2nd ECCOMAS Thematic Conference on Mechanical Response of Composites
Imperial College London, UK, 1–3 April 2009

NUMERICAL SIMULATION OF PROGRESSIVE GROWTH OF DELAMINATIONS WITH CURVED FRONTS

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Keywords: Delamination, Laminates, Finite Element Method, Fracture Mechanics.

Abstract. *A numerically efficient approach for the prediction of consecutive growth of delaminations with curved fronts is developed. The approach is based on the assumption that the delamination grows from its current front to a predefined advanced front and that the delamination front remains smooth during the entire growth process. For computation of the load required to propagate the delamination the finite element method and the virtual crack closure technique are employed and incremental growth is predicted. As an example, delamination growth in a laminated plate is analyzed.*

1 Introduction

Laminates made of fiber reinforced polymers have become increasingly important over the past years due to their potential for weight saving. One of the critical failure modes in such laminates is delamination, as it can change the load carring capacity of such structures considerably.

For the analysis of delamination growth various methods based on linear elastic fracture mechanics have been developed. For the prediction of consecutive growth of delaminations with straight fronts a semi–analytical approach has bee proposed [6, 7]. Within the Finite Element Method (FEM) the Virtual Crack Closure Technique (VCCT) [2] is used successfully for the prediction of the onset of delamination growth. The VCCT is based on the assumption of self similar growth and requires meshes that are locally orthogonal to the delamination front. The latter requirement cannot be satisfied if consecutive growth of delaminations with curved fronts is analyzed by means of standard FEM. Consequently, the VCCT is not suitable for such analysis. To circumvent these problems moving mesh technology can be used [4]. However, such technology is numerically very expensive and the achievement of convergence is tricky if unstable delamination growth takes place.

An alternative approach for the prediction of growth of delaminations with curved fronts is the deployment of cohesive zone elements [1]. However, their usage is numerically expensive as fine meshes and small load increments are required.

To overcome these problems, a numerically more efficient approach for prediction of progressive growth of delaminations with curved fronts is proposed. Instead of evaluating a growth criterion at individual points along the delamination front a *total delamination front criterion* that predicts growth along the entire front is defined. If it is satisfied the delamination is assumed to grow from a current shape to a predefined advanced shape. For computation of the load required to propagate the delamination the FEM is employed.

2 Total Delamination Front Criterion

According to Griffith a delamination will grow if the energy released is equal to or greater than the energy required to create new delaminated area. Within the proposed approach the energy released along the entire delamination front is considered and if it reaches some critical value the delamination is assumed to grow from its current front to a predefined advanced front. The advanced delamination front is chosen to be continuous, smooth, and geometrically compatible with respect to the current delamination front, generic examples are shown in Fig. 1 (left). Hence, growth along the entire front is assumed to take place and the delamination front is enforced to remain smooth during the entire growth process. Note that the delamination size can never decrease. As a consequence all points of the advanced delamination front needs to be at or ahead of the position of the current delamination front. All advanced delamination fronts that fullfill this requirement are geometrically compatible with respect to the current one. Various advanced delamination fronts can be defined and the delamination will grow from its current shape to one of these advanced shapes if

$$\lambda^2 \frac{\sum_{i=1}^{N} \mathcal{G}_i A_i}{\sum_{i=1}^{N} \mathcal{G}_{\mathrm{c},\,i}\, A_i} = 1 \quad , \tag{1}$$

where $\mathcal{G}_i$ is the energy release rate at node i computed by means of the VCCT for a prescribed loading scenario. N is the total number of nodes along the delamination front. A_i is the area associated to each node for delamination progress. It is defined by the current and the advanced delamination front, see Fig. 1 for an example. λ is the load factor by which the magnitude of the prescribed loading scenario has to be increased to propagate the delamination. The load factor needs to be computed for each advanced delamination front individually. $\mathcal{G}_{\mathrm{c},\,i}$ is the critical energy release rate at node i, which depends on the critical energy release rate for pure mode

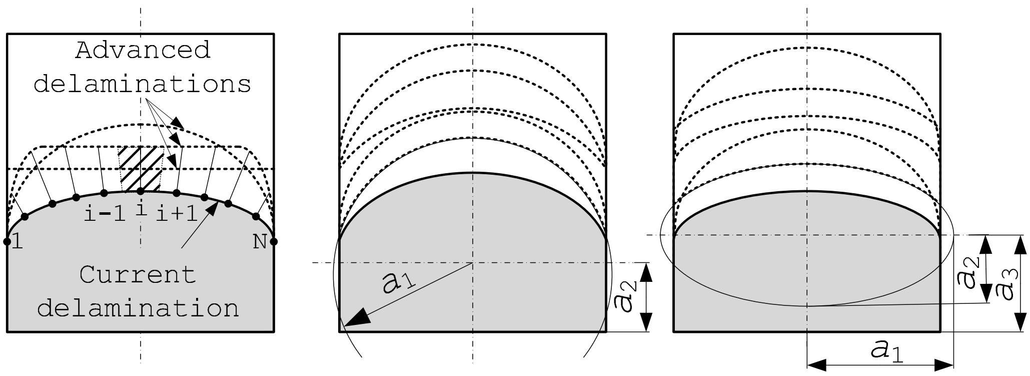

Figure 1: Advanced delamination fronts geometrically compatible with the current delamination front; area associated to node i for delamination progress (left); delamination shapes described by two (center) and three (right) delamination coordinates.

I, mode II, and mode III loading as well as on the actual mode mix at node i. The latter is computed by means of the VCCT, see [5] for details.

For definition of delamination fronts it is assumed that their shape can be described by a limited number of delamination coordinates a_i. In Fig. 1 two generic examples are shown; circular delamination fronts (middle) described by two delamination coordinates (a_1 and a_2) and elliptical delamination fronts (right) described by three delamination coordinates (a_1, a_2, and a_3). A range of delamination coordinates is selected in a way that all delaminations fronts of interest can be described. The range of coordinates is then discretized and a parameter field is defined. Each combination of discrete coordinates, i.e. each point in the parameter field, describes one possible shape of the delamination front.

For computation of the energy release rates and the fracture modes an FEM model parametrized in the delamination coordinates is set up employing the FEM package *ABAQUS/Standard/V6.6 (ABAQUS Inc., Pawtucket, RI, USA)*. For each shape an FEM model with a mesh locally orthogonal to the front is generated and the energy release rate as well as the fracture mode are computed for all nodes along the delamination front by means of the VCCT. The generation of the models and all computations are done within a fully automated procedure and the results are stored in a database. Consecutive delamination growth is predicted by postprocessing of these data, using the data processing capability of *MATLAB R2007b (The MarthWork Inc.)*.

For a given starting delamination Eq. (1) is applied for all geometrically compatible advanced delamination fronts analyzed. Out of these delaminations the one for which the load factor is smallest is chosen as the new current delamination front. The load required to propagate the delamination is computed from the load applied in the prescribed loading scenario and the load factor computed. Repeating these considerations allows to predict incremental delamination growth and the corresponding structural response. The size of the incrementation in delamination size is set by the incrementation chosen for discretization of the delamination coordinates.

2.1 Quality Check of the Predefined Delamination Front

Consecutive growth for a prescribed loading scenario is predicted by selecting delamination fronts out of a set of predefined ones. The question whether or not the predefined shapes allow for a proper approximation of the delamination front is discussed in the following. In Eq. (1) only the global energy balance at delamination growth is considered. Locally the energy release rate may be greater or smaller than the critical energy release rate. The ratio between energy release rate and critical energy release rate at each node, called *energy ratio*, provides information whether or not the assumed shape allows for a satisfying approximation of the delamination front. First, the energy ratios at each point along the delamination front are normalized by their mean value. In sections where the normalized energy ratio is greater than unity, delamination growth is underpredicted, in sections where it is smaller than unity, delamination growth is overpredicted. A proper description of the shape of the delamination front is found if the normalized energy ratios are equal to unity, or close to unity, along the entire delamination front.

Second, the standard deviation of the normalized energy ratios,

$$s = \frac{N}{\sum_{i=1}^{N} \frac{\mathcal{G}_i}{\mathcal{G}_{\mathrm{c},\,i}}} \sqrt{\frac{1}{N} \sum_{i=1}^{N} \left(\frac{\mathcal{G}_i}{\mathcal{G}_{\mathrm{c},\,i}} - 1 \right)^2} \quad , \tag{2}$$

is defined and used to compare individual shapes of the delamination front. The smaller the standard deviation is the better is the description of the shape of the delaminated front. A proper description is found if the standard deviation is close to zero.

Table 1: Material and interface properties of plies made of a unidirectional carbon fiber reinforced epoxy layer *T300/976* taken from [3].

Elastic constants					Critical energy release rates	
E_l	$E_q = E_n$	$G_{lq} = G_{ln}$	$\nu_{lq} = \nu_{ln}$	ν_{qn}	$\mathcal{G}_{\mathrm{Ic}}$	$\mathcal{G}_{\mathrm{IIc}}$
139.3 GPa	9.72 GPa	5.59 GPa	0.29	0.40	193 J/m^2	455 J/m^2

3 Example – Delamination in a laminated plate

Consecutive growth of a delamination in a laminated plate made of plies of a carbon fiber reinforced epoxy resin is analyzed. Material data and interface properties are taken from [3] and are given in Table 1. For the considered material no reliable experimental data for the critical energy release rate for mode III is available. Hence, it is assumed that the critical energy release rate for mode III and mode II are equal (i.e. $\mathcal{G}_{\mathrm{IIIc}} = \mathcal{G}_{\mathrm{IIc}}$). Details concerning geometry and loading of the laminated plate are shown in Fig. 2. A $[+45°_{10}/-45°_{10}]$ layup is considered, where the orientation of the plies is measured with respect to the *x*–direction. The ply thickness is 0.1 mm and. The structure contains a circular delamination at the midplane of the laminate with a radius of 20 mm centered at one corner of the plate. The upper sublaminate is loaded at this corner by a prescribed displacement in *z*–direction, the lower sublaminate is fixed in *z*–direction at this corner. Appropriate boundary conditions are applied to prohibit rigid body motions but without applying constraints otherwise.
For the FEM computations each sublaminate is modeled using four–noded Kirchhoff shell elements with full integration, the element length is about 1 mm. In the region that is not yet delaminated the translational degrees of freedom between the nodes of the two sublaminates are coupled accordingly. Note that such models do not allow to analyze free edge effects.

Shape of the Delamination Front Various geometries (line, circle, ellipse) are considered to describe the shape of the delamination front during the growth process and the quality check is applied to find out which one is best suited. Comparison of the standard deviation of the normalized energy ratios for all geometries considered shows, that the more delamination coordinates (i.e. more free variables) are used, the better is the description of the shape of the delamination front. However, the numerical effort for evaluating all feasible combinations of

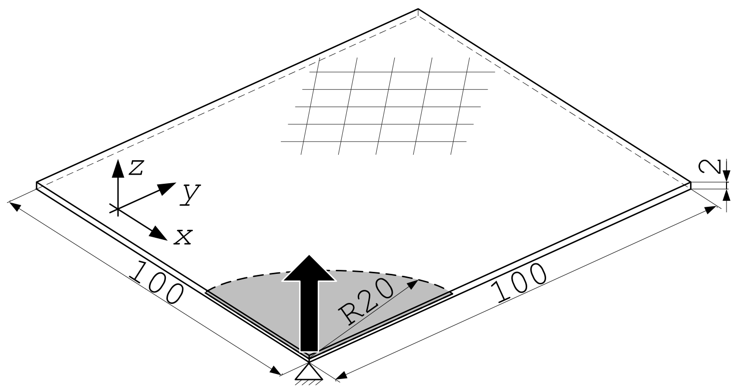

Figure 2: Delamination in a laminated plate; definition of geometry, layup, and loading.

delamination coordinates increases enormously if the number of delamination coordinates is increased. Based on these findings, circles centered somewhere at the diagonal symmetryline are selected for description of the shape of the delamination front. These circles are described by two delamination coordinates (e.g. position of the center and the radius).

Consecutive Delamination Growth Consecutive growth of the starting delamination shown in Fig. 2 is predicted. For numerical evaluation a parameter field of discrete values of the delamination coordinates is defined and each combination of the coordinates is analyzed in an automated procedure.
Growth is now predicted by postprocessing of the data generated. Beginning from the starting delamination one shape after the other is selected from all analyzed predefined shapes. In Fig. 3 the starting delamination (dark gray region), some intermediate delaminations (dashed lines), and the final delamination (gray region) are shown together with the standard deviation of the normalized energy ratios computed for each delamination front. The standard deviations are small for all delaminations predicted, hence, it can be concluded that the delamination is sufficiently well described by a circle during the entire growth process.
The predicted structural response is shown in Fig. 4 in terms of the prescribed displacement and the reaction force at the loading point. At a load level of about 4.8 mm delamination growth starts. Further increase of the prescribed displacement leads to stable delamination growth up to the point where the delamination front connects the lateral corners of the plate. Somewhat beyond this point delamination growth becomes unstable and the load required to cause delamination growth decreases as the size of the delamination increases. At the final point of the force–displacement curve a small not yet delaminated region remains only and the analysis is stopped.

4 Summary

A numerically efficient approach for the prediction of consecutive growth of delaminations with curved fronts is proposed. The approach is based on the assumption that the delamination grows from its current front to a predefined advanced front and that the delamination front remains smooth during the entire growth process. Some “quality check” is developed that allows to determine whether or not the assumed shape allows for a proper approximation of the advanced delamination front. If not, it guides the way to systematic improvement of the

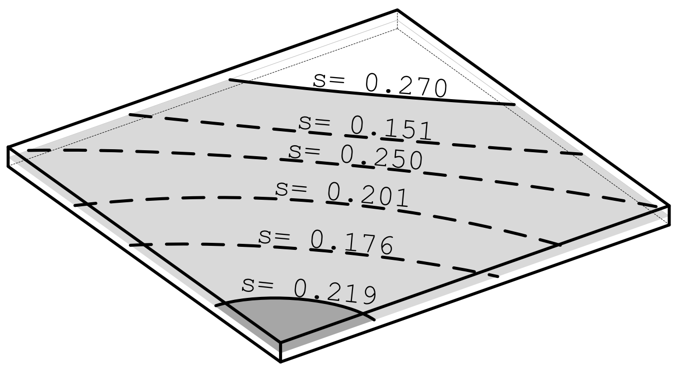

Figure 3: Predicted consecutive delamination growth in the laminated plate; starting delamination (dark gray region), intermediate delamination fronts (dashed lines), final delamination (gray region); Standard deviation of the normalized energy release rates, s, shown for each delamination.

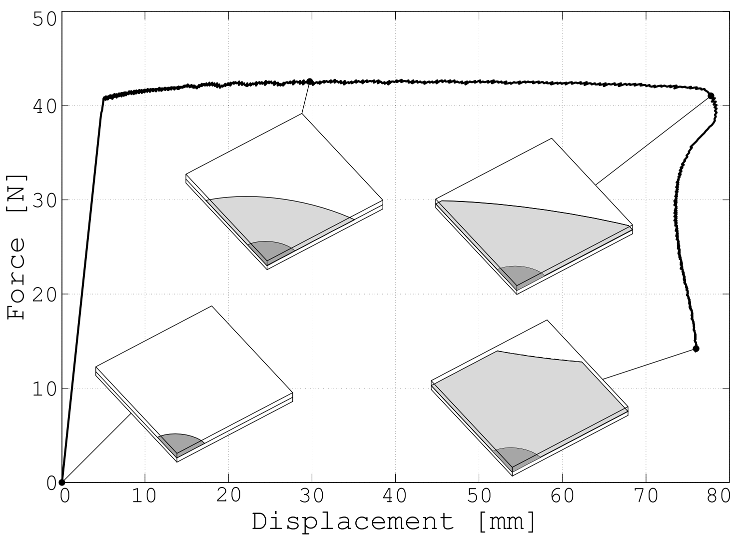

Figure 4: Predicted structural response of the plate subject to a vertical displacement load.

delamination shape. The proposed approach is used for the prediction of consecutive growth of a delamination in a laminated plate. The growth process as well as the non–linear structural response are determined successfully.

Acknowledgment The funding of the Austrian Aeronautics Research (AAR)/Network for Materials and Engineering by the Austrian Federal Ministry of Economics and Labor is gratefully acknowledged.

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