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Center vortex influence on the Dirac spectrum

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We study the influence of center vortices on the low-lying eigenmodes of the Dirac operator, in both the overlap and asqtad formulations. In particular we suggest a solution to a puzzle raised some years ago by Gattnar et al. [Nucl. Phys. B 716, 105 (2005)], who noted the absence of low-lying Dirac eigenmodes required for chiral symmetry breaking in center-projected configurations. We show that the low-lying modes are present in the staggered (asqtad) formulation, but not for overlap, and we argue that this is due to the absence of a field-independent chiral symmetry on the very rough center-projected configurations for ovlap and "chirally improved" fermions. We also confirm and extend the results of Kovalenko et al. [Phys. Lett. B 648, 383 (2007)]: we find strong correlations between center vortex locations, and the scalar density of low-lying Dirac eigenmodes and find that both asqtad and overlap eigenmodes have their largest concentrations in point-like regions, rather than on submanifolds of higher dimensionality.

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1. Introduction

Center vortices explain quark confinement and the Casher argument [1] implies that a force strong enough to confine quarks is also generally expected to break chiral symmetry. The Banks-Casher relation [2] on the other hand, relates chiral symmetry breaking (χSB) with a finite density of near-zero eigenmodes of the chiral-invariant Dirac operator. Several years ago, however, Gattnar et al. [3] reported a puzzling result, concerning the low-lying eigenvalue spectrum of a chirally-improved version of the Dirac operator due to Gattringer [4], which approximates Ginsparg-Wilson fermions. They found a large gap around zero in the spectrum for center-projected configurations, which contain only thin vortex excitations and which *are* confining, implying zero chiral condensate and therefore no χSB. We suggest that this large gap found by Gattnar et al. is related to the way in which chiral symmetry is realized on the lattice. The Casher argument [1] is based on the usual $SU(N_f)_L \times SU(N_f)_R$ symmetry of the continuum theory with massless fermions. Center-projected configurations are, however, maximally discontinous; plaquette variables make a sudden transition from the trivial center element outside the thin vortex, to a non-trivial center element inside. The chirally-improved Dirac operator is not necessarily chirally symmetric, even approximately, in such backgroundsand there is no reason to expect spontaneous symmetry breaking.

We will reinforce these arguments in section 2, looking at the spectra of the overlap [5] and asqtad [13] Dirac operators, when evaluated on normal, vortex-only (i.e. center-projected), and vortex-removed lattices. Our results even support the view that center vortices alone can induce both confinement *and* chiral symmetry breaking.[1] In section 3 we report on other correlations between center-vortex location, and the density distribution of low-lying Dirac eigenmodes, following the earlier work by Kovalenko et al. [9]. These correlations support the picture advocated by Engelhardt and Reinhardt [10], in which topological charge is concentrated at points where vortices either intersect, or twist about themselves ("writhe") in a certain way. Dirac zero modes are concentrated where the topological charge density is large, therefore one would expect that the eigenmode densities of low-lying eigenmodes would be peaked in point-like regions. We provide some supporting evidence for this type of concentration and conclude our results in Section 4. Throughout this article we work with lattices generated by lattice Monte Carlo simulation of the tadpole improved Lüscher-Weisz pure-gauge action, mainly at coupling $\beta_{LW} = 3.3$ (lattice spacing $a = 0.15$ fm) for the $SU(2)$ gauge group [6]. Center-Projection is performed by Direct Maximal Center Gauge (adjoint Landau gauge), maximizing the squared trace of link variables $U_\mu(x)$ by the over-relaxation method. The mapping to link variables on the center-projected (or "vortex-only") lattice, for the $SU(2)$ gauge group, is given by $U_\mu(x) \to Z_\mu(x) = \mathrm{signTr}[U_\mu(x)]$ and the link variables on vortex-removed lattices are defined as $U'_\mu(x) = Z_\mu(x) U_\mu(x)$.

2. Thin Vortices and Near-Zero Modes

In Fig. 1 we present the first twenty overlap eigenvalues for a 16^4 lattice at $\beta_{LW} = 3.3$. There is a big gap around zero for center-projected data, indicating zero chiral condensate. Looking closer at the center-projected eigenvalues one spots only five of the twenty eigenvalues. This indicates a

[1] Similar results have been obtained previously by Gubarev et al. [7] and by Bornyakov et al. [8].

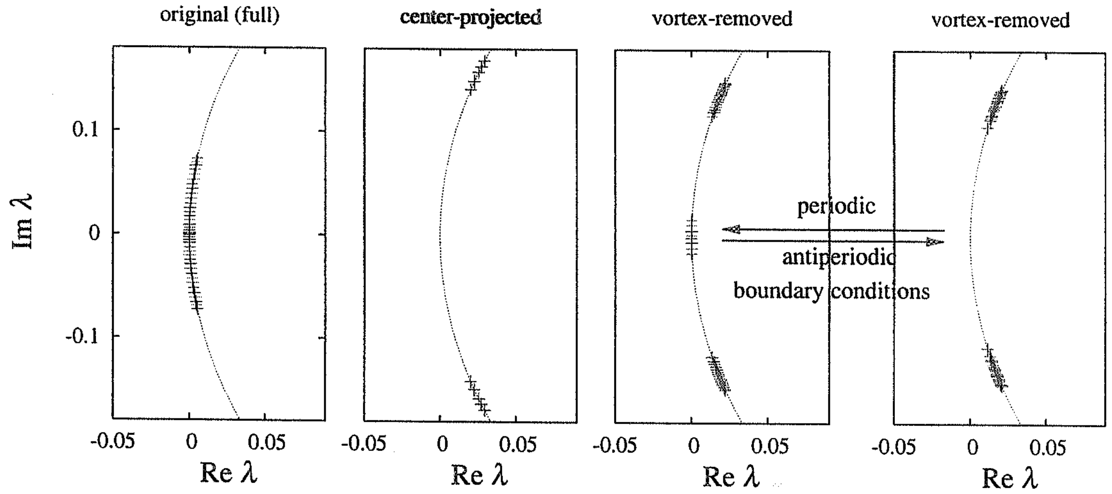

Figure 1: The first twenty overlap Dirac eigenvalue pairs on the Ginsparg-Wilson circle for a 16^4 lattice at $\beta_{LW} = 3.3$. The center-projected configurations show a four-fold degeneracy. Zero-modes in vortex-removed configurations disappear for antiperiodic boundary conditions.

degenerency of four, caused by the real trivial link variables ($\pm\mathbb{1}_2$), where the two colors decouple and the eigenvalue equation $D\psi_n = \lambda_n\psi_n$ is invariant under charge conjugation.[2] The vortex-removed data shows four near-zero modes for each chirality, which can be interpreted as real zero modes since they disappear in case of antiperiodic boundary conditions and therefore are irrelevant to χSB. We speculated that the reason for the large gap in the vortex-only case was connected with the lack of smoothness of center-projected lattices. Of course the overlap operator, in contrast to the chirally-improved operator, *does* have an exact global symmetry, but the symmetry transformations are gauge-field dependent [12], and only approximate the $SU(N_f)_L \times SU(N_f)_R$ chiral symmetry transformations of the continuum theory for configurations which vary slowly at the scale of the lattice spacing. Center-projected configurations are not even close to smooth, and the Casher argument, relating confinement to χSB need not apply. However, the overlap operator should produce a more reasonable answer when applied to a smoother version of the center-projected lattice. Therefore we perform an interpolation between full (gauged) and projected configurations, reducing the angle between the vector representing group element $U_\mu(x)$ in maximal center gauge, and the vector representing the $SU(2)$ center element $Z_\mu(x)I_2$ by some fixed percentage. In Fig. 2 we show the low-lying eigenvalues for partial projections together with the unprojected and center-projected lattices. We see that there is no really obvious gap in the partially-projected lattices, even at 85% projection. This agrees with our conjecture that applying the overlap operator to a smoother version of the vortex-only vacuum would give a result consistent with χSB and the Banks-Casher relation. Staggered and asqtad fermions, on the other hand, do not require a smooth configuration to preserve a subgroup of the usual continuum $SU(N_f)_L \times SU(N_f)_R$ symmetry, and by the Casher argument [1] one would expect this remaining symmetry to be spontaneously broken by any confining gauge configuration. Indeed, ref. [14] already reported that $\langle\bar{\psi}\psi\rangle > 0$ for staggered fermions on a center-projected lattice. Fig. 3 shows the first twenty asqtad eigenvalues, which distribute very differently now. The low eigenmode density (chiral condensate) increases for center-projected compared to

[2]Given that the Dirac operator has the Wilson or overlap (but not staggered) form. Thus, if ψ_n is an eigenstate with eigenvalue λ_n, then $C^{-1}\psi_n^*$ is also an eigenstate, with the same eigenvalue [11].

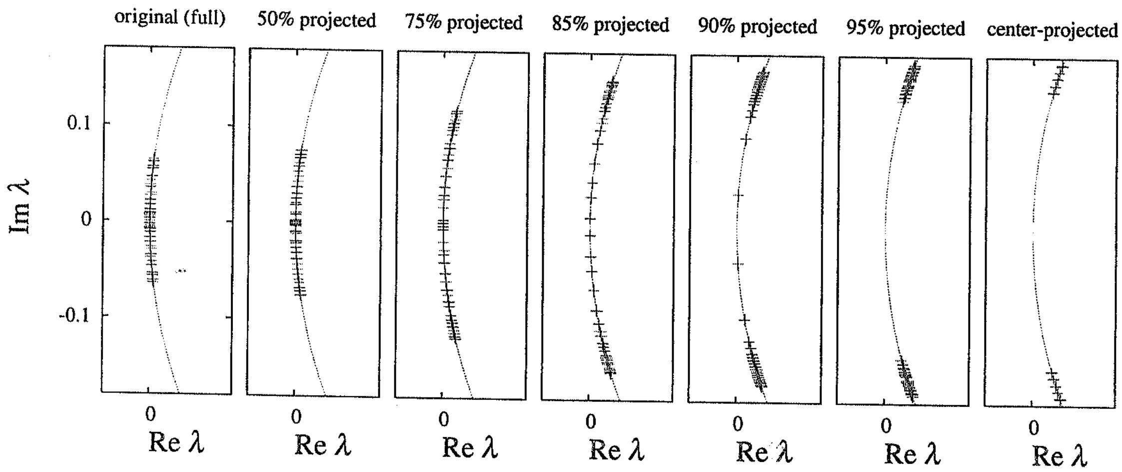

Figure 2: The first twenty overlap Dirac eigenvalue pairs from a single configuration on a 16^4 lattice, antiperiodic boundary conditions at $\beta_{LW} = 3.3$, for interpolated fields.

full (original) data. Thus, for the asqtad operator, we have found exactly what was expected prior to the results of Gattnar et al.: the vortex excitations of the vortex-only lattice carry not only the information about confinement, but are also responsible for χSB via the Banks-Casher relation. The vortex-removed data develops a central band around Im$\lambda = 0$ of eight doubly degenerate eigenmodes per chirality, which are a remanent of the 32 free-field zero modes (four zero modes for each of four "tastes" times two colors), and play no role in χSB. In fact, these modes again disappear using antiperiodic boundary conditions in one direction.

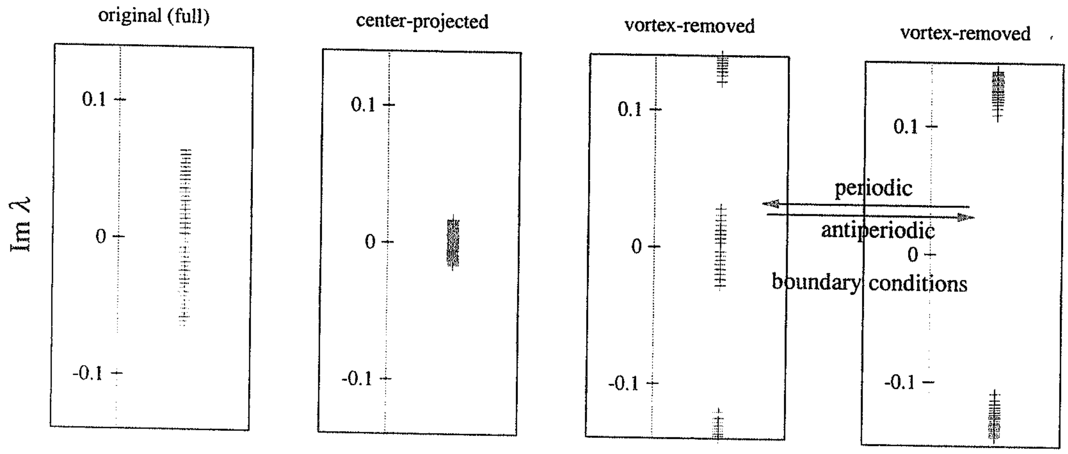

Figure 3: The first twenty asqtad Dirac eigenvalue pairs from a 16^4 lattice at $\beta_{LW} = 3.3$. The center-projected configurations show no gap around zero. Zero-modes in vortex-removed configurations disappear for antiperiodic boundary conditions.

3. Vortex surfaces and Dirac eigenmode densities

In order to clarify the role of the vortices in the topological structure of the vacuum, the correlator C_λ between the density of the eigenmode λ and the vortex surface is investigated. The correlator depends on the eigenvalue and on the local geometry of the vortex. The vortex points P_i live on the dual lattice and they are correlated to the averaged scalar eigenmode density $\rho_\lambda(x)$ over the 16 vertices x of the 4d hypercube, H, dual to P_i. [9]

$$C_\lambda(N_v) = \frac{\sum_{P_i}\sum_{x\in H}(V\rho_\lambda(x)-1)}{\sum_{P_i}\sum_{x\in H}1} \tag{3.1}$$

In Fig. 4 we display the data for $C_\lambda(N_v)$ vs. N_v computed for eigenmodes of the asqtad Dirac operator in the full and center-projected configurations. We find that the values of $C_\lambda(N_v)$ obtained from eigenmodes in the full configurations are only about a factor of four smaller than the corresponding values in the center-projected configurations the figures look much the same. The most important feature, in our opinion, is the fact that the correlator increases steadily with increasing number of the vortex plaquettes N_v, attached to a point P_i where the Dirac eigenmode density seems to be significantly enhanced. This fact is at least compatible with the general picture advanced by Engelhardt and Reinhardt. Our results for eigenmodes of the overlap operator are consistent with the results reported by Kovalenko et al. in Ref. [9].

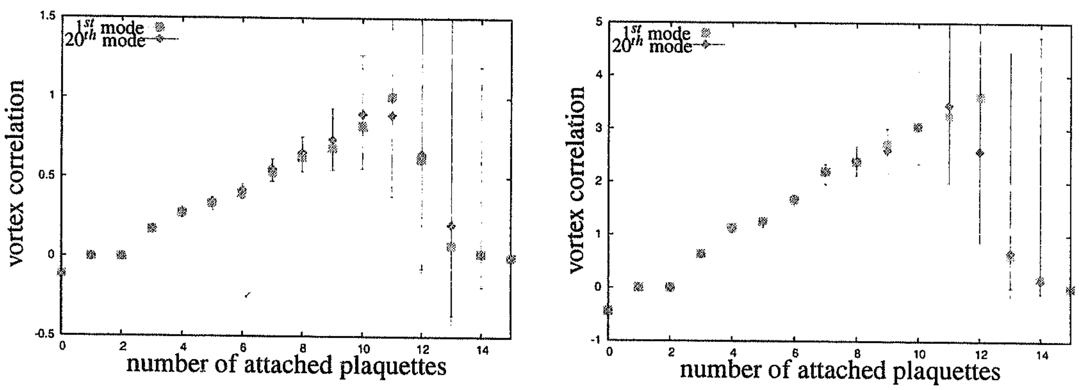

Figure 4: Vortex correlation $C_\lambda(N_v)$ for asqtad staggered eigenmodes on a 20^4 lattice at $\beta_{LW} = 3.3$, full (left) and center-projected (right) configurations.

The correlations provide some degree of evidence that low-lying Dirac eigenmodes concentrate preferentially at regions on the center vortex surface where there are self-intersections or "writhing"-points. So it is natural to ask whether there is any supporting evidence that the eigenmode density is especially concentrated in point-like regions. Therefore we simply *look* at sample plots of $\rho_\lambda(x)$ throughout the lattice volume. In Fig. 5 we display our data for the lowest eigenmode of the asqtad Dirac operator, in some two-dimensional slices of the four-dimensional lattice volume taken in the neighborhood of the point where $\rho_\lambda(x)$ is largest. Each lattice, unprojected (left) and center-projected (right), contains several sharp peaks of this kind; it is obvious that the concentration of eigenmode density is in a point-like region, rather than being spread over a submanifold of higher dimensionality. Figure 6 shows the same type of data for a zero mode of the overlap Dirac operator on 16^4 lattices. For full configurations the eigenmode density again is concentrated in a point-like region, whereas in center-projected configurations, instead of having a sharp peak, the eigenmode concentration is extending over most of the lattice volume.

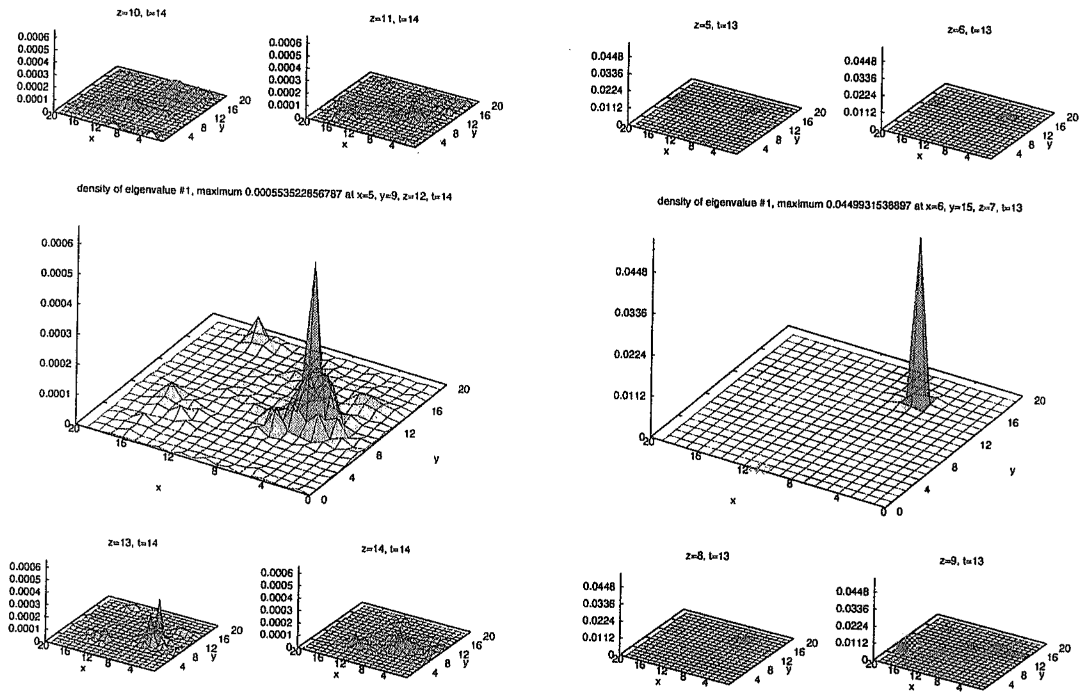

Figure 5: Maximum density peak (center) of the first **asqtad** eigenmode on a 20^4-lattice at $\beta_{LW} = 3.3$ with upper (above) and lower (below) z-slices of the same t-slice. Eigenmodes are computed on (full (left) and center-projected (right) lattices (notice different scales!).

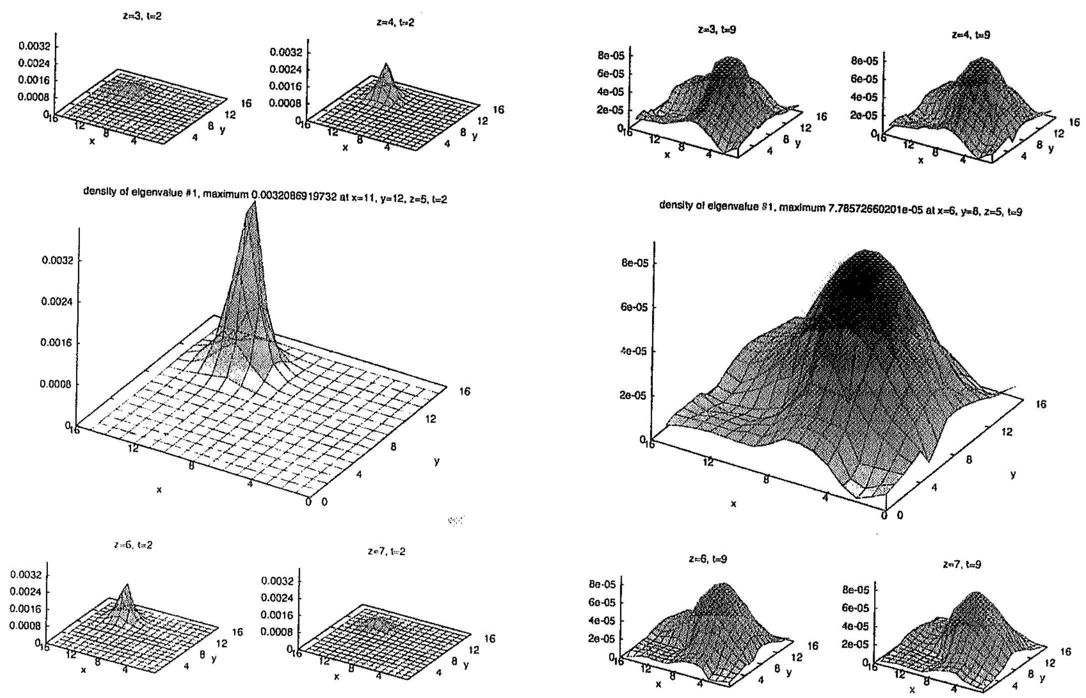

Figure 6: Maximum density peak (center) of the first **overlap** eigenmode on a 16^4-lattice at $\beta_{LW} = 3.3$ with upper (above) and lower (below) z-slices of the same t-slice. Eigenmodes are computed on (a) full lattices, and (b) center-projected lattices (notice different scales!).

4. Conclusions

We find that the thin vortices found in center projection give rise to a low-lying spectrum of Dirac eigenmodes, providing that the chiral symmetry of the Dirac operator does not depend on the smoothness of the lattice configuration. Thus, the vortex excitations of the vortex-only lattice carry not only the information about confinement, but are also responsible for χSB via the Banks-Casher relation. There are significant correlations between center vortices and the low-lying modes of both the asqtad and overlap Dirac operators. These eigenmodes have their largest concentrations in point-like regions, rather than on submanifolds of higher dimensionality. Taken together, correlations and dimensionality support the picture of topological charge from center vortices. Our results indicate that center vortices have a strong effect on the existence and properties of low-lying eigenmodes of the Dirac operator. (C.f. [15])

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