Time-mean description of turbulent bluff-body separation in the high-Reynolds-number limit

B. Scheichl[†⋆], A. Kluwick[†], and F. T. Smith[‡]

[†]Institute of Fluid Mechanics and Heat Transfer, Vienna University of Technology, A-1040 Vienna, Austria,
[⋆]Austrian Center of Competence for Tribology, A-2700 Wiener Neustadt, Austria,
[‡]Department of Mathematics, University College London, London WC1E 6BT, UK
bernhard.scheichl at tuwien.ac.at

1 Motivation

A most reliable prediction of the position of time-mean gross separation of incompressible turbulent boundary layer (BL) flow from the smooth impervious surface of a rigid and more-or-less blunt body not only still defies theoreticians but, needless to say, is also of great interest from an engineering point of view. The undeniable vital progress in computational techniques made in the recent past does not master this challenge presently in the form of sufficiently accurate solutions of the full (unsteady) Navier–Stokes equations. This is largely due to the fact that for practical applications, e.g. in aerodynamics, the relevant Reynolds numbers are still too high to be dealt with adequately.

It is desirable to gain a profound insight into two fundamental aspects, constituting the core problem: (i) the behaviour of the nominally two-dimensional and steady flow in the vicinity of separation, and (ii) how the local theory, describing (i), fits into the global picture of the flow past the obstacle under consideration. In the following, all flow quantities are non-dimensional, respectively, with the speed $\tilde{U}$ of the unperturbed oncoming uniform flow, a typical body dimension $\tilde{L}$, the (constant) kinematic fluid viscosity $\tilde{\nu}$, and the (constant) fluid density. As the basic assumption, the globally formed Reynolds number $Re := \tilde{U}\tilde{L}/\tilde{\nu}$ takes on arbitrarily large values. In the first instance, analytical methods, such as matched asymptotic expansions, then provide the appropriate means of choice to establish a rational theory.

The asymptotic splitting of the initially attached turbulent BL in the limit $Re \to \infty$ that aims at a local description of the separation process has already been tackled by other researchers; for references and discussion of the, as we feel, apparent formal shortcomings of these approaches see [1, 2]. As demonstrated in [3, 4], a fully self-consistent flow structure that provides a match of the BL region with the (asymptotically small) region of pronounced laminar–turbulent transition near the leading edge of the obstacle essentially agrees with the well-known picture of a two-tiered turbulent BL: it consists of a fully turbulent outer main region that exhibits a small defect of the streamwise velocity component u with respect to its value u_e imposed by the external potential flow and where the Reynolds shear stress dominates over its viscous counterpart, and the viscous wall layer, where both stresses are of comparable magnitude. However, here that classical asymptotic structure is relaxed insofar as also to account for "underdeveloped" turbulence, i.e. for a BL having a level of turbulence intensity below that referring to fully developed turbulence.

The asymptotic concept which allows for describing this type of a "transitional" BL on the basis of the time- or Reynolds-averaged Navier–Stokes equations, i.e. the Reynolds equations, was originally proposed in [1]. In its form adopted here, two perturbation parameters are employed: a measure for the velocity defect, ϵ, and one, δ, for the BL thickness. By setting $\gamma := 1/\ln Re$, we deal with "underdeveloped" turbulent flow if $\delta \ll \gamma \lesssim \epsilon \ll 1$, which eventually assumes its fully developed form if both δ and ϵ are of $O(\gamma)$. However, as has been pointed out recently in [3, 4], the process of laminar–turbulent transition provides a source of hampering the BL from becoming a fully developed turbulent one. Self-consistency is confirmed by considering the local asymptotic splitting of the BL close to separation: a rigorous description of the locally strong viscous/inviscid interaction process requires the von Kármán number δ^+, namely, the ratio of the inner and the outer layer thicknesses, to vary merely algebraically with δ rather than exponentially, as in the limiting classical case. The detailed analysis in [4] of a turbulent BL evolving from the leading edge towards the location of separation indicates that the first situation applies by considering a specific distinguished double limit $\epsilon \to 0$, $\delta/\epsilon \to 0$ as $Re \to \infty$, determining the maximum turbulence intensity level possible.

To be more precise, turbulent separation is found to be associated with a quite complex interplay of an "outer" and an "inner" mechanism of local viscous/inviscid interaction. Below we present some recent results regarding the former mechanism, which is of paramount importance for the understanding of the drastic change of the flow in the wall layer as it undergoes separation, governed by the "inner" interaction. Here we only note that the latter gives rise to a novel internal triple-deck structure, detected at the base of the BL.

2 Asymptotic picture of the flow near separation

We analyse the flow in the vicinity of the point $\mathcal{O}$ where in the inviscid limit $Re \to \infty$ the irrotational free-stream flow separates tangentially from the body surface, cf. Fig. 1 (a): here x, y, ψ, p denote natural coordinates along and perpendicular to the body surface, respectively, with origin in $\mathcal{O}$, the stream function, and the pressure. We write $[\psi, p] \sim [\psi_0, p_0](x, y) + O(\epsilon\delta)$, $Re \to \infty$, so that the subscript 0 indicates the inviscid-flow limit. It is characterised by the free streamline $\mathcal{S}$ that separates the oncoming irrotational flow from a cavity or an inviscid backflow eddy in the slipstream of the body for $x > 0$.

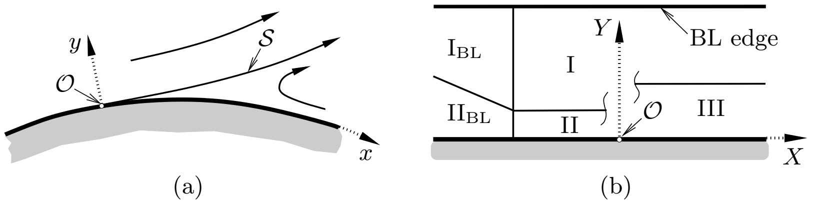

Fig. 1. (a) global situation: arrows on streamlines (*solid*) indicate flow direction, (b) local splitting of small-defect tier: regions $\mathrm{I_{BL}}$, $\mathrm{II_{BL}}$ refer to incident BL flow.

Close to $\mathcal{O}$, the potential flow is conveniently described in terms of polar coordinates $r := (x^2 + y^2)^{1/2}$, $\theta := \arctan(y/x)$, $\pi \geq \theta > 0$, cf. [3, 4]:

$$\psi_0/u_{\mathcal{O}} \sim r\sin\theta + \hat{\psi}(r,\theta) + O(r^2)\ , \quad \hat{\psi} := -(4k/3)r^{3/2}\cos(3\theta/2)\ , \quad r \to 0\ . \quad (1)$$

Herein, $u_{\mathcal{O}}$ denotes the value of the surface speed $u_e = \partial_y\psi_0|_{y=0}$ at $\mathcal{O}$, and the value of the positive parameter k depends on the a priori unknown position of $\mathcal{O}$ and measures the strength of the well-known Brillouin–Villat (BV) singularity there, expressed by (1). Accordingly, $u_e/u_{\mathcal{O}} \sim 1 + 2k(-x)^{1/2} + O(-x)$, $x \to 0_-$. For $x \to 0_+$, the expansion (1) breaks down passively for $y = O(y_{\mathcal{S}})$ such that $x, y_{\mathcal{S}} \sim (4k/3)x^{3/2} + O(x^{5/2})$ gives the position of $\mathcal{S}$ where $\psi_0 = 0$.

For the subsequent considerations we refer to Fig. 1 (b). The analysis carried out in [4] indicates that in the outer tier of the BL $\partial_y p_0$ and $\partial_x p_0$ become of the same order of magnitude when $x = O(\delta)$ and, consequently, the conventional BL approximation ceases to be valid. In addition, the BL thickness is found to remain of $O(\delta)$ there (region I). We introduce suitable local variables $[X, Y, \Psi] := [x, y, \psi/u_{\mathcal{O}}]/\delta$. Inspection of the Reynolds equations and matching the asymptotic representations of the velocity defect in the oncoming BL and in region I then shows that in the latter the expansion

$$\Psi \sim Y + \delta^{1/2}\Psi_{\mathrm{ir},1}(X,Y) + \epsilon\Psi_{\mathrm{BL}}(Y) + \delta\Psi_{\mathrm{ir},2}(X,Y) + \epsilon\delta^{1/2}\Psi_{\mathrm{rot}}(X,Y) + O(\epsilon\delta) \quad (2)$$

holds. It describes a predominantly inviscid flow: the Reynolds stresses are of $O(\epsilon\delta)$ and thus only affect terms of the same magnitude in (2). Furthermore, the subscripts "ir" and "rot" in expansion (2) refer to contributions that account for, respectively, the imposed irrotational flow and induced rotational flow: the latter results from the interaction of the external potential flow with the locally "frozen" velocity defect of the oncoming BL, captured by the term of $O(\epsilon)$ in (2). It comprises the celebrated logarithmic law of the wall,

$$\Psi_{\mathrm{BL}} \sim \kappa^{-1} Y \ln Y + cY + O(Y^2 \ln Y)\ , \quad Y \to 0\ , \quad (3)$$

where κ denotes the von Kármán and c a further (flow-dependent) constant.

By substitution of (2) into the Reynolds equations and elimination of the pressure in standard manner one readily obtains $\Delta\Psi_{\mathrm{ir},1} = 0$, $\Delta := \partial_{XX} + \partial_{YY}$. Let $\Psi^*(X,Y) := \Psi_{\mathrm{ir},1} - \hat{\psi}(R,\theta)$, $R := (X^2 + Y^2)^{1/2}$, with $\theta = \arctan(Y/X)$ according to (1), such that $\Delta\Psi^* = 0$. Hence, matching with the attached portion of the near-wall flow demands $\Psi^*(X,0)=0$ for $X \le 0$. Moreover, $\Psi^* = o(R^{3/2})$ as $R \to 0$, since the BV singularity can only be avoided by taking into account viscous/inviscid interaction if the representation of the Euler flow near $\mathcal{O}$ is not more singular than in the non-interactive case. Finally, we consider the irrotational velocity perturbations provoked in the ambient free-stream flow: consistency of their pressure feedback in the oncoming BL with the original small-defect structure of the latter requires $\delta^{3/2}\Psi^* = o(\delta^2)$ for $r = O(1)$, giving $\Psi^* = o(R^{-1/2})$ as $R \to \infty$. One then infers from the above properties of Ψ^*, by adopting methods of potential theory, that $\Psi^* \equiv 0$, i.e. $\Psi_{\mathrm{ir},1} = \hat{\psi}(R,\theta)$. In turn, exploitation of the Reynolds equations and the conditions of matching with the oncoming flow yields the crucial result that the turbulence-induced inviscid vortex flow is governed by the Poisson equation

$$\Delta\Psi_{\mathrm{rot}} = \Psi_{\mathrm{BL}}'''(Y)\,\hat{\psi}(R,\theta)\;, \quad Y > 0\;. \tag{4}$$

The investigation of (4) reveals a contribution adding to the logarithmic portion of the velocity profile given by (3) upstream of separation, which is superseded by a stronger singular behaviour immediately downstream:

$$Y \to 0: \quad \Psi_{\mathrm{rot}} \sim -\frac{2k}{3\kappa}\begin{cases} 3(-X)^{1/2}\,Y\ln Y + O(Y)\;, & X < 0\;, \\ 2X^{3/2}\ln Y + O(Y\ln Y)\;, & X > 0\;. \end{cases} \tag{5}$$

For $X < 0$, in the wall layer u varies quite rapidly with u_e according to $u/u_e \sim \epsilon u^+(y^+) = O(1)$, $y^+ := Y/\delta^+$, with $u^+ \sim \kappa^{-1}\ln y^+ + O(1)$, $y^+ \to \infty$; see [4]. The apparent mismatch with region I requires the introduction of the sublayer II where $Y = O(\delta)$. On the other hand, for $X > 0$ the separated-flow region III where $Y = O(\delta^{1/2})$ has to be considered. Note that the asymptotic structure outlined so far closely resembles that of the turbulent BL flow past the trailing edge of an inclined flat plate in uniform stream, studied first in [5].

3 Current research and further outlook

The gradual transition between the two limiting forms given in (5) is analysed by considering the behaviour of Ψ_{rot} in the limit $R \to 0$, θ kept fixed. Once found, this determines the extent of a further region close to $\mathcal{O}$, which is of salient importance for the understanding of the conversion of region II into region III and accounts for the "inner" (nonlinear) interaction process. Most important, the latter is expected to fix the dependence of ϵ and δ on Re.

References

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