A MULTIPLICATIVE WEIGHT PERTURBATION SCHEME FOR DISTRIBUTED BEAMFORMING IN WIRELESS RELAY NETWORKS WITH 1-BIT FEEDBACK

Peter Fertl[*], *Ari Hottinen*[†], *and Gerald Matz*[*]

[*]Institute of Communications and Radio-Frequency Engineering, Vienna University of Technology
Gusshausstrasse 25/389, A-1040 Vienna, Austria, email: {pfertl,gmatz}@nt.tuwien.ac.at

[†]Nokia Research Center, P.O. Box 407, FIN-00045 Nokia Group, Finland, email: ari.hottinen@nokia.com

ABSTRACT

This paper focuses on perturbation-based distributed beamforming with 1-bit feedback for wireless amplify-and-forward relay networks. We propose to use multiplicative perturbations based on Givens rotations to adapt the beamforming weights while guaranteeing a sum power constraint for the relays. This perturbation scheme is shown to be computationally efficient and easy to design, thus allowing for low-complexity relay nodes. An adaptation of the Givens rotation angle allows to approach optimum performance arbitrarily close. Numerical simulations demonstrate noticeable performance gains over additive perturbation schemes that have been exclusively considered up to now.

***Index Terms*—** relay network, amplify-and-forward, distributed beamforming, Givens rotation, feedback

1. INTRODUCTION

Motivation. Distributed beamforming with half-duplex amplify-and-forward (AF) relays has recently attracted much attention due to its ability to exploit spatial diversity in a distributed fashion [1–3]. However, this approach imposes stringent requirements on the availability of channel state information (CSI) at the relay nodes; either global CSI (i.e., all channels) or local CSI (i.e., each relay's own backward and forward channel) is required. The requirement for CSI at the relays can be avoided by using feedback from the destination to the relays in order to adaptively adjust the beamforming weights. Perturbation-based beamforming (PB-BF) is a well known example for such an approach, applicable to centralized arrays with co-located antennas [4, 5]. In [6], we proposed deterministic, additive vector perturbations and 1-bit feedback to extend PB-BF to relay networks. This scheme has the potential to approach optimum performance without any CSI at the relays. However, to satisfy a sum power constraint each update of the beamforming weights involves a vector normalization at each relay.

Contributions and Paper Organization. In this paper, we propose multiplicative perturbations in terms of elementary Givens rotations [7] for PB-BF in wireless relay networks. The relays receive 1-bit feedback from the destination to adapt their beamforming weights with the goal of maximizing the signal-to-noise ratio (SNR) at the destination. Our multiplicative perturbation scheme has the advantage that it inherently maintains constant sum power and has a computational complexity at each relay which is independent of the number of relays in the network. This is in striking contrast to additive vector perturbation techniques (cf. [6]). We further show how the adaptation behavior of our proposed scheme can be controlled in an intuitive manner and provide numerical results that illustrate the performance of our method in comparison to additive PB-BF.

The rest of the paper is organized as follows. Section 2 provides the background for distributed beamforming in relay networks. The proposed multiplicative perturbation scheme is introduced and discussed in detail in Section 3. Simulation results are shown in Section 4. Finally, conclusions are given in Section 5.

2. NETWORK BEAMFORMING

System Model. We consider a half-duplex wireless relay network with single antenna nodes where a single source $\mathcal{S}$ communicates with a single destination $\mathcal{D}$ via R amplify-and-forward relays $\mathcal{R}_i$, $i = 1, \ldots, R$ (cf. Fig. 1). There is no direct link between $\mathcal{S}$ and $\mathcal{D}$, and we assume perfect synchronization among the nodes. The half-duplex constraint necessitates transmission in two hops. In the first hop, $\mathcal{S}$ transmits the signal $\sqrt{P_\mathcal{S}}\, s$ to the relays which receive

$$x_i = \sqrt{P_\mathcal{S}}\, h_i s + w_i. \tag{1}$$

Here, s is the transmit symbol normalized as $\mathbb{E}\{|s|^2\} = 1$ ($\mathbb{E}\{\cdot\}$ denotes expectation), $P_\mathcal{S}$ denotes the average transmit power of the source $\mathcal{S}$, h_i is the complex fading coefficient of the "backward" channel[1], and $w_i \sim \mathcal{CN}(0, N_0)$ denotes i.i.d. circularly complex Gaussian noise. In the second hop, each relay applies a complex beamforming weight α_i to the signal it has received and forwards

$$r_i = \sqrt{\frac{P}{P_{x_i}}}\, \alpha_i^*\, x_i\,, \tag{2}$$

to the destination $\mathcal{D}$; here, $P_{x_i} = \mathbb{E}\{|x_i|^2 | h_i\}$ is the average receive power at relay $\mathcal{R}_i$ and complex conjugation (superscript *) will simplify notation later on. Throughout this paper, we assume a sum power constraint $\sum_{i=1}^R \mathbb{E}\{|r_i|^2 | h_i\} = P$. With the power normalization in (2), this requires the beamforming vector $\boldsymbol{\alpha} = (\alpha_1 \ldots \alpha_R)^T$ to have unit Euclidean norm, $\|\boldsymbol{\alpha}\| = 1$. The destination receives $y = \sum_{i=1}^R g_i r_i + v$, where g_i denotes the complex fading coefficient of the "forward" channel between $\mathcal{R}_i$ and $\mathcal{D}$ and $v \sim \mathcal{CN}(0, N_0)$ is circularly complex Gaussian noise. Combining this with (1) and (2) yields the compound channel model[2]

$$y = \xi s + \eta, \quad \text{with } \xi \triangleq \boldsymbol{\alpha}^H \bar{\mathbf{h}}, \quad \eta \triangleq \boldsymbol{\alpha}^H \bar{\mathbf{G}}\mathbf{w} + v. \tag{3}$$

In this expression, $\bar{\mathbf{h}} \triangleq (\bar{h}_1 \ldots \bar{h}_R)^T$ with $\bar{h}_i \triangleq h_i g_i \sqrt{P_\mathcal{S} P / P_{x_i}}$, $\bar{\mathbf{G}} \triangleq \text{diag}(\bar{g}_1, \ldots, \bar{g}_R)$ with $\bar{g}_i \triangleq g_i \sqrt{P/P_{x_i}}$, and $\mathbf{w} \triangleq (w_1 \ldots w_R)^T$. Note that the beamforming vector $\boldsymbol{\alpha}$ not only affects the effective channel gain ξ in (3) but also the noise η.

This work was supported by the STREP project MASCOT (IST-026905) within the Sixth Framework Programme of the European Commission and by the FWF project "Information Networks" (S10606-N13) within the National Research Network SISE.

[1]Note that we do not assume a specific channel statistics.

[2]Superscript ^{T} (^{H}) denotes (Hermitian) transposition; $\text{diag}(x_1, \ldots, x_m)$ is the $m \times m$ diagonal matrix with diagonal elements $x_1, \ldots, x_m$.

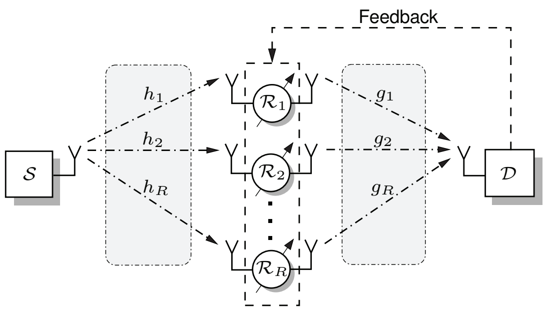


Fig. 1. Wireless relay network with feedback.

The receive SNR at the destination $\mathcal{D}$ can be obtained from (3) as ($\mathbf{I}$ is the identity matrix)

$$\rho(\boldsymbol{\alpha}) \triangleq \frac{\mathbb{E}\{|\xi s|^2 | \bar{\mathbf{h}}\}}{\mathbb{E}\{|\eta|^2 | \bar{\mathbf{G}}\}} = \frac{1}{N_0} \frac{|\boldsymbol{\alpha}^H \bar{\mathbf{h}}|^2}{\|(\mathbf{I}+\bar{\mathbf{G}}\bar{\mathbf{G}}^H)^{1/2}\boldsymbol{\alpha}\|^2}. \quad (4)$$

Note that the SNR is invariant to a common phase factor, i.e., $\rho(\boldsymbol{\alpha}) = \rho(e^{j\psi}\boldsymbol{\alpha})$. Up to this phase ambiguity, $\rho(\boldsymbol{\alpha})$ can be shown to have a unique global maximum $\rho_{\max}$ (and no local maxima). The beamforming weights $\boldsymbol{\alpha}$ can be designed to maximize the received SNR $\rho(\boldsymbol{\alpha})$ in (4) subject to the sum power constraint $\|\boldsymbol{\alpha}\|^2 = 1$. This leads to the optimum beamforming vector [1, 2]

$$\boldsymbol{\alpha}_{\text{opt}} = \frac{(\mathbf{I}+\bar{\mathbf{G}}\bar{\mathbf{G}}^H)^{-1}\bar{\mathbf{h}}}{\|(\mathbf{I}+\bar{\mathbf{G}}\bar{\mathbf{G}}^H)^{-1}\bar{\mathbf{h}}\|}, \quad (5)$$

which is unique up to a phase factor. However, calculating the optimum beamforming weights can be shown to require either global CSI at all relays or local CSI for each relay and global CSI at $\mathcal{D}$ with feedback of a scalar normalization factor to the relays.

Perturbation-based Beamforming. Motivated by beamforming techniques for co-located arrays [4, 5], we proposed distributed BF for relay networks based on *additive* weight perturbations in [6]. These techniques circumvent the need for CSI at the relays and allow to approach the maximum of the objective function[3] $\rho(\boldsymbol{\alpha})$.

We next briefly review the transmission principle, termed *take/reject (T/R) perturbation* (see [6] for more details). The idea is to approximate the optimum beamforming vector $\boldsymbol{\alpha}_{\text{opt}}$ by iteratively updating the beamforming weights at the relays according to 1-bit feedback provided by the destination. To this end, the source transmits frames that consist of two training blocks $\tilde{\mathcal{B}}_k^{(\text{p})}$ and $\mathcal{B}_k^{(\text{p})}$, and a data block $\mathcal{B}_k^{(\text{d})}$ (k is the frame index). The relays forward these frames according to (2), using the currently best beamforming vector (denoted $\boldsymbol{\alpha}_k$) for $\mathcal{B}_k^{(\text{p})}$ and $\mathcal{B}_k^{(\text{d})}$, while using a perturbed version $\tilde{\boldsymbol{\alpha}}_k$ for the training portion $\tilde{\mathcal{B}}_k^{(\text{p})}$. Up to now, only the following additive perturbations have been considered (cf. [6]):

$$\tilde{\boldsymbol{\alpha}}_k = \frac{\boldsymbol{\alpha}_k + \mu\, \mathbf{q}_{k \bmod \tilde{N}}}{\|\boldsymbol{\alpha}_k + \mu\, \mathbf{q}_{k \bmod \tilde{N}}\|}. \quad (6)$$

Here, μ is a step-size parameter and $\mathbf{q}_n$ denotes the additive perturbation vector taken cyclically from a deterministic $R \times \tilde{N}$ matrix $(\mathbf{q}_0 \ldots \mathbf{q}_{\tilde{N}-1})$ with $\tilde{N} \geq 4R$ [6]. Note that (6) involves a normalization which is necessary to satisfy the sum power constraint and requires that all beamforming weights are tracked at each relay. This implies that the computation of (6) in general needs $10R$ real flops and one square root operation per relay.

[3]Note that the receive power $P_{\mathcal{D}}(\boldsymbol{\alpha}) \triangleq \mathbb{E}\{|\xi s|^2 | \bar{\mathbf{h}}\} = |\boldsymbol{\alpha}^H \bar{\mathbf{h}}|^2$ can as well be used as objective function (cf. [6]).

The destination evaluates the effectiveness of the beamforming weights $\boldsymbol{\alpha}_k$ and $\tilde{\boldsymbol{\alpha}}_k$ with regard to the objective function $\rho(\boldsymbol{\alpha})$ within $\mathcal{B}_k^{(\text{p})}$ and $\tilde{\mathcal{B}}_k^{(\text{p})}$, respectively. It then provides a single bit c_k of feedback to the relays, indicating which weights perform better, i.e., $c_k = 0$ if $\rho(\boldsymbol{\alpha}_k) \geq \rho(\tilde{\boldsymbol{\alpha}}_k)$ and $c_k = 1$ if $\rho(\boldsymbol{\alpha}_k) < \rho(\tilde{\boldsymbol{\alpha}}_k)$. The relays update the beamforming vector $\boldsymbol{\alpha}_{k+1}$ to be used in the next frame according to $\boldsymbol{\alpha}_{k+1} = \boldsymbol{\alpha}_k$ ("reject" $\tilde{\boldsymbol{\alpha}}_k$) if $c_k = 0$ and $\boldsymbol{\alpha}_{k+1} = \tilde{\boldsymbol{\alpha}}_k$ ("take" $\tilde{\boldsymbol{\alpha}}_k$) if $c_k = 1$. The new vector $\boldsymbol{\alpha}_{k+1}$ will then be the basis for the next perturbation and the whole process continues in an iterative manner. The first frame is initialized by setting $\rho_0 = 0$ and $\tilde{\boldsymbol{\alpha}}_0 = \boldsymbol{\alpha}_0$, where $\boldsymbol{\alpha}_0$ can be an arbitrary vector with $\|\boldsymbol{\alpha}_0\|^2 = 1$.

3. PROPOSED PERTURBATION SCHEME

Beamforming Manifold. This section proposes to replace (6) with a multiplicative weight perturbation scheme. This is motivated by the fact that the sum power constraint $\|\boldsymbol{\alpha}\|^2 = 1$ implies that the real-valued representation $(\text{Re}\{\boldsymbol{\alpha}^T\}\ \text{Im}\{\boldsymbol{\alpha}^T\})^T$ of admissible beamforming vectors lies on a $(2R-1)$-dimensional hypersphere in the $2R$-dimensional (real) Euclidean space. In addition, the phase invariance of our objective function means that there are disjoint one-dimensional equivalence classes of beamforming vectors within which the SNR $\rho(\boldsymbol{\alpha})$ remains constant. It is sufficient to consider only one representative of each equivalence class, which reduces the number of degrees of freedom by one. Without loss of generality, we choose this representative to be $\boldsymbol{\alpha}' = e^{-j\arg(\alpha_R)}\boldsymbol{\alpha}$ such that $\text{Re}\{\alpha'_R\} = |\alpha_R| \geq 0$ and $\text{Im}\{\alpha'_R\} = 0$. In the following, we thus restrict to the beamforming vectors

$$\mathbf{a} = \big(\text{Re}\{\boldsymbol{\alpha}^T\}\ \text{Im}\{\alpha_1\}\ \ldots\ \text{Im}\{\alpha_{R-1}\}\big)^T$$

which have length $\bar{R} = 2R-1$ but, due to the constraint $\|\mathbf{a}\|^2 = 1$, lie on a $(2R-2)$-dimensional hyper-hemisphere $\mathcal{H}$. Note that

$$\boldsymbol{\alpha} = (a_1 \ldots a_R)^T + j(a_{R+1} \ldots a_{2R-1}\ 0)^T.$$

Rewriting the cost function $\rho(\boldsymbol{\alpha})$ in terms of $\mathbf{a}$ reveals that it has a unique global maximum on $\mathcal{H}$, i.e., without phase ambiguity.

As compared to the Euclidean perspective underlying (6), we have reduced the problem dimension by two. Furthermore, (6) is intended to approximate Euclidean-space steepest ascent (gradient) techniques but does not account for the manifold structure of the hypersphere (this necessitates the renormalization). Specifically, the natural notion of a translation on the hypersphere is *rotation*, amounting to *multiplication* by a matrix $\mathbf{Q}$ belonging to the *special orthogonal group* $\text{SO}(\bar{R})$ [8], defined by $\mathbf{Q}^T\mathbf{Q} = \mathbf{I}$ and $\det(\mathbf{Q}) = 1$.

Multiplicative Perturbation. In the light of the foregoing discussion we propose to replace the additive perturbation (6) with

$$\tilde{\mathbf{a}}_k = \mathbf{Q}_{k \bmod N}\, \mathbf{a}_k\,, \quad (7)$$

where $\mathbf{Q}_n \in \text{SO}(\bar{R})$ denotes orthogonal matrices cyclically taken from an appropriately chosen set $\mathcal{I} \triangleq \{\mathbf{Q}_0, \ldots, \mathbf{Q}_{N-1}\}$ of size $|\mathcal{I}| = N$. The set $\mathcal{I}$ is known to all relays so that each relay can keep track of all beamforming weights. Note that by construction $\|\tilde{\mathbf{a}}_k\| = \|\mathbf{a}_k\| = 1$. Particularly simple and useful examples for orthogonal matrices are *Givens rotations* [7] by an angle $\phi \in (-\pi, \pi]$ within the (l, m)-plane (with $l \neq m$):

$$\boldsymbol{\Gamma}_{lm}(\phi) = (\mathbf{e}_l\, \mathbf{e}_m)\begin{pmatrix} c & s \\ -s & c \end{pmatrix}(\mathbf{e}_l\, \mathbf{e}_m)^T + \Big[\mathbf{I} - (\mathbf{e}_l\, \mathbf{e}_m)(\mathbf{e}_l\, \mathbf{e}_m)^T\Big], \quad (8)$$

where $c = \cos(\phi)$, $s = \sin(\phi)$, and $\mathbf{e}_l$ denotes the lth canonical unit vector. Applying $\boldsymbol{\Gamma}_{lm}(\phi)$ to a vector performs a clockwise rotation of the lth and mth element by the angle ϕ (first term in (8)), while all other elements remain unaffected (last term in (8)).

For any given initial vector $\mathbf{a}_0$, there is an orthogonal matrix $\mathbf{Q}'$ that rotates $\mathbf{a}_0$ into the optimum beamforming vector $\mathbf{a}_{\text{opt}}$ in one step. This matrix can be factored into $\bar{R}-1$ Givens rotations [7] as

$$\mathbf{Q}' = \prod_{l=1}^{\bar{R}-1} \mathbf{\Gamma}_{l,l+1}(\phi_l), \qquad (9)$$

with properly chosen angles ϕ_l. In our distributed beamforming setup, the angles ϕ_l and hence $\mathbf{Q}'$ is not available. Nonetheless, it appears promising to perform the multiplicative perturbation (7) using a set $\mathcal{I}$ consisting of appropriately chosen Givens rotations.

Givens Perturbations. We next show that perturbations based on Givens rotations have the advantage of being intuitive, computationally efficient, and simple to design.

Intuition. As argued previously, rotations are the natural translation on the hyper-(hemi)sphere $\mathcal{H}$ and thus more intuitive than additive perturbations. Specifically, additive perturbations can have arbitrary orientation relative to $\mathcal{H}$, thereby hindering an interpretation of the parameter μ in (6) as step size. In the extreme case where $\mathbf{q}_{k \bmod \tilde{N}} = \boldsymbol{\alpha}_k$, the perturbation is orthogonal to $\mathcal{H}$ at $\boldsymbol{\alpha}_k$, resulting in $\tilde{\boldsymbol{\alpha}}_k = \boldsymbol{\alpha}_k$, i.e., no perturbation at all. In contrast, the angle of the Givens rotation gives a clear indication of the amount of perturbation on $\mathcal{H}$. Fig. 2 illustrates this behavior in two dimensions. Starting from the initial vector $\mathbf{a}_0$, the Givens perturbations continually rotate the beamforming vector (marked with bullets) closer to the optimum weights $\mathbf{a}_{\text{opt}}$ while retaining the sum power constraint. In contrast, additive perturbations (marked with crosses) suffer from strongly varying step sizes (e.g., in the first perturbation) and require normalization.

Design. The action of Givens rotations is geometrically intuitive and simplifies the design of the set $\mathcal{I}$. In particular, any Givens rotation $\mathbf{\Gamma}_{lm}(\phi)$ is completely specified in terms of the index pair (l, m) and the angle ϕ. Thus, instead of specifying the set $\mathcal{I}$ in terms of N orthogonal matrices of dimension $\bar{R} \times \bar{R}$, it is sufficient to specify the corresponding N index pairs and angles. For the moment, we consider a fixed choice of the rotation angle. Then, there are $\bar{R}(\bar{R}-1)/2$ different index pairs and corresponding rotation planes in total. However, following (9), the minimum number of rotation planes is given by $\bar{R}-1$ (in this case, each index has to occur at least once in the list). For our T/R scheme, we have to allow for clockwise and counter-clockwise rotations within each rotation plane, yielding a set $\mathcal{I}$ of maximum size $N = \bar{R}(\bar{R}-1)$ and minimum size $N = 2(\bar{R}-1)$. Note that counter-clockwise rotations can be achieved by swapping indices, i.e., $\mathbf{\Gamma}_{ml}(\phi) = \mathbf{\Gamma}_{lm}(-\phi)$. Choosing a large perturbation set increases the chance of picking a rotation plane that allows a perturbation within the direction of the steepest gradient; however, it potentially requires more trials until the "right" rotation plane is getting used. With small N, each rotation plane is tested more frequently but certain rotations not available within the perturbation set can only be approximated over several iterations.

Complexity. Any Givens rotation involves only two elements of $\mathbf{a}_k$, i.e., $\tilde{a}_{k,l} = c\, a_{k,l} - s\, a_{k,m}$, $\tilde{a}_{k,m} = s\, a_{k,l} + c\, a_{k,m}$, and $\tilde{a}_{k,i} = a_{k,i}$ for $i \neq l, m$. This means that the beamforming weights of at most two relays are updated within each iteration, with each update requiring only 6 flops per relay. In contrast to additive perturbation, the complexity per relay of our multiplicative perturbation scheme thus is *independent of the number of relays*. Furthermore, similarities between our perturbation scheme and CORDIC algorithms [9] can be exploited to reduce complexity even further via appropriate choice of the rotation angle.

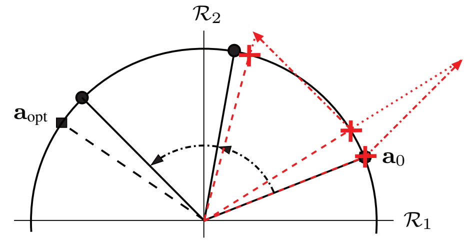


Fig. 2. Example for perturbations in two dimensions under a unit-length constraint (bullets: multiplicative, crosses: additive).

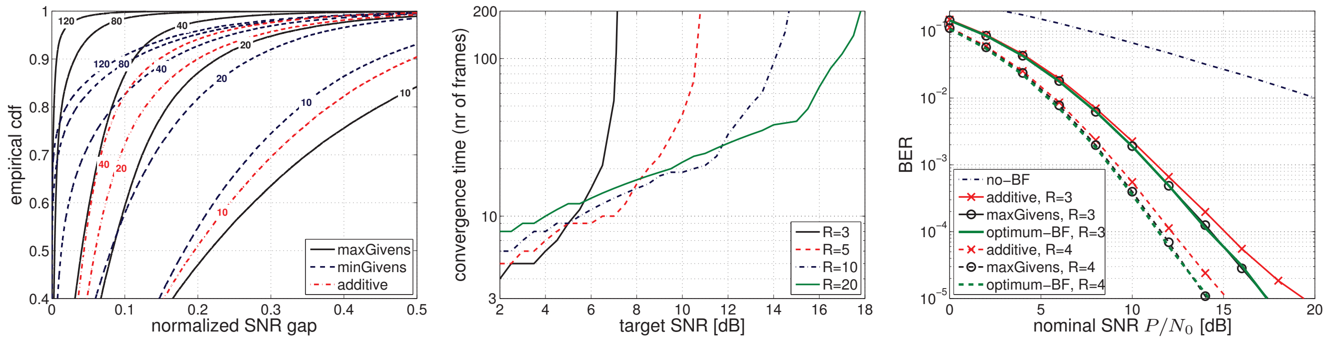
Angle Adaptation. In the following, we present a modification of our Givens rotations based perturbation scheme which is motivated by the fact that the method will get stuck as soon as the angular distance between the current beamforming vector $\mathbf{a}_k$ and the optimum vector $\mathbf{a}_{\text{opt}}$ is less than the fixed angle ϕ. In this case none of the available rotations further improves the objective function. An obvious way to evade this deadlock is a reduction of the rotation angle. Specifically, we propose that all relays count the number of successive Givens perturbations that have been rejected by the destination because they did not improve $\rho(\boldsymbol{\alpha})$; this essentially amounts to accumulating the number of successive feedback bits equal to zero. Whenever this number is larger than a certain integer $M \leq N$, all relays switch to a smaller angle (e.g., according to $\phi \leftarrow \gamma\, \phi$, $\gamma < 1$). It may be advantageous to shrink the rotation angle even before all available rotations have been rejected. Note that the perturbation index set $\mathcal{I}$ remains unchanged, however. It follows from the properties of the objective function that our Givens perturbation scheme with angle adaptation asymptotically achieves optimum performance. Yet, the convergence speed depends on the specific choice of initial angle, angle reduction, and M.

4. SIMULATION RESULTS

We next study the performance of the multiplicative perturbation scheme with angle adaptation via numerical simulations and provide a comparison with the additive PB-BF scheme proposed in [6]. In our simulations, all channels were chosen static i.i.d. Rayleigh fading. The source $\mathcal{S}$ transmitted BPSK symbols with power $P_{\mathcal{S}} = P$. The destination $\mathcal{D}$ had perfect knowledge of the compound channel ξ to perform ML detection. We further ensured exact evaluation of the objective funcion (in practice, the SNR is estimated at the destination using the training blocks, cf. [6]) and error-free 1-bit feedback. All results shown were obtained using 10^5 fading realizations. Unless stated otherwise, we chose an initial rotation angle of $\phi = 45°$, an angle reduction factor of $\gamma = 0.25$, and M equal to the size of the perturbation set for the multiplicative scheme. For the additive scheme, we used a constant step-size $\mu = 0.5$ and a perturbation set of size $\tilde{N} = 4R$ based on a discrete Fourier transform matrix (cf. [6]).

Convergence Behavior. We first study the convergence rate for a network of $R = 3$ relays at nominal SNR $P/N_0 = 14$ dB. Fig. 3(a) shows the empirical cumulative distribution function (cdf) of the normalized SNR gap $\frac{\rho_{\max} - \rho(\boldsymbol{\alpha}_k)}{\rho_{\max}}$ that remains after a fixed number of transmission frames (shown as curve labels). Results are shown for additive perturbation and for Givens perturbations with perturbation sets[4] of minimum size $N = 2(\bar{R}-1) = 8$ (labeled 'minGivens') and of maximum size $N = \bar{R}(\bar{R}-1) = 20$ ('maxGivens'). We note that no

[4]Here, we chose the sets such that all the different rotations planes first undergo clockwise and then counter-clockwise rotations.



(a) (b) (c)

Fig. 3. Illustration of PB-BF performance: (a) cdf of the normalized SNR gap after a fixed number of frames ($R=3$, $P/N_0=14$ dB), (b) convergence time (in frames) versus target SNR ($P/N_0=14$ dB) for different network sizes, and (c) BER versus nominal SNR ($R=3,4$).

noticeable improvements are observed for additive perturbations beyond 40 transmission frames (iterations). It is seen that initially (after 10 iterations) minGivens performs best; here, rapid improvement is achieved since only a few rotations have to be checked. However, if a small SNR gap has to be ensured with high probability, maxGivens is preferable. For example, to achieve a normalized SNR gap less than 11% in 90% of the cases, minGivens and maxGivens require 40 and 80 iterations, respectively. After 120 frames, an SNR gap less than 5% is achieved in 99% of the cases with maxGivens but only in 85% of the cases with minGivens. With additive perturbations, initial convergence is poorer than with minGivens and the SNR gap after many iterations is larger than with maxGivens.

Network Size. Next, we analyze the impact of the network size (i.e., number of relays R) on the convergence of PB-BF with minGivens perturbations ($N=2(\bar{R}-1)$) from the perspective of rapidly achieving a certain target SNR at the destination given that $P/N_0=14$ dB (note that the sum relay power is independent of the network size). Fig. 3(b) shows the convergence time versus target SNR. Here, we define convergence time as the minimum number of frames required to achieve the target SNR in 98% of the cases. Obviously, the achievable SNR is higher for larger R due to increasing array gain (i.e., about 7.5 dB for $R=3$ and about 15 dB for $R=10$). However, very large convergence times are implied if the target SNR approaches the SNR limit. It is further seen that the curves for different R intersect, and hence, for a given target SNR there is an optimum network size minimizing the convergence time. For example, 7 dB target SNR can most rapidly be obtained using $R=5$ relays whereas the optimum number of relays to achieve 10 dB is $R=10$.

BER Performance. Bit error rate (BER) versus nominal SNR P/N_0 for the case of $R=3$ and $R=4$ relays is shown in Fig. 3(c). Since close-to-optimum performance is desired, we here use maxGivens (20 and 42 rotation planes, respectively) and compare the results with additive perturbation. As ultimate benchmarks, we also include the results for optimum beamforming using the weights in (5) (labeled 'optimum-BF') and a scheme without beamforming ('no-BF'), i.e., uniform power allocation $\alpha_i=\sqrt{P/R}$ and no coherent combining. For each fading realization, we excluded the initial convergence phase (first 60 frames) from the BER evaluation. It can be seen that the angle adaptation allows maxGivens to closely approach optimum performance and to outperform additive PB-BF in the high-SNR regime (e.g., 0.8 dB SNR gain at a BER of 10^{-4} for $R=3$), even though the complexity of maxGivens is smaller than that of additive perturbations. Also, the curves show that our scheme is able to fully exploit the spatial diversity offered by cooperative relaying and offers significant gains over the no-BF case (e.g., 13 dB SNR improvement at a BER of 10^{-2} for $R=3$).

5. CONCLUSION

We have proposed to use multiplicative perturbations based on Givens rotations for distributed beamforming in wireless relay networks with 1-bit feedback. These perturbations are much better matched to the non-Euclidean manifold underlying the problem setup than additive perturbations considered previously. It further allows direct step-size control in terms of rotation angles and is computationally very efficient. In fact, the per-relay complexity is independent of the network size. Numerical simulations showed that our scheme approaches optimum performance arbitrarily close at a satisfactory convergence speed.

REFERENCES

[1] P. Larsson, "Large-scale cooperative relaying network with optimal coherent combining under aggregate relay power constraints," in *Proc. Future Telecom. Conf. (FTC)*, 2003.

[2] I. Hammerström, M. Kuhn, and A. Wittneben, "Impact of relay gain allocation on the performance of cooperative diversity networks," in *Proc. IEEE VTC-2004 (fall)*, Los Angeles, USA, Sept. 2004, pp. 1815–1819.

[3] Y. Jing and H. Jafarkhani, "Network beamforming using relays with perfect channel information," in *Proc. IEEE ICASSP-2007*, April 2007, pp. III–473–III–476.

[4] B. C. Banister and J. R. Zeidler, "A simple gradient sign algorithm for transmit antenna weight adaptation with feedback," *IEEE Trans. Signal Processing*, vol. 51, no. 5, pp. 1156–1171, May 2003.

[5] B. Raghothaman, "Deterministic perturbation gradient approximation for transmission subspace tracking in FDD-CDMA," in *Proc. IEEE ICC-2003*, May 2003, vol. 4, pp. 2450–2454.

[6] P. Fertl, A. Hottinen, and G. Matz, "Perturbation-based distributed beamforming for wireless relay networks," in *Proc. IEEE GLOBECOM-2008*, New Orleans, USA, Nov./Dec. 2008.

[7] G. H. Golub and C. F. Van Loan, *Matrix Computations*, Johns Hopkins University Press, Baltimore, 3rd edition, 1996.

[8] Larry C. Grove, *Classical Groups and Geometric Algebra*, vol. 39 of *Graduate studies in mathematics*, American Mathematical Society, Providence, Rhode Island, 2002.

[9] Jack E. Volder, "The CORDIC trigonometric computing technique," *IRE Trans. Electron. Comput.*, vol. EC-8, pp. 330–334, Sept. 1959.