International Journal of Modern Physics A
Vol. 22, No. 25 (2007) 4651–4666

ON NONLEPTONIC $\Sigma^+ \to p\pi^0$ DECAY IN THE EFFECTIVE QUARK MODEL WITH CHIRAL U(3) × U(3) SYMMETRY

A. YA. BERDNIKOV,[†] YA. A. BERDNIKOV,[†] M. FABER,[*] A. N. IVANOV,[*,†,‡] V. A. IVANOVA,[†] A. V. NIKITCHENKO[†] and N. I. TROITSKAYA[†]

[*] *Atominstitut der Österreichischen Universitäten, Technische Universität Wien, Wiedner Hauptstrasse 8-10, A-1040 Wien, Austria*

and

[†] *State Polytechnic University, Polytechnicheskaya 29, 195251, St. Petersburg, Russian Federation, Russia*

[‡] *ivanov@kph.tuwien.ac.at*

Received 5 May 2007
Revised 18 June 2007

Within the effective quark model with chiral U(3) × U(3) symmetry we calculate the S-wave and P-wave amplitudes of the nonleptonic decay $\Sigma^+ \to p\pi^0$, the partial width and the dynamical polarization of the proton in the dependence on the polarization of the Σ^+-hyperon. The theoretical results agree well with the experimental data.

Keywords: Quark; chiral; hyperon.

PACS numbers: 13.30.Eg, 13.75.Ev, 13.88.+e, 14.20.Jn

1. Introduction

The theoretical analysis of the polarization properties of Σ-hyperons both in the laboratory frame and in the rest frame of the Σ-hyperon is meaningful in connection with the theoretical and experimental investigations of a mechanism of the production of the Σ-hyperons in high-energy heavy-ion collisions.

The Σ^+-hyperon possesses the nonleptonic mode $\Sigma^+ \to p\pi^0$ with the probability $B(\Sigma^+ \to p\pi^0)_{\rm exp} = (0.516 \pm 0.003)$ and the asymmetry $\alpha^{\rm exp}_{p\pi^0} = -0.980^{+0.017}_{-0.015}$.[1] This makes the mode $\Sigma^+ \to p\pi^0$ to be the most attractive from the point of view of the experimental investigation of the polarization properties of Σ^+-hyperons produced in ultrarelativistic heavy-ion collisions. Indeed, the appearance of unpolarized Σ^+-hyperons can be a signal for the quark–gluon plasma in the intermediate state of ultrarelativistic heavy-ion collisions.

[‡]Permanent Address: State Polytechnic University, Department of Nuclear Physics, 195251 St. Petersburg, Russian Federation.

The effective low-energy Lagrangian responsible for the nonleptonic decays of the Σ^+-hyperon is[2,3,a]

$$\mathcal{L}_{\rm weak}(x) = -\frac{G_F}{\sqrt{2}}\,V^*_{ud}V_{us}(c_+O_+(x) + c_-O_-(x)) + {\rm h.c.}\,, \tag{1.1}$$

where $G_F = 1.166 \times 10^{-11}$ MeV^{-2} is the Fermi constant, $V^*_{ud}V_{us} = 0.218$ are Cabibbo–Kobayashi–Maskawa quark-mixing matrix elements[1] $c_+ = 2.164$ and $c_- = 0.680$ are Wilson's coefficients, calculated to leading order in gluon exchanges at the normalization scale $\mu = 1$ GeV,[2,3] which is of order of $\Lambda_\chi = 0.94$ GeV, the scale of the spontaneous breaking of chiral symmetry in the effective quark model with chiral U(3) × U(3) symmetry.[4–11] The operators $O_+(x)$ and $O_-(x)$ are expressed in terms of the quark fields and take the form[2]

$$\begin{aligned} O_+(x) &= \frac{1}{2}\left\{[\bar{u}_\ell(x)\gamma_\mu(1-\gamma^5)s_\ell(x)][\bar{d}_{\ell'}(x)\gamma^\mu(1-\gamma^5)u_{\ell'}(x)]\right.\\ &\quad\left. + [\bar{u}_\ell(x)\gamma_\mu(1-\gamma^5)u_\ell(x)][\bar{d}_{\ell'}(x)\gamma^\mu(1-\gamma^5)s_{\ell'}(x)]\right\},\\ O_-(x) &= \frac{1}{2}\left\{[\bar{u}_\ell(x)\gamma_\mu(1-\gamma^5)s_\ell(x)][\bar{d}_{\ell'}(x)\gamma^\mu(1-\gamma^5)u_{\ell'}(x)]\right.\\ &\quad\left. - [\bar{u}_\ell(x)\gamma_\mu(1-\gamma^5)u_\ell(x)][\bar{d}_{\ell'}(x)\gamma^\mu(1-\gamma^5)s_{\ell'}(x)]\right\}, \end{aligned} \tag{1.2}$$

where $u_\ell(x)$, $d_\ell(x)$ and $s_\ell(x)$ are current quark fields. The summation over the color indices $\ell(\ell') = 1, 2, 3$ is assumed. The operator $O_-(x)$ is responsible for the $\Delta I = 1/2$ transitions, whereas the operator $O_+(x)$ describes both the $\Delta I = 1/2$ and $\Delta I = 3/2$ transitions.[2]

The paper is organized as follows. In Sec. 2 we define the amplitude of the $\Sigma^+ \to p\pi^0$ in terms of the matrix element of the effective Lagrangian (1.1) and apply the soft-pion technique to the reduction of the π^0-meson. We represent the matrix element of the $\Sigma^+ \to p$ in the factorized form and find that it can be saturated by the contribution of the Δ^{++}-resonance. In Sec. 3 we apply the effective quark model with chiral U(3) × U(3) symmetry to the calculation of the matrix element of the $\Sigma^+ \to \Delta^{++}$ transition. In Sec. 4 we calculate the S-wave and P-wave amplitudes and the partial width of the $\Sigma^+ \to p\pi^0$ decay. We analyze also the angular distribution of the decay rate in dependence on the polarizations of baryons and calculate the dynamical polarization vector of the proton. In the conclusion (Sec. 6) we discuss the obtained results. The effective Lagrangian of low-energy weak interactions contains the contribution of quark–gluon interactions, which provide an appearance of effective four-quark interactions called as QCD-penguin operators.[2,3] In App. A we show that the contribution of QCD-penguin operators to the amplitude of the $\Sigma^+ \to p\pi^0$ decay, calculated in our effective quark model, vanishes. In App. B we give a detailed calculation of the momentum integrals, defining the matrix element of the $\Sigma^+ \to \Delta^{++}$ transition.

[a]See Ref. 2 p. 104.

2. Amplitude of the $\Sigma^+ \to p\pi^0$ Decay

The amplitude of the $\Sigma^+ \to p\pi^0$ decay is defined by the matrix element

$$M(\Sigma^+ \to p\pi^0) = -C_+ \frac{G_F}{\sqrt{2}} V^*_{ud} V_{us} \big\langle \pi^0(k) p(\mathbf{k}_p, \sigma_p) \big| \big[\bar{d}_{\ell'}(0)\gamma^\mu(1-\gamma^5) u_{\ell'}(0)\big] \times \big[\bar{u}_\ell(0)\gamma_\mu(1-\gamma^5) s_\ell(0)\big] \big| \Sigma^+(\mathbf{k}_{\Sigma^+}, \sigma_{\Sigma^+}) \big\rangle\,. \tag{2.1}$$

The coefficient $C_+ = (c_+ + 2c_-)/3 = 1.175$ is obtained by a Fierz transformation with the account for the *color* degrees of freedom of quarks.[2,5,6]

The amplitude of the $\Sigma^+ \to p\pi^0$ decay can be written in the following general form:[b]

$$M(\Sigma^+(\mathbf{k}_{\Sigma^+}, \sigma_{\Sigma^+}) \to p(\mathbf{k}_p, \sigma_p)\pi^0(\mathbf{k})) = \bar{u}_p(\mathbf{k}_p, \sigma_p)(A_{p\pi^0} - B_{p\pi^0}\gamma^5) u_{\Sigma^+}(\mathbf{k}_{\Sigma^+}, \sigma_{\Sigma^+})\,, \tag{2.2}$$

where $A_{p\pi^0}$ and $B_{p\pi^0}$ are the constants related to the contribution of the $p\pi^0$ pair coupled in the S- and P-wave states, and $\bar{u}_p(\mathbf{k}_p, \sigma_p)$ and $u_{\Sigma^+}(\mathbf{k}_{\Sigma^+}, \sigma_{\Sigma^+})$ are bispinorial wave functions of the proton and the Σ^+-hyperon.

We propose to calculation the amplitude of the $\Sigma^+ \to p\pi^0$ decay in soft-pion limit[12] or differently to leading order in Chiral Perturbation Theory (ChPT)[14–20] (see also Ref. 11).

In the soft-pion limit the amplitude of the $\Sigma^+ \to p\pi^0$ decay (2.2) is defined as follows:[12]

$$M(\Sigma^+ \to p\pi^0) = -i\frac{C_+}{F_\pi}\frac{G_F}{\sqrt{2}} V^*_{ud} V_{us} \times \langle p(\mathbf{k}_p, \sigma_p)| \big[Q^3_5(0), [\bar{d}_{\ell'}(0)\gamma^\mu(1-\gamma^5) u_{\ell'}(0)] \times [\bar{u}_\ell(0)\gamma_\mu(1-\gamma^5) s_\ell(0)]\big] |\Sigma^+(\mathbf{k}_{\Sigma^+}, \sigma_{\Sigma^+})\rangle\,, \tag{2.3}$$

where $F_\pi = 92.4$ MeV is the PCAC constant of the π^--meson and $Q^3_5(0)$ is the axial-charge operator. In terms of the current quark fields it is defined by

$$Q^3_5(0) = \int d^3x\, \frac{1}{2}\, [u^\dagger_\ell(0,\mathbf{x})\gamma^5 u_\ell(0,\mathbf{x}) - d^\dagger_\ell(0,\mathbf{x})\gamma^5 d_\ell(0,\mathbf{x})]\,. \tag{2.4}$$

Using canonical anticommutation relations for the current quark fields we get

$$M(\Sigma^+ \to p\pi^0) = -i\,\frac{C_+}{2F_\pi}\,\frac{G_F}{\sqrt{2}} V^*_{ud} V_{us} \times \langle p(\mathbf{k}_p, \sigma_p)| [\bar{d}_{\ell'}(0)\gamma^\mu(1-\gamma^5) u_{\ell'}(0)] \times [\bar{u}_\ell(0)\gamma_\mu(1-\gamma^5) s_\ell(0)] |\Sigma^+(\mathbf{k}_{\Sigma^+}, \sigma_{\Sigma^+})\rangle\,. \tag{2.5}$$

[b]See Ref. 1 p. 864.

For the subsequent analysis of this matrix element we propose to insert the complete set of intermediate states[21,22]

$$\sum_X |X\rangle\langle X| = 1\,, \tag{2.6}$$

where X is a state with a baryon number $B = 1$. This transcribes the right-hand side (r.h.s.) of (2.5) as follows:

$$M(\Sigma^+ \to p\pi^0) = -i\frac{C_+}{2F_\pi}\frac{G_F}{\sqrt{2}}\,V^*_{ud}V_{us}\sum_X \langle p(\mathbf{k}_p,\sigma_p)|[\bar{d}_{\ell'}(0)\gamma^\mu(1-\gamma^5)u_{\ell'}(0)]|X\rangle$$
$$\times\,\langle X|[\bar{u}_\ell(0)\gamma_\mu(1-\gamma^5)s_\ell(0)]|\Sigma^+(\mathbf{k}_{\Sigma^+},\sigma_{\Sigma^+})\rangle\,. \tag{2.7}$$

Since the lowest state is the $\Delta(1232)$-resonance, i.e. $X = \Delta^{++}$, we propose to saturate the r.h.s. of (2.5) with the intermediate state $X = \Delta^{++}$. This gives

$$M(\Sigma^+ \to p\pi^0) = -i\frac{C_+}{2F_\pi}\frac{G_F}{\sqrt{2}}\,V^*_{ud}V_{us}$$
$$\times \sum_{\sigma=\pm1/2,\pm3/2}\int\frac{d^3Q}{(2\pi)^3 2E_\Delta(\mathbf{Q})}$$
$$\times\,\langle p(\mathbf{k}_p,\sigma_p)|[\bar{d}_{\ell'}(0)\gamma^\mu(1-\gamma^5)u_{\ell'}(0)]|\Delta^{++}(\mathbf{Q},\sigma)\rangle$$
$$\times\,\langle \Delta^{++}(\mathbf{Q},\sigma)|[\bar{u}_\ell(0)\gamma_\mu(1-\gamma^5)s_\ell(0)]|\Sigma^+(\mathbf{k}_{\Sigma^+},\sigma_{\Sigma^+})\rangle\,. \tag{2.8}$$

The matrix element of the $V - A$ quark current between the proton states can be expressed in terms of the form factor[22,23]

$$\langle p(\mathbf{k}_p,\sigma_p)|J^{1-i2}_\mu(0) - J^{1-i2}_{5\mu}(0)|\Delta^{++}(\mathbf{Q},\sigma)\rangle = -\sqrt{2}g_A F_A(\mathbf{q}^2)\bar{u}_p(\mathbf{k}_p,\sigma_p)u^\mu_\Delta(\mathbf{Q},\sigma)\,, \tag{2.9}$$

where $u_{\Delta^{++}\nu}(\mathbf{Q},\sigma)$ is a spinorial wave function of the Δ^{++}-resonance.[24–26] We take the form factor in the dipole form[24–26] (see also Ref. 9):

$$F_A(\mathbf{q}^2) = \frac{1}{\left(1+\frac{\mathbf{q}^2}{M_A^2}\right)^2}\,. \tag{2.10}$$

The slope parameter we set equal $M_A = 1096$ MeV.[9] It agrees well with the slope parameters $M_A = (1050 \pm 140)$ MeV, obtained from the ν_μ-reactions,[26] and $M_A^{\exp} = (1026 \pm 21)$ MeV and $M_A^{\exp} = (1069 \pm 16)$ MeV obtained from neutrino scattering and electroproduction, respectively.[27]

3. Matrix Element $\langle \Delta^{++}(\mathbf{Q},\boldsymbol{\sigma})|\cdots|\Sigma^+(\mathbf{k}_{\Sigma^+},\boldsymbol{\sigma}_{\Sigma^+})\rangle$

The calculation of the matrix element $\langle \Delta^{++}(\mathbf{Q},\sigma)|[\bar{u}_\ell(0)\gamma_\mu(1-\gamma^5)s_\ell(0)]$ $|\Sigma^+(\mathbf{k}_{\Sigma^+},\sigma_{\Sigma^+})\rangle$ we carry out within the effective quark model with chiral U(3) × U(3) symmetry.[4–11] After the application of the reduction technique

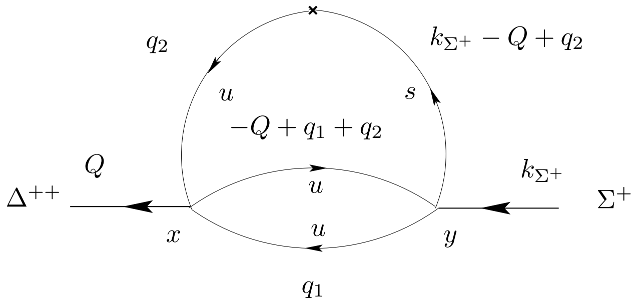

Fig. 1. The Feynman diagram of the $\Sigma^+ \to \Delta^{++}$ transition in the effective quark model with chiral $\mathrm{U}(3) \times \mathrm{U}(3)$ symmetry.

and the equations of motion, the matrix element $\langle \Delta^{++}(\mathbf{Q},\sigma)|[\bar{u}_\ell(0)\gamma_\mu(1 - \gamma^5)s_\ell(0)]|\Sigma^+(\mathbf{k}_{\Sigma^+},\sigma_{\Sigma^+})\rangle$ is

$$\begin{aligned}
&\langle \Delta^{++}(\mathbf{Q},\sigma)|[\bar{u}_\ell(0)\gamma_\mu(1-\gamma^5)s_\ell(0)]|\Sigma^+(\mathbf{k}_{\Sigma^+},\sigma_{\Sigma^+})\rangle \\
&\quad = -\frac{g_B^2}{\sqrt{2}} \int d^4x\, d^4y\, e^{iQ\cdot x - ik_{\Sigma^+}\cdot y}\bar{u}_{\Delta\nu}(\mathbf{Q},\sigma) \\
&\qquad \times \langle 0|\mathrm{T}(\eta^\nu_{\Delta^{++}}(x)[\bar{u}_\ell(0)\gamma_\mu(1-\gamma^5)s_\ell(0)]\bar{\eta}_{\Sigma^+}(y))|0\rangle_c u_{\Sigma^+}(\mathbf{k}_{\Sigma^+},\sigma_{\Sigma^+})\,, \quad (3.1)
\end{aligned}$$

where T is a time-ordering operator, the index c in $\langle 0|\cdots|0\rangle_c$ means the calculation of the *connected* part of the vacuum expectation value, then, $\eta_{\Sigma^+}(x)$ and $\eta_{\Delta^{++}}(y)$ are the three-quark densities[4]

$$\begin{aligned}
\eta_{\Sigma^+}(x) &= \varepsilon^{ijk}[\bar{u^c}_i(x)\gamma_\mu u_j(x)]\gamma^\mu\gamma^5 s_k(x)\,, \\
\eta^\nu_{\Delta^{++}}(x) &= \varepsilon^{ijk}[\bar{u^c}_i(x)\gamma^\nu u_j(x)]u_k(x)
\end{aligned} \quad (3.2)$$

coupled to the Σ^+-hyperon and the Δ^{++}-resonance as[4]

$$\mathcal{L}_{\mathrm{eff}}(x) = \frac{g_B}{\sqrt{2}}\bar{\Sigma}^+(x)\eta_{\Sigma^+}(x) + g_B\bar{\Delta}^{++}_\mu(x)\eta^\mu_{\Delta^{++}}(x) + \mathrm{h.c.} \quad (3.3)$$

The three-quark density $\bar{\eta}_{\Sigma^+}(x)$ is equal to

$$\bar{\eta}_{\Sigma^+}(x) = -\varepsilon^{ijk}\bar{s}_i(x)\gamma_\mu\gamma^5[\bar{u}_j(x)\gamma^\mu u^c_k(x)]\,. \quad (3.4)$$

The coupling constant g_B has been calculated in Refs. 4 and 8: $g_B = 1.38 \times 10^{-4}\ \mathrm{MeV}^{-2}$.

The matrix element Eq. (3.1) is defined by the Feynman diagram in Fig. 1. In the coordinate representation the analytical expression of the matrix element Eq. (3.1) is

$$\begin{aligned}
&\langle \Delta^{++}(\mathbf{Q},\sigma)|[\bar{u}_\ell(0)\gamma_\mu(1-\gamma^5)s_\ell(0)]|\Sigma^+(\mathbf{k}_{\Sigma^+},\sigma_{\Sigma^+})\rangle \\
&\quad = 3\sqrt{2}g_B^2 \int d^4x\, d^4y\, e^{iQ\cdot x - ik_{\Sigma^+}\cdot y}\,\mathrm{tr}\left\{\gamma_\nu S^{(u)}_F(x-y)\gamma_\beta S^{(u)}_F(y-x)\right\}
\end{aligned}$$

$$\times\, \bar{u}^{\nu}_{\Delta}(\mathbf{Q},\sigma) S^{(u)}_F(x)\gamma_{\mu}(1-\gamma^5) S^{(s)}_F(-y)\gamma^{\beta}\gamma^5 u_{\Sigma^+}(\mathbf{k}_{\Sigma^+},\sigma_{\Sigma^+})$$
$$- 3\sqrt{2} g^2_B \int d^4x\, d^4y\, e^{iQ\cdot x - ik_{\Sigma^+}\cdot y} \bar{u}^{\nu}_{\Delta}(\mathbf{Q},\sigma) S^{(u)}_F(x-y)\gamma_{\beta} S^{(u)}_F(y-x)$$
$$\times\, \gamma_{\nu} S^{(u)}_F(x)\gamma_{\mu}(1-\gamma^5) S^{(s)}_F(-y)\gamma^{\beta}\gamma^5 u_{\Sigma^+}(\mathbf{k}_{\Sigma^+},\sigma_{\Sigma^+})\,. \quad (3.5)$$

In the momentum representation the matrix element Eq. (3.5) reads

$$\langle \Delta^{++}(\mathbf{Q},\sigma)|[\bar{u}_{\ell}(0)\gamma_{\mu}(1-\gamma^5)s_{\ell}(0)]|\Sigma^+(\mathbf{k}_{\Sigma^+},\sigma_{\Sigma^+})\rangle$$
$$= 3\sqrt{2} g^2_B \int \frac{d^4q_1}{(2\pi)^4 i} \int \frac{d^4q_2}{(2\pi)^4 i}\, \bar{u}^{\nu}_{\Delta}(\mathbf{Q},\sigma)$$
$$\times \left[\frac{1}{m_u - \hat{q}_2}\gamma_{\mu}(1-\gamma^5)\frac{1}{m_s - \hat{q} - \hat{q}_2}\gamma^{\beta}\gamma^5\, \mathrm{tr}\left\{\gamma_{\nu}\frac{1}{m_u - \hat{q}_1}\gamma_{\beta}\frac{1}{m_u + \hat{Q} - \hat{q}_1 - \hat{q}_2}\right\}\right]$$
$$\times\, u_{\Sigma^+}(\mathbf{k}_{\Sigma^+},\sigma_{\Sigma^+}) - 3\sqrt{2} g^2_B \int \frac{d^4q_1}{(2\pi)^4 i} \int \frac{d^4q_2}{(2\pi)^4 i} \bar{u}^{\nu}_{\Delta}(\mathbf{Q},\sigma)$$
$$\times \left[\frac{1}{m_u - \hat{q}_1}\gamma_{\beta}\frac{1}{m_u + \hat{Q} - \hat{q}_1 - \hat{q}_2}\gamma_{\nu}\frac{1}{m_u - \hat{q}_2}\gamma_{\mu}(1-\gamma^5)\frac{1}{m_s - \hat{q} - \hat{q}_2}\gamma^{\beta}\gamma^5\right]$$
$$\times\, u_{\Sigma^+}(\mathbf{k}_{\Sigma^+},\sigma_{\Sigma^+})\,, \quad (3.6)$$

where we have set $q = k_{\Sigma^+} - Q$.

The integrations over virtual momenta are restricted from above by the $\Lambda_{\chi} = 940$ MeV.[4–11] In this region of relative momenta of quark–quark interactions chiral symmetry is spontaneously broken and quarks become converted into constituents quarks with dynamical masses of order $O(m)$, where $m \sim 330$ MeV is a dynamical quark mass.[4–11] According to effective quark models,[28–34] in such a region of relative quark–quark interactions hadronic interactions are described only by quark loops with constituent quarks. The contribution of gluon interactions is taken effectively in the form of effective constants of low-energy interactions,[35–37] constituent quark mass calculated in the chiral limit[38–40] and quark condensate.[38]

The calculation of the momentum integrals we carry out in the heavy-baryon limit, used for the analysis of baryon exchanges in the Chiral Perturbation Theory (ChPT)[14–20,41–45] (see also Refs. 4 and 22). Following the procedure for the calculation of the momentum integrals proposed in Ref. 4, we get (see App. B)

$$\langle \Delta^{++}(\mathbf{Q},\sigma)|[\bar{u}_{\ell}(0)\gamma_{\mu}(1-\gamma^5)s_{\ell}(0)]|\Sigma^+(\mathbf{k}_{\Sigma^+},\sigma_{\Sigma^+})\rangle$$
$$= \frac{g^2_B\langle\bar{u}u\rangle}{24\sqrt{2}m_N}\bar{u}^{\nu}_{\Delta}(\mathbf{Q},\sigma)\gamma_{\mu}\gamma_{\nu}$$
$$\times \left\{\frac{(2m_s + m_u)\langle\bar{u}u\rangle - (2m_u + m_s)\langle\bar{s}s\rangle}{m^2_s - m^2_u}\right.$$
$$\left. - \frac{(2m_s - m_u)\langle\bar{u}u\rangle - (2m_u - m_s)\langle\bar{s}s\rangle}{m^2_s - m^2_u}\gamma^5\right\} u_{\Sigma^+}(\mathbf{k}_{\Sigma^+},\sigma_{\Sigma^+})\,, \quad (3.7)$$

where $\langle \bar{q}q \rangle$ is the quark condensate[10]

$$\langle \bar{q}q \rangle = -\frac{3m_q}{4\pi^2}\left[\Lambda_\chi^2 - m_q^2 \ell n\left(1 + \frac{\Lambda_\chi^2}{m_q^2}\right)\right] \tag{3.8}$$

and $m_d = m_u = 330$ MeV and $m_s = 465$ MeV are the mass of the constituent u and s quarks.[10,21] The constituent quark masses of $q = u$, d and s quarks are different, since the SU(3)-flavor symmetry is broken. According to Refs. 4–11 and 28–34, the masses of constituent u, d and s quarks are defined by $m_u = m + m_{0u}$, $m_d = m + m_{0d}$ and $m_s = m + m_{0s}$, where $m_{0u} = 4$ MeV, $m_{0d} = 7$ MeV and $m_{0s} = 135$ MeV are current quark masses, obtained at the normalization scale $\mu = 1$ GeV,[46] which is of order $\Lambda_\chi = 0.94$ GeV the scale of spontaneously broken chiral symmetry. Since the contribution of current quark masses to the masses of constituent u and d quarks is about 2%, we have set $m_u \simeq m_d = m$.

4. Partial Width of the $\Sigma^+ \to p\pi^0$ Decay

Using the matrix elements, calculated in Sec. 3, we can obtain the parameters A and B, defining the amplitude of the $\Sigma^+ \to p\pi^0$ decay. We get

$$\begin{aligned} A_{p\pi^0} &= -iC_+ G_F V_{ud}^* V_{us}\,\frac{g_A g_B^2 \langle \bar{u}u \rangle}{96\sqrt{2}\,\pi F_\pi m_N} \\ &\quad \times \frac{(2m_s + m_u)\langle \bar{u}u \rangle - (2m_u + m_s)\langle \bar{s}s \rangle}{m_s^2 - m_u^2}\, M_A^3 = i3.0 \times 10^{-7}\,, \\ B_{p\pi^0} &= +iC_+ G_F V_{ud}^* V_{us}\,\frac{g_A g_B^2 \langle \bar{u}u \rangle}{96\sqrt{2}\,\pi F_\pi m_N} \\ &\quad \times \frac{(2m_s - m_u)\langle \bar{u}u \rangle - (2m_u - m_s)\langle \bar{s}s \rangle}{m_s^2 - m_u^2}\, M_A^3 = -i23.7 \times 10^{-7}\,. \end{aligned} \tag{4.1}$$

The factors M_A^3 appear (4.1) due to the integration of the form factors (2.10) over $\mathbf{Q}$.

The partial width of the $\Sigma^+ \to p\pi^0$ decay is equal to

$$\begin{aligned} \Gamma(\Sigma^+ \to p\pi^0) &= \frac{k}{2\pi}\,\frac{m_p}{m_{\Sigma^+}}\left\{|A_{p\pi^0}|^2 + \frac{k^2}{4m_p^2}\,(|A_{p\pi^0}|^2 + |B_{p\pi^0}|^2)\right\} \\ &= 3.80 \times 10^{-12}\ \text{MeV}\,, \end{aligned} \tag{4.2}$$

where $k = 189$ MeV is the relative momentum of the $p\pi^0$ pair.[1] For the calculation of the partial width $\Gamma(\Sigma^+ \to p\pi^0)$ we have used the experimental values of the masses of interacting particles.[1]

The theoretical value agrees with the experimental data[1]

$$\Gamma_{\text{exp}}(\Sigma^+ \to p\pi^0) = (4.23 \pm 0.03) \times 10^{-12}\ \text{MeV} \tag{4.3}$$

within an accuracy better than 10%.

5. *P*-Wave Amplitude and Baryon Polarization Properties

Using the parameters $A_{p\pi^0}$ and $B_{p\pi^0}$, defined by (4.1), we can calculate the values of the S-wave and P-wave amplitudes. They are determined by[b]

$$s_{p\pi^0} = A_{p\pi^0} = i3.0 \times 10^{-7}\,, \qquad p_{p\pi^0} = \frac{k}{E_p + m_p} B_{p\pi^0} = -i2.4 \times 10^{-7}\,. \quad (5.1)$$

For the ratio of the P-wave and S-wave amplitude we obtain

$$R_{p\pi^0} = \frac{p_{p\pi^0}}{s_{p\pi^0}} = -0.80\,. \quad (5.2)$$

This result agrees well with the experimental data $R^{\exp}_{p\pi^0} = -0.82 \pm 0.07$. The result is extracted from the experimental data on the asymmetry parameter $\alpha^{\exp}_{p\pi^0} = -0.980^{+0.017}_{-0.015}$.[1]

For the calculation of the dynamical polarization vector of the proton $\mathbf{P}_p$ we can use the results obtained in Ref. 9. In the laboratory frame the dynamical polarization vector $\mathbf{P}_p$ is

$$\begin{aligned}\mathbf{P}_p = &\bigg\{2\mathcal{R}e(A^*_{p\pi^0} B_{p\pi^0})\bigg[E_{\Sigma^+}(\mathbf{k}_{\Sigma^+})\mathbf{k}_p - m_p\mathbf{k}_{\Sigma^+} - \frac{(\mathbf{k}_{\Sigma^+}\cdot\mathbf{k}_p)\mathbf{k}_p}{E_p(\mathbf{k}_p) + m_p}\bigg] \\ &- \Big[(k_p\cdot k_{\Sigma^+} + m_p m_{\Sigma^+})|A_{p\pi^0}|^2 - (k_p\cdot k_{\Sigma^+} - m_p m_{\Sigma^+})|B_{p\pi^0}|^2\Big] \\ &\times \bigg[-\boldsymbol{\zeta}_{\Sigma^+} + \frac{(\mathbf{k}_{\Sigma^+}\cdot\boldsymbol{\zeta}_{\Sigma^+})\mathbf{k}_p}{E_{\Sigma^+}(\mathbf{k}_{\Sigma^+})m_p} - \frac{(\mathbf{k}_p\cdot\boldsymbol{\zeta}_p)\mathbf{k}_p}{m_p(E_p(\mathbf{k}_p)+m_p)}\bigg] \\ &+ (|A_{p\pi^0}|^2 - |B_{p\pi^0}|^2)\bigg[\frac{E_{\Sigma^+}(\mathbf{k}_{\Sigma^+})}{m_p}\mathbf{k}_p - \mathbf{k}_{\Sigma^+} - \frac{(\mathbf{k}_{\Sigma^+}\cdot\mathbf{k}_p)\mathbf{k}_p}{m_p(E_p(\mathbf{k}_p)+m_p)}\bigg]\bigg\} \\ &\times \big\{(k_p\cdot k_{\Sigma^+} + m_p m_{\Sigma^+})|A_{p\pi^0}|^2 + (k_p\cdot k_{\Sigma^+} - m_p m_{\Sigma^+})|B_{p\pi^0}|^2 \\ &- 2\mathcal{R}e(A^*_{p\pi^0}B_{p\pi^0})m_{\Sigma^+}(k_p\cdot\zeta_{\Sigma^+})\big\}^{-1}\,, \end{aligned} \quad (5.3)$$

where

$$\zeta^\mu_B = \left(\frac{\mathbf{k}_B\cdot\boldsymbol{\xi}_B}{m_B}\,, \boldsymbol{\xi}_B + \frac{\mathbf{k}_B(\mathbf{k}_B\cdot\boldsymbol{\xi}_B)}{m_Y(E_B(\mathbf{k}_B)+m_B)}\right). \quad (5.4)$$

The polarization vector ζ^μ_B satisfies the constraints $\zeta^2_B = -1$ and $k_B\cdot\zeta_B = 0$. The angular distribution of decay rate $\Sigma^+ \to p\pi^+$ is defined by[9]

$$\begin{aligned}&4\pi\frac{dB(\Sigma^+\to p\pi^0)(\boldsymbol{\xi}_{\Sigma^+})}{d\Omega_{\Sigma^+p}} \\ &= B(\Sigma^+\to p\pi^0)\bigg[1 - \frac{2\mathcal{R}e(A^*_{p\pi^0}B_{p\pi^0})m_{\Sigma^+}(k_p\cdot\zeta_{\Sigma^+})}{(k_p\cdot k_{\Sigma^+} + m_p m_{\Sigma^+})|A_{p\pi^0}|^2 + (k_p\cdot k_{\Sigma^+} - m_p m_{\Sigma^+})|B_{p\pi^0}|^2}\bigg],\end{aligned} \quad (5.5)$$

where $B(\Sigma^+ \to p\pi^0) = (0.5157 \pm 0.0030)$.[1]

In the rest frame of the Σ^+-hyperon the dynamical polarization vector $\mathbf{P}_p$ reduces to the form[b]

$$\mathbf{P}_p = \frac{\alpha_{p\pi^0}\mathbf{n}_p + \gamma_{p\pi^0}\boldsymbol{\xi}_{\Sigma^+} + (1-\gamma_{p\pi^0})(\mathbf{n}_p\cdot\boldsymbol{\xi}_{\Sigma^+})\mathbf{n}_p}{1+\alpha_{p\pi^0}\mathbf{n}_p\cdot\boldsymbol{\xi}_{\Sigma^+}}\,, \tag{5.6}$$

where $\mathbf{n}_p = \mathbf{k}_p/|\mathbf{k}_p|$. For the derivation of (5.6) we have neglected the contribution of the term $|s|^2(\mathbf{k}_p^2/4m_p^2)$. The parameters $\alpha_{p\pi^0}$ and $\gamma_{p\pi^0}$ are equal to[b]

$$\begin{aligned}\alpha_{p\pi^0} &= \frac{2\mathcal{R}e(s^*_{p\pi^0}p_{p\pi^0})}{|s_{p\pi^0}|^2+|p_{p\pi^0}|^2} = -0.98\,,\\ \gamma_{p\pi^0} &= \frac{|s_{p\pi^0}|^2-|p_{p\pi^0}|^2}{|s_{p\pi^0}|^2+|p_{p\pi^0}|^2} = 0.22\,.\end{aligned} \tag{5.7}$$

The theoretical value $\alpha_{p\pi^0} = -\,0.980$ agrees well with the experimental one $\alpha^{\mathrm{exp}}_{p\pi^0} = -0.980^{+0.017}_{-0.015}$.[1]

In the rest frame of the Σ^+-hyperon the angular distribution of the decay rate (5.5) reduces to the from

$$4\pi\frac{dB(\Sigma^+\to p\pi^0)(\boldsymbol{\xi}_{\Sigma^+})}{d\Omega_{\Sigma^+p}} = B(\Sigma^+\to p\pi^0)(1+\alpha_{p\pi^0}\mathbf{n}_p\cdot\boldsymbol{\xi}_{\Sigma^+})\,. \tag{5.8}$$

Hence, the parameter $\alpha_{p\pi^0}$ characterizes the asymmetry of the $\Sigma^+ \to p\pi^0$ decay for the unpolarized protons.[1] The angular distribution has a maximum for the Σ^+-hyperons polarized antiparallel to the momentum of the proton.

6. Conclusion

Following the soft-pion technique, the procedure for the calculation of the matrix elements of four-quark operators developed in Ref. 9 and the effective quark model with chiral U(3) × U(3) symmetry, we have calculated the S-wave and P-wave amplitudes and the partial width of the nonleptonic decay mode $\Sigma^+ \to p\pi^0$. All theoretical results agree well with the experimental data. We have shown that the violation of the SU(3)-flavor symmetry plays an important role for the correct description of the experimental data. The saturation of the amplitude of the $\Sigma^+ \to p\pi^0$ decay by the Δ^{++}-resonance agrees well with the results obtained by Borasoy and Holstein for the analysis of the contribution of resonances to the nonleptonic decays of hyperons.[41–45]

Since the asymmetry of the decay mode $\Sigma^+ \to p\pi^0$ is very high $\alpha_{p\pi^0} = -0.980$, this modes seems to be the most attractive for the analysis of the polarization properties of the Σ^+-hyperon produced in ultrarelativistic heavy-ion collisions. Indeed, the analysis of the angular distribution of the probability of the $\Sigma^+ \to p\pi^0$ decay shows that the maximum of the angular distribution can be reached only for Σ^+-hyperons polarized antiparallel to the momentum of the proton. The appearance of unpolarized Σ^+-hyperons can serve as a signal for the existence of the quark–gluon plasma as well as the unpolarized Λ^0-hyperons.[7]

Appendix A. Contribution of the QCD-Penguin Operators

According to Refs. 2 and 3, the effective weak interactions, defining the transitions with $\Delta S = 1$, contain the contribution of the so-called QCD-penguin operators

$$\mathcal{L}_{\rm weak}^{\rm QCD\text{-}penguin}(x) = -\frac{G_F}{\sqrt{2}}\, V^*_{ud}V_{us}(c_3O_3(x) + c_4O_4(x) + c_5O_5(x) + c_6O_6(x)) + {\rm h.c.}\,, \tag{A.1}$$

where the operators $O_i(x)$ for $i = 3, \ldots, 6$ are expressed in terms of the quark fields and take the form[2,3]

$$\begin{aligned}
O_3(x) &= \left[\bar{d}_\ell(x)\gamma_\mu(1-\gamma^5)s_\ell(x)\right] \sum_{q=u,d,s} \left[\bar{q}_{\ell'}(x)\gamma^\mu(1-\gamma^5)q_{\ell'}(x)\right],\\
O_4(x) &= \left[\bar{d}_\ell(x)\gamma_\mu(1-\gamma^5)s_{\ell'}(x)\right] \sum_{q=u,d,s} \left[\bar{q}_{\ell'}(x)\gamma^\mu(1-\gamma^5)q_{\ell}(x)\right],\\
O_5(x) &= \left[\bar{d}_\ell(x)\gamma_\mu(1-\gamma^5)s_\ell(x)\right] \sum_{q=u,d,s} \left[\bar{q}_{\ell'}(x)\gamma^\mu(1+\gamma^5)q_{\ell'}(x)\right],\\
O_6(x) &= \left[\bar{d}_\ell(x)\gamma_\mu(1-\gamma^5)s_{\ell'}(x)\right] \sum_{q=u,d,s} \left[\bar{q}_{\ell'}(x)\gamma^\mu(1+\gamma^5)q_{\ell}(x)\right].
\end{aligned} \tag{A.2}$$

The Wilson's coefficients c_i for $i = 3, \ldots, 6$ are calculated to leading order in gluon exchanges at the normalization scale $\mu = 1$ GeV,[2,3] which is of order $\Lambda_\chi = 0.94$ GeV, the scale of the spontaneous breaking of chiral symmetry in the effective quark model with chiral $U(3) \times U(3)$ symmetry.[4–11]

In the soft-pion limit the contribution of the QCD-penguin operators $\delta M(\Sigma^+ \to p\pi^0)$ to the amplitude of the $\Sigma^+ \to p\pi^0$ decay is defined by

$$\begin{aligned}
\delta M(\Sigma^+ \to p\pi^0) = {}& -\frac{i}{F_\pi}\frac{G_F}{\sqrt{2}}\, V^*_{ud}V_{us}\, \langle p(\mathbf{k}_p, \sigma_p)|\\
&\times \left[Q^3_5(0), \sum_{i=3}^{6} c_iQ_i(0)\right] |\Sigma^+(\mathbf{k}_{\Sigma^+}, \sigma_{\Sigma^+})\rangle\,.
\end{aligned} \tag{A.3}$$

Since the equal-time commutator is equal to

$$\left[Q^3_5(0), \sum_{i=3}^{6} c_iQ_i(0)\right] = \sum_{i=3}^{6} c_iQ_i(0)\,, \tag{A.4}$$

the contribution of the QCD-penguin operators to the amplitude $\delta M(\Sigma^+ \to p\pi^0)$ of the $\Sigma^+ \to p\pi^0$ decay is

$$\delta M(\Sigma^+ \to p\pi^0) = -\frac{i}{2F_\pi}\frac{G_F}{\sqrt{2}}\, V^*_{ud}V_{us} \sum_{i=3}^{6} c_i\langle p(\mathbf{k}_p, \sigma_p)|Q_i(0)|\Sigma^+(\mathbf{k}_{\Sigma^+}, \sigma_{\Sigma^+})\rangle\,. \tag{A.5}$$

As we have made in Sec. 2, we insert the complete set of intermediate states Eq. (2.6) and keep the contributions only the lowest states, which are $|X\rangle = |p\rangle$ and $|\Delta^+\rangle$. This gives

$$\begin{aligned}
\delta M(\Sigma^+ \to p\pi^0) = &-\frac{i}{2F_\pi}\,\frac{G_F}{\sqrt{2}}\,V^*_{ud}V_{us}\\
&\times\left\{C_{LL}\sum_{\sigma=\pm 1/2}\int\frac{d^3Q}{(2\pi)^3 2E_p(Q)}\sum_{q=u,d,s}\right.\\
&\times\langle p(\mathbf{k}_p,\sigma_p)|[\bar{q}_{\ell'}(0)\gamma^\mu(1-\gamma^5)q_{\ell'}(0)]|p(\mathbf{Q},\sigma)\rangle\\
&\times\langle p(\mathbf{Q},\sigma)|[\bar{d}_\ell(0)\gamma_\mu(1-\gamma^5)s_\ell(0)]|\Sigma^+(\mathbf{k}_{\Sigma^+},\sigma_{\Sigma^+})\rangle\\
&+C_{LL}\sum_{\sigma=\pm 1/2,\pm 3/2}\int\frac{d^3Q}{(2\pi)^3 2E_\Delta(Q)}\sum_{q=u,d,s}\\
&\times\langle p(\mathbf{k}_p,\sigma_p)|[\bar{q}_{\ell'}(0)\gamma^\mu(1-\gamma^5)q_{\ell'}(0)]|\Delta^+(\mathbf{Q},\sigma)\rangle\\
&\times\langle \Delta^+(\mathbf{Q},\sigma)|[\bar{d}_\ell(0)\gamma_\mu(1-\gamma^5)s_\ell(0)]|\Sigma^+(\mathbf{k}_{\Sigma^+},\sigma_{\Sigma^+})\rangle\\
&+C_{LR}\sum_{\sigma=\pm 1/2}\int\frac{d^3Q}{(2\pi)^3 2E_p(Q)}\sum_{q=u,d,s}\\
&\times\langle p(\mathbf{k}_p,\sigma_p)|[\bar{q}_{\ell'}(0)\gamma^\mu(1+\gamma^5)q_{\ell'}(0)]|p(\mathbf{Q},\sigma)\rangle\\
&\times\langle p(\mathbf{Q},\sigma)|[\bar{d}_\ell(0)\gamma_\mu(1-\gamma^5)s_\ell(0)]|\Sigma^+(\mathbf{k}_{\Sigma^+},\sigma_{\Sigma^+})\rangle\\
&+C_{LR}\sum_{\sigma=\pm 1/2,\pm 3/2}\int\frac{d^3Q}{(2\pi)^3 2E_\Delta(Q)}\sum_{q=u,d,s}\\
&\times\langle p(\mathbf{k}_p,\sigma_p)|[\bar{q}_{\ell'}(0)\gamma^\mu(1+\gamma^5)q_{\ell'}(0)]|\Delta^+(\mathbf{Q},\sigma)\rangle\\
&\left.\times\langle \Delta^+(\mathbf{Q},\sigma)|[\bar{d}_\ell(0)\gamma_\mu(1-\gamma^5)s_\ell(0)]|\Sigma^+(\mathbf{k}_{\Sigma^+},\sigma_{\Sigma^+})\rangle\right\}. \qquad \text{(A.6)}
\end{aligned}$$

The coefficients C_{LL} and C_{LR} are defined by

$$C_{LL} = c_3 + \frac{1}{3}\,c_4\,, \qquad C_{LR} = c_5 + \frac{1}{3}\,c_6\,, \qquad \text{(A.7)}$$

for the definition of which we have taken into account that hadrons are colorless states.

Using the results obtained in Refs. 4 and 8 and in Sec. 2 one can show that the matrix elements of the operator $[\bar{d}_\ell(0)\gamma_\mu(1-\gamma^5)s_\ell(0)]$ between states $\langle X(\mathbf{Q},\sigma)|$, where $X = p$ or Δ^+, and $|\Sigma^+(\mathbf{k}_{\Sigma^+}, \sigma_{\Sigma^+})\rangle$ are proportional to the following momentum integrals:

$$\begin{aligned}
&\langle p(\mathbf{Q},\sigma)|[\bar{d}_\ell(0)\gamma_\mu(1-\gamma^5)s_\ell(0)]|\Sigma^+(\mathbf{k}_{\Sigma^+},\sigma_{\Sigma^+})\rangle \\
&\quad \propto \int \frac{d^4q_1}{(2\pi)^4 i}\int \frac{d^4q_2}{(2\pi)^4 i}\bar{u}_p(\mathbf{Q},\sigma)\gamma^\alpha\gamma^5\,\frac{1}{m_d-\hat{q}_2} \\
&\qquad \times \gamma_\mu(1-\gamma^5)\frac{1}{m_s-\hat{q}-\hat{q}_2}\gamma^\beta\gamma^5 u_{\Sigma^+}(\mathbf{k}_{\Sigma^+},\sigma_{\Sigma^+}) \\
&\qquad \times \mathrm{tr}\left\{\gamma_\alpha\frac{1}{m_u-\hat{q}_1}\gamma_\beta\frac{1}{m_u+\hat{Q}-\hat{q}_1-\hat{q}_2}\right\}, \\
&\langle \Delta^+(\mathbf{Q},\sigma)|[\bar{d}_\ell(0)\gamma_\mu(1-\gamma^5)s_\ell(0)]|\Sigma^+(\mathbf{k}_{\Sigma^+},\sigma_{\Sigma^+})\rangle \\
&\quad \propto \int \frac{d^4q_1}{(2\pi)^4 i}\int \frac{d^4q_2}{(2\pi)^4 i}\,\bar{u}^\nu_\Delta(\mathbf{Q},\sigma)\frac{1}{m_d-\hat{q}_2} \\
&\qquad \times \gamma_\mu(1-\gamma^5)\frac{1}{m_s-\hat{q}-\hat{q}_2}\gamma^\beta\gamma^5 u_{\Sigma^+}(\mathbf{k}_{\Sigma^+},\sigma_{\Sigma^+}) \\
&\qquad \times \mathrm{tr}\left\{\gamma_\nu\frac{1}{m_u-\hat{q}_1}\gamma_\beta\frac{1}{m_u+\hat{Q}-\hat{q}_1-\hat{q}_2}\right\}.
\end{aligned} \tag{A.8}$$

In the heavy-baryon approximation to leading order in the large m_B expansion one gets[4,8]

$$\begin{aligned}
&\int \frac{d^4q_1}{(2\pi)^4 i}\,\mathrm{tr}\left\{\gamma_\alpha\frac{1}{m_u-\hat{q}_1}\gamma_\beta\frac{1}{m_u+\hat{Q}-\hat{q}_1-\hat{q}_2}\right\} \\
&\qquad = -\frac{1}{3}\,\frac{\langle\bar{q}q\rangle}{m_B^2}\,\mathrm{tr}\{\gamma_\alpha\gamma_\beta\hat{Q}\} = 0\,,
\end{aligned} \tag{A.9}$$

where $Q^2 = m_B^2$ and $m_B = m_N$ or m_Δ. The integral Eq. (A.9) vanishes as $\mathrm{tr}\{\gamma_\alpha\gamma_\beta\hat{Q}\} = 0$.

Thus, in our approach the contribution of the QCD-penguin operators to the amplitude of the $\Sigma^+ \to p\pi^0$ decay vanishes.

Appendix B. Calculation of Matrix Element Eq. (3.6)

In this appendix we give a detailed calculation of the momentum integrals, defining the matrix element of the $\Sigma^+ \to \Delta^{++}$ transition. As has been shown in App. A, the momentum integral containing the trace of Green functions of the constituent quarks, vanishes to leading order in the heavy-baryon approximation. Thus, in the heavy-baryon approximation the matrix element of the $\Sigma^+ \to \Delta^{++}$ transition is defined by the following momentum integral

$$\begin{aligned}&\langle \Delta^{++}(\mathbf{Q},\sigma)|[\bar u_\ell(0)\gamma_\mu(1-\gamma^5)s_\ell(0)]|\Sigma^+(\mathbf{k}_{\Sigma^+},\sigma_{\Sigma^+})\rangle\\ &= -3\sqrt{2}g_B^2\int\frac{d^4q_1}{(2\pi)^4i}\int\frac{d^4q_2}{(2\pi)^4i}\bar u^\nu_\Delta(\mathbf{Q},\sigma)\\ &\quad\times\left[\frac{1}{m_u-\hat q_1}\gamma_\beta\frac{1}{m_u+\hat Q-\hat q_1-\hat q_2}\right.\\ &\quad\left.\times\gamma_\nu\frac{1}{m_u-\hat q_2}\gamma_\mu(1-\gamma^5)\frac{1}{m_s-\hat q-\hat q_2}\gamma^\beta\gamma^5\right]u_{\Sigma^+}(\mathbf{k}_{\Sigma^+},\sigma_{\Sigma^+}),\end{aligned}\qquad (B.1)$$

where we have set $q = k_{\Sigma^+} - Q$.

Following the technique in Ref. 4 and keeping only the leading we reduce the r.h.s. of Eq. (B.1) to the form

$$\begin{aligned}&\langle \Delta^{++}(\mathbf{Q},\sigma)|[\bar u_\ell(0)\gamma_\mu(1-\gamma^5)s_\ell(0)]|\Sigma^+(\mathbf{k}_{\Sigma^+},\sigma_{\Sigma^+})\rangle\\ &= -3\sqrt{2}\frac{g_B^2}{m_N^2}\int\frac{d^4q_1}{(2\pi)^4i}\int\frac{d^4q_2}{(2\pi)^4i}\\ &\times\bar u^\nu_\Delta(\mathbf{Q},\sigma)\left[\frac{1}{m_u-\hat q_1}\gamma_\beta\hat Q\gamma_\nu\frac{1}{m_u-\hat q_2}\gamma_\mu(1-\gamma^5)\frac{1}{m_s-\hat q_2}\gamma^\beta\gamma^5\right]u_{\Sigma^+}(\mathbf{k}_{\Sigma^+},\sigma_{\Sigma^+}).\end{aligned}\qquad (B.2)$$

Integration over q_1 gives[9–11]

$$\int\frac{d^4q_1}{(2\pi)^4i}\,\frac{1}{m_u-\hat q_1} = \frac{m_u}{16\pi^2}\left[\Lambda_\chi^2 - m_u^2\ell n\left(1+\frac{\Lambda_\chi^2}{m_u^2}\right)\right] = -\frac{1}{12}\,\langle\bar u u\rangle\,. \qquad (B.3)$$

Using Eq. (B.3) we transcribe the matrix element Eq. (B.2) as follows:

$$\begin{aligned}&\langle \Delta^{++}(\mathbf{Q},\sigma)|[\bar u_\ell(0)\gamma_\mu(1-\gamma^5)s_\ell(0)]|\Sigma^+(\mathbf{k}_{\Sigma^+},\sigma_{\Sigma^+})\rangle\\ &= \frac{g_B^2\langle\bar u u\rangle}{2\sqrt{2}m_N^2}\int\frac{d^4q_2}{(2\pi)^4i}\,\bar u^\nu_\Delta(\mathbf{Q},\sigma)\\ &\quad\times\left[\gamma_\beta\hat Q\gamma_\nu\frac{1}{m_u-\hat q_2}\gamma_\mu(1-\gamma^5)\frac{1}{m_s-\hat q_2}\gamma^\beta\gamma^5\right]u_{\Sigma^+}(\mathbf{k}_{\Sigma^+},\sigma_{\Sigma^+}).\end{aligned}\qquad (B.4)$$

Integrating over directions of four-vector q_2, we get

$$\begin{aligned}&\langle \Delta^{++}(\mathbf{Q},\sigma)|[\bar u_\ell(0)\gamma_\mu(1-\gamma^5)s_\ell(0)]|\Sigma^+(\mathbf{k}_{\Sigma^+},\sigma_{\Sigma^+})\rangle\\ &= \frac{g_B^2\langle\bar u u\rangle}{4\sqrt{2}m_N^2}\int\frac{d^4q_2}{(2\pi)^4i}\,\bar u^\nu_\Delta(\mathbf{Q},\sigma)\gamma_\beta\hat Q\gamma_\nu\gamma_\mu\gamma^\beta\\ &\quad\times\left[\frac{2m_um_s+q_2^2}{(m_u^2-q_2^2)(m_s^2-q_2^2)}+\frac{2m_um_s-q_2^2}{(m_u^2-q_2^2)(m_s^2-q_2^2)}\gamma^5\right]u_{\Sigma^+}(\mathbf{k}_{\Sigma^+},\sigma_{\Sigma^+}).\end{aligned}\qquad (B.5)$$

Due to algebra of the Dirac matrices $\gamma_\beta \hat{Q} \gamma_\nu \gamma_\mu \gamma^\beta = -2\gamma_\mu \gamma_\nu \hat{Q}$, we obtain

$$\begin{aligned}
&\langle \Delta^{++}(\mathbf{Q}, \sigma)|[\bar{u}_\ell(0)\gamma_\mu(1-\gamma^5)s_\ell(0)]|\Sigma^+(\mathbf{k}_{\Sigma^+}, \sigma_{\Sigma^+})\rangle \\
&\quad = -\frac{g_B^2 \langle \bar{u}u \rangle}{2\sqrt{2} m_N^2} \int \frac{d^4 q_2}{(2\pi)^4 i} \bar{u}^\nu_\Delta(\mathbf{Q}, \sigma) \gamma_\mu \gamma_\nu \hat{Q} \left[\frac{2 m_u m_s + q_2^2}{(m_u^2 - q_2^2)(m_s^2 - q_2^2)} \right. \\
&\qquad \left. + \frac{2 m_u m_s - q_2^2}{(m_u^2 - q_2^2)(m_s^2 - q_2^2)} \gamma^5 \right] u_{\Sigma^+}(\mathbf{k}_{\Sigma^+}, \sigma_{\Sigma^+}) \\
&\quad = -\frac{g_B^2 \langle \bar{u}u \rangle}{2\sqrt{2} m_N} \int \frac{d^4 q_2}{(2\pi)^4 i} \bar{u}^\nu_\Delta(\mathbf{Q}, \sigma) \gamma_\mu \gamma_\nu \left[\frac{2 m_u m_s + q_2^2}{(m_u^2 - q_2^2)(m_s^2 - q_2^2)} \right. \\
&\qquad \left. - \frac{2 m_u m_s - q_2^2}{(m_u^2 - q_2^2)(m_s^2 - q_2^2)} \gamma^5 \right] u_{\Sigma^+}(\mathbf{k}_{\Sigma^+}, \sigma_{\Sigma^+}) \,, \qquad \text{(B.6)}
\end{aligned}$$

where we have used the Dirac equation $\hat{Q} u_{\Sigma^+}(\mathbf{k}_{\Sigma^+}, \sigma_{\Sigma^+}) = m_N u_{\Sigma^+}(\mathbf{k}_{\Sigma^+}, \sigma_{\Sigma^+})$ valid to leading order in the heavy-baryon and chiral expansion.

The momentum integrals are equal to

$$\begin{aligned}
&\int \frac{d^4 q_2}{(2\pi)^4 i} \frac{2 m_u m_s + q_2^2}{(m_u^2 - q_2^2)(m_s^2 - q_2^2)} \\
&\quad = \frac{2m_s + m_u}{m_s^2 - m_u^2} \int \frac{d^4 q_2}{(2\pi)^4 i} \frac{m_u}{m_u^2 - q_2^2} - \frac{2m_u + m_s}{m_s^2 - m_u^2} \int \frac{d^4 q_2}{(2\pi)^4 i} \frac{m_s}{m_s^2 - q_2^2} \\
&\quad = \frac{2m_s + m_u}{m_s^2 - m_u^2} \left[-\frac{\langle \bar{u}u \rangle}{12} \right] - \frac{2m_u + m_s}{m_s^2 - m_u^2} \left[-\frac{\langle \bar{s}s \rangle}{12} \right] \\
&\quad = -\frac{1}{12} \frac{(2m_s + m_u)\langle \bar{u}u \rangle - (2m_u + m_s)\langle \bar{s}s \rangle}{m_s^2 - m_u^2} \,, \\
&\int \frac{d^4 q_2}{(2\pi)^4 i} \frac{2 m_u m_s - q_2^2}{(m_u^2 - q_2^2)(m_s^2 - q_2^2)} \qquad \text{(B.7)} \\
&\quad = \frac{2m_s - m_u}{m_s^2 - m_u^2} \int \frac{d^4 q_2}{(2\pi)^4 i} \frac{m_u}{m_u^2 - q_2^2} - \frac{2m_u - m_s}{m_s^2 - m_u^2} \int \frac{d^4 q_2}{(2\pi)^4 i} \frac{m_s}{m_s^2 - q_2^2} \\
&\quad = \frac{2m_s - m_u}{m_s^2 - m_u^2} \left[-\frac{\langle \bar{u}u \rangle}{12} \right] - \frac{2m_u - m_s}{m_s^2 - m_u^2} \left[-\frac{\langle \bar{s}s \rangle}{12} \right] \\
&\quad = -\frac{1}{12} \frac{(2m_s - m_u)\langle \bar{u}u \rangle - (2m_u - m_s)\langle \bar{s}s \rangle}{m_s^2 - m_u^2} \,.
\end{aligned}$$

Substituting Eq. (B.7) into Eq. (B.6) we get the expression

$$\langle \Delta^{++}(\mathbf{Q},\sigma)|[\bar{u}_\ell(0)\gamma_\mu(1-\gamma^5)s_\ell(0)]|\Sigma^+(\mathbf{k}_{\Sigma^+},\sigma_{\Sigma^+})\rangle$$

$$= \frac{g_B^2\langle\bar{u}u\rangle}{24\sqrt{2}m_N}\bar{u}^\nu_\Delta(\mathbf{Q},\sigma)\gamma_\mu\gamma_\nu\left\{\frac{(2m_s+m_u)\langle\bar{u}u\rangle-(2m_u+m_s)\langle\bar{s}s\rangle}{m_s^2-m_u^2}\right.$$

$$\left. - \frac{(2m_s-m_u)\langle\bar{u}u\rangle-(2m_u-m_s)\langle\bar{s}s\rangle}{m_s^2-m_u^2}\gamma^5\right\}u_{\Sigma^+}(\mathbf{k}_{\Sigma^+},\sigma_{\Sigma^+})\,. \qquad \text{(B.8)}$$

This confirms our result Eq. (3.7).

We would like to emphasize that the parameter $\Lambda_\chi = 940$ MeV has a meaning of the energy scale at which chiral symmetry is spontaneously broken. The physical coupling constants such as the leptonic decay constant $F_\pi = 92.4$ MeV, which is also the PCAC of pseudoscalar mesons, can be expressed in terms of $\Lambda_\chi = 940$ MeV and $m = 330$ MeV, the constituent quark mass calculated in the chiral limit[9–11] (see also Refs. 28–34). The constant F_π is defined by logarithmically divergent momentum integral and depends on $\ell n(1+\Lambda_\chi^2/m^2)$. Therefore, for the definition of other physical observables we have to keep the contributions proportional to $\ell n(1+\Lambda_\chi^2/m_q^2)$. This explains the appearance of such a term in the definition of the quark condensate.

We would also like to mention that the heavy-baryon approximation used in our effective quark model corresponds to the large N_C expansion,[47] which provides a nonperturbative analysis of low-energy interactions of hadrons.[47–58]

References

1. The Particle Data Group (S. Eidelman *et al.*), *Phys. Lett. B* **595**, 1 (2004).
2. A. J. Buras, Les Houches Lectures, Table 5, pp. 95–97, hep-ph/9806471.
3. A. J. Buras, P. Gambino and U. A. Haisch, *Nucl. Phys. B* **570**, 117 (2000).
4. A. N. Ivanov, M. Nagy and N. I. Troitskaya, *Phys. Rev. C* **59**, 451 (1999).
5. Ya. A. Berdnikov *et al.*, *Phys. Rev. C* **60**, 015201 (1999).
6. A. Ya. Berdnikov *et al.*, *Phys. Rev. D* **64**, 014027 (2001).
7. A. Ya. Berdnikov *et al.*, *Acta Phys. Hung. A* **22**, 139 (2005), nucl-th/0312045.
8. A. Ya. Berdnikov *et al.*, *Int J. Mod. Phys. A* **22**, 1835 (2007).
9. A. N. Ivanov, M. Nagy and N. I. Troitskaya, *Int. J. Mod. Phys. A* **7**, 7305 (1992).
10. A. N. Ivanov, *Int. J. Mod. Phys. A* **8**, 853 (1993).
11. A. N. Ivanov, N. I. Troitskaya and M. Nagy, *Int. J. Mod. Phys. A* **8**, 2027 (1993).
12. S. L. Adler and R. Dashen, in *Current Algebras and Applications to Particle Physics* (Benjamin, New York, 1968).
13. J. Gasser, *Nucl. Phys. B* (*Proc. Suppl.*) **86**, 257 (2000), and references therein.
14. H. Leutwyler, *PiN Newslett.* **15**, 1 (1999).
15. Ulf-G. Meißner, *PiN Newslett.* **13**, 7 (1997).
16. H. Leutwyler, *Ann. Phys.* **235**, 165 (1994).
17. G. Ecker, *Prog. Part. Nucl. Phys.* **36**, 71 (1996).
18. J. Gasser, *Nucl. Phys. B* **279**, 65 (1987).
19. J. Gasser and H. Leutwyler, *Nucl. Phys. B* **250**, 465 (1985).
20. J. Gasser and H. Leutwyler, *Ann. Phys.* **158**, 142 (1984).

21. A. N. Ivanov *et al.*, *Eur. Phys. J. A* **23**, 79 (2005), nucl-th/0406053.
22. T. E. O. Ericson and A. N. Ivanov, *Phys. Lett. B* **634**, 39 (2006), hep-ph/0503277.
23. T. R. Hemmert, B. R. Holstein and N. C. Mukhopadhyay, *Phys. Rev. D* **51**, 158 (1995).
24. M. M. Nagels *et al.*, *Nucl. Phys. B* **147**, 189 (1979).
25. O. Dumbrajs *et al.*, *Nucl. Phys. B* **216**, 277 (1983).
26. T. E. O. Ericson and W. Weise, in *Pions and Nuclei* (Clarendon Press, Oxford, 1988).
27. V. Bernard, L. Elouadrhiri and Ulf-G. Meißner, *J. Phys. G* **28**, R1 (2002), hep-ph/0107088.
28. T. Eguchi, *Phys. Rev. D* **14**, 2755 (1976).
29. K. Kikkawa, *Progr. Theor. Phys.* **56**, 947 (1976).
30. H. Kleinert, *Proc. Int. Summer School of Subnuclear Physics*, Erice, Italy, 23 Jul.–8 Aug. 1976, ed. A. Zichichi, p. 289.
31. A. Dhar, R. Shankar and S. R. Wadia, *Phys. Rev. D* **31**, 3256 (1984).
32. D. Ebert and H. Reinhardt, *Nucl. Phys. B* **271**, 188 (1986).
33. M. Wakamatsu, *Ann. Phys.* (*N.Y.*) **193**, 287 (1989).
34. J. Bijnens, C. Bruno and E. de Rafael, *Nucl. Phys. B* **390**, 501 (1993).
35. T. Goldman and R. W. Haymaker, *Phys. Rev. D* **24**, 724 (1981).
36. R. W. Haymaker and T. Goldman, *Phys. Rev. D* **24**, 743 (1981).
37. M. Baker, J. S. Bell and F. Zachariasen, *Phys. Rev. D* **34**, 3894 (1986).
38. A. N. Ivanov *et al.*, *Mod. Phys. Lett. A* **8**, 1021 (1993).
39. A. N. Ivanov *et al.*, *Nuovo Cimento A* **107**, 1667 (1994).
40. A. N. Ivanov *et al.*, *Phys. Lett. B* **336**, 555 (1994).
41. B. Borasoy and B. Holstein, *Eur. Phys. J. C* **6**, 85 (1999).
42. B. Borasoy and B. Holstein, *Phys. Rev. D* **59**, 094025 (1999).
43. B. Borasoy and B. Holstein, *Phys. Rev. D* **60**, 054021 (1999).
44. R. P. Springer, *Phys. Lett. B* **461**, 167 (1999).
45. A. Abd El-Hady and J. Tandean, *Phys. Rev. D* **61**, 114014 (2000).
46. J. Gasser and H. Leutwyler, *Phys. Rep.* **87**, 77 (1982).
47. E. Witten, *Nucl. Phys. B* **160**, 57 (1979).
48. E. Jenkins, *Phys. Lett. B* **315**, 431 (1993).
49. E. Jenkins, *Phys. Lett. B* **315**, 441 (1993).
50. E. Jenkins, *Phys. Lett. B* **315**, 447 (1993).
51. E. Jenkins, *Phys. Rev. D* **53**, 2625 (1996).
52. D. B. Kaplan and A. V. Manohar, *Phys. Rev. D* **56**, 76 (1997).
53. D. B. Kaplan and A. V. Manohar, *Ann. Rev. Nucl. Part. Sci.* **48**, 81 (1998).
54. R. Flores-Mendieta, Ch. P. Hofmann and E. Jenkins, *Phys. Rev. D* **61**, 116014 (2000).
55. R. Flores-Mendieta, Ch. P. Hofmann, E. Jenkins and A. V. Manohar, *Phys. Rev. D* **62**, 034001 (2000).
56. R. Kaiser and H. Leutwyler, *Eur. Phys. J. C* **17**, 623 (2000).
57. M. L. M. Lutz and E. E. Kolomeitsev, *Found. Phys.* **31**, 1671 (2001).
58. E. Jenkins, X. Li and A. V. Manohar, *Phys. Rev. Lett.* **89**, 242001 (2002).