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Basic quantum irreversibility in neutron interferometry

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Abstract
The transition between the quantum and classical world is a topical problem in quantum physics, which can be investigated by neutron interferometric methods. Here we discuss unavoidable quantum losses as they appear in neutron phase-echo and spin rotation experiments and we show how entanglement effects in a single-particle system demonstrate quantum contextuality, i.e. an entanglement between external and internal degrees of freedom in single-particle systems. This contextuality phenomenon also shows that a quantum system carries much more information than usually extracted. In all cases of an interaction, parasitic beams are produced which cannot be recombined completely with the original beam. This means that a complete reconstruction of the original state is, in principle, impossible which causes a kind of intrinsic irreversibility. Even small interaction potentials can have huge effects when they are applied in quantum Zeno-like experiments. The path towards advanced neutron quantum optics will be discussed.

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(Some figures in this article are in colour only in the electronic version.)

1. Introduction

Neutron interferometry has proven its capacity to proof quantum phenomena of massive particles. Pure single-particle effects exist in any neutron experiment due to the extremely low-phase-space density of neutron beams (10^{-14}). Coherent beam separations up to several centimetres can be achieved by perfect silicon interferometers and these beams can be influenced by nuclear, magnetic and gravitational interactions, but also by topological effects. Perfect crystal neutron interferometry invented in 1974 [1] and its status until the year 2000 has been summarized in a book [2]. It can be stated in a rather general form that the wave function behind the interferometer is composed by wave functions from both beam paths and contains much more information than can be extracted by standard experimental techniques. In most cases one focuses on one parameter, but there are always additional effects which may be of interest in the discussion about basic quantum entanglement, irreversibility and decoherencing effects. The question of entanglement and decoherence becomes an essential issue for the understanding of quantum mechanics and especially of the quantum measurement process. Several books and review articles deal with this topic [3–5]. Here we will show that not only do dissipative interactions cause an irreversible change of the wave function, but that deterministic ones can cause such a change too.

The well-known Zeno phenomenon (paradox) can be taken as a characteristic example of a temporal or spatial evolution of a quantum system which is kept under frequent observation and which becomes frozen in the initial state [6–9], but shows an essential different behaviour when realistic situations are considered [10–18]. The topic is closely related to the non-exponential decay of quantum states for very short times where a quadratic time dependence is expected and to so-called 'interaction-free' measurements [15, 19].

Additionally, it must be considered that quantum entanglement between external and internal degrees of freedom exist and this means that any measurement of one observable influences the outcome of a later measurement of another observable, which denotes quantum contextuality [20–22].

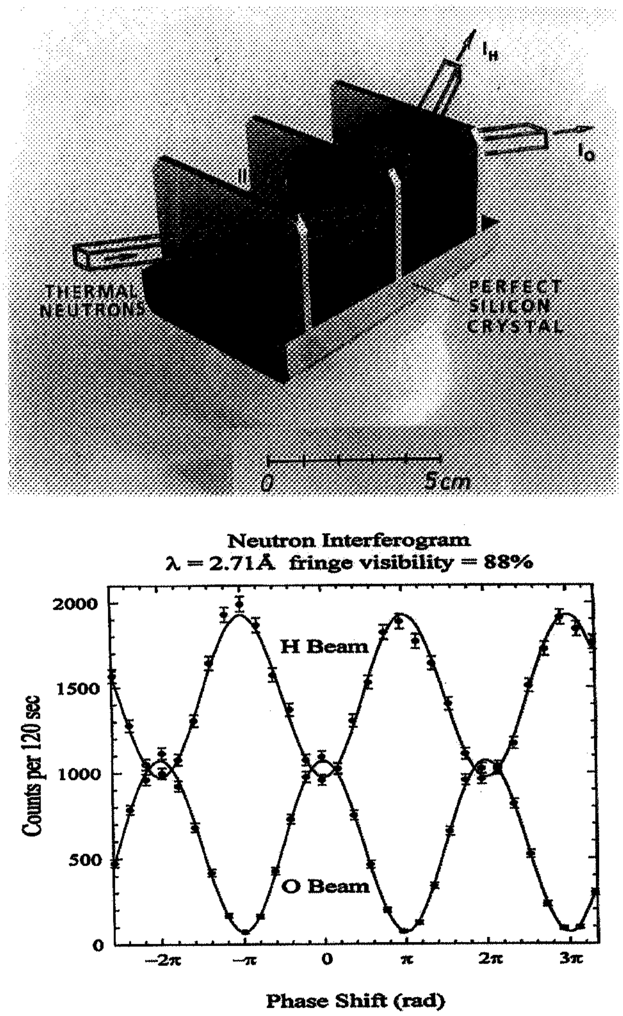

Figure 1. Sketch of a perfect crystal neutron interferometer and a typical interference pattern.

2. Basic relations

The wave function for the beam in forward direction (0) behind a Mach–Zehnder interferometer (figure 1) is given by the superposition of wave functions arising from the right and the left beam paths. They are transmitted–reflected–transmitted (trr) and reflected–reflected–transmitted (rrt), respectively, and they are equal in amplitude and phase due to symmetry reasons ($\psi_{\rm trr} = \psi_{\rm rrt}$).

When parts of the beams are exposed to an interaction, a phase shift occurs

$$\chi = \oint k_c \, \mathrm{d}s, \tag{1}$$

where k_c denotes the canonical momentum of the neutrons along the beam paths. Conservative interactions cause a change of the kinematical momentum k, which can be described by an index of refraction (e.g. [1])

$$n = \frac{k}{k_0} = \sqrt{1 - \frac{\bar{V}}{E}}. \tag{2}$$

$\bar{V}$ denotes the mean optical interaction potential within the phase shifter which amounts to $\bar{V} = 2\pi\lambda^2 N b_c/m$ for nuclear interaction and to $\bar{V} = \pm\mu B$ for magnetic interaction. m and μ denote the mass and the magnetic moment of the neutron, N the particle density and b_c the coherent neutron scattering length of the phase shifter. When a purely magnetic interaction is considered, equation (2) describes the longitudinal Zeeman splitting.

The different kinematical momentum of the beam inside the phase shifter also causes a spatial shift Δ of the wave packets and the phase shift can also be written as

$$\chi = (1 - n)\, kD = \Delta \cdot k. \tag{3}$$

A realistic description of a neutron beam can only be achieved by a wave packet formalism, which, for a stationary and one-dimensional situation, is

$$\psi(r) \propto \int a(k) \mathrm{e}^{ikr} \, \mathrm{d}k, \tag{4}$$

where $a(k)$ denotes the amplitude function, which is related to the momentum distribution function as $g(k) = |a(k)|^2$. It represents a coherent superposition of plane waves, which are non-local and which may be responsible for the non-local feature of quantum mechanics [23]. The interference pattern follows as

$$\begin{aligned} I(\Delta) \propto |\psi(0) + \psi(\Delta)|^2 &= \left|\psi_{\rm rrt} + \psi_{\rm trr} \mathrm{e}^{\mathrm{i}\Delta\cdot k}\right|^2 \\ &= 1 + |\Gamma(\Delta)| \cos(\Delta \cdot k), \end{aligned} \tag{5}$$

where $\Gamma(\Delta)$ denotes the coherence function which is given by the autocorrelation function of the wave function (e.g. [24])

$$|\Gamma(\Delta)| = |\langle\psi(0)\psi(\Delta)\rangle| \propto \left|\int g(k) \mathrm{e}^{ik\Delta} \mathrm{d}k\right|. \tag{6}$$

$|\Gamma(\Delta)|$ can be determined for three dimensions by the measured visibility of the interference pattern at large phase shifts [25]. For Gaussian wave packets the characteristic widths Δ_c of the coherence function and the momentum spread of the packets δk fulfil the minimum Heisenberg uncertainty relation ($\Delta_c \delta k = \frac{1}{2}$) and

$$|\Gamma(\Delta)| = \exp[-(\Delta \cdot \delta k)^2/2]. \tag{7}$$

When several phase shifts act onto the beams χ or Δ are additive quantities (equations (1) and (3)), which has been demonstrated in a dedicated phase-echo experiment (figure 2) [26]

$$\chi = \chi_1 + \chi_2 \quad \text{or} \quad \Delta = \Delta_1 + \Delta_2. \tag{8}$$

In this experiment a large phase shift, i.e. larger than the coherence length, has been applied by a Bi- and alternatively by a Ti-phase shifter. The visibility of the interference pattern nearly disappeared completely, in agreement with (5) and (8) as shown in figure 2. Bi and Ti cause an opposite phase shift due to a positive and negative coherent scattering length and, therefore, the interference pattern can be retrieved when both phase shifters are applied simultaneously and $\chi_1 + \chi_2 \cong 0$ [26]. These experiments can be improved when magnetic fields are used which do not cause any absorption or incoherent scattering effects. The analogy of the phase-echo concept to the spin-echo concept should be emphasized [27, 28].

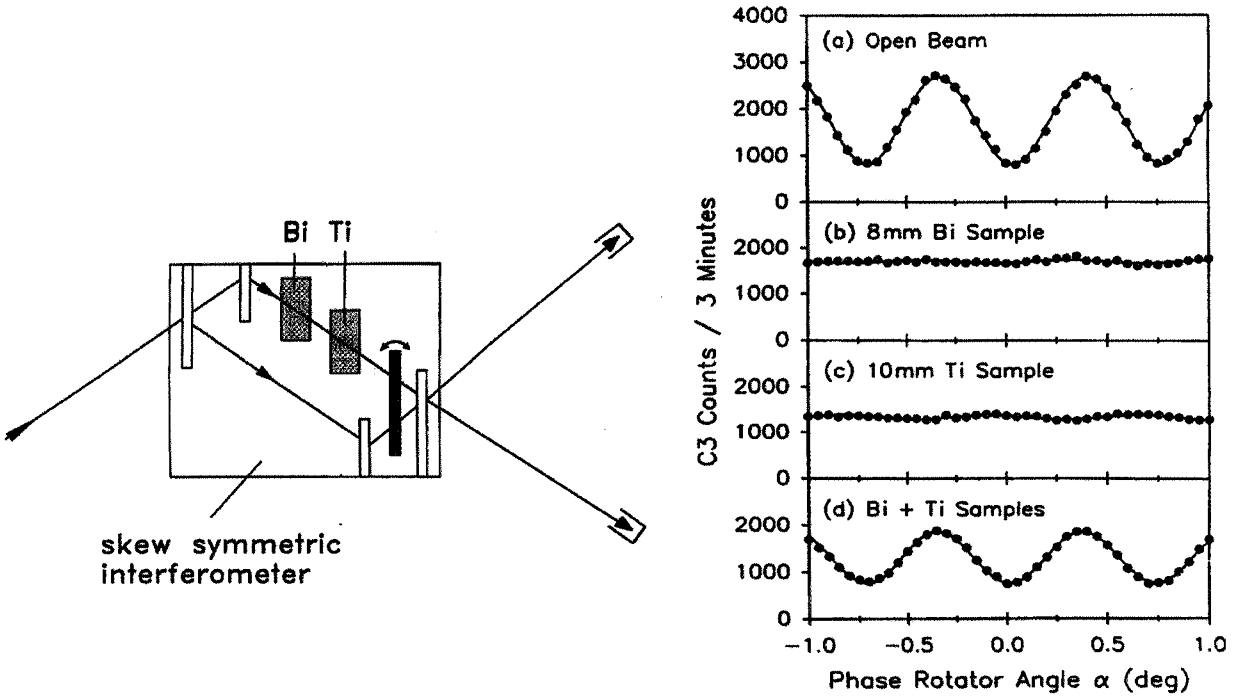

Figure 2. Phase-echo experiment using phase shifters with positive (Bi) and negative (Ti) coherent scattering lengths [26].

3. Quantum irreversibility effects

In the phase-echo experiments the question arises whether a complete retrieval of the interference pattern is feasible or not. First of all it should be mentioned that any phase shift is caused by an interaction which changes the momentum of the neutron (2) and such interaction potentials do not only cause a phase shift but also a back-reflection and/or back- and forth-reflections of parts of the wave function, as shown schematically in figure 3. One notices additionally that the phase shifts of the direct transmitted beam are additive whereas all other partial waves have much larger phase shifts and their amplitudes vary as well. A complete retrieval seems to become impossible [29]. The losses are at least in the order of $(V_0/E)^2$ and can become much larger when resonance effects are considered as well (e.g. super-mirrors) [30, 31].

In this connection quantum Zeno-like experiments have been proposed when neutrons cross a magnetic field many times within a perfect neutron resonator [10, 16, 19, 32]. Such experiments are related to experiments which investigate a non-exponential behaviour of quantum state transitions. Due to unavoidable quantum losses a complete freezing of the initial state becomes impossible.

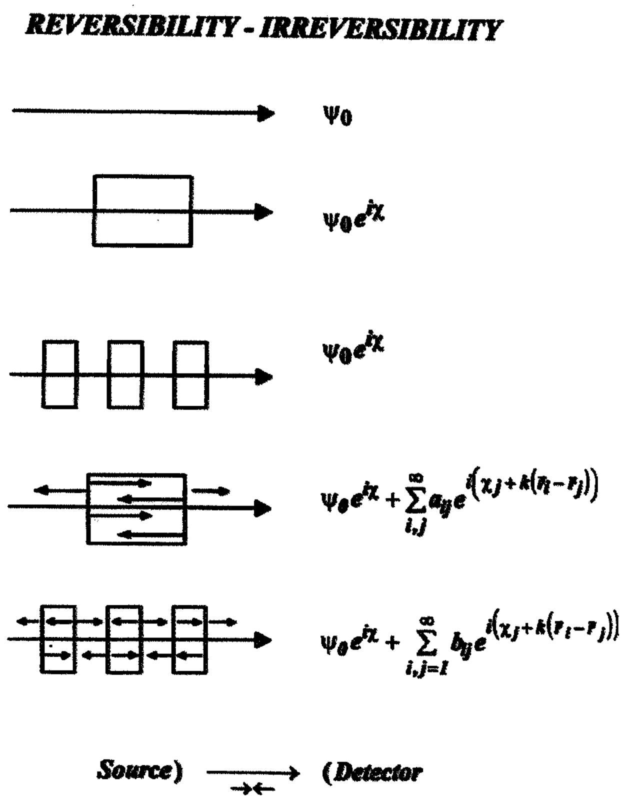

Figure 3. Approximate and complete wave functions when different phase shifters causing the same total phase shift are used.

4. Neutron post-selection experiments

Equations (5) and (7) show that the interference pattern at high interference order vanishes when the phase shift becomes larger than the coherence length. When the interference in ordinary space disappears, a modulation of the momentum distribution appears. This is shown schematically in figure 4 [23].

In light optics [33, 34] and more recently in neutron optics [35], it has been shown that interference can be retrieved when a momentum post-selection method is applied (figure 5). In this case, the wave packets in ordinary space become enlarged and overlap with the reference packet. This figure also shows that interferences can be observed

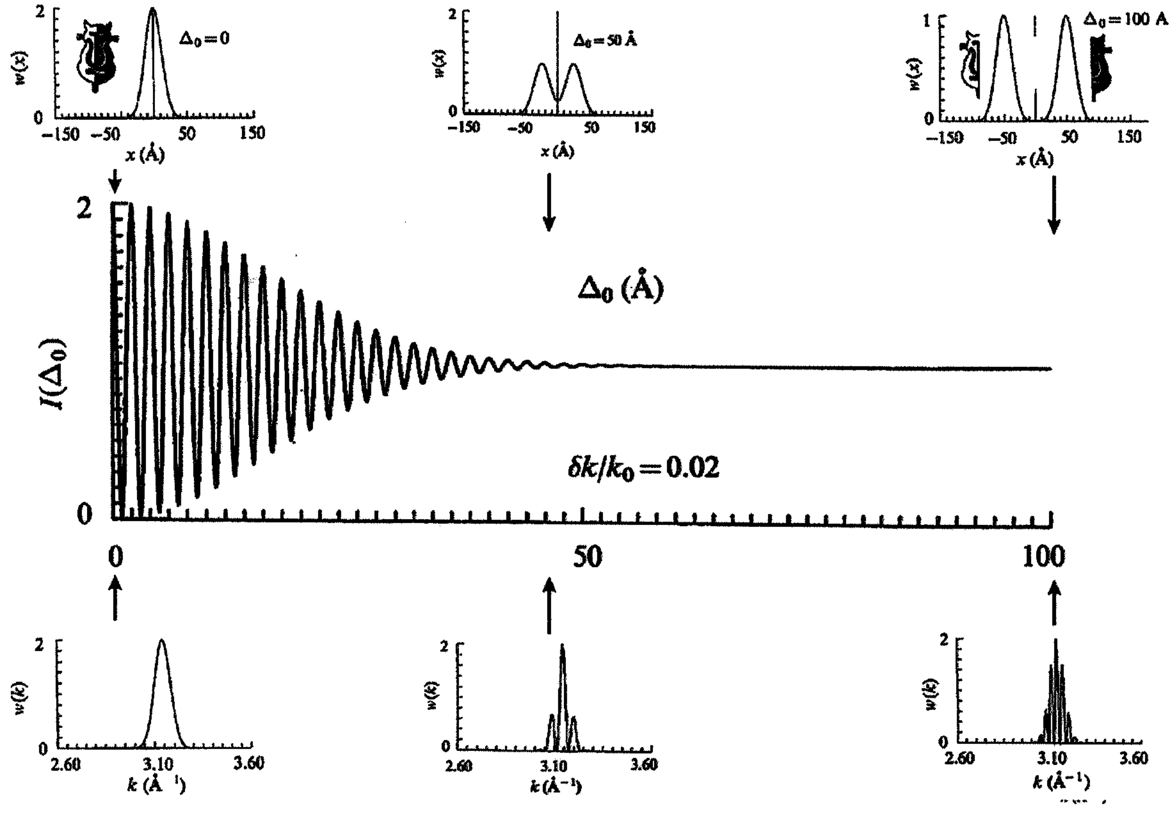

Figure 4. Schematic contrast reduction at higher interference order (middle) and wave packets in ordinary and momentum space at 0th, 50th and 100th interference order.

with beam detectors no. $1 \ldots n$, whereas a simultaneous measurement of the total count rate or the numerical addition of all count rates does not show any interference pattern. In terms of a wave-particle dualism, several authors describe the wave properties by the square of the fringe visibility (V^2) and the particle properties by diagonal terms of the density operator (P_D, which can also be seen as path distinguishability) [36–39]

$$P_D^2 + V^2 = 1. \tag{9}$$

A relation which has been tested by means of a two-loop interferometer, where a distinct quantum state can be prepared with the first loop and analysed by the second one (figure 6). The measured values agree fairly well with the predictions [40–42].

5. Quantum engineered neutron states

The post-selection experiments have shown that non-classical neutron states can be produced by means of interference experiments, e.g. Schrödinger's cat-like states where the neutron is dislocated at two local separated regions. Such states are most clearly described by means of Wigner functions [24, 43, 44]. The Wigner quasi-distribution function is given by

$$W_s(k, x) = \frac{1}{\pi} \int_{-\infty}^{+\infty} e^{ikx'} \psi^* \left(x + \frac{x'}{2} \right) \psi \left(x - \frac{x'}{2} \right) dx', \tag{10}$$

and has the features that a spatial integration gives the momentum distribution and a momentum integration the spatial distribution.

A negative part of the Wigner function indicates a non-classical state. Quantum state reconstruction experiments based on quadrature measurements of position and momentum yield the Wigner function or the density operator of a quantum state [45–47]. An example of a neutron spin quantum state experiment can be found in [48].

Figure 7 shows the Wigner function at low and high interference order [44]. When any fluctuations and parasitic side effects are included a complete retrieval by means of phase-echo methods becomes impossible (see chapter 3). This has been treated in detail for neutron spin-echo systems which are also interference systems of two beams in the forward direction but with beams split by the Zeeman effect [49].

With a double-loop interferometer even more complicated and advanced non-classical states can be produced and measured. Figure 8 shows such a double-loop interferometer and the related Wigner function at the exit of such an advice [44]. A triple humped spatial wave function can be produced which observes the physical situation at a position r within a sample at time t_0, $t_1 = \Delta_1/v$ and $t_2 = (\Delta_1 + \Delta_2)/v$, i.e. it provides the basis of measuring condensed matter triple correlation functions.

6. Contexuality or self-entangled neutron states

Entanglement of pairs of photons or material particles is a well-known phenomenon (e.g. [50, 51]). Contextuality

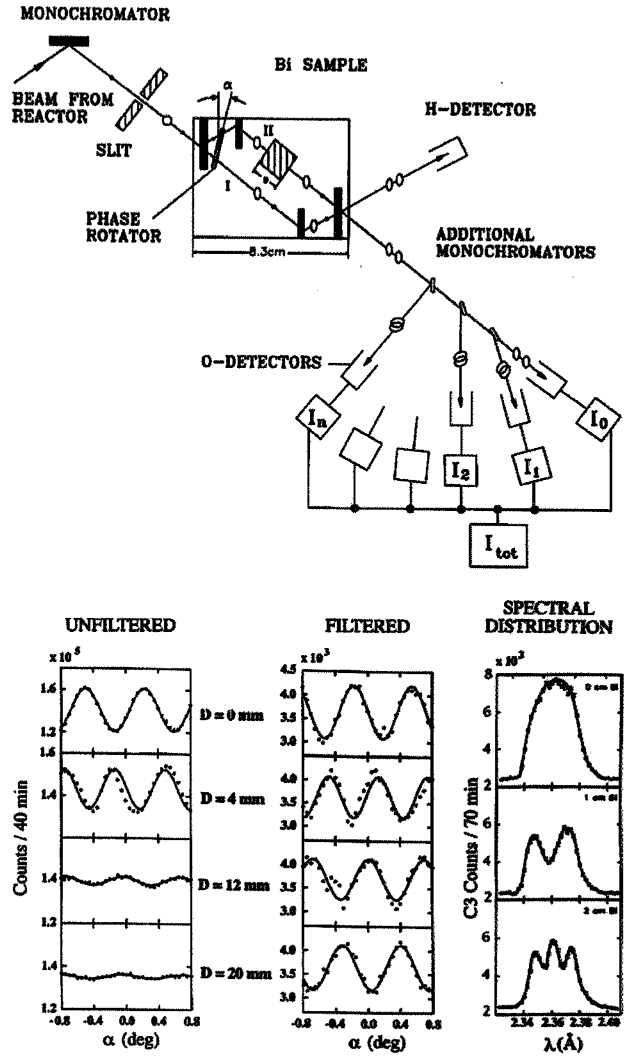

Figure 5. Sketch of a momentum post-selection set-up and interference pattern of the overall beam (left, detector I_0), the filtered beam (middle, detector I_n) and the related momentum distribution by rocking the analyser crystal [35].

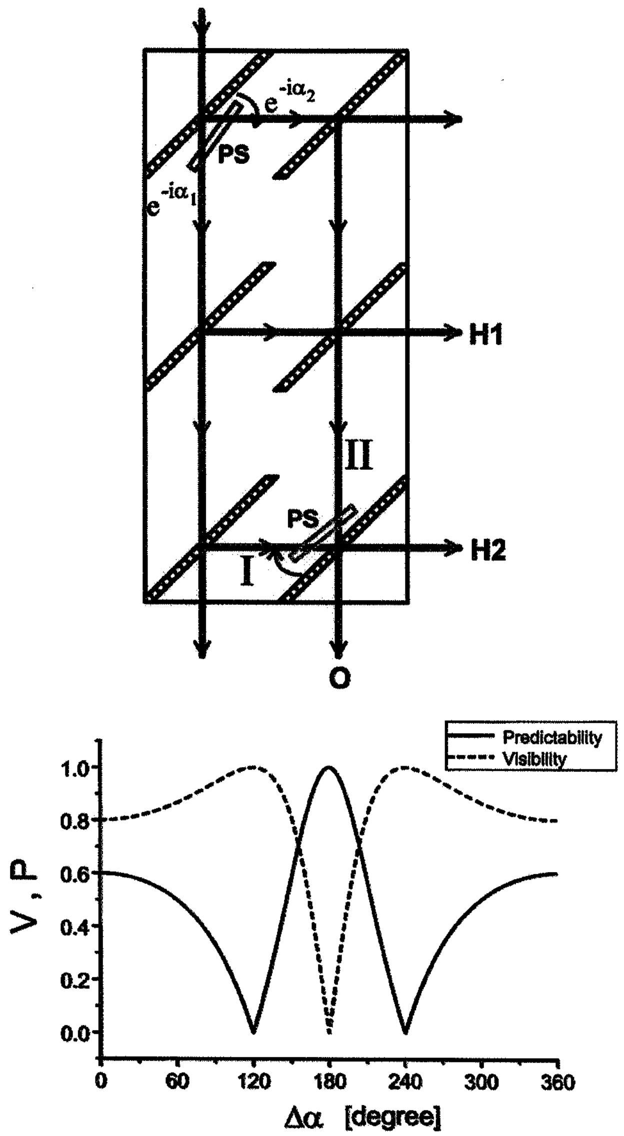

Figure 6. Double-loop interferometer used to measure wave-particle properties (top) and sketch of the experimental results (bottom) [41].

means a quantum entanglement between different degrees of freedom in a single particle. In a related neutron experiment we made a joint measurement of the commuting observables of the spin and of the beam path through the interferometer [52]. The related entangled state can be produced within the interferometer when a polarized incident beam is split coherently into the two beam paths (I and II) and the spin in one beam path is rotated by Larmor precession to the $-y$ and in the other beam path to the $+y$ direction (figure 9). The entangled state reads as

$$|\psi\rangle = (|\rightarrow\rangle \otimes |\mathrm{I}\rangle + |\leftarrow\rangle \otimes |\mathrm{II}\rangle). \tag{11}$$

In this case Bell-like inequalities can be formulated, where the expectation values can be measured when the phase shift χ between the beams and the spin rotation angle α are chosen as given in the formulas.

$$-2 \leqslant S \leqslant 2,$$
$$S = E(\alpha_1, \chi_1) + E(\alpha_1, \chi_2) - E(\alpha_2, \chi_1) + E(\alpha_2, \chi_2),$$
$$E(\alpha, \chi) = \frac{N(\alpha, \chi) + N(\alpha+\pi, \chi+\pi) - N(\alpha, \chi+\pi) - N(\alpha+\pi, \chi)}{N(\alpha, \chi) + N(\alpha+\pi, \chi+\pi) + N(\alpha, \chi+\pi) + N(\alpha+\pi, \chi)}. \tag{12}$$

The maximal violation of this inequality due to quantum mechanics happens for the following parameters: $\alpha_1 = 0$, $\alpha_2 = \pi/2$, $\chi_1 = \pi/4$ and $\chi_2 = -\pi/4$ and amounts to $S = 2\sqrt{2} = 2.82$. The first experiment in this direction yielded $S = 2.051 \pm 0.019$ indicating a violation of the Bell inequality and contradicting classical hidden variable theories [52].

More recently related measurements dealt with the Kochen–Specker theorem [53] and the Mermin inequalities [54], which showed even a stronger violation of classical hidden theories [55]. Thus that different degrees of freedom in a single particle have to be considered as entangled opens new possibilities when such states are

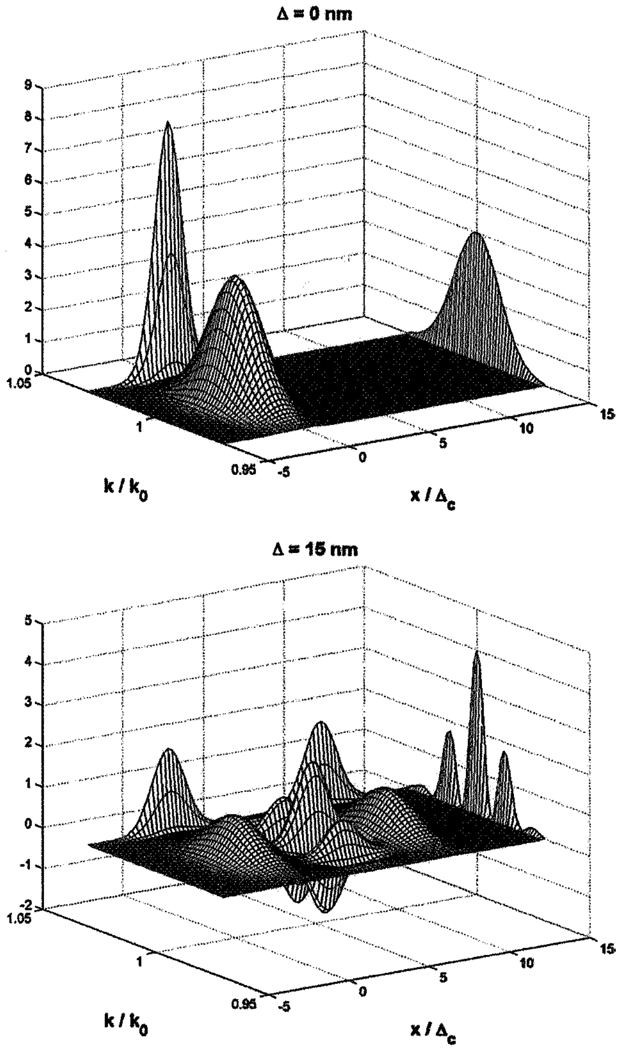

Figure 7. Interferometric Wigner functions at zero- and higher interference order.

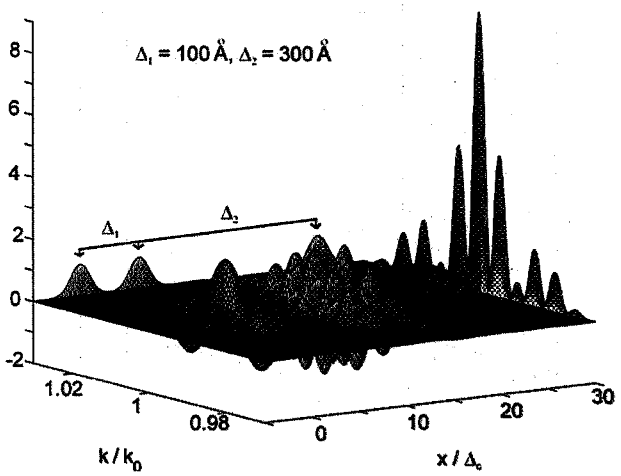

Figure 8. Triple humped Wigner function behind a double-loop interferometer.

used in spectroscopy since a measurement on one degree of freedom influences the outcome of a subsequent measurement of the other commuting observable.

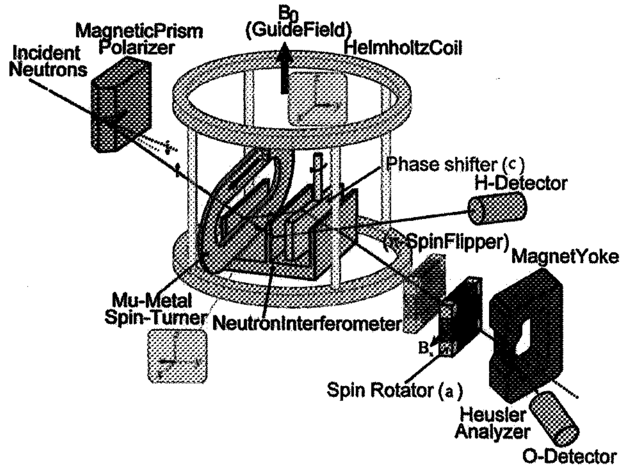

Figure 9. Experimental set-up for the production of spin-path entangled neutron states used for the proof of quantum contextuality [52].

7. Discussion

Due to their variety of interactions neutrons are proper tools for test experiments in quantum mechanics. It has been shown that any interaction of a neutron beam with a potential produces unavoidable parasitic beams which are characteristic for just this kind of interaction. Multiple potential barriers do not only produce an additive effect of losses, but also show enhanced losses due to various resonance effects. Therefore, the Zeno phenomenon, which is based on a repetitive interaction and observation of a quantum system, has to be discussed in a new light including unavoidable quantum losses. In this respect, the imperfect Zeno phenomenon and the limit of a non-completely frozen state for an infinite number of stages can be seen as a fingerprint of quantum mechanics. Various shapes of wave packets can be produced which can provide a basis for the measurement of higher order condensed matter correlation functions. For the first time an entanglement in a single-particle system has been proved which verifies quantum contextuality. The demonstrated single-particle entanglement effect may have consequences for the interpretation of many-particle entanglement experiments as well.

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