Making a georeferenced mosaic of historical map series using constrained polynomial fit

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Present day GIS software packages make it possible to handle several hundreds of rasterised map sheets.
For proper usage of such datasets we usually have two requirements: First these map sheets should be georeferenced, secondly these georeferenced maps should fit properly together, without overlap and short.

Both requirements can be fulfilled easily, if the geodetic background for the map series is accurate, and the projection of the map series is known. In this case the individual map sheets should be georeferenced in the projected coordinate system of the map series. This means every individual map sheets are georeferenced using overprinted coordinate grid or image corner projected coordinates as ground control points (GCPs). If after this georeferencing procedure the map sheets do not fit together (for example because of using different projection for every map sheet, as it is in the case of Third Military Survey) a common projection can be chosen, and all the georeferenced maps should be transformed to this common projection using a map-to-map transformation.

If the geodetic background is not so strong, ie. there are distortions inside the map sheets, a polynomial (linear quadratic or cubic) polynomial fit can be used for georeferencing the map sheets. Finding identical surface objects (as GCPs) on the historical map and on a present day cartographic map, let us to determine a transformation between raw image coordinates (x,y) and the projected coordinates (Easting, Northing, E,N). This means, for all the map sheets, several GCPs should be found, (for linear, quadratic of cubic transformations at least 3, 5 or 10 respectively) and every map sheets should be transformed to a present day coordinate system individually using these GCPs. The disadvantage of this method is that, after the transformation, the individual transformed map sheets not necessarily fit together properly any more. To overcome this problem neither the reverse order of procedure helps: if we make the mosaic first (eg. graphically) and we try the polynomial fit of this mosaic afterwards, neither using this can we reduce the error of internal inaccuracy of the map-sheets.

We can overcome this problem by calculating the transformation parameters of polynomial fit with constrains (Mikhail, 1976). The constrain is that the common edge of two neighboring map-sheets should be transformed identically, ie. the right edge of the left image and the left edge of the right image should fit together after the transformation. This condition should fulfill for all the internal (not only the vertical, but also for the horizontal) edges of the mosaic.

Constrains are expressed as a relationship between parameters: The parameters of the polynomial transformation should fulfill not only the least squares adjustment criteria but also the constrain: the transformed coordinates should be identical on the image edges. (With the example mentioned above, for image points of the rightmost column of the left image the transformed coordinates should be the same a for the image points of the leftmost column of the right image, and these transformed coordinates can depend on the line number image coordinate of the raster point.)

The normal equation system can be calculated with Lagrange-multipliers. The resulting set of parameters for all map-sheets should be applied on the transformation of the images. This parameter set can not been directly applied in GIS software for the transformation. The simplest solution applying this parameters is 'simulating' GCPs for every image, and applying these simulated GCPs for the georeferencing of the individual map sheets.

This method is applied on a set of map-sheets of the First military Survey of the Habsburg Empire with acceptable results.

Reference: Mikhail, E. M.: Observations and Least Squares. IEP—A Dun-Donnelley Publisher, New York, 1976. 497 pp.