pi\, G} \int \mathrm{d}^3x \sqrt{-g}\, R$$

Equations of motion:

$$R_{\mu\nu} = 0$$

Ricci-flat and therefore Riemann-flat – locally trivial!

Properties of `Einstein-Hilbert`

- No gravitons (recall: in D dimensions $D(D-3)/2$ gravitons)
- No BHs
- `Einstein-Hilbert` in 3D is too simple for us!

Attempt 2: Topologically massive gravity

Deser, Jackiw and Templeton found a way to introduce gravitons!

Let us now add a gravitational Chern–Simons term. TMG action:

$$I_{\text{TMG}} = I_{\text{EH}} + \frac{1}{16\pi\,G}\int \mathrm{d}^3x\sqrt{-g}\,\frac{1}{2\mu}\,\varepsilon^{\lambda\mu\nu}\,\Gamma^\rho{}_{\lambda\sigma}\big(\partial_\mu\Gamma^\sigma{}_{\nu\rho} + \frac{2}{3}\,\Gamma^\sigma{}_{\mu\tau}\Gamma^\tau{}_{\nu\rho}\big)$$

Equations of motion:

$$R_{\mu\nu} + \frac{1}{\mu}\,C_{\mu\nu} = 0$$

with the Cotton tensor defined as

$$C_{\mu\nu} = \frac{1}{2}\,\varepsilon_\mu{}^{\alpha\beta}\nabla_\alpha R_{\beta\nu} + (\mu\leftrightarrow\nu)$$

Attempt 2: Topologically massive gravity

Deser, Jackiw and Templeton found a way to introduce gravitons!

Let us now add a gravitational Chern–Simons term. TMG action:

$$I_{\text{TMG}} = I_{\text{EH}} + \frac{1}{16\pi\, G} \int \mathrm{d}^3x \sqrt{-g}\, \frac{1}{2\mu}\, \varepsilon^{\lambda\mu\nu}\, \Gamma^\rho{}_{\lambda\sigma}\big(\partial_\mu \Gamma^\sigma{}_{\nu\rho} + \frac{2}{3}\, \Gamma^\sigma{}_{\mu\tau}\Gamma^\tau{}_{\nu\rho}\big)$$

Equations of motion:

$$R_{\mu\nu} + \frac{1}{\mu}\, C_{\mu\nu} = 0$$

with the Cotton tensor defined as

$$C_{\mu\nu} = \frac{1}{2}\, \varepsilon_\mu{}^{\alpha\beta} \nabla_\alpha R_{\beta\nu} + (\mu \leftrightarrow \nu)$$

Properties of TMG

- Gravitons! Reason: third derivatives in Cotton tensor!
- No BHs
- TMG is slightly too simple for us!

Attempt 3: Einstein–Hilbert–AdS

Bañados, Teitelboim and Zanelli (and Henneaux) taught us how to get 3D BHs

Add negative cosmological constant to `Einstein-Hilbert` action:

$$I_{\Lambda\mathrm{EH}} = \frac{1}{16\pi\,G}\int \mathrm{d}^3x\sqrt{-g}\,\big(R + \frac{2}{\ell^2}\big)$$

Equations of motion:

$$G_{\mu\nu} = R_{\mu\nu} - \frac{1}{2}\,g_{\mu\nu}R - \frac{1}{\ell^2}\,g_{\mu\nu} = 0$$

Particular solutions: BTZ BH with line-element

$$\mathrm{d}s^2_{\mathrm{BTZ}} = -\frac{(r^2-r_+^2)(r^2-r_-^2)}{\ell^2 r^2}\,\mathrm{d}t^2 + \frac{\ell^2 r^2}{(r^2-r_+^2)(r^2-r_-^2)}\,\mathrm{d}r^2 + r^2\,\big(\mathrm{d}\phi - \frac{r_+r_-}{\ell r^2}\,\mathrm{d}t\big)^2$$

Attempt 3: Einstein–Hilbert–AdS

Bañados, Teitelboim and Zanelli (and Henneaux) taught us how to get 3D BHs

Add negative cosmological constant to `Einstein-Hilbert` action:

$$I_{\Lambda\text{EH}} = \frac{1}{16\pi\, G} \int \mathrm{d}^3x\sqrt{-g}\, \big(R + \frac{2}{\ell^2}\big)$$

Equations of motion:

$$G_{\mu\nu} = R_{\mu\nu} - \frac{1}{2}\, g_{\mu\nu} R - \frac{1}{\ell^2}\, g_{\mu\nu} = 0$$

Particular solutions: BTZ BH with line-element

$$\mathrm{d}s^2_{\text{BTZ}} = -\frac{(r^2-r_+^2)(r^2-r_-^2)}{\ell^2 r^2}\, \mathrm{d}t^2 + \frac{\ell^2 r^2}{(r^2-r_+^2)(r^2-r_-^2)}\, \mathrm{d}r^2 + r^2\, \big(\mathrm{d}\phi - \frac{r_+ r_-}{\ell r^2}\, \mathrm{d}t\big)^2$$

Properties of `Einstein-Hilbert-AdS`

- No gravitons
- Rotating BH solutions that asymptote to AdS_3!
- Adding a negative cosmological constant produces BH solutions!

Cosmological topologically massive gravity

CTMG is a 3D theory with BHs and gravitons!

We want a 3D theory with gravitons and BHs and therefore take CTMG action

$$I_{\mathrm{CTMG}} = \frac{1}{16\pi\, G}\,\int \mathrm{d}^3x\sqrt{-g}\,\Big[R + \frac{2}{\ell^2} + \frac{1}{2\mu}\,\varepsilon^{\lambda\mu\nu}\,\Gamma^\rho{}_{\lambda\sigma}\big(\partial_\mu\Gamma^\sigma{}_{\nu\rho} + \frac{2}{3}\,\Gamma^\sigma{}_{\mu\tau}\Gamma^\tau{}_{\nu\rho}\big)\Big]$$

Equations of motion:

$$G_{\mu\nu} + \frac{1}{\mu}\,C_{\mu\nu} = 0$$

Cosmological topologically massive gravity

CTMG is a 3D theory with BHs and gravitons!

We want a 3D theory with gravitons and BHs and therefore take CTMG action

$$I_{\mathrm{CTMG}} = \frac{1}{16\pi\, G} \int \mathrm{d}^3x\sqrt{-g}\,\Big[R + \frac{2}{\ell^2} + \frac{1}{2\mu}\,\varepsilon^{\lambda\mu\nu}\,\Gamma^\rho{}_{\lambda\sigma}\big(\partial_\mu\Gamma^\sigma{}_{\nu\rho} + \frac{2}{3}\,\Gamma^\sigma{}_{\mu\tau}\Gamma^\tau{}_{\nu\rho}\big)\Big]$$

Equations of motion:

$$G_{\mu\nu} + \frac{1}{\mu}\,C_{\mu\nu} = 0$$

Properties of CTMG

- Gravitons!
- BHs!
- CTMG is just perfect for us. Study this theory!

Einstein sector of the classical theory

Solutions of Einstein's equations

$$G_{\mu\nu} = 0 \qquad \leftrightarrow \qquad R = -\frac{6}{\ell^2}$$

also have vanishing Cotton tensor

$$C_{\mu\nu} = 0$$

and therefore are solutions of CTMG.

Einstein sector of the classical theory

Solutions of Einstein's equations

$$G_{\mu\nu} = 0 \qquad \leftrightarrow \qquad R = -\frac{6}{\ell^2}$$

also have vanishing Cotton tensor

$$C_{\mu\nu} = 0$$

and therefore are solutions of CTMG.
This sector of solutions contains

- BTZ BH
- Pure AdS

Einstein sector of the classical theory

Solutions of Einstein's equations

$$G_{\mu\nu} = 0 \qquad \leftrightarrow \qquad R = -\frac{6}{\ell^2}$$

also have vanishing Cotton tensor

$$C_{\mu\nu} = 0$$

and therefore are solutions of CTMG.
This sector of solutions contains

- BTZ BH
- Pure AdS

Line-element of pure AdS:

$$\mathrm{d}s^2_{\mathrm{AdS}} = \bar{g}_{\mu\nu}\,\mathrm{d}x^\mu\,\mathrm{d}x^\nu = \ell^2\big(-\cosh^2\rho\,\mathrm{d}\tau^2 + \sinh^2\rho\,\mathrm{d}\phi^2 + \mathrm{d}\rho^2\big)$$

Isometry group: $SL(2,\mathbb{R})_L \times SL(2,\mathbb{R})_R$
Useful to introduce light-cone coordinates $u = \tau + \phi$, $v = \tau - \phi$

AdS$_3$-algebra of Killing vectors

A technical reminder

The $SL(2,\mathbb{R})_L$ generators

$$L_0 = i\partial_u$$

$$L_{\pm 1} = ie^{\pm iu}\Big[\frac{\cosh 2\rho}{\sinh 2\rho}\partial_u - \frac{1}{\sinh 2\rho}\partial_v \mp \frac{i}{2}\,\partial_\rho\Big]$$

obey the algebra

$$[L_0, L_{\pm 1}] = \mp L_{\pm 1}\,, \qquad [L_1, L_{-1}] = 2L_0$$

and have the quadratic Casimir

$$L^2 = \frac{1}{2}(L_1 L_{-1} + L_{-1} L_1) - L_0^2$$

The $SL(2,\mathbb{R})_R$ generators $\bar{L}_0, \bar{L}_{\pm 1}$ obey same algebra, but with

$$u \leftrightarrow v\,, \qquad L \leftrightarrow \bar{L}$$

Cotton sector of the classical theory

Solutions of CTMG with

$$G_{\mu\nu} \neq 0$$

necessarily have also non-vanishing Cotton tensor

$$C_{\mu\nu} \neq 0$$

Few exact solutions of this type are known.

Cotton sector of the classical theory

Solutions of CTMG with

$$G_{\mu\nu} \neq 0$$

necessarily have also non-vanishing Cotton tensor

$$C_{\mu\nu} \neq 0$$

Few exact solutions of this type are known.
Perhaps most interesting solution:

- Warped AdS (stretched/squashed), see `Bengtsson & Sandin`

Line-element of space-like warped AdS:

$$\mathrm{d}s^2_{\text{warped AdS}} = \frac{\ell^2}{\nu^2+3}\left(-\cosh^2\rho\,\mathrm{d}\tau^2 + \frac{4\nu^2}{\nu^2+3}\,(\mathrm{d}u + \sinh\rho\,\mathrm{d}\tau)^2 + \mathrm{d}\rho^2\right)$$

Sidenote: null-warped AdS in holographic duals of cold atoms:

$$\mathrm{d}s^2_{\text{null warped AdS}} = \ell^2\left(\frac{dy^2 + 2\,\mathrm{d}x^+\,\mathrm{d}x^-}{y^2} \pm \frac{(\mathrm{d}x^-)^2}{y^4}\right)$$

Stationarity plus axi-symmetry:

- Two commuting Killing vectors

CTMG as particle mechanics problem

Stationary and axi-symmetric solutions

Stationarity plus axi-symmetry:

- Two commuting Killing vectors
- Effectively reduce 2+1 dimensions to 1+0 dimensions

CTMG as particle mechanics problem
Stationary and axi-symmetric solutions

Stationarity plus axi-symmetry:

- Two commuting Killing vectors
- Effectively reduce 2+1 dimensions to 1+0 dimensions
- Like particle mechanics, but with up to three time derivatives

CTMG as particle mechanics problem
Stationary and axi-symmetric solutions

Stationarity plus axi-symmetry:

- Two commuting Killing vectors
- Effectively reduce 2+1 dimensions to 1+0 dimensions
- Like particle mechanics, but with up to three time derivatives
- Still surprisingly difficult to get exact solutions!

CTMG as particle mechanics problem

Stationary and axi-symmetric solutions

Stationarity plus axi-symmetry:

- Two commuting Killing vectors
- Effectively reduce $2+1$ dimensions to $1+0$ dimensions
- Like particle mechanics, but with up to three time derivatives
- Still surprisingly difficult to get exact solutions!
- Known solutions: AdS, BTZ, warped AdS

CTMG as particle mechanics problem
Stationary and axi-symmetric solutions

Stationarity plus axi-symmetry:

- Two commuting Killing vectors
- Effectively reduce 2+1 dimensions to 1+0 dimensions
- Like particle mechanics, but with up to three time derivatives
- Still surprisingly difficult to get exact solutions!
- Known solutions: AdS, BTZ, warped AdS

Reduced action (`Clement`):

$$I_{\mathrm{C}}[\zeta, X^i] \sim \int \mathrm{d}\rho \left[\frac{\zeta}{2}\,\dot{X}^i\dot{X}^j\eta_{ij} - \frac{2}{\zeta\ell^2} + \frac{\zeta^2}{2\mu}\,\epsilon_{ijk}\,X^i\dot{X}^j\ddot{X}^k\right]$$

Here ζ is a Lagrange-multiplier and $X^i = (T, X, Y)$ a Lorentzian 3-vector

CTMG as particle mechanics problem

Stationary and axi-symmetric solutions

Stationarity plus axi-symmetry:

- Two commuting Killing vectors
- Effectively reduce 2+1 dimensions to 1+0 dimensions
- Like particle mechanics, but with up to three time derivatives
- Still surprisingly difficult to get exact solutions!
- Known solutions: AdS, BTZ, warped AdS

Reduced action (`Clement`):

$$I_{\mathrm{C}}[\zeta, X^i] \sim \int \mathrm{d}\rho \left[\frac{\zeta}{2}\,\dot{X}^i\dot{X}^j\eta_{ij} - \frac{2}{\zeta\ell^2} + \frac{\zeta^2}{2\mu}\,\epsilon_{ijk}\,X^i\dot{X}^j\ddot{X}^k\right]$$

Here ζ is a Lagrange-multiplier and $X^i = (T, X, Y)$ a Lorentzian 3-vector

> It could be rewarding to investigate this mechanical problem systematically and numerically!

CTMG at the chiral point

...abbreviated as CCTMG

Definition: CTMG at the chiral point is CTMG with the tuning

$$\mu\,\ell = 1$$

between the cosmological constant and the Chern–Simons coupling.

CTMG at the chiral point

...abbreviated as CCTMG

Definition: CTMG at the chiral point is CTMG with the tuning

$$\mu\,\ell = 1$$

between the cosmological constant and the Chern–Simons coupling.
Why special?

CTMG at the chiral point
...abbreviated as CCTMG

Definition: CTMG at the chiral point is CTMG with the tuning

$$\mu\,\ell = 1$$

between the cosmological constant and the Chern–Simons coupling.
Why special?
Calculating the central charges of the dual boundary CFT yields

$$c_L = \frac{3}{2G}\,\big(1 - \frac{1}{\mu\,\ell}\big)\,, \qquad c_R = \frac{3}{2G}\,\big(1 + \frac{1}{\mu\,\ell}\big)$$

Thus, at the chiral point we get

$$c_L = 0\,, \qquad c_R = \frac{3}{G}$$

CTMG at the chiral point
...abbreviated as CCTMG

Definition: CTMG at the chiral point is CTMG with the tuning

$$\mu\,\ell = 1$$

between the cosmological constant and the Chern–Simons coupling.
Why special?
Calculating the central charges of the dual boundary CFT yields

$$c_L = \frac{3}{2G}\left(1 - \frac{1}{\mu\,\ell}\right), \qquad c_R = \frac{3}{2G}\left(1 + \frac{1}{\mu\,\ell}\right)$$

Thus, at the chiral point we get

$$c_L = 0\,, \qquad c_R = \frac{3}{G}$$

Notes:

- Abbreviate “CTMG at the chiral point” as CCTMG
- CCTMG is also known as “chiral gravity”

Gravitons around AdS_3 in CTMG

Linearization around AdS background

$$g_{\mu\nu} = \bar{g}_{\mu\nu} + h_{\mu\nu}$$

Gravitons around AdS$_3$ in CTMG

Linearization around AdS background

$$g_{\mu\nu} = \bar{g}_{\mu\nu} + h_{\mu\nu}$$

leads to linearized EOM that are third order PDE

$$G^{(1)}_{\mu\nu} + \frac{1}{\mu}\, C^{(1)}_{\mu\nu} = (\mathcal{D}^R \mathcal{D}^L \mathcal{D}^M h)_{\mu\nu} = 0 \tag{1}$$

with three mutually commuting first order operators

$$(\mathcal{D}^{L/R})_\mu{}^\nu = \delta_\mu^\nu \pm \ell\, \varepsilon_\mu{}^{\alpha\nu} \bar{\nabla}_\alpha\,, \qquad (\mathcal{D}^M)_\mu{}^\nu = \delta_\mu^\nu + \frac{1}{\mu} \varepsilon_\mu{}^{\alpha\nu} \bar{\nabla}_\alpha$$

Gravitons around AdS_3 in CTMG

Linearization around AdS background

$$g_{\mu\nu} = \bar{g}_{\mu\nu} + h_{\mu\nu}$$

leads to linearized EOM that are third order PDE

$$G^{(1)}_{\mu\nu} + \frac{1}{\mu}\, C^{(1)}_{\mu\nu} = (\mathcal{D}^R \mathcal{D}^L \mathcal{D}^M h)_{\mu\nu} = 0 \tag{1}$$

with three mutually commuting first order operators

$$(\mathcal{D}^{L/R})_\mu{}^\nu = \delta^\nu_\mu \pm \ell\, \varepsilon_\mu{}^{\alpha\nu} \bar{\nabla}_\alpha\,, \qquad (\mathcal{D}^M)_\mu{}^\nu = \delta^\nu_\mu + \frac{1}{\mu} \varepsilon_\mu{}^{\alpha\nu} \bar{\nabla}_\alpha$$

Three linearly independent solutions to (1):

$$\big(\mathcal{D}^L h^L\big)_{\mu\nu} = 0\,, \qquad \big(\mathcal{D}^R h^R\big)_{\mu\nu} = 0\,, \qquad \big(\mathcal{D}^M h^M\big)_{\mu\nu} = 0$$

Gravitons around AdS$_3$ in CTMG

Linearization around AdS background

$$g_{\mu\nu} = \bar{g}_{\mu\nu} + h_{\mu\nu}$$

leads to linearized EOM that are third order PDE

$$G^{(1)}_{\mu\nu} + \frac{1}{\mu}\, C^{(1)}_{\mu\nu} = (\mathcal{D}^R \mathcal{D}^L \mathcal{D}^M h)_{\mu\nu} = 0 \tag{1}$$

with three mutually commuting first order operators

$$(\mathcal{D}^{L/R})_\mu{}^\nu = \delta^\nu_\mu \pm \ell\, \varepsilon_\mu{}^{\alpha\nu} \bar{\nabla}_\alpha\,, \qquad (\mathcal{D}^M)_\mu{}^\nu = \delta^\nu_\mu + \frac{1}{\mu} \varepsilon_\mu{}^{\alpha\nu} \bar{\nabla}_\alpha$$

Three linearly independent solutions to (1):

$$\big(\mathcal{D}^L h^L\big)_{\mu\nu} = 0\,, \qquad \big(\mathcal{D}^R h^R\big)_{\mu\nu} = 0\,, \qquad \big(\mathcal{D}^M h^M\big)_{\mu\nu} = 0$$

At chiral point left (L) and massive (M) branches coincide!

Degeneracy at the chiral point

Will be quite important later!

`Li, Song & Strominger` found all solutions of linearized EOM.

- Primaries: $L_0, \bar{L}_0$ eigenstates $\psi^{L/R/M}$ with

$$L_1\psi^{R/L/M} = \bar{L}_1\psi^{R/L/M} = 0$$

Degeneracy at the chiral point

Will be quite important later!

Li, Song & Strominger found all solutions of linearized EOM.

- Primaries: $L_0, \bar{L}_0$ eigenstates $\psi^{L/R/M}$ with
$$L_1\psi^{R/L/M} = \bar{L}_1\psi^{R/L/M} = 0$$
- Descendants: act with L_{-1} and $\bar{L}_{-1}$ on primaries

Degeneracy at the chiral point

Will be quite important later!

`Li, Song & Strominger` found all solutions of linearized EOM.

- Primaries: $L_0, \bar{L}_0$ eigenstates $\psi^{L/R/M}$ with

$$L_1\psi^{R/L/M} = \bar{L}_1\psi^{R/L/M} = 0$$

- Descendants: act with L_{-1} and $\bar{L}_{-1}$ on primaries
- General solution: linear combination of $\psi^{R/L/M}$

Degeneracy at the chiral point

Will be quite important later!

`Li, Song & Strominger` found all solutions of linearized EOM.

- Primaries: $L_0, \bar{L}_0$ eigenstates $\psi^{L/R/M}$ with

$$L_1 \psi^{R/L/M} = \bar{L}_1 \psi^{R/L/M} = 0$$

- Descendants: act with L_{-1} and $\bar{L}_{-1}$ on primaries
- General solution: linear combination of $\psi^{R/L/M}$
- Linearized metric is then the real part of the wavefunction

$$h_{\mu\nu} = \mathrm{Re}\, \psi_{\mu\nu}$$

Degeneracy at the chiral point

Will be quite important later!

`Li, Song & Strominger` found all solutions of linearized EOM.

- Primaries: $L_0, \bar{L}_0$ eigenstates $\psi^{L/R/M}$ with

$$L_1 \psi^{R/L/M} = \bar{L}_1 \psi^{R/L/M} = 0$$

- Descendants: act with L_{-1} and $\bar{L}_{-1}$ on primaries
- General solution: linear combination of $\psi^{R/L/M}$
- Linearized metric is then the real part of the wavefunction

$$h_{\mu\nu} = \mathrm{Re}\,\psi_{\mu\nu}$$

- At chiral point: L and M branches degenerate. Get new solution (`DG & Johansson`)

$$\psi_{\mu\nu}^{\text{new}} = \lim_{\mu\ell \to 1} \frac{\psi_{\mu\nu}^{M}(\mu\ell) - \psi_{\mu\nu}^{L}}{\mu\ell - 1}$$

with property

$$\left(\mathcal{D}^L \psi^{\text{new}}\right)_{\mu\nu} = \left(\mathcal{D}^M \psi^{\text{new}}\right)_{\mu\nu} \neq 0\,, \qquad \left((\mathcal{D}^L)^2 \psi^{\text{new}}\right)_{\mu\nu} = 0$$

Sign oder nicht sign?

That is the question. Choosing between Skylla and Charybdis.

- With signs defined as in this talk: BHs positive energy, gravitons negative energy

Sign oder nicht sign?

That is the question. Choosing between Skylla and Charybdis.

- With signs defined as in this talk: BHs positive energy, gravitons negative energy
- With signs as defined in `Deser-Jackiw-Templeton` paper: BHs negative energy, gravitons positive energy

Sign oder nicht sign?

That is the question. Choosing between Skylla and Charybdis.

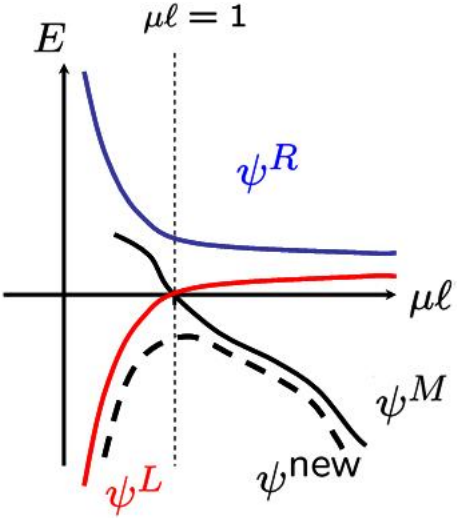
- With signs defined as in this talk: BHs positive energy, gravitons negative energy
- With signs as defined in `Deser-Jackiw-Templeton` paper: BHs negative energy, gravitons positive energy
- Either way need a mechanism to eliminate unwanted negative energy objects – either the gravitons or the BHs

Sign oder nicht sign?

That is the question. Choosing between Skylla and Charybdis.

- With signs defined as in this talk: BHs positive energy, gravitons negative energy
- With signs as defined in `Deser-Jackiw-Templeton` paper: BHs negative energy, gravitons positive energy
- Either way need a mechanism to eliminate unwanted negative energy objects – either the gravitons or the BHs
- Even at chiral point the problem persists because of the logarithmic mode. See Figure. (Figure: thanks to `N. Johansson`)

Energy for all branches:



Outline

Why lower-dimensional gravity?

Which 2D theory?

Which 3D theory?

How to quantize 3D gravity?

What next?

Witten's attempt

Different approach (without gravitons!):

Witten's attempt

Different approach (without gravitons!):

- Naive remark 1: 3D gravity is trivial

Witten's attempt

Different approach (without gravitons!):

- Naive remark 1: 3D gravity is trivial
- Naive remark 2: 3D gravity is non-renormalizable

Witten's attempt

Different approach (without gravitons!):

- Naive remark 1: 3D gravity is trivial
- Naive remark 2: 3D gravity is non-renormalizable
- Synthesis of naive remarks: 3D quantum gravity may exist as non-trivial theory

Witten's attempt

Different approach (without gravitons!):

- Naive remark 1: 3D gravity is trivial
- Naive remark 2: 3D gravity is non-renormalizable
- Synthesis of naive remarks: 3D quantum gravity may exist as non-trivial theory
- Positive cosmological constant: impossible?

Witten's attempt

Different approach (without gravitons!):

- Naive remark 1: 3D gravity is trivial
- Naive remark 2: 3D gravity is non-renormalizable
- Synthesis of naive remarks: 3D quantum gravity may exist as non-trivial theory
- Positive cosmological constant: impossible?
- Vanishing cosmological constant: S-matrix, but no gravitons!

Witten's attempt

Different approach (without gravitons!):

- Naive remark 1: 3D gravity is trivial
- Naive remark 2: 3D gravity is non-renormalizable
- Synthesis of naive remarks: 3D quantum gravity may exist as non-trivial theory
- Positive cosmological constant: impossible?
- Vanishing cosmological constant: S-matrix, but no gravitons!
- Therefore introduce negative cosmological constant

Witten's attempt

Different approach (without gravitons!):

- Naive remark 1: 3D gravity is trivial
- Naive remark 2: 3D gravity is non-renormalizable
- Synthesis of naive remarks: 3D quantum gravity may exist as non-trivial theory
- Positive cosmological constant: impossible?
- Vanishing cosmological constant: S-matrix, but no gravitons!
- Therefore introduce negative cosmological constant
- Define quantum gravity by its dual CFT at the AdS boundary

Witten's attempt

Different approach (without gravitons!):

- Naive remark 1: 3D gravity is trivial
- Naive remark 2: 3D gravity is non-renormalizable
- Synthesis of naive remarks: 3D quantum gravity may exist as non-trivial theory
- Positive cosmological constant: impossible?
- Vanishing cosmological constant: S-matrix, but no gravitons!
- Therefore introduce negative cosmological constant
- Define quantum gravity by its dual CFT at the AdS boundary
- Constructing this CFT still a "monstrous" effort...

Witten's attempt

Different approach (without gravitons!):

- Naive remark 1: 3D gravity is trivial
- Naive remark 2: 3D gravity is non-renormalizable
- Synthesis of naive remarks: 3D quantum gravity may exist as non-trivial theory
- Positive cosmological constant: impossible?
- Vanishing cosmological constant: S-matrix, but no gravitons!
- Therefore introduce negative cosmological constant
- Define quantum gravity by its dual CFT at the AdS boundary
- Constructing this CFT still a "monstrous" effort...

`Maloney & Witten`: taking into account all known contributions to path integral leads to non-sensible result for partition function Z.
In particular, no holomorphic factorization:

$$Z_{\text{MW}} \neq Z_L \cdot Z_R$$

Witten's attempt

Different approach (without gravitons!):

- Naive remark 1: 3D gravity is trivial
- Naive remark 2: 3D gravity is non-renormalizable
- Synthesis of naive remarks: 3D quantum gravity may exist as non-trivial theory
- Positive cosmological constant: impossible?
- Vanishing cosmological constant: S-matrix, but no gravitons!
- Therefore introduce negative cosmological constant
- Define quantum gravity by its dual CFT at the AdS boundary
- Constructing this CFT still a "monstrous" effort...

`Maloney & Witten`: taking into account all known contributions to path integral leads to non-sensible result for partition function Z.
In particular, no holomorphic factorization:

$$Z_{\mathrm{MW}} \neq Z_L \cdot Z_R$$

Various suggestions to interpret this problem: need cosmic strings, need sum over complex geometries, 3D quantum gravity does not exist by itself

Li, Song & Strominger attempt
Is CCTMG dual to a chiral CFT?

Interesting observations:

1. If left-moving sector is trivial, $Z_L = 1$, then problem of holomorphic factorization
$$Z = Z_L \cdot Z_R = Z_R$$
is solved.

Li, Song & Strominger attempt

Is CCTMG dual to a chiral CFT?

Interesting observations:

1. If left-moving sector is trivial, $Z_L = 1$, then problem of holomorphic factorization

$$Z = Z_L \cdot Z_R = Z_R$$

is solved.

2. CCTMG has

$$c_L = 0$$

Li, Song & Strominger attempt
Is CCTMG dual to a chiral CFT?

Interesting observations:

1. If left-moving sector is trivial, $Z_L = 1$, then problem of holomorphic factorization
$$Z = Z_L \cdot Z_R = Z_R$$
is solved.
2. CCTMG has
$$c_L = 0$$
3. Massive graviton degenerates with left boundary graviton: $\psi^M = \psi^L$

Li, Song & Strominger attempt
Is CCTMG dual to a chiral CFT?

Interesting observations:

1. If left-moving sector is trivial, $Z_L = 1$, then problem of holomorphic factorization
$$Z = Z_L \cdot Z_R = Z_R$$
is solved.
2. CCTMG has
$$c_L = 0$$
3. Massive graviton degenerates with left boundary graviton: $\psi^M = \psi^L$

Thus, dual CFT chiral? If yes, we are done!

Li, Song & Strominger attempt
Is CCTMG dual to a chiral CFT?

Interesting observations:

1. If left-moving sector is trivial, $Z_L = 1$, then problem of holomorphic factorization
$$Z = Z_L \cdot Z_R = Z_R$$
is solved.
2. CCTMG has
$$c_L = 0$$
3. Massive graviton degenerates with left boundary graviton: $\psi^M = \psi^L$

Thus, dual CFT chiral? If yes, we are done!

Suggestive to interpret LSS results as absence of gravitons

Li, Song & Strominger attempt
Is CCTMG dual to a chiral CFT?

Interesting observations:

1. If left-moving sector is trivial, $Z_L = 1$, then problem of holomorphic factorization
$$Z = Z_L \cdot Z_R = Z_R$$
is solved.
2. CCTMG has
$$c_L = 0$$
3. Massive graviton degenerates with left boundary graviton: $\psi^M = \psi^L$

Thus, dual CFT chiral? If yes, we are done!

Suggestive to interpret LSS results as absence of gravitons

But:

Disagrees with results by `Carlip, Deser, Waldron & Wise`!

Gravitons in CCTMG

Is CCTMG dual to a logarithmic CFT?

New mode resolves apparent contradiction between LSS and CDWW.

Gravitons in CCTMG

Is CCTMG dual to a logarithmic CFT?

New mode resolves apparent contradiction between LSS and CDWW.

Interesting property:

$$L_0 \begin{pmatrix} \psi^{\text{new}} \\ \psi^L \end{pmatrix} = \begin{pmatrix} 2 & \frac{1}{2} \\ 0 & 2 \end{pmatrix} \begin{pmatrix} \psi^{\text{new}} \\ \psi^L \end{pmatrix},$$

$$\bar{L}_0 \begin{pmatrix} \psi^{\text{new}} \\ \psi^L \end{pmatrix} = \begin{pmatrix} 0 & \frac{1}{2} \\ 0 & 0 \end{pmatrix} \begin{pmatrix} \psi^{\text{new}} \\ \psi^L \end{pmatrix}.$$

Gravitons in CCTMG

Is CCTMG dual to a logarithmic CFT?

New mode resolves apparent contradiction between LSS and CDWW.

Interesting property:

$$L_0 \begin{pmatrix} \psi^{\text{new}} \\ \psi^L \end{pmatrix} = \begin{pmatrix} 2 & \frac{1}{2} \\ 0 & 2 \end{pmatrix} \begin{pmatrix} \psi^{\text{new}} \\ \psi^L \end{pmatrix},$$

$$\bar{L}_0 \begin{pmatrix} \psi^{\text{new}} \\ \psi^L \end{pmatrix} = \begin{pmatrix} 0 & \frac{1}{2} \\ 0 & 0 \end{pmatrix} \begin{pmatrix} \psi^{\text{new}} \\ \psi^L \end{pmatrix}.$$

Such a Jordan form of $L_0, \bar{L}_0$ is defining property of a logarithmic CFT!

Note: called "logarithmic CFT" because some correlators take the form

$$\langle \psi^{\text{new}}(z) \psi^{\text{new}}(0) \rangle \sim \ln z + \dots$$

Gravitons in CCTMG

Is CCTMG dual to a logarithmic CFT?

New mode resolves apparent contradiction between LSS and CDWW.

Interesting property:

$$L_0 \begin{pmatrix} \psi^{\text{new}} \\ \psi^L \end{pmatrix} = \begin{pmatrix} 2 & \frac{1}{2} \\ 0 & 2 \end{pmatrix} \begin{pmatrix} \psi^{\text{new}} \\ \psi^L \end{pmatrix},$$

$$\bar{L}_0 \begin{pmatrix} \psi^{\text{new}} \\ \psi^L \end{pmatrix} = \begin{pmatrix} 0 & \frac{1}{2} \\ 0 & 0 \end{pmatrix} \begin{pmatrix} \psi^{\text{new}} \\ \psi^L \end{pmatrix}.$$

Such a Jordan form of $L_0, \bar{L}_0$ is defining property of a logarithmic CFT!

Note: called "logarithmic CFT" because some correlators take the form

$$\langle \psi^{\text{new}}(z) \psi^{\text{new}}(0) \rangle \sim \ln z + \dots$$

- Logarithmic CFT: not unitary and not chiral!

Gravitons in CCTMG

Is CCTMG dual to a logarithmic CFT?

New mode resolves apparent contradiction between LSS and CDWW.

Interesting property:

$$L_0 \begin{pmatrix} \psi^{\text{new}} \\ \psi^L \end{pmatrix} = \begin{pmatrix} 2 & \frac{1}{2} \\ 0 & 2 \end{pmatrix} \begin{pmatrix} \psi^{\text{new}} \\ \psi^L \end{pmatrix},$$

$$\bar{L}_0 \begin{pmatrix} \psi^{\text{new}} \\ \psi^L \end{pmatrix} = \begin{pmatrix} 0 & \frac{1}{2} \\ 0 & 0 \end{pmatrix} \begin{pmatrix} \psi^{\text{new}} \\ \psi^L \end{pmatrix}.$$

Such a Jordan form of $L_0, \bar{L}_0$ is defining property of a logarithmic CFT!

Note: called "logarithmic CFT" because some correlators take the form

$$\langle \psi^{\text{new}}(z)\psi^{\text{new}}(0)\rangle \sim \ln z + \dots$$

- Logarithmic CFT: not unitary and not chiral!
- Either logarithmic or chiral CFT dual (or none)

Gravitons in CCTMG

Is CCTMG dual to a logarithmic CFT?

New mode resolves apparent contradiction between LSS and CDWW.

Interesting property:

$$L_0 \begin{pmatrix} \psi^{\text{new}} \\ \psi^L \end{pmatrix} = \begin{pmatrix} 2 & \frac{1}{2} \\ 0 & 2 \end{pmatrix} \begin{pmatrix} \psi^{\text{new}} \\ \psi^L \end{pmatrix} ,$$

$$\bar{L}_0 \begin{pmatrix} \psi^{\text{new}} \\ \psi^L \end{pmatrix} = \begin{pmatrix} 0 & \frac{1}{2} \\ 0 & 0 \end{pmatrix} \begin{pmatrix} \psi^{\text{new}} \\ \psi^L \end{pmatrix} .$$

Such a Jordan form of $L_0, \bar{L}_0$ is defining property of a logarithmic CFT!

Note: called "logarithmic CFT" because some correlators take the form

$$\langle \psi^{\text{new}}(z) \psi^{\text{new}}(0) \rangle \sim \ln z + \dots$$

- Logarithmic CFT: not unitary and not chiral!
- Either logarithmic or chiral CFT dual (or none)
- Currently unknown which of these alternatives is realized!

Viability of the logarithmic mode, part 1

Explicit solution for logarithmic mode (DG & Johansson)

Is the logarithmic mode really there?

Collect in the following suggestions how the logarithmic mode could drop out of the physical spectrum and show that none of them is realized.

Viability of the logarithmic mode, part 1

Explicit solution for logarithmic mode (DG & Johansson)

Is the logarithmic mode really there?

Collect in the following suggestions how the logarithmic mode could drop out of the physical spectrum and show that none of them is realized.

Before starting, here is the explicit form of the logarithmic mode:

$$
\begin{aligned}
h_{\mu\nu}^{\text{new}} = & \frac{\sinh\rho}{\cosh^3\rho}\left(c\,\tau - s\ln\cosh\rho\right)\begin{pmatrix} 0 & 0 & 1 \\ 0 & 0 & 1 \\ 1 & 1 & 0 \end{pmatrix}_{\mu\nu} \\
& - \tanh^2\rho\left(s\,\tau + c\ln\cosh\rho\right)\begin{pmatrix} 1 & 1 & 0 \\ 1 & 1 & 0 \\ 0 & 0 & -a^2 \end{pmatrix}_{\mu\nu}
\end{aligned} \tag{2}
$$

with

$$
c = \cos(2u)\,, \qquad s = \sin(2u)\,, \qquad a = \frac{1}{\sinh\rho\,\cosh\rho}
$$

Viability of the logarithmic mode, part 2

Physical mode with negative energy

Suggestion 1

The logarithmic mode is pure gauge?

Viability of the logarithmic mode, part 2

Physical mode with negative energy

Suggestion 1

The logarithmic mode is pure gauge? No!

h^{new} does not solve linearized Einstein equations. Thus is not pure gauge.
Note: confirmed by `Sachs` who considered logarithmic quasi-normal modes

Viability of the logarithmic mode, part 2

Physical mode with negative energy

Suggestion 1

The logarithmic mode is pure gauge? No!

$h^{\rm new}$ does not solve linearized Einstein equations. Thus is not pure gauge.
Note: confirmed by Sachs who considered logarithmic quasi-normal modes

Suggestion 2

Logarithmic mode has infinite energy and thus must be discarded?

Viability of the logarithmic mode, part 2

Physical mode with negative energy

Suggestion 1

The logarithmic mode is pure gauge? No!

h^{new} does not solve linearized Einstein equations. Thus is not pure gauge.
Note: confirmed by `Sachs` who considered logarithmic quasi-normal modes

Suggestion 2

Logarithmic mode has infinite energy and thus must be discarded? No!

$$E^{\mathrm{new}} = -\frac{47}{1152 G\, \ell^3}$$

Energy is finite and negative.
Thus logarithmic mode leads to instability but cannot be discarded.

Viability of the logarithmic mode, part 3

Boundary conditions beyond Brown–Henneaux

Suggestion 3

New mode is not a small perturbation?

Viability of the logarithmic mode, part 3

Boundary conditions beyond Brown–Henneaux

Suggestion 3

New mode is not a small perturbation? It is!

h^{new} diverges asymptotically like ρ, but AdS background diverges asymptotically like $e^{2\rho}$. Thus h^{new} is really a small perturbation.

Viability of the logarithmic mode, part 3

Boundary conditions beyond Brown–Henneaux

Suggestion 3

New mode is not a small perturbation? It is!

h^{new} diverges asymptotically like ρ, but AdS background diverges asymptotically like $e^{2\rho}$. Thus h^{new} is really a small perturbation.

Suggestion 4

New mode is not asymptotically AdS?

Viability of the logarithmic mode, part 3
Boundary conditions beyond Brown–Henneaux

Suggestion 3

New mode is not a small perturbation? It is!

h^{new} diverges asymptotically like ρ, but AdS background diverges asymptotically like $e^{2\rho}$. Thus h^{new} is really a small perturbation.

Suggestion 4

New mode is not asymptotically AdS? It is!

Solution is asymptotically AdS

$$\mathrm{d}s^2 = \mathrm{d}\rho^2 + \left(\gamma_{ij}^{(0)} e^{2\rho/\ell} + \gamma_{ij}^{(1)} \rho + \gamma_{ij}^{(0)} + \gamma_{ij}^{(2)} e^{-2\rho/\ell} + \dots\right) \mathrm{d}x^i \, \mathrm{d}x^j$$

but violates Brown-Henneaux boundary conditions! $(\gamma_{ij}^{(1)}\big|_{\mathrm{BH}} = 0)$
Henneaux et al. showed precedents where this may happen in 3D
New boundary conditions replacing Brown-Henneaux (DG & Johansson)

Viability of the logarithmic mode, part 4

Brown–York boundary stress tensor

Suggestion 5

New mode leads to ill-defined `Brown-York` boundary stress tensor?

Viability of the logarithmic mode, part 4

Brown–York boundary stress tensor

Suggestion 5

New mode leads to ill-defined Brown-York boundary stress tensor? No!

Total action including boundary terms (Kraus & Larsen)

$$I_{\text{total}} = I_{\text{CTMG}} + \frac{1}{8\pi G} \int \mathrm{d}^2x \sqrt{-\gamma} \left(K - \frac{1}{\ell}\right)$$

Its first variation leads to Brown-York boundary stress-tensor:

$$\delta I_{\text{total}}\big|_{\text{EOM}} = \frac{1}{32\pi G} \int \mathrm{d}^2x \sqrt{-\gamma^{(0)}}\, T^{ij}\, \delta\gamma_{ij}^{(0)}$$

DG & Johansson: T_{ij} is finite, traceless and chiral:

$$T_{ij} = -\frac{\ell}{16\pi\, G} \begin{pmatrix} 1 & 1 \\ 1 & 1 \end{pmatrix}_{ij}$$

Note: coincides with Brown-York boundary stress-tensor of global AdS_3

Viability of the logarithmic mode, part 5

Artifact of linearization?

Suggestion 6

Maybe some non-linear “magic” kills the new mode?

Viability of the logarithmic mode, part 5

Artifact of linearization?

Suggestion 6

Maybe some non-linear "magic" kills the new mode? Unlikely!

DG, Jackiw & Johansson: classical phase space analysis of CCTMG

$$N = \frac{1}{2}\left(2 \times D - 2 \times N_1 - N_2\right) = \frac{1}{2}\left(2 \times 18 - 2 \times 14 - 6\right) = 1$$

- N: number of physical degrees of freedom (per point)
- D: number of canonical pairs in full phase space
- $N_{1(2)}$: number of linearly independent first (second) class constraints

confirmed in more general calculation by Carlip

Viability of the logarithmic mode, part 5

Artifact of linearization?

Suggestion 6

Maybe some non-linear “magic” kills the new mode? Unlikely!

`DG, Jackiw & Johansson`: classical phase space analysis of CCTMG

$$N = \frac{1}{2}\left(2 \times D - 2 \times N_1 - N_2\right) = \frac{1}{2}\left(2 \times 18 - 2 \times 14 - 6\right) = 1$$

- N: number of physical degrees of freedom (per point)
- D: number of canonical pairs in full phase space
- $N_{1(2)}$: number of linearly independent first (second) class constraints

confirmed in more general calculation by `Carlip`

- Conclusion 1: logarithmic mode passed all tests so far
- Conclusion 2: CCTMG is unstable; dual CFT probably logarithmic

Outline

Chiral vs. logarithmic

Pivotal open question: does dual CFT exist? is it chiral or logarithmic?

To Do

- Chiral route: must show consistency of truncation!
- Logarithmic: must show consistency of 2^{nd} order perturbations!

Chiral vs. logarithmic

Pivotal open question: does dual CFT exist? is it chiral or logarithmic?

To Do

- Chiral route: must show consistency of truncation!
- Logarithmic: must show consistency of 2^{nd} order perturbations!

ad chiral:

- restricting to Brown-Henneaux boundary conditions does not help
- Giribet, Kleban & Porrati showed that descendent of new mode

$$\bar{L}_{-1}\psi_{\mu\nu}^{\mathrm{new}} = Y_{\mu\nu} = X_{\mu\nu} + \mathcal{L}_\xi \bar{g}_{\mu\nu}$$

  after a diffeomorphism ξ obeys Brown-Henneaux boundary conditions
- Descendants of logarithmic mode are there even when boundary conditions are restricted beyond requiring variational principle!

Chiral vs. logarithmic

Pivotal open question: does dual CFT exist? is it chiral or logarithmic?

To Do

- Chiral route: must show consistency of truncation!
- Logarithmic: must show consistency of 2nd order perturbations!

ad chiral:

- restricting to Brown-Henneaux boundary conditions does not help
- Giribet, Kleban & Porrati showed that descendent of new mode

$$\bar{L}_{-1}\psi^{\text{new}}_{\mu\nu} = Y_{\mu\nu} = X_{\mu\nu} + \mathcal{L}_\xi \bar{g}_{\mu\nu}$$

  after a diffeomorphism ξ obeys Brown-Henneaux boundary conditions
- Descendants of logarithmic mode are there even when boundary conditions are restricted beyond requiring variational principle!
- Need different mechanism of truncation!

Chiral vs. logarithmic

Pivotal open question: does dual CFT exist? is it chiral or logarithmic?

To Do

- Chiral route: must show consistency of truncation!
- Logarithmic: must show consistency of 2^{nd} order perturbations!

ad logarithmic:

- straightforward but somewhat lengthy calculation
- expand metric around AdS background up to second order:

$$g_{\mu\nu} = \bar{g}_{\mu\nu} + h^{\text{new}}_{\mu\nu} + h^{(2)}_{\mu\nu}$$

  EOM lead to linear PDE for $h^{(2)}_{\mu\nu}$:

$$\mathcal{D}^{(3)}\, h^{(2)} = f\big((h^{\text{new}}_{\mu\nu})^2\big)$$

- Check if $h^{(2)}$ really is smaller than $h^{\text{new}}_{\mu\nu}$

Chiral vs. logarithmic

Pivotal open question: does dual CFT exist? is it chiral or logarithmic?

To Do

- Chiral route: must show consistency of truncation!
- Logarithmic: must show consistency of 2nd order perturbations!

ad logarithmic:

- straightforward but somewhat lengthy calculation
- expand metric around AdS background up to second order:

$$g_{\mu\nu} = \bar{g}_{\mu\nu} + h^{\text{new}}_{\mu\nu} + h^{(2)}_{\mu\nu}$$

  EOM lead to linear PDE for $h^{(2)}_{\mu\nu}$:

$$\mathcal{D}^{(3)}\, h^{(2)} = f\big((h^{\text{new}}_{\mu\nu})^2\big)$$

- Check if $h^{(2)}$ really is smaller than $h^{\text{new}}_{\mu\nu}$
- Might be rewarding exercise for a student

Which groundstate?

Two observations:

- Global AdS_3 has mass and angular momentum in (C)CTMG

$$M_{\text{AdS}_3} = \mu\, J_{\text{AdS}_3} = -\frac{1}{8G}$$

Which groundstate?

Two observations:

- Global AdS_3 has mass and angular momentum in (C)CTMG
$$M_{\mathrm{AdS}_3} = \mu\, J_{\mathrm{AdS}_3} = -\frac{1}{8G}$$
- If AdS_3 is unstable in CCTMG because of logarithmic mode, where does it run to?

Which groundstate?

Two observations:

- Global AdS_3 has mass and angular momentum in (C)CTMG

$$M_{\mathrm{AdS}_3} = \mu\, J_{\mathrm{AdS}_3} = -\frac{1}{8G}$$

- If AdS_3 is unstable in (C)CTMG because of massive graviton mode, where does it run to?

Both observations suggest that there might be a ground state different from pure AdS_3 in (C)CTMG.

Which groundstate?

Two observations:

- Global AdS_3 has mass and angular momentum in (C)CTMG

$$M_{\mathrm{AdS}_3} = \mu\, J_{\mathrm{AdS}_3} = -\frac{1}{8G}$$

- If AdS_3 is unstable in (C)CTMG because of massive graviton mode, where does it run to?

Both observations suggest that there might be a ground state different from pure AdS_3 in (C)CTMG.

> Consider other possible ground states with less symmetry

Which groundstate?

Two observations:

- Global AdS$_3$ has mass and angular momentum in (C)CTMG

$$M_{\mathrm{AdS}_3} = \mu \, J_{\mathrm{AdS}_3} = -\frac{1}{8G}$$

- If AdS$_3$ is unstable in (C)CTMG because of massive graviton mode, where does it run to?

Both observations suggest that there might be a ground state different from pure AdS$_3$ in (C)CTMG.

> Consider other possible ground states with less symmetry

Example: warped AdS has four Killing vectors with $U(1)_L \times SL(2,\mathbb{R})_R$

Which groundstate?

Two observations:

- Global AdS_3 has mass and angular momentum in (C)CTMG

$$M_{\mathrm{AdS}_3} = \mu\, J_{\mathrm{AdS}_3} = -\frac{1}{8G}$$

- If AdS_3 is unstable in (C)CTMG because of massive graviton mode, where does it run to?

Both observations suggest that there might be a ground state different from pure AdS_3 in (C)CTMG.

> Consider other possible ground states with less symmetry

Example: warped AdS has four Killing vectors with $U(1)_L \times SL(2,\mathbb{R})_R$

> `Strominger et al. :`
>
> Suggestive to consider warped AdS as possible groundstate of (C)CTMG

Most crucial question we would like to answer

Does 3D quantum gravity exist with no strings attached?

Most crucial question we would like to answer

Does 3D quantum gravity exist with no strings attached?

Perhaps a win-win situation!

- Consider the possible outcomes to this question:

Most crucial question we would like to answer

Does 3D quantum gravity exist with no strings attached?

Perhaps a win-win situation!

- Consider the possible outcomes to this question:
- If yes: we would have an interesting quantum theory of gravity with BHs and gravitons to get conceptual insight into quantum gravity

Most crucial question we would like to answer

Does 3D quantum gravity exist with no strings attached?

Perhaps a win-win situation!

- Consider the possible outcomes to this question:
- If yes: we would have an interesting quantum theory of gravity with BHs and gravitons to get conceptual insight into quantum gravity
- if no: potentially exciting news for string theory

Most crucial question we would like to answer

Does 3D quantum gravity exist with no strings attached?

Perhaps a win-win situation!

- Consider the possible outcomes to this question:
- If yes: we would have an interesting quantum theory of gravity with BHs and gravitons to get conceptual insight into quantum gravity
- if no: potentially exciting news for string theory

Thank you for your attention!

Some literature

- D. Grumiller, W. Kummer, and D. Vassilevich, "Dilaton gravity in two dimensions," *Phys. Rept.* **369** (2002) 327–429, hep-th/0204253.
- T. Hartman and A. Strominger, "Central charge for AdS_2 quantum gravity," [arXiv:0803.3621 [hep-th]].
- A. Castro, D. Grumiller, F. Larsen, and R. McNees, "Holographic Description of AdS_2 Black Holes," [arXiv:0809.4264 [hep-th]].
- W. Li, W. Song and A. Strominger, JHEP **0804** (2008) 082, 0801.4566.
- S. Carlip, S. Deser, A. Waldron and D. Wise, Phys.Lett. **B666** (2008) 272, 0807.0486, 0803.3998
- D. Grumiller and N. Johansson, JHEP **0807** (2008) 134, 0805.2610.
- G. Giribet, M. Kleban and M. Porrati, JHEP **0810** (2008) 045, 0807.4703.
- D. Grumiller, R. Jackiw and N. Johansson, 0806.4185.

Thanks to Bob McNees for providing the LaTeX beamerclass!