FUNCTIONAL ANALYTIC FRAMEWORK FOR A REDUCED MODEL IN THIN-FILM MICROMAGNETICS

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Abstract

The steady state of a magnetization $\mathbf{M}$ of a ferromagnetic sample Ω was first described by Landau and Lifschitz as the solution of a certain minimization problem, which is nowadays accepted as the relevant model to describe micromagnetic phenomena. For uniaxial material with easy axis e_1, the free micromagnetic energy reads

$$E(\mathbf{M}) = d^2 \int_\Omega |\nabla \mathbf{M}|^2 \, dx + Q \int_\Omega (\mathbf{M}_2^2 + \mathbf{M}_3^2) \, dx + \int_{\mathbb{R}^3} |\nabla U|^2 \, dx - 2 \int_\Omega \mathbf{F} \cdot \mathbf{M} \, dx. \tag{1}$$

Here, $\nabla \mathbf{M}$ denotes the Jacobian of $\mathbf{M}$ and $|\cdot|$ the Euclidian norm of the matrix $\nabla \mathbf{M}$ and the vector ∇U, respectively. The problem of micromagnetics is to find a local minimizer $\mathbf{M}^*$ of E which satisfies the non-convex constraint $|\mathbf{M}| = 1$.

The first term of the energy penalizes spatial variations of $\mathbf{M}$. The second energy term is related to the crystalline structure of the ferromagnetic material and favors magnetizations which are aligned with the easy axis e_1. The third term, called the stray-field energy, involves the magnetostatic potential U which solves the Maxwell equation

$$\operatorname{div}(-\nabla U + \chi_\Omega \mathbf{M}) = 0 \in \mathcal{D}'(\mathbb{R}^3) \tag{2}$$

stated here in distributional form. The fourth energy contribution favors alignment of the magnetization with an applied exterior field $\mathbf{F}$. Finally, the constants $d, Q > 0$ are material dependant parameters.

The full micromagnetic problem is, from a numerical point of view, quite complex. Besides the non-convex contraint $|\mathbf{M}| = 1$, the minimization problem is also non-local and the presence of various length scales, which have to be resolved, make the simulation very expensive from a computational point of view. In [3], a reduced model for thin-film micromagnetics has been introduced and mathematically studied. It is consistent with the prior works [1] and [4]. In contrast to [3], where the focus is on a distributional point of view, we state a proper functional analytic framework which is more suitable to develop a sophisticated numerical analysis and to discuss discretization schemes. Well-posedness of the model problem in this setting is studied and also certain uniqueness results are proven. First numerical experiments conclude the talk.

References

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