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Wealth Induced Multiple Equilibria in Small Open Economy Versions of the Ramsey Model

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Abstract The small open economy version of the Ramsey model exhibits the counterfactual outcome that consumption tends to zero in the long run in case that domestic residents are relatively impatient. It is shown that incorporating either absolute or relative wealth preferences allows for multiple interior steady states, while retaining the standard steady state (no matter how much wealth is appreciated); moreover, both frameworks can be observationally equivalent despite substantial differences in preferences. Convergence to an interior steady state arises if the initial level of nonhuman wealth exceeds an endogenously determined threshold value. This property of a history dependent evolution is surprising for a strict concave framework (without externalities in case of absolute wealth preferences).

Keywords Open economy, Ramsey model, wealth effects, history dependence

1. Introduction

The small open economy version of the Ramsey model implies that the entire capital stock and all future labor incomes are asymptotically mortgaged if domestic residents are impatient, i.e., if their constant subjective rate of time preference exceeds the exogenously given world interest rate. Barro and Sala-i-Martin (1995, chapter 3), Turnovsky (1995, chapter 12), and Turnovsky (1997, chapter 2) discuss several modifications in order to eliminate this para-

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doxical result such as the introduction of (i) a constraint on international borrowing, (ii) a variable subjective rate of time preference, (iii) a variable effective rate of time preference due to aggregation effects in a model with finite time horizon, or (iv) a variable domestic interest rate due to a debt-dependent risk premium.

The goal of this paper is to offer two alternative modifications of the standard model in which the following results can be obtained under quite general specifications of preferences: (i) there are multiple interior steady states, in which both total wealth (= nonhuman wealth + human wealth) and consumption are positive, (ii) the steady state of the standard model is retained, (iii) the long-run outcome is history-dependent, where convergence to an interior steady state arises if the initial level of nonhuman wealth exceeds an endogenously determined threshold value. Both extensions yield a variable effective rate of time preference (resp. a variable effective rate of return on assets) that depends on own consumption and own nonhuman wealth, in spite of the fact that the subjective rate of time preference and the foreign interest are constant and exogenously given.

The first extension of the standard model is analyzed in section 2. There, it is assumed that the flow of utility of the representative consumer depends not only on own consumption, but also on own nonhuman wealth. This specification (henceforth, the absolute wealth approach) implies that wealth per se is appreciated for various reasons, e.g., greed, intrinsic, non-consumptive values, insurance, signals (who likes to be considered 'poor'?), and large debts can be life threatening if owed to the Mafia or to a drug dealer. The second extension, a relative wealth approach, is studied in section 3. It is based on the assumption that the instantaneous utility is a function of consumption and social status, in which the latter is determined by a comparison of own nonhuman wealth with average wealth in the economy. For convenience, it is assumed that agents are identical in every respect which implies that attention can be restricted to symmetric equilibria. The relative wealth approach differs from the absolute wealth approach in two respects: (i) the accumulation of own assets now creates negative externalities for the other agents, and the decentralized solution is no longer socially optimal, (ii) the specification of the status function ensures that the flow of utility is independent of the level of wealth in a symmetric equilibrium.[1] However, in spite of these significant differences, it is possible that an economy with relative wealth preferences is observationally equivalent to an economy with absolute wealth preferences. In this paper, observational equivalence studied in section 4 means that the decentralized solutions for the time paths of consumption and nonhuman

[1] The result that the introduction of relative wealth preferences may allow for multiple interior long-run equilibria has been shown in Corneo and Jeanne (2001a) in a closed economy framework. Our paper differs, however, in several respects that will be described in section 3.

wealth coincide in the two economies.

Scientists who either think that one should restrict the analysis to the simplest possible framework (e.g. Ockham and Einstein) or dislike the status approach could argue that the absolute wealth approach is sufficient for the derivation of the main results, i.e., multiple interior equilibria and history dependence. However, an increasing number of researches stresses the importance of status preference and of relative magnitudes for economic decisions and happiness[2], so that the status approach is also introduced, briefly discussed and linked to the absolute wealth approach through observational equivalence.

2. Extension #1: absolute wealth preferences

Our model extends the standard small open economy Ramsey model by allowing for nonhuman wealth to affect the flow of utility; Krawczyk and Shimomura (2003) is a recent example that attributes these kind of preferences to capitalists in a game between capitalists and work within an AK-growth model, yet their reason for multiple equilibria - multiplicity of Markov perfect equilibria in dynamic games - is different from ours. To simplify, we will abstract from population growth, depreciation of physical capital and technological progress, and employ an additively separable instantaneous utility function. The infinitely-lived representative consumer maximizes overall utility as given by

$$\int_0^\infty e^{-\rho t}\left[u\left(c\right)+v(a)\right]dt, \tag{1}$$

subject to the budget constraint, the initial condition, and the no-Ponzi-game condition

$$\dot{a}=r^{*}a+w-c, \qquad a\left(0\right)=a_0, \qquad \lim_{t\to\infty}a\left(t\right)e^{-r^{*}t}\geq 0, \tag{2}$$

respectively, where the following notation is used: ρ = constant subjective rate of time preference, c = per capita consumption, a = per capita stock of nonhuman wealth, r^* = world interest rate (= real rental rate in world capital market), w = domestic real wage. The function $u\left(c\right)$ satisfies $u'>0$, $u''<0$, and the standard Inada conditions, $\lim_{c\to 0}u'\left(c\right)=\infty$, $\lim_{c\to\infty}u'\left(c\right)=0$. The increasing and concave function $v\left(a\right)$, which measures the flow of utility from nonhuman wealth a, is defined for $a\in\left(\bar{a},\infty\right)$; a discussion about the level $\bar{a}$ follows below. The special case $v=0$ yields the standard model.

[2] The importance of relative aspects was already stressed by J. Duesenberry, F. Hirsch and T. Scitovsky. Empirical evidence for the significance of relative magnitudes is found in Frank (1985, 1997), Easterlin (1974, 1995) and in many experiments, which indicate that people hate to fall below the average, see e.g. the recent survey in Fehr and Fischbacher (2002).

Labor supply is exogenous and normalized to unity. The domestic real wage w, which is taken as given by each individual, is determined in the perfectly competitive domestic labor market. Since physical capital and loans are perfect substitutes as stores of value, they must yield the same real return r^* that is exogenously given for the small open economy. Following Barro and Sala-i-Martin (1995), we assume that r^* is constant over time and less than the subjective rate of time preference, $r^* < \rho$, i.e., domestic residents are relatively impatient.

The current value Hamiltonian,

$$H = u(c) + v(a) + \lambda\left[r^* a + w - c\right],$$

of the optimization problem (1) and (2), is obviously concave in state and control. Therefore, the necessary optimality conditions

$$H_c = u'(c) - \lambda = 0, \tag{3}$$

$$\dot{\lambda} = \rho\lambda - H_a = (\rho - r^*)\lambda - v'(a), \tag{4}$$

are also sufficient if the limiting transversality condition $\lim_{t\to\infty} e^{-\rho t}\lambda a = 0$ holds. Substituting $u'(c) = \lambda$ and $u''(c)\dot{c} = \dot{\lambda}$ into (4), and rearranging yields $\dot{c} = \left[(\rho - r^*)u'(c) - v'(a)\right]/u''(c)$. This Euler equation for consumption can be rewritten as

$$\dot{c}/c = \sigma(c)\left[r^* - \rho + m(c,a)\right], \tag{5}$$

where $\sigma(c) \equiv -u'(c)/\left[cu''(c)\right]$ is the elasticity of intertemporal substitution and

$$m(c,a) \equiv \frac{v'(a)}{u'(c)}, \quad \text{with } m_c = -\frac{v'u''}{(u')^2}, \qquad m_a = \frac{v''}{u'}, \tag{6}$$

is the marginal rate of substitution (MRS) of nonhuman wealth a for consumption c. In the non-standard case in which $v' > 0$ and $v'' < 0$, the MRS depends positively on c and negatively on a, $m_c > 0$ and $m_a < 0$. In (5),

$$\rho^e \equiv \rho - m(c,a), \tag{7}$$

and

$$r^e \equiv r^* + m(c,a), \tag{8}$$

can be interpreted as the *effective* rate of time preference (ρ^e) and as the *effective* rate of return on nonhuman wealth (r^e), respectively. First, the normality properties $m_c > 0$ and $m_a < 0$ imply that the effective rate of time preference depends negatively on c and positively on a.[3] In Blanchard's (1985) model in which people or dynasties die off according to a Poisson process, the *effective* rate of time preference is positively related to the ratio of nonhuman wealth to consumption due to aggregation effects. Second, the effective rate of return depends positively on c and negatively on a. The latter property is assumed a priori in models that account for debt-dependent risk premia [e.g. Turnovsky (1997, subsection 2.6)].

Using the definition of the marginal rate of substitution, (6), the transversality condition can be written as

$$\lim_{t\to\infty}\left\{a(t)\exp\left[-\int_0^t \left[r^* + m(c(v), a(v))\right]dv\right]\right\} = 0. \tag{9}$$

In the decentralized equilibrium of the small open economy that is socially efficient due to the lack of externalities, physical capital per capita k and the real wage rate w are determined by the well-known conditions $r^* = f'(k)$ and $w = f(k) - kf'(k)$, where f is a neoclassical production function in intensive form. Since the exogenously given r^* is constant over time, the variables k and w do not exhibit any transitional behavior, and equal their steady state values, $\tilde{k}$ and $\tilde{w}$, respectively. Therefore, the equilibrium evolution of nonhuman wealth a and consumption c is governed by the generalized Euler equation (5), the domestic economy's wealth accumulation equation

$$\dot{a} = r^* a + \tilde{w} - c, \tag{10}$$

the initial condition $a(0) = a_0$, and the generalized transversality condition (9).

In the standard model in which $m(c, a) = 0$ for all $c > 0$ and $a > \bar{a}$, the Euler equation and the transversality condition simplify to $\dot{c}/c = \sigma(c)(r^* - \rho)$ and $\lim_{t\to\infty} ae^{-r^* t} = 0$. The isoelastic instantaneous utility function $u(c) = (1-\theta)^{-1}\left(c^{1-\theta} - 1\right)$, in which $\sigma(c) = 1/\theta$ for all c, used in Barro and Sala-i-Martin (1995), yields the following explicit solution:

$$a(t) = -\frac{\tilde{w}}{r^*} + \left(a_0 + \frac{\tilde{w}}{r^*}\right)e^{-(1/\theta)(\rho - r^*)t}, \tag{11}$$

$$c(t) = \left[r^* + (1/\theta)(\rho - r^*)\right]\left[a(t) + \frac{\tilde{w}}{r^*}\right], \tag{12}$$

with $\lim_{t\to\infty} a(t) = -\tilde{w}/r^* < 0$, $\lim_{t\to\infty}\left[a(t) + \tilde{w}/r^*\right] = 0$, $\lim_{t\to\infty} c(t) = 0$. That is, an impatient country ($\rho > r^*$) asymptotically mortgages all of its

[3] This contrasts with the approach chosen in Gootzeit, Schneider and Smith (2002) in which the *subjective* rate of time preference ρ depends on c and a due to the assumption that ρ is a function of saving, $\rho = \rho(\dot{a})$.

capital and all of its labor income so that both total wealth [nonhuman wealth a + human wealth $\tilde{w}/r^*$] and consumption converge to zero. As will become obvious below, this unpleasant long-run property holds also for more general specifications of $u(c)$. In order to avoid constraining the set of possible steady states in our extended model *a priori*, we assume that the domain of $v(a)$ includes the steady state of the standard model, $\bar{a} \leq -\tilde{w}/r^*$.

In contrast to the standard model, our extension allows for steady states $(\tilde{a}, \tilde{c})$ in which $\tilde{c} > 0$. In terms of phase diagrams [in the (a, c)–plane], such steady states are the points of intersection between the $\dot{a} = 0$ isocline and the $\dot{c} = 0$ isocline, see Fig. 1. The $\dot{a} = 0$ isocline is given by the a straight line $c = r^* a + \tilde{w}$ [see (10)], where c is positive for $a > -\tilde{w}/r$. Along this isocline the level of consumption equals the sum of net interest income and wage income. Therefore, any point above the $\dot{a} = 0$ line corresponds to a level of consumption that induces a run down of the stock of assets, while a consumption below this isocline allows for asset accumulation; see the laws of motion shown by the arrows in Fig. 1. The $\dot{c} = 0$ isocline is implicitly defined by $m(c, a) = \rho - r$. From the definition of the MRS, m, given in (6) and the assumptions made above with respect to u', v', and $\rho - r^*$, it follows that

$$\lim_{c \to 0} m(c, a) = 0 < \rho - r^* < \infty = \lim_{c \to \infty} m(c, a), \quad \forall a \in (\bar{a}, \infty). \tag{13}$$

This, in turn, implies that a unique positive $c = c(a, \rho - r^*)$ exists solving $m(c, a) = \rho - r^*$ for all $a \in (\bar{a}, \infty)$. In other words, the $\dot{c} = 0$ isocline does not intersect the horizontal axis. From $m_c > 0$ and $m_a < 0$, it follows that the slope of the $\dot{c} = 0$ curve, $-m_a/m_c$, is positive. Further properties of this isocline depend on the third derivatives u''' and v''':

$$c_{aa}\big|_{\dot{c}=0} = -\left(\frac{u'}{u''}\right)\left(\frac{v''}{v'}\right)^2 \left[\frac{u'''/u''}{u''/u'} - \frac{v'''/v''}{v''/v'}\right]. \tag{14}$$

Although differently shaped (but always increasing) isoclines are possible, the following discussion focuses on specifications of u and v, in which the $\dot{c} = 0$ isocline is globally strictly convex. This is ensured if the expression between the brackets, which involves four elasticities, is positive (trivially if $u''' > 0$ and $v''' \leq 0$). Below, we will give two complying examples employing simple and widely used specifications of u and v, yet non-trivial ones due to $v''' > 0$.

A strictly convex shape of $\dot{c} = 0$ implies generically either two intersections with the $\dot{a} = 0$ isocline or none. The laws of motions of consumption follow from the Euler equation (5) and are displayed in Fig. 1. The effective rate of return, $r^e = r^* + m(c, a)$, equals the subjective rate of time preference, ρ, along the $\dot{c} = 0$ isocline. An increase in consumption (i.e. any point above $\dot{c} = 0$) increases the effective rate of return (due to $m_c > 0$), which implies (similar to other growth models) growth in consumption, $\dot{c} > 0$; the converse holds for consumption levels below this isocline.

In the remainder of the paper we will restrict our attention to specifications of the instantaneous utility function $u(c)$ in which (like for the standard CRRA specification)

$$\lim_{c \to 0} [c\sigma(c)] = -\lim_{c \to 0} [u'(c)/u''(c)] = 0 \tag{15}$$

holds. Condition (15) implies that the horizontal axis, $c = 0$, is an asymptotic $\dot{c} = 0$ locus in both the standard and the extended model. In the standard model, (15) implies that

$$\lim_{c \to 0} \dot{c}(c) = \lim_{c \to 0} [c\sigma(c)(r^* - \rho)] = 0.$$

In the extended model, it follows from (15) and $\lim_{c \to 0} m(c,a) = 0$ [see (13)] that

$$\lim_{c \to 0} \dot{c}(c,a) = \lim_{c \to 0} \{c\sigma(c)[r^* - \rho + m(c,a)]\} = 0, \quad \forall a \in (\bar{a}, \infty).$$

Consequently, $(-\tilde{w}/r^*, 0)$ is a steady state in both versions. From the phase diagram analysis given in Fig. 1 it follows that $(0, -\tilde{w}/r^*)$ exhibits the saddle point property in the extended model. The same result can be easily obtained in the standard model.[4]

A linear approximation around an interior steady state with $\tilde{c} > 0$ yields

$$\begin{pmatrix} \dot{a} \\ \dot{c} \end{pmatrix} = \mathbf{J} \begin{pmatrix} a - \tilde{a} \\ c - \tilde{c} \end{pmatrix},$$

where the Jacobian $\mathbf{J}$ and its determinant $\det(\mathbf{J})$ are given by

$$\begin{aligned} \mathbf{J} &\equiv \begin{pmatrix} r^* & -1 \\ \tilde{c}\sigma(\tilde{c})\, m_a(\tilde{c},\tilde{a}) & \tilde{c}\sigma(\tilde{c})\, m_c(\tilde{c},\tilde{a}) \end{pmatrix} \\ &= \begin{pmatrix} r^* & -1 \\ -v''(\tilde{a})/u''(\tilde{c}) & (\rho - r^*) \end{pmatrix}, \end{aligned} \tag{16}$$

[4] The corresponding phase diagram for the standard case can be derived from Fig. 1. by removing the positively sloped $\dot{c} = 0$ curve as well as all upward pointing arrows.

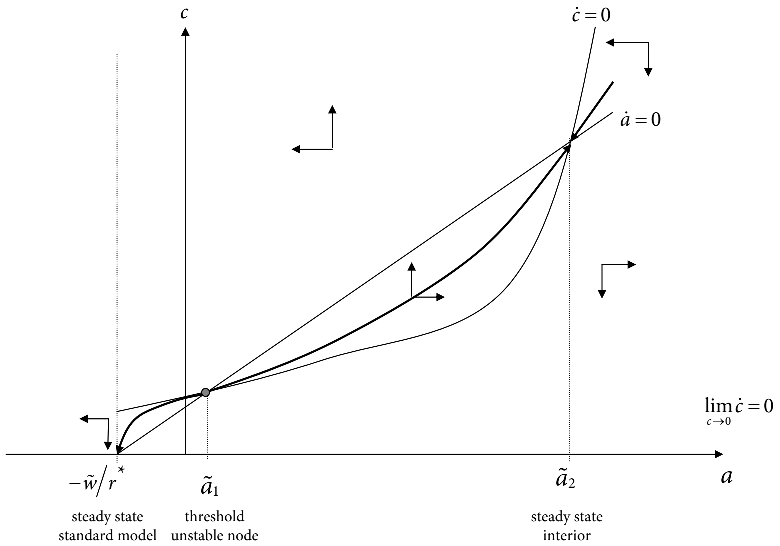

Fig. 1 — Phase portrait (wealth a and consumption c) and optimal paths contingent on initial wealth $a(0)$

$$\det(\mathbf{J}) = \tilde{c}\sigma(\tilde{c})\left[r^* m_c(\tilde{c},\tilde{a}) + m_a(\tilde{c},\tilde{a})\right] = r^*(\rho - r^*) - \frac{v''(\tilde{a})}{u''(\tilde{c})}. \tag{17}$$

The sign of $\det(\mathbf{J})$ is, in general, ambiguous: While m_c is positive, m_a is negative. Alternatively, while $r^*(\rho - r^*)$ is positive, $-v''(\tilde{a})/u''(\tilde{c})$ is negative. Therefore, an instability can arise; compare Wirl and Feichtinger (2005). In fact it will be shown that multiple stationary equilibria and consequently unstable steady states exist. $\det(\mathbf{J}) > 0$ implies an unstable node since the eigenvalues are real due to a positive discriminant, $[\mathrm{tr}(\mathbf{J})]^2 - 4\det(\mathbf{J}) = (2r^* - \rho)^2 + 4\left[v''(\tilde{a})/u''(\tilde{c})\right] > 0$. Conversely, if $\det(\mathbf{J}) < 0$, then the steady state is a saddlepoint. In terms of the phase diagram, saddlepoint stability arises iff the $\dot{c} = 0$ locus cuts the $\dot{a} = 0$ line at $(\tilde{a},\tilde{c})$ from below since it can be easily verified that $\det(\mathbf{J}) < 0 \Leftrightarrow -m_a(\tilde{c},\tilde{a})/m_c(\tilde{c},\tilde{a}) > r^*$. In the phase diagram shown in Fig. 1, the convex $\dot{c} = 0$ isocline intersects the $\dot{a} = 0$ isocline twice at $(\tilde{a}^1,\tilde{c}^1)$ and $(\tilde{a}^2,\tilde{c}^2)$ with $\tilde{a}^1 < \tilde{a}^2$ and $\tilde{c}^1 < \tilde{c}^2$ at levels of a exceeding the minimum feasible level, $-\tilde{w}/r^*$. Since the steady state of the standard model is also a steady state of the extended model, either three steady states result in total, or only the standard one (e.g. if $\dot{c} = 0$ remains above $\dot{a} = 0$).

From above (see Fig. 1) we know that the lowest steady state, i.e., the long-run outcome of the standard model, $(-\tilde{w}/r^*, 0)$, is a saddlepoint. The highest of the three steady states depicted in Fig. 1, $\left(\tilde{a}^2, \tilde{c}^2\right)$, is also a saddlepoint, since $\dot{c} = 0$ cuts $\dot{a} = 0$ from below, while the intermediate one, $\left(\tilde{a}^1, \tilde{c}^1\right)$, is an unstable node. This unstable node separates the domains of attractions of the two stable steady states:

- If $a_0 > \tilde{a}^1$, then $(a, c) \to \left(\tilde{a}^2, \tilde{c}^2\right)$ as $t \to \infty$. That is, the counterfactual long-run behavior of the standard model is avoided.
- If $a_0 < \tilde{a}^1$, then $(a, c) \to (-\tilde{w}/r^*, 0)$ as $t \to \infty$, and this convergence must be asymptotic due to the Inada conditions. Although this long-run outcome equals that of the standard model, the transient behavior is different.

The consumption policy, albeit history dependent, is everywhere unique (since the optimization problem is strictly concave), and continuous in the state, in particular, at the unstable steady state $\tilde{a}^1$ (= threshold). As a consequence, $\left(\tilde{a}^1, \tilde{c}^1\right)$ itself is optimal (see Fig. 1).

Example 1 The isoelastic function $u(c) = (1-\theta)^{-1}\left(c^{1-\theta} - 1\right)$, $\theta > 0$, and the CARA-type function

$$v(a) = -(\alpha/\beta)\, e^{-\beta a}, \qquad \forall a; \alpha > 0, \beta > 0, \tag{18}$$

satisfy all assumptions made in the paper with respect to u and v. This representation of preferences yields $\sigma = 1/\theta$ and $m(c, a) = \alpha e^{-\beta a} c^{\theta}$. The Euler equation becomes $\dot{c}/c = (1/\theta)\left(r^* - \rho + \alpha e^{-\beta a} c^{\theta}\right)$ so that the $\dot{c} = 0$ curve is given by the exponential function

$$c = \left(\frac{\rho - r^*}{\alpha}\right)^{1/\theta} e^{(\beta/\theta)a}. \tag{19}$$

This isocline maps the real numbers into the positive real numbers. It is increasing and strictly convex so that only two different generic outcomes are possible: no interior steady state or two interior steady states.

The above example is now used to verify the existence of multiple equilibria for such an economy and to perform bifurcation analyses in model parameters to show the two different generic long-run outcomes: either the one known from the standard model is the only long-run equilibrium, or the possibility of an albeit history-contingent interior long-run equilibrium.

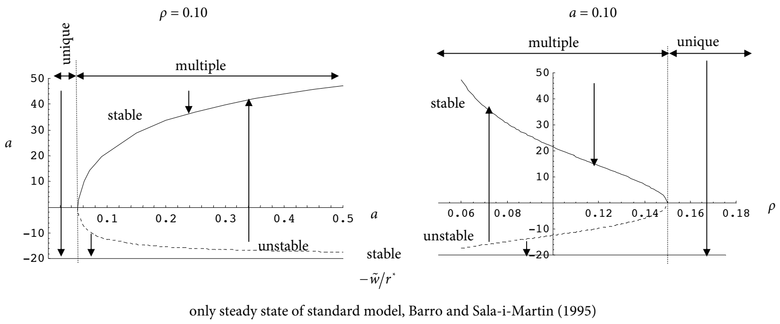

Fig. 2 — Bifurcation diagrams: Steady state levels of assets (a) as functions of model parameters α, and ρ, respectively, for $r = 0.05$, $\theta = 2$, $\beta = 0.1$, and $\tilde{w} = 1$ (arrows refer to the attracting steady state)

1. No interior long-run equilibrium exists if and only if

   $$\rho > \rho^m \equiv r^* + \alpha\beta^{-\theta}\left(\theta r^*\right)^{\theta} e^{\beta\left(\tilde{w}/r^*\right)-\theta}.$$

   In the bifurcation diagram for the parameter ρ (the right-hand side panel of Fig. 2) $\rho^m = 0.15$. Hence, for subjective discount rates exceeding 15%, the only steady state is that of the standard model. An alternative example of a bifurcation is to vary the parameter α, the weight attached to the utility from wealth, see the panel on the left-hand side of Fig. 2. There exists a corresponding critical level $\alpha^m = 0.05$ such that the standard outcome is retained for smaller weights.

2. If either $\rho < \rho^m = 0.15$ or $\alpha > \alpha^m = 0.05$, three steady states result. Of the two interior steady states, $c > 0$, the larger one is stable, the other is unstable. Interestingly, the domain of attraction of the interior and stable steady state becomes quickly dominating so that given either sufficient patience or appreciation of wealth, only highly indebted countries will converge to the standard, corner solution.

In example 1 the $\dot{c} = 0$ curve is globally strictly convex for all admissible preference parameters, i.e., for all $\theta > 0$, $\alpha > 0$, and $\beta > 0$. We will now give a second example in which the properties of the $\dot{c} = 0$ curve depend on the parameters of u and v.

Example 2 If u is of the isoelastic form used in example 1 and v takes the form

$$v(a) = \gamma(1-\xi)^{-1}\left[(a-\bar{a})^{1-\xi} - 1\right] \qquad \text{for } a > \bar{a};\ \ \gamma > 0, \xi > 0, \tag{20}$$

then $\sigma = 1/\theta$ and $m(c,a) = \gamma c^{\theta} (a - \bar{a})^{-\xi}$. This implies for the $\dot{c} = 0$ isocline:

$$c = \left(\frac{\rho - r^*}{\gamma}\right)^{1/\theta} (a - \bar{a})^{\xi/\theta}, \qquad \text{for } a > \bar{a}. \tag{21}$$

Obviously, it is globally strictly convex if and only if $\xi > \theta$. In this case there exist two points of intersection between the $\dot{a} = 0$ and $\dot{c} = 0$ curves if and only if $\rho < \rho^m$, where

$$\rho^m \equiv r^* + \gamma (r^*)^{\theta} \theta^{\theta} \xi^{-\xi} (\xi - \theta)^{\xi - \theta} \left(-\frac{\tilde{w}}{r^*} - \bar{a}\right)^{-(\xi - \theta)}.$$

Conversely, if $\rho > \rho^m$, then there is no interior long-run equilibrium.

What are the mathematical and economic reasons for the possibility of multiple equilibria? This can be answered by drawing on the phase diagram depicted in Fig. 1 and on the properties of the modified Euler equation [see (5)–(8)] and the asset accumulation equation. If the initial level of assets, a_0, is low ($a_0 < \tilde{a}^2$), but exceeds the threshold $\tilde{a}^1$, then it is possible to choose an initial value of consumption $c(0)$ that allows for both $\dot{a} > 0$ and $\dot{c} > 0$ along the optimal path. On the one hand, $\dot{a} > 0$ requires that $(a_0, c(0))$ lies below the $\dot{a} = 0$ line so that consumption is less than the sum of wage income and interest income. On the other hand, $\dot{c} > 0$ requires that $(a_0, c(0))$ lies above the $\dot{c} = 0$ curve (where $\rho^e \equiv \rho - m(c,a) = r^*$) so that $\rho^e < r^*$ holds due to $\rho^e_c < 0$, i.e., consumers are patient enough due to a sufficiently high level of consumption. Obviously, both requirements can be met if $a_0 > \tilde{a}^1$. This allows for convergence to the interior saddlepoint-stable steady state $(\tilde{a}^2, \tilde{c}^2)$. However, if $a_0 < \tilde{a}^1$, then it is impossible to satisfy both requirements: First, an initial level of consumption $c(0)$ that induces sufficient effective patience is incompatible with asset accumulation. Second, a level of $c(0)$ that leads to asset accumulation is incompatible with $\dot{c} > 0$ due to insufficient effective patience. In this case $\dot{a} < 0$ and $\dot{c} < 0$ hold along the optimal path and the economy converges to the standard steady state.

In Fig. 1 the strict convexity of the $\dot{c} = 0$ curve plays a crucial role for the existence of multiple equilibria. What does this property of this isocline imply for the underlying preferences?[5] From the discussions above it is well known that along $\dot{c} = 0$ the MRS of nonhuman wealth a for consumption c is constant and equal to $\rho - r^*$. This is not true for the $\dot{a} = 0$ curve, which assigns to each level of nonhuman wealth a the corresponding stationary consumption level $r^* a + \tilde{w}$. In example 1, for instance, in which the $\dot{c} = 0$

[5] The following interpretation was inspired by one of the referee's comments.

curve is unambiguously strictly convex, we have $m(c,a) = \alpha e^{-\beta a} c^{\theta}$. Hence, along the $\dot{a} = 0$ curve, the MRS is given by

$$Z(a) \equiv m(r^* a + \tilde{w}, a) = \alpha e^{-\beta a} (r^* a + \tilde{w})^{\theta}.$$

It is easily verified that Z is a hump-shaped function of a: $Z'(a) > 0$ for $a < \hat{a}$ and $Z'(a) < 0$ for $a > \hat{a}$, where $\hat{a} \equiv (\theta/\beta) - (\tilde{w}/r^*)$. In other words, as wealth a (and, hence, also consumption $c = r^* a + \tilde{w}$) increases from low levels, the willingness to sacrifice consumption in order to obtain an extra unit of wealth initially rises, but then decreases as wealth exceeds the critical value $\hat{a}$.[6] If, in addition, $\rho < \rho^m$ holds, where ρ^m is defined in example 1, then $Z(\hat{a}) > \rho - r^*$ so that we obtain two interior steady states $(\tilde{a}_1, r^* \tilde{a}_1 + \tilde{w})$ and $(\tilde{a}_2, r^* \tilde{a}_2 + \tilde{w})$ in which $Z(\tilde{a}_1) = Z(\tilde{a}_2) = \rho - r^*$. Similar results can be derived in example 2 for the case in which $\xi > \theta$. Hence, in both illustrations multiple interior equilibria arise from two features of preferences: i) the subjective rate of time preference is less than a critical level, and ii) in situations in which consumption equals its stationary level, i.e., along $\dot{a} = 0$, there is a hump-shaped relation between the MRS and nonhuman wealth. Since $\rho^e \equiv \rho - m(c,a)$ and $r^e \equiv r^* + m(c,a)$, the latter property implies that along $\dot{a} = 0$ the effective rate of time preference ρ^e is a U-shaped function of wealth, while the relation between the effective rate of return r^e and a is hump-shaped. As wealth a and stationary consumption $c = r^* a + \tilde{w}$ increase from low levels, both the effective rate of return and the effective patience of agents initially rise, but then decrease as wealth exceeds a critical value.

3. Extension #2: relative wealth (status)

This extension serves two purposes: i) to account for a growing literature, theoretical, empirical and experimental, that emphasizes relative magnitudes, ii) to highlight that interior equilibria result even if the level of wealth does not matter. We will assume that the flow of utility depends on both own consumption and social status which in turn is determined by a comparison of own wealth and average wealth in the economy. In other words, we will assume that individuals endeavor to keep up with the Joneses in terms of wealth. In the macroeconomic literature on status the relative wealth approach is, for instance, employed in Corneo and Jeanne (1997, 2001a,b), Rauscher (1997a), Futagami and Shibata (1998), Fisher (2004), Van Long and Shimomura (2004a, b), and Fisher and Hof (2005). In the relative consumption approach used by Boskin and Sheshinski (1978), Galí (1994), Persson

[6] Recall that $m_c > 0$ and $m_a < 0$. Hence, as we move to the northeast along $\dot{a} = 0$, the rise in consumption c causes the MRS to increase, while the rise in nonhuman wealth a causes the MRS to fall. If $a < \hat{a}$, the positive effect more than offsets the negative effect so that the MRS increases, while the opposite is true for $a > \hat{a}$.

(1995), Harbaugh (1996), Rauscher (1997b), Grossmann (1998), Ljungqvist and Uhlig (2000), Fisher and Hof (2000), Dupor and Liu (2003), Abel (2005), and Liu and Turnovsky (2005), the determination of status involves a comparison of own consumption and average consumption in the economy. It can be shown that the relative consumption approach does not eliminate the unpleasant properties of the standard model. For this reason it will not be considered in this paper. A recent and related application of relative aspects is to social interactions and dynamics, e.g. Durlauf and Young (2001) and Glaeser, Sacerdote and Scheinkman (2002).

In the following, the overall utility function (1) used in the preceding section will now be replaced by

$$\int_0^\infty e^{-\rho t}\left[u(c)+V(s)\right]dt, \qquad s=s(a,A),$$

where u is as above and V (increasing and concave) evaluates the benefit from social status s in which A denotes average wealth in the domestic population. We will assume that i) status s depends positively on own wealth a and negatively on average wealth in the economy A, and ii) marginal status of own assets s_a is decreasing in a (for any given value of A)

$$s_a>0,\ s_A<0,\ s_{aa}<0, \qquad \forall\,(a,A)\in(\bar{a},\infty)\times(\bar{a},\infty), \tag{22}$$

where $\bar{a}<-\tilde{w}/r^*$ by assumption. In the rest of this section we will use the simplifying assumption that agents are identical in every respect. Under this assumption it is common practice in the literature to focus attention on symmetric equilibria in which identical agents make identical choices so that $a=A$ holds at any point in time along an equilibrium path. With respect to symmetric states we will assume that

$$s(a,a)=\chi, \qquad \forall a\in(\bar{a},\infty), \qquad \chi=\text{ constant}, \tag{23}$$

$$\frac{ds_a(a,a)}{da}=s_{aa}(a,a)+s_{aA}(a,a)<0, \qquad \forall a\in(\bar{a},\infty) \tag{24}$$

hold. (23) implies that, contrary to the absolute wealth approach studied in the preceding section, the flow of utility is now independent of the level of wealth a along any (symmetric) equilibrium path, $u(c)+V(s(a,a))=u(c)+V(\chi)$, $\forall a\in(\bar{a},\infty)$. Inequality (24) states that the marginal status with respect to own wealth a declines if the economy moves from one symmetric situation to another in which each individual has a higher level of wealth.

In the existing literature mentioned above, it is common practice to employ one of the following specifications of the status function:

$$s(a,A)=\varphi\left(\frac{a}{A}\right), \quad \varphi'>0, \varphi''\le 0, \tag{25}$$

$$s(a,A)=\varphi(a-A), \quad \varphi'>0, \varphi''\le 0. \tag{26}$$

The ratio specification (25) satisfies both assumptions with respect to symmetric states, (23) and (24). It also satisfies both (23) and (22) as long as $a > 0$ and $A > 0$. However, in our open economy framework in which a and A may become negative even in a symmetric equilibrium, the specification (25) violates two conditions given in (22): First, from $s_a = (1/A)\,\varphi'$, it follows that $s_a < 0$ for $A < 0$. Second, $s_A = -\left(a/A^2\right)\varphi'$ yields $s_A > 0$ for $a < 0$. For these reasons, the standard ratio specification (25) cannot be used in this open economy model. In example 3, however, we will adopt a modified version of the ratio specification employed in Fisher and Hof (2001), that satisfies all assumptions made with respect to the status function.[7]

The difference specification (26) used, for instance, in Corneo and Jeanne (2001a) satisfies all our assumptions about s with the single exception of (24), since $ds_a\,(a,a)\,/da = 0$ holds due to $s_a\,(a,a) = \varphi'\,(0)$. The consequences of this violation of (24) will be discussed in detail at the end of section 3.

A crucial feature of the decentralized optimization problem under status preferences is that the representative household takes the time path of average wealth A as given. In other words, each individual is small enough to neglect her own contribution to the average wealth level. It can be easily verified that in a symmetric macroeconomic equilibrium in which $A = a$ holds, the dynamic evolution of consumption is governed by the following modified Euler equation:

$$\dot{c}/c = \sigma\,(c)\left[r^* - \rho + M(c,a)\right], \tag{27}$$

where

$$M\,(c,a) \equiv \frac{V'(\chi)s_a\,(a,a)}{u'\,(c)} = \left.\frac{V'(s\,(a,A))s_a\,(a,A)}{u'\,(c)}\right|_{a=A} \tag{28}$$

is the marginal rate of substitution of own net assets a for consumption c as perceived by the consumer in symmetric situations (henceforth, the symmetric private MRS). The capital letter M is used to differentiate it from its pendant m used in the absolute wealth approach. The asset accumulation equation and the initial condition are identical to those used in the preceding section and in the transversality condition, (9), m has to be replaced by M. These properties imply that—in spite of the induced externality—there is no need for a separate dynamic analysis of the relative wealth case. All results can be easily obtained by substituting M for m in the dynamic analysis of the absolute wealth approach.

[7] The focus of the analysis in Fisher and Hof (2005) is on the comparative dynamic behavior of the small open economy, rather than issues such as existence of multiple equilibria or observational equivalence between absolute wealth and relative wealth preferences.

Recall that in the absolute wealth model $m_c > 0$ and $m_a < 0$. In the relative wealth framework, it follows from (28) and $u'' < 0$, that the symmetric private MRS depends positively on consumption, $M_c > 0$. In addition, M depends negatively on the level of net assets a, i.e., $M_a < 0$ due to (24). This in turn, implies that the $\dot{c} = 0$ locus is positively sloped, $-M_a/M_c > 0$, just as in the absolute wealth model $(-m_a/m_c > 0)$. Further properties of the $\dot{c} = 0$ locus depend on u''' and $d^2 s_a(a,a)/da^2$; in particular, $c_{aa}\,|_{\dot{c}=0}$ can be obtained by substituting in (14): $s_a(a,a)$ for v', $ds_a(a,a)/da$ for v'', and $d^2 s_a(a,a)/da^2$ for v'''. Since there exist specifications of relative wealth preferences that yield a strictly convex $\dot{c} = 0$ locus (similar to that given in Fig. 1), the status approach also allows for multiple interior equilibria and history-dependent evolution. In the following example we obtain results that are analogous to those derived in examples 1 and 2 for absolute wealth preferences: Multiple interior equilibria arise if i) the subjective rate of time preference is below a critical level, and ii) the symmetric private MRS is a hump-shaped function of nonhuman wealth a along the $\dot{a} = 0$ curve.

Example 3 Assume that preferences are represented by $u(c) + V(s(a,A))$, where $u(c) = (1-\theta)^{-1}\left(c^{1-\theta} - 1\right)$, V is increasing and concave in s, and

$$s(a,A) = \varphi\left(\frac{a-\bar{a}}{A-\bar{a}}\right), \qquad a, A > \bar{a};\ \varphi' > 0, \varphi'' < 0. \tag{29}$$

These specifications imply that $M(c,a) = V'(\varphi(1))\varphi'(1)(a-\bar{a})^{-1} c^{\theta}$, and $\sigma(c) = 1/\theta$. The resulting $\dot{c} = 0$ curve,

$$c = \left[\frac{\rho - r^*}{V'(\varphi(1))\varphi'(1)}\right]^{1/\theta} (a-\bar{a})^{1/\theta},$$

is unambiguously positively sloped. Obviously, it is globally strictly convex if $\theta < 1$. Since $\bar{a} < -\tilde{w}/r^*$ by assumption, it follows that if the $\dot{c} = 0$ curve is strictly convex, then it will intersect the $\dot{a} = 0$ line twice provided that $(\rho - r^*)/[V'(\varphi(1))\varphi'(1)]$ is sufficiently small. An alternative proof of this result is based on the fact that along the $\dot{a} = 0$ curve the symmetric private MRS is given by

$$Z(a) \equiv M(r^* a + \tilde{w}, a) = V'(\varphi(1))\varphi'(1)(a-\bar{a})^{-1}(r^* a + \tilde{w})^{\theta}$$

If $\theta < 1$, then Z is a hump-shaped function of a, i.e., $Z'(a) > 0$ for $a < \hat{a}$ and $Z'(a) < 0$ for $a > \hat{a}$, where $\hat{a} \equiv \bar{a} - (1-\theta)^{-1}(\bar{a} + \tilde{w}/r^*) > -\tilde{w}/r^*$. If, in addition, the subjective rate of time preference ρ is less than a critical level, then $Z(\hat{a}) > \rho - r^*$ and we obtain two interior steady states in which $Z(\tilde{a}_1) = Z(\tilde{a}_2) = \rho - r^*$.

The result that the introduction of relative wealth preferences may allow for multiple interior steady states has already been shown in Corneo and Jeanne (2001a). Our paper differs, however, in several respects from their work: First, while Corneo and Jeanne consider a closed economy in which the interest rate is endogenously determined, we examine a small open economy in which the interest rate equals the exogenously given world interest rate. Second, the focus of these authors is whether the quest for social status can be an engine of economic growth. In the present paper, we are not interested in growth without bound, since in this case the domestic economy would eventually accumulate enough assets to violate the small-country assumption. Third, they do not examine whether their results could also be obtained in a pure absolute wealth approach (i.e., in the absence of status concerns). In our paper we will finally also study the conditions for the observational equivalence between economies with absolute and relative wealth preferences.

It turns out that two assumptions in Corneo and Jeanne (2001a) are crucial for unbounded growth: The first one is that the marginal productivity of capital net of depreciation remains positive as capital increases. In terms of our notation, the second one is equivalent to bounding $s_a(a, a)$ from below by a strictly positive number.[8] Obviously, the second assumption is satisfied for the difference specification (26), $s(a, A) = \varphi(a - A)$, due to $s_a(a, a) = \varphi'(0)$. For convenience, Corneo and Jeanne restrict their attention to this simplifying specification. When the status-seeking motive is weak, i) there are two steady states $\left(\tilde{k}^1, \tilde{c}^1\right)$ and $\left(\tilde{k}^2, \tilde{c}^2\right)$ with $\tilde{k}^2 > \tilde{k}^1$, where $\left(\tilde{k}^1, \tilde{c}^1\right)$ is saddlepoint-stable, while $\left(\tilde{k}^2, \tilde{c}^2\right)$ is an unstable node, and ii) long-run growth only appears if the initial stock of capital $k(0)$ exceeds $\tilde{k}^2$. However, when the status-seeking motive is strong, long-run growth arises irrespective of the initial condition. The growth process exhibits the interesting, yet 'extreme' property that the capital stock and the output level grow without bound, while consumption converges to a finite value.[9]

If the difference specification of the status function (26), $s(a, A) = \varphi(a - A)$, were used in the present paper, it would yield a horizontal $\dot{c} = 0$ line, $-M_a/M_c = 0$, intersecting the $\dot{a} = 0$ line from above. The resulting unique interior steady state would be an unstable node so that long-run convergence to an interior steady state would be impossible irrespective of the initial condition. Obviously, our result that the quest for status may allow for

[8] See Corneo and Jeanne (2001a, p. 356 and the footnote given on page 352).

[9] 'The motive behind this indefinite accumulation of capital is not to provide for future consumption. … It is accumulated purely for the sake of social status. … Although individuals would be collectively better off consuming more of this accumulated wealth, it is rational for each individual to hoard all additional income in order to maintain his social status' [Corneo and Jeanne (2001a, p. 356)].

long-run convergence to an interior steady state, without being an engine of growth, hinges crucially on the assumption that $s_a(a,a)$ depends negatively on a. Note however, that this property of the status function, which at least in our view is a quite natural one, is necessary, yet not sufficient. For instance, for $\theta > 1$ the specification of relative wealth preferences used in example 3 yields a strictly concave $\dot{c} = 0$ curve that intersects the $\dot{a} = 0$ line from above. The corresponding unique interior steady state is an unstable node, just like the steady state that results from the difference specification of the status function (26).

4. Digression on observational equivalence

From the above analysis it is also clear that the decentralized solutions for a and c obtained in the economy with absolute wealth preferences, represented by $u(c) + v(a)$, are identical to those of an economy with relative wealth preferences, given by $u(c) + V(s(a,A))$, if $M(c,a) = m(c,a)$ holds for all $c > 0$ and $a \in (\bar{a}, \infty)$. That is, both economies are observationally equivalent, although the behavior of individuals is governed by different psychological motivations. However, there remains a crucial difference in spite of observational equivalence: in the model with relative wealth preferences, the decentralized solution deviates from the socially optimal one due to wealth externalities. It is easily verified that the socially optimal solution coincides with the solution of the standard model.

The condition for observational equivalence can be rewritten as

$$V'(\chi)s_a(a,a) = v'(a) \qquad \forall a \in (\bar{a}, \infty). \tag{30}$$

For any economy with well-behaved relative wealth preferences there exists an observational equivalent economy with well-behaved absolute wealth preferences and vice versa. In this context the word 'well-behaved' means that the utility functions u, v, and V as well as the status function s satisfy all assumptions made in the paper. These assertions are easily verified as follows:

First, for given functions V and s, any function $v(a)$ defined as

$$v(a) \equiv V'(\chi) \int_{\check{a}}^{a} s_a(z,z)\, dz,$$

where $\check{a}$ is an arbitrary constant with $\check{a} > \bar{a}$, satisfies the condition for observational equivalence (30) as well as $v' > 0$ and $v'' < 0$. This follows from

$$v'(a) = V'(\chi)s_a(a,a) > 0, \qquad a \in (\bar{a}, \infty),$$

$$v''(a) = V'(\chi)\left[s_{aa}(a,a) + s_{aA}(a,a)\right] < 0, \qquad a \in (\bar{a}, \infty),$$

where the inequalities hold due to $V' > 0$ and the assumptions made in (22) and (24).

Second, for given v, the simplest way to construct an observational equivalent economy with relative wealth preferences is to assume that i) the status function takes the additively separable form

$$s(a, A) \equiv v(a) - v(A) + \chi,$$

where χ is an arbitrary constant, and ii) V is an arbitrary increasing and concave function satisfying $V'(\chi) = 1$. Obviously, V and s satisfy the condition for observational equivalence (30) and all assumptions made in the paper due to

$$s_a(a, A) = v'(a) > 0, \quad s_A(a, A) = -v'(A) < 0, \quad (a, A) = v''(a) < 0,$$

$$s(a, a) = \chi, \qquad s_{aa}(a, a) + s_{aA}(a, a) = v''(a) < 0.$$

In addition to this more or less trivial specification of relative wealth preferences, there may also exist non-trivial ones. This is illustrated by means of examples 2 and 3, in which the condition for observational equivalence, $m(c, a) = M(c, a)$, for all $c > 0$ and $a \in (\bar{a}, \infty)$, becomes

$$\gamma c^{\theta} (a - \bar{a})^{-\xi} = V'(\varphi(1))\varphi'(1)\, c^{\theta} (a - \bar{a})^{-1} \text{ for all } c > 0 \text{ and } a \in (\bar{a}, \infty).$$

Obviously, in the special case $\xi \to 1$ in which $v(a)$ simplifies to the logarithmic specification

$$v(a) = \gamma \ln(a - \bar{a}) = \lim_{\xi \to 1} \gamma (1 - \xi)^{-1} \left[(a - \bar{a})^{1-\xi} - 1\right],$$

non-trivial observational equivalent relative wealth preferences are given by those of example 3, provided that V satisfies the condition $V'(\varphi(1))\varphi'(1) = \gamma$.

5. Concluding Remarks

The small open economy version of the Ramsey model exhibits the property that both total wealth and consumption tend to zero in the long run in case that the constant subjective rate of time preference of the domestic residents exceeds the exogenously given world interest rate. This counterfactual outcome initiated various modifications and extensions of the standard model in which the unpleasant standard steady state was replaced by a unique interior steady state. In this paper it was shown that incorporating either absolute or relative wealth preferences may allow for multiple interior steady states, while retaining the standard steady state. Convergence to an interior steady state then arises if the initial level of nonhuman wealth exceeds an endogenously determined threshold, which coincides with an unstable steady state. All these results depend crucially on the fact that both extensions yield a variable effective rate of time preference (resp. a variable effective rate of return on assets) that depends on own consumption and own nonhuman wealth. The possibility of multiple interior steady states and history-dependent evolutions implies that different attitudes towards wealth per se and status, respectively, could provide (at least partial) explanations of the persistent differences between debt ridden (e.g. Latin America throughout the twentieth century) and wealth accumulating economies (e.g. Singapore, Taiwan, South Korea).

One might argue that it is not surprising that the incorporation of absolute wealth preferences may allow for interior steady states such that impatient agents accumulate net assets (instead of asymptotically mortgaging all of their capital and labor income). Yet this objection overlooks that absolute wealth preferences need not eliminate the unpleasant standard steady state. On the contrary, it can still be the only saddlepoint-stable long-run outcome and in fact is always a stable long-run equilibrium in our examples, no matter how much wealth is appreciated. However, maybe more surprising is that multiple interior steady states may also arise under relative wealth preferences in which the level of wealth does not affect the flow of utility in a symmetric equilibrium. The reason is that the marginal rate of substitution of own net assets for consumption as perceived by the consumers (who take average wealth as given) depends on both variables even in symmetric states. Hence, just as under absolute wealth preferences, the effective rate of time preference (resp. effective rate of return on assets) depends on both consumption and own nonhuman wealth. This property even allows for the observational equivalence of an economy with absolute wealth preferences and an economy with relative wealth preferences. In this context, a simple condition was given which ensures that the equilibrium time paths of consumption and wealth are identical in the two economies.

The time path of the economy may be history-dependent under both relative and absolute wealth preferences, and thus in the latter case in a concave economy without externalities. A particularly surprising finding is that the policy function is continuous in the state, in particular, at the unstable steady state that equals the threshold. These (technical) properties contrast with a large amount of the existing literature that introduces (locally) convex relations (often attributed to increasing returns, recent examples of macroeconomic approaches are Greiner and Semmler (1999) and Semmler and Sieveking (1999)), and sometimes, in addition, an externality (see e.g. Krugman

(1991)), in order to obtain similar hystereses effects. Furthermore, these models with history-dependent evolutions are characterized by controls that are indeterminate and discontinuous at the threshold.[10] To sum up, the two alternative wealth approaches studied in this paper provide additional and by and large overlooked reasons for history dependent outcomes.[11] As a consequence, history dependence applies for a larger domain than so far emphasized in the existing literature with its focus on convex-concave relations.

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References

Abel, A. B. (2005), Optimal Taxation When Consumers Have Endogenous Benchmark Levels of Consumption, *Review of Economic Studies* 72: 21–42.

Barro, R. J. and X. Sala-i-Martin (1995), *Economic Growth*, Mc Graw Hill.

Blanchard, O. J. (1985), Debt, Deficits, and Finite Horizons, *Journal of Political Economy* 93: 223–47.

Boskin, M. J. and Sheshinski, E.(1978), Optimal Redistributive Taxation when Individual Welfare Depends on Relative Income, *The Quarterly Journal of Economics* 92, 589–601.

Brock, W. A. and Dechert, D. W. (1985), Dynamic Ramsey Pricing, *International Economic Review* 26, 569–591.

Corneo, G. and Jeanne, O (1997), On Relative Wealth Effects and the Optimality of Growth, *Economics Letters* 54: 87–92.

Corneo, G. and Jeanne, O. (2001a), On Relative Wealth Effects and Long-Run Growth, *Research in Economics* 55: 349–358.

Corneo, G., and Jeanne, O. (2001b), Status, the Distribution of Wealth, and Growth, *Scandinavian Journal of Economics* 103: 283–293.

Dechert, D. W. and Nishimura, K. (1983), Complete Characterization of Optimal Growth Paths in an Aggregative Model with a Non-Concave Production Function, *Journal of Economic Theory* 31: 332–354.

Dupor, B. and Liu, W. F. (2003) Jealousy and Equilibrium Overconsumption, *American Economic Review* 93: 423–428.

[10] Compare the theoretical papers of Skiba (1978) and Dechert-Nishimura (1983) and of the many following applications we refer to Brock and Dechert (1985) as an early example and to Ladron-de-Guevara, Ortigueira and Santos, (1999), and Mäler (2000) for recent examples.

[11] An additional reason is given in Feichtinger et al. (2001) that obtains history dependence without an externality but within a relative adjustment cost framework that exploits a different mechanism.

Durlauf, S. N. and Peyton Young, H. (eds.) (2001), *Social Dynamics*, MIT Press.

Easterlin, R. A. (1974) Does Economic Growth Improve the Human Lot? Some Empirical Evidence, in P. A. David and M. W. Reder (eds.) *Nations and Households in Economic Growth: Essays in Honor of Moses Abramowitz*, Academic Press.

Easterlin, R. A. (1995), Will Raising the Incomes of All Increase the Happiness of All?, *Journal of Economic Behavior and Organization* 27: 35–47.

Fehr, E. and Fischbacher, U. (2002), Why Social Preferences Matter – The Impact of Non-Selfish Motives on Competition, Cooperation and Incentives, *Economic Journal* 112: C1–C34.

Feichtinger G., Hartl, R. F., Kort, P. and Wirl, F. (2001), The Dynamics of a Simple Relative Adjustment-Cost Framework, *German Economic Review* 2: 255–268.

Fisher, W. H. (2004), Status Preference, Wealth, and Dynamics in the Open Economy, *German Economic Review* 5: 335–355.

Fisher, W. H. and Hof, F. X. (2000), Relative Consumption, Economic Growth, and Taxation, *Journal of Economics* 72: 241–62.

Fisher, W. H. and Hof, F. X. (2005), Status Seeking in the Small Open Economy, *Journal of Macroeconomics* 27: 209–232.

Frank, R. H. (1985) *Choosing the Right Pond – Human Behavior and the Quest for Status*, Oxford University Press.

Frank, R. H. (1997) The Frame of Reference as a Public Good, *Economic Journal* 107: 1832–47.

Futagami, K. and Shibata, A. (1998), Keeping One Step Ahead of the Joneses: Status, the Distribution of Wealth, and Long Run Growth, *Journal of Economic Behavior and Organization* 36: 93–111.

Galí, J., (1994), Keeping Up with the Joneses: Consumption Externalities, Portfolio Choice, and Asset Prices, *Journal of Money, Credit, and Banking* 26: 1–8.

Glaeser, E. L., Sacerdote, B. I., and J. A. Scheinkman (2002), The Social Multiplier, http://www.nber.org/papers/w9153.

Gootzeit, M., Schneider, J., and Smith, W. (2002) Marshallian Recursive Preferences and Growth, *Journal of Economic Behavior and Organization* 49: 381–404.

Greiner A. and Semmler, W. (1999), Monetary Policy, Multiple Equilibria and Hysteresis Effects on the Labour Market, paper prepared for the conference on: Computing in Economic and Finance, Barcelona, June 2000, mimeo.

Grossmann, V. (1998), Are Status Concerns Harmful for Growth?, *FinanzArchiv* 55: 357–73.

Harbaugh, R. (1996), Falling Behind the Joneses: Relative Consumption and the Growth-Savings Paradox, *Economics Letters* 53: 297–304.

Krawczyk J. and Shimomura, K. (2003) Why Countries with the same Technology and Preferences can have different Growth Rates, *Journal of Economic Dynamics and Control* 27: 1899–1916.

Krugman, P. (1991), History versus Expectations, *Quarterly Journal of Economics* 56: 651–667.

Ladron-de-Guevara, A., Ortigueira, S., and Santos, M. S. (1999) A Two Sector Model of Endogenous Growth with Leisure, *Review of Economic Studies* 66: 609–631.

Liu, W.F. and Turnovsky, S. J. (2005), Consumption Externalities, Production Externalities, and the Accumulation of Capital, *Journal of Public Economics* 89: 1097–1129.

Ljungqvist, L. and Uhlig, H. (2000), Tax Policy and Aggregate Demand Management Under Catching Up with the Joneses, *The American Economic Review* 90: 356–66.

Mäler, K. G. (2000), Development, Ecological Resources and their Management: A Study of Complex Dynamic Systems, Joseph Schumpeter Lecture, *European Economic Review* 44: 645–665.

Persson, M. (1995), Why are Taxes so High in Egalitarian Societies?, *Scandinavian Journal of Economics* 97: 569–80.

Rauscher, M. (1997a), Protestant Ethic, Status Seeking, and Economic Growth, *Discussion Paper*, University of Rostock, Germany.

Rauscher, M. (1997b), Conspicuous Consumption, Economic Growth, and Taxation, *Journal of Economics* 66: 35–42.

Semmler W. and Sieveking, M. (1999), Using Vector Field Analysis for Studying Debt Dynamics, paper prepared for the North American Winter Meeting of the Econometric Society, Chicago. mimeo, University of Bielefeld.

Skiba, A. K. (1978), Optimal Growth with a Convex-Concave Production Function, *Econometrica* 46: 527–539.

Turnovsky, S. J. (1995), *Methods of Macroeconomic Dynamics*, MIT Press.

Turnovsky, S. J. (1997), *International Macroeconomic Dynamics*, MIT Press.

Van Long, N. and Shimomura, K. (2004a) Relative Wealth, Status-Seeking, and Catching-Up, *Journal of Economic Behavior and Organization* 53: 529–542.

Van Long, N. and Shimomura, K. (2004b) Relative Wealth, Catching-up, and Economic Growth, in S. Dowrick, R. Pitchford, and S.J. Turnovsky (eds), *Economic Growth and Macroeconomic Dynamics: Recent Developments in Economic Theory*, Cambridge University Press.

Wirl, F. and Feichtinger, G. (2005), History Dependence in Concave Economies, *Journal of Economic Behavior and Organization* 57: 390–407.