High-order dispersion in chirped-pulse oscillators

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Abstract: The effects of high-order dispersion on a chirped-pulse oscillator operating in the positive dispersion regime were studied both theoretically and experimentally. It was found that odd and negative even high-order dispersions impair the oscillator stability owing to resonance with the dispersion waves, but can broaden the spectrum as in the case of continuum generation in the fibers. Positive fourth-order dispersion enhances the stability and shifts the stability range into negative dispersion. The destabilization mechanism was found to be a parametrical instability which causes noisy mode locking around zero dispersion.

OCIS codes: (140.4050) Mode-locked lasers; (190.5530) Pulse propagation and temporal solitons

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1. Introduction

Generation of over-100 nJ femtosecond pulses at MHz repetition rates directly from a laser oscillator is of interest for numerous applications, including frequency conversion, gas sensing, metrology, micro-machining etc. One can approach such energy frontiers by stretching an oscillator cavity, which allows energy scalability $\propto T_{cav}$ (T_{cav} is the cavity period) without extra amplification [1, 2]. The main problem is that a long-cavity oscillator has low stability owing to enhanced nonlinear effects at high peak power. To shift the instability threshold to higher energies, one has to stretch the pulse, i.e. decrease its peak power. In the negative dispersion regime (NDR), pulse stretching requires a fair amount of net negative group-delay dispersion (GDD) [3, 4]. Since the pulse is chirp-free in the NDR, such stretching is irreversible and requires the use of an extra-cavity nonlinear compression technique to achieve sub-100 fs pulse duration (τ).

An alternative technique providing sub-100 fs pulses with over-100 nJ energies uses an oscillator operating in the net positive dispersion regime (PDR realized in a chirped-pulse oscillator, CPO) [5]. In that case the pulse has a large chirp ψ [6] that results in substantial pulse stretching (up to few picoseconds), reduction of its peak power and, thereby, stabilization [7]. Since $\psi \gg 1$, the pulse spectrum is broad (the spectral width Δ in a mode-locked Ti:sapphire oscillator operating in the PDR can approach 100 nm [9]), that provides its compressibility down to $T \approx 2/\Delta$ ($<$40 fs in a Ti:sapphire oscillator [8, 9]).

A vague parallel to such a technique is provided by a similariton fiber oscillator operating in the PDR [10]: substantial pulse stretching prevents instabilities with energy growth and strong chirp provides pulse compressibility down to few optical-cycle duration. As was pointed out in Ref. [11], the similariton fiber oscillator characteristics (such as the spectrum profile, the dependence of pulse duration, spectral width, stability etc. on GDD and energy) are interrelated (on average) with those of a solid-state CPO. Such an interrelation of a solid-state CPO and a similariton laser motivates additional study of the PDR.

Since the spectrum in the PDR is broad enough, the effect of high-order dispersion (HOD) is present and can substantially transform the spectral shape [7]. Such an effect in the NDR has been the subject of a great deal of speculation. Analysis has shown that the Schrödinger soliton [12] developing in the NDR is unstable in the presence of third-order dispersion (TOD) if GDD is close to zero [13, 14, 15, 16], but can be stabilized in the presence of stimulated Raman scattering and nonlinear dispersion [17, 18]. In an oscillator, TOD i) shifts the spectrum to the range where GDD is more negative [19], ii) broadens the spectrum owing to dispersion wave radiation [20, 21, 22, 23] the underlying mechanism is analogous to the Cherenkov radiation [24]), iii) increases the minimum pulse width [22, 26, 27, 28, 29], iv) decreases the energy [29], and v) enhances the pulse instability (in particular, as a result of multipulsing and bounded perturbations) [19, 25, 26, 27, 28, 29]. For relatively large negative GDD, TOD can reduce the pulse duration owing to extra broadening of the spectrum [29, 30] and even enhance the stability owing to negative feedback provided by the spectral loss of the side-bands [29].

The effect of TOD on the chirped solitary pulse developing in the PDR [31, 32, 33] is not so well investigated. It is known that TOD causes asymmetry of the spectrum [6, 7] as well as pulse broadening and leads to higher GDD values being necessary for stable operation in the so-called similariton regime referred to mode-locking in a fiber oscillator [30, 11].

The effect of fourth-order dispersion (FOD) in the NDR substantially differs from that of

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TOD. If FOD is positive (i.e. GDD grows towards the edges of the pulse spectrum), the dispersion waves (the side-bands) radiate when the net GDD approaches zero [24, 34, 35, 36]. These waves take part of the pulse energy (spectral recoil). The energy transfer between the pulse and the dispersive waves is more effective than that for the TOD case [35] and can lead to pulse splitting [28]. Nevertheless, soliton propagation without energy transfer to the dispersive waves is possible, if the nonlinearity is saturable [37]. The situation substantially differs if FOD is negative: there is a stable radiationless soliton-like pulse [36, 38], and a certain minimum FOD provides its stabilization in the vicinity of zero GDD [39]. In such a regime, FOD can provide additional spectral broadening [23].

The FOD effect on the CPO remains largely unexplored. It was found, in particular, that FOD is the source of "M-shaped" spectra [7], and there are regimes, in which the parts of the spectrum can be located within both the negative and positive GDD regions [23]. Also, a soliton with modulated envelope can develop in the presence of FOD when GDD is positive [16].

In this paper we present a systematic numerical study of HOD effects in the CPO. We found that HOD plays a crucial role in the oscillator stability and, in particular, show the stabilizing effect of positive FOD as well as the strong destabilizing effect of negative FOD and TOD. We demonstrate that the nature of the instability in the vicinity of zero GDD substantially changes in the presence of HOD: strong irregular oscillation of the pulse occurs instead of CW excitation. As a result, the "chaotic mode locking" appears in the vicinity of zero GDD. Experiments with a Ti:sapphire CPO verify the existence of such a regime. The interconnection between the PDR and noisy mode-locking regimes is not clear, and so they both have to be further studied.

2. Heavily-chirped pulses in the PDR

When an oscillator operates in the PDR, pulses with strong chirp develop [5, 31, 40]. Such a regime can be described on the basis of the distributed generalized complex cubic-quintic nonlinear Ginzburg-Landau model [7, 32, 33, 40, 41]:

$$\frac{\partial A}{\partial z} = \left[\sigma A + \alpha \frac{\partial^2 A}{\partial t^2} - \sum_{k=2}^{N} i^{k+1} \beta_k \frac{\partial^k A}{\partial t^k}\right] + \left[(\kappa - i\gamma) P - \kappa \zeta P^2\right] A. \tag{1}$$

Here $A(z,t)$ is the slowly varying field amplitude, $P = |A|^2$ is the power, t is the local time, z the propagation distance, α is the square of the inverse spectral filter bandwidth, and σ is the spectrally independent saturated net gain. The parameters β_k describe the kth-order net dispersion (up to N^{th} order), and γ and κ are the self-phase and self-amplitude modulation coefficients, respectively. Parameter ζ describes saturation of the self-amplitude modulation. In a broadband solid-state oscillator, for example, parameter α is the square of the inverse gain band width multiplied by the saturated gain coefficient; γ is the nonlinear coefficient of the active medium; parameters κ and ζ are defined by the Kerr-lens mode locking mechanism [7]. The distance z is normalized to the cavity length L_{cav}, so that all parameters are calculated over L_{cav}.

It was found [32, 33, 40], that a heavily-chirped pulse can be described as the soliton-like solution of Eq. (1) for N =2. For high-energy pulses the quintic nonlinear term ζ becomes important. Such pulses suffer from the collapsing instability in the framework of the reduced (i.e. cubic nonlinear [31, 40]) version of Eq. (1) [42]. Physically, such an instability means that the Kerr-lensing inside an active medium is not limited by the pump- and laser-mode overlap, which forms the so-called soft aperture [43].

The existence of soliton-like pulses in the PDR can be explained by the combined action of two mechanisms: a pure phase mechanism and a frequency-dissipative one. The first mecha-

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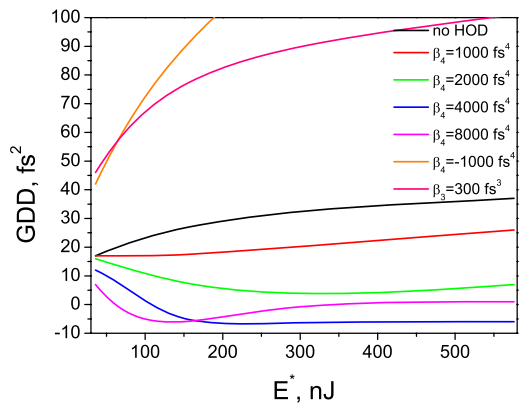

Fig. 1. Dependence of the stability threshold on the energy and higher-order dispersion.

nism is the balance of phase contributions from the pulse envelope $A(t)$: $\beta_2 \partial^2 A(t)/\partial t^2$ and the time-dependent phase $\phi(t)$: $-\beta_2 A(t)\left[\partial\phi(t)/\partial t\right]^2$ (see Eq. (1)). In the cubic nonlinear Ginzburg-Landau equation without HOD, such a balance is possible if the pulse chirp is equal to $\psi^2 = 2 + \gamma P_0/\beta_2\tau^2$ (P_0 is the peak pulse power, $\beta_2 > 0$). However, a pure phase balance is not sufficient for pulse stabilization as the pulse spreads (just as it takes place in the similariton regime [10]), i.e. some dissipative effects are required to form a quasi-soliton [40]. As was pointed out in Ref. [31], the pulse lengthening due to GDD can be compensated by its shortening owing to frequency filtering if the pulse is chirped. The chirp results in frequency deviation at the pulse front and tail, and the filter cuts off the high- and low-frequency wings of the pulse, thus shortening it.

The analysis demonstrates that without HOD, the heavily-chirped pulse is stable within the range of its existence, i.e. in the range of $\sigma < 0$ [33]. Negativity of σ, i.e. stability against the CW amplification, can be provided by a certain minimum positive GDD growing with energy. Even in the case of σ <0, the heavily-chirped pulse possesses rich dynamics [44, 45], which can be gained by HOD. As a result, the stability conditions can be changed dramatically in the presence of HOD.

3. Stability of the CPO in the presence of HOD

To study the effect of HOD on the CPO stability, we solve Eq. (1) numerically by the means of the split-step Fourier method [12]. The parameters used correspond to those in Ref. [33], i.e. a Ti:sapphire CPO: $\alpha = 1.1$ fs^2, $\kappa = 0.04\gamma$, $\zeta = 0.6\gamma$, $\gamma = 4.55$ MW^{-1}; the local time step equals to 1 fs (2^{15} points); the longitudinal step is $\Delta z = 10^{-3}L_{cav}$, and the simulation interval exceeds $10^4 L_{cav}$. The energy dependence of the net gain parameter σ is linearized in the vicinity of 0: $\sigma(E) \approx \frac{d\sigma}{dE}\big|_{E=E^*} E^*\left(\frac{E}{E^*} - 1\right)$, where E^* corresponds to the energy stored inside an oscillator in the CW regime. Parameter $\delta \equiv \frac{d\sigma}{dE}\big|_{E=E^*} E^* = -0.05$ defines the response of the active medium to the pulse energy E and can be expressed through the gain and loss coefficients only [33]. The initial conditions are: $a = a_0\,\mathrm{sech}[(t-t')/\theta]$, θ =200 fs, $a_0^2 = 2.5\times 10^{-3}/\gamma$, and $\sigma = -\delta$. The seeding pulse is located in the center of the time window at t'. In the absence of HOD, the numerical solution converges to the analytical one of Ref. [33] during less than 1000 round-trips. Variations of the initial a_0 and θ affect only the convergence time.

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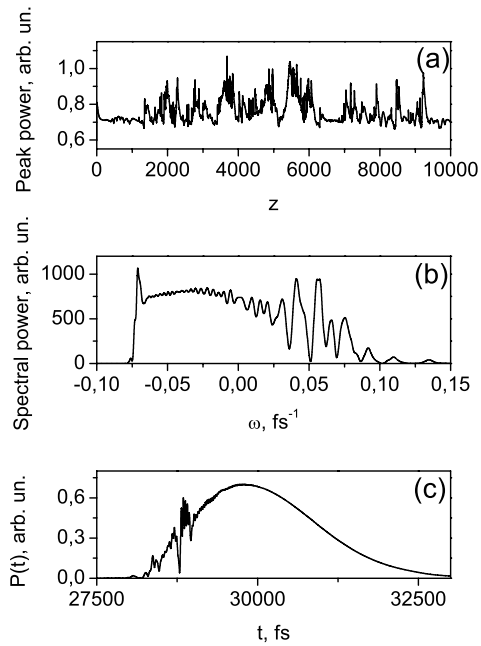

Fig. 2. The pulse peak power evolution (a), spectral (b) and temporal (c) profiles (at $z = 10^4 L_{cav}$) of the strongly perturbed pulse, E^* =288 nJ, β_2=90 fs^2, β_3 =-300 fs^2.

The simulations demonstrate that, as in the case without HOD, there exists some minimum GDD (β_2) for a given energy (E^*) and a given HOD (β_k, $k > 2$), which provides stability for the CPO. We varied the HOD terms separately, i.e. only a single β_k term was nonzero for $k > 2$. The corresponding dependences of the threshold GDD (β_2), which provides the CPO stability, are shown in Fig. 1.

Odd HOD destabilizes the pulse so that a large positive GDD (larger β_2) is required for pulse stabilization (in Fig. 1 only one case with $\beta_3 \neq 0$ is shown). As in the case without HOD, the stabilization threshold monotonically increases with the energy. The source of destabilization in the presence of odd HOD is the excitation of dispersive waves, which is caused by resonance with the CW perturbation. Since the spectrum of the chirped pulse is sufficiently broad (and it broadens as β_2 decreases), the resonance frequency (defined from the dispersion relation [12, 41]) shifts into the pulse spectrum. The appearing resonance with the dispersive wave causes strong parametric instability and pulse auto-oscillations (Fig. 2(a)) [6, 11]. As a result of this instability, the fragmentation of spectral and temporal envelopes occurs (Figs. 2(b,c)). Hence, the destabilization does not have the form of a CW excitation (i.e. $\sigma > 0$) but looks like pulse fission. An additional source of destabilization can be enhancement of the asymmetric internal perturbation modes analyzed in Ref. [45].

The effect of FOD has to be considered separately. The sign of the β_4 term is substantial in this case. When $\beta_4 < 0$, i.e FOD decreases the total dispersion toward the edges of the spectrum, the pulse is unstable within a wide range of net dispersion (see Fig. 1). Such destabilization is understandable on the basis of the hypothesis about the role of resonance between a solitary pulse and linear waves. Negative FOD results in two resonances of the solitary pulse with the linear waves [12, 33, 41]: $-\beta_2(\Delta/2)^2 = \beta_2\omega^2 + \beta_3\omega^3 + \beta_4\omega^4 + ...$, where $\Delta/2$ is the frequency

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of the spectral truncation measured from the carrier frequency. Since the pulse cannot be stabilized by the spectral self-shift, as it happens in the case of pure TOD, the destabilization is stronger for $\beta_4 < 0$. Again, the destabilization looks like the strong auto-oscillations of the pulse peak power (like those in Fig. 2(a)) and the fragmentation of the pulse envelope. The fragmentation is similar to that illustrated by Fig. 2, but it is approximately symmetric in both the temporal and spectral domains owing to the symmetry of the two dispersive resonances.

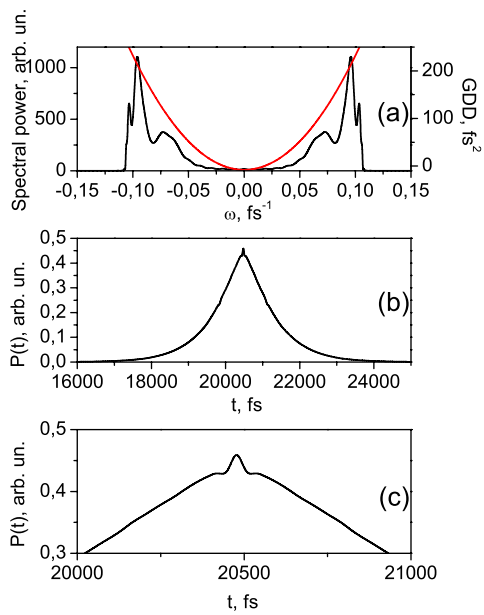

Fig. 3. Pulse spectrum (a), temporal profile (b) and enlarged temporal profile (c) at the stability border ($z = 10^4 L_{cav}$). E =144 nJ, β_4=4000 fs^4. The red curve is the GDD profile $\beta_2 + 6\beta_4\omega^2$ [7].

FOD of opposite sign ($\beta_4 > 0$) forbids resonance with the linear waves within the range $\beta_2 > 0$. An important feature of the regime with $\beta_4 > 0$ is that it is very stable in the vicinity of $\beta_2 = 0$. The parameter β_2 that stabilizes the regime decreases with positive FOD (Fig. 1) and can "penetrate" even into the negative dispersion range. Within the range $\beta_2 < 0$, two resonances with the linear waves are possible if $\beta_4 > 0$. As a result, the destabilization arises. Therefore, there exists a certain maximum positive FOD providing a stable pulse within the widest range of parameter β_2. Higher values of FOD destabilize the pulse. It should be noted that for sufficiently large positive FOD the dependence of the stabilization threshold on energy exists only for comparatively low energies (Fig. 1). When the positive FOD and energy values are not too large, the destabilization mechanism is the CW amplification ($\sigma > 0$). Growth of both energy and positive FOD results in a change of the destabilization mechanism, which is pulse splitting.

4. Temporal and spectral shapes of the chirped-pulse in the presence of HOD

The chirped pulse under the influence of HOD substantially differs from the usual chirped solitary pulse. If $\beta_4 > 0$, the spectral components are pushed out of the spectral range, where β_2 is close to zero (Fig. 3(a)). Such spectra are widely-observable in the experiment (Fig. 4). A

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strong asymmetry of the dispersion curve relative to its minimum resulted from a contribution of odd-order HODs can cause the spectrum asymmetry.

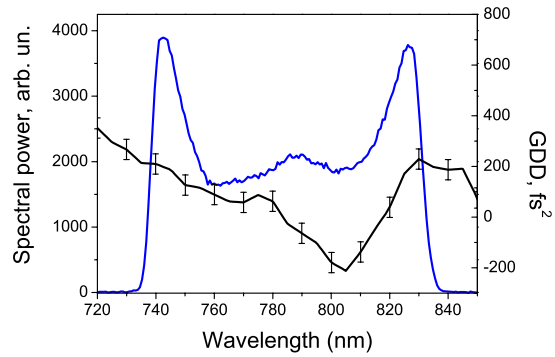

Fig. 4. Broadest experimental spectrum for the 10.7 MHz Ti:sapphire oscillator (blue curve) and net GDD (black curve). $\beta_2 \approx$55 fs^2, $\beta_3 \approx$126 fs^3, $\beta_4 \approx$4500 fs^4.

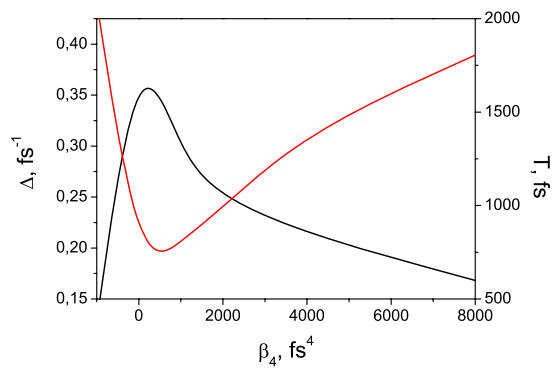

Fig. 5. Dependence of the spectral Δ (black curve) and temporal T (red curve) widths on β_4 at the stability threshold. E =144 nJ, $z = 10^4 L_{cav}$.

In contrast to the chirped pulse without HOD [32], the pulse is Λ-shaped (Fig. 3(b)) with the profile peak (Fig. 3(c)) if $\beta_4 > 0$. The effect resembles optical wave breaking [12] and will be analyzed elsewhere.

For the variable parameter β_4, the variations of the temporal and spectral pulse widths are shown in Fig. 5. The minimum pulse and maximum spectral widths correspond to $\beta_2 \approx 0$. This is caused by shift of the spectral components to the edges of the spectrum if $\beta_4 \neq 0$. The stability of the regime is provided by the both spectral splitting and shifting out of the zero GDD point. As a result, spectral dissipation grows and narrows the pulse spectrum. The pulse parameter variations with FOD are much smaller if $\beta_4 > 0$.

In combination with the generation of dispersive waves, the odd HOD terms allow the spectral shape to be controlled. In particular, the spectrum can be extra-broadened (see Fig. 6, where the full spectral widths are $\approx$0.53 fs^{-1} (black curve) vs. $\approx$0.36 fs^{-1} for β_5 =0 (red curve)). One

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can see that the fifth-order dispersion provides the zero net-GDD at $\omega \approx 0.15$ fs^{-1}. As a result, the intensive dispersive wave develops around the zero-GDD as well as in the negative dispersion range. The mechanism of broadening is similar to that in fibers, where a spectral continuum appears [46]. Such extra broadening is possible in a similariton fiber oscillator operating in the PDR [30] and in a solid-state oscillator operating in the NDR [29], as well.

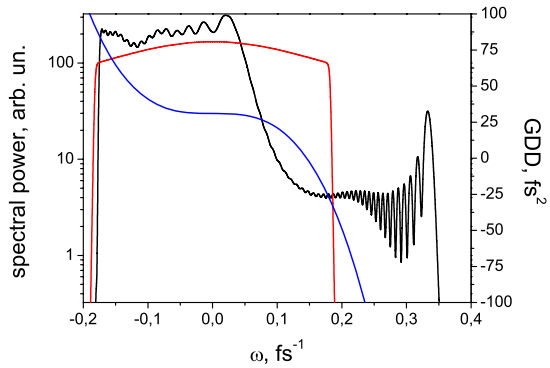

Fig. 6. Spectrum at $z = 10^4 L_{cav}$ for E =144 nJ, β_2=31 fs^2, $\beta_5 = -10^3$ fs^5 (black) and 0 fs^5 (red). The blue curve is the GDD profile.

5. Chaotic mode-locking in the vicinity of zero GDD

As it was described above, positive FOD stabilizes the chirped pulse in the vicinity of zero GDD against CW amplification. Stable operation at $\beta_2 \approx 0$ (but not over the entire dispersion range) was reported in Ref. [6].

Our experiments have demonstrated that an oscillator in the vicinity of zero GDD can suffer from auto-oscillation instability (so-called chaotic mode-locking [44, 45]; Fig. 7(a,c)). Even in the NDR, the chaotic mode-locked spectrum can mimic that in the PDR (Fig. 7(b)): it is truncated but not too abruptly. Smoothing of the spectrum edges also takes place in the PDR when β_2 approaches zero, thereby violating the condition $\alpha \ll \beta_2$ [32, 33]. Hence, these regimes cannot be unambiguously spectrally distinguished.

The simulations on the basis of Eq. (1) reproduce the chaotic mode locking in the presence of positive FOD. The standard deviation of the peak pulse power set (that is $\sqrt{\sum_{i=1}^{N} (P_i - \bar{P})^2 / (N-1)}/\bar{P}$, where $\bar{P}$ is the average peak power, P_i is the peak power at i^{th}-round-trip; $N = 10^4$) at the stability threshold as a function of β_4 is shown in Fig. 8. The pulse auto-oscillations grow with β_4. Nevertheless, the pulse is not destroyed and there is no CW amplification in spite of the strong chaotic auto-oscillations of the pulse peak power.

One can see, that the auto-oscillations are strongly related to the β_2-parameter value (for nonzero β_4). Figure 9 demonstrates that there is a dispersion (β_2) range (down to ≈ -8 fs^2, then CW grows), where the pulse exists but its peak power auto-oscillates.

Such an instability substantially differs from multipulsing or CW amplification when σ crosses the zero-level [47], and is similar to the parametrical instability causing the picosecond supercontinuum generation in fibers [46].

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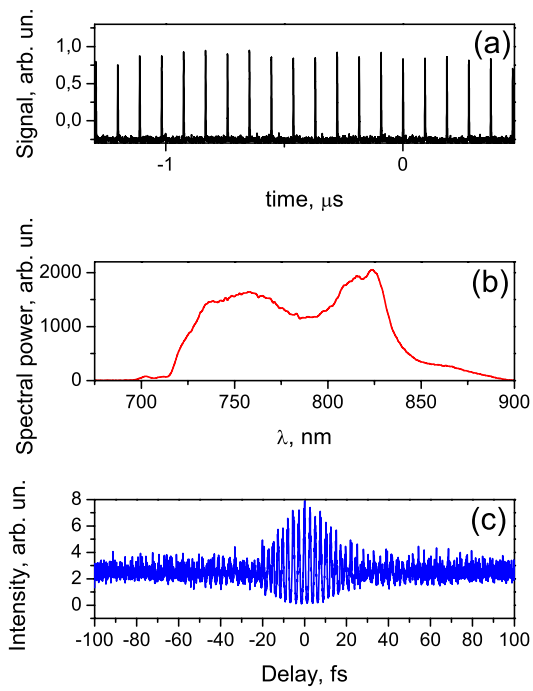

Fig. 7. The pulse train (a), generated spectrum (b), and autocorrelation trace (c) realized in the Ti:sapphire chaotic mode-locked oscillator.

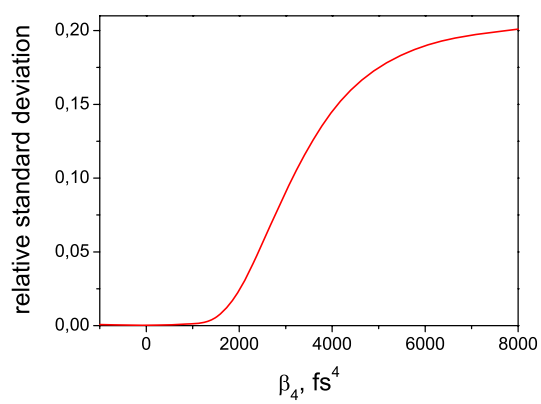

Fig. 8. Relative standard deviation of the peak power evolution in dependence on β_4 at the stability threshold. E =144 nJ.

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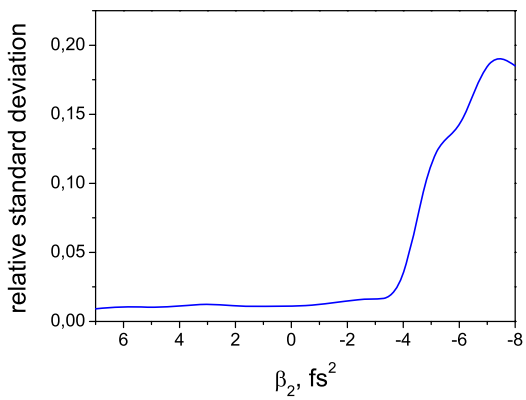

Fig. 9. Relative standard deviation of the peak power evolution as a function of β_2. E =144 nJ, β_4 =4000 fs^4.

6. Conclusion

Effects of HOD on CPO characteristics have been studied both theoretically and experimentally. It is found that odd HOD destabilizes the pulse so that larger positive GDD is required for pulse stabilization. The mechanism of the pulse destabilization is the parametric instability caused by resonance with dispersive waves when the resonance frequencies reach the heavily-chirped pulse spectrum.

The effect of FOD depends on the sign of β_4: negative FOD strongly destabilizes the CPO, while positive FOD enhances the pulse stability, suppresses its energy dependence, and allows stable operation for both positive and negative GDD values. Growth of FOD results in the pulse broadening and its spectrum narrowing, while for positive FOD this dependence is weaker.

The chaotic mode-locking regime was also studied. It was found that in the presence of positive FOD low GDD leads to irregular oscillations of the pulse peak power. Such auto-oscillations grow with FOD. Thus, there is a range in the vicinity of zero GDD where the chirped pulse exists, but its parameters oscillate. Such a range can spread into the negative GDD range.

The main results were obtained by studying a solid-state CPO, where a chirped solitary pulse develops. However, a solitary pulse is a particular case of a similariton. Therefore, the effects of HOD in a similariton fiber laser can be similar, so that the main conclusions of this paper can be valid for fiber oscillators, as well.

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