Mixed Conforming Elements for the Large-Body Limit in Micromagnetics

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We consider the large-body limit of the stationary Landau-Lifshitz minimization problem introduced by DeSimone in 1993, [1]: Find a minimizer $\mathbf{m} : \Omega \to \mathbb{R}^d$ with $|\mathbf{m}| \leq 1$ a.e. of the bulk energy

(1) $$E(\mathbf{m}) = \frac{1}{2} \int_{\mathbb{R}^d} |\nabla u|^2 \, dx + \int_\Omega \varphi^{**}(\mathbf{m}) \, dx - \int_\Omega \mathbf{f} \cdot \mathbf{m} \, dx.$$

Here, $\Omega \subset \mathbb{R}^d$, for $d = 2, 3$, is the spatial domain of the ferromagnetic material, φ^{**} is the (convexified) anisotropy density, and $\mathbf{f} : \Omega \to \mathbb{R}^d$ is an applied exterior field. The magnetic potential $u : \mathbb{R}^d \to \mathbb{R}$ is related to the magnetization $\mathbf{m}$ by the magnetostatic Maxwell equation, which reads in distributional form

(2) $$\operatorname{div}(-\nabla u + \mathbf{m}) = 0 \quad \text{in } \mathcal{D}'(\mathbb{R}^d).$$

For the numerical treatment of this minimization problem we have to deal with the following issues:

a) The side constraint $|\mathbf{m}| \leq 1$: We enforce this by using a penalization strategy, which means that we add an appropriate penalization term to (1).

b) The side constraint given by Maxwell's equation (2): We append (2) to the energy functional by a Lagrange multiplier. To ensure stability of the discrete saddle point problem, we add a consistent stabilization term.

c) The full space problems involved in the first integral in (1) and in Maxwell's equation (2): The energy contribution
$$\int_{\mathbb{R}^d \setminus \Omega} |\nabla u|^2 \, dx$$
in (1) is realized by a boundary integral term. For the Maxwell equation (2) we use a similar idea. The numerical realization of the boundary integral terms then results in a FEM-BEM coupling.

In this talk, we discuss the well-posedness of the discrete problem and present an *a priori* error analysis, where we show optimal convergence rates under sufficient regularity assumptions. We illustrate our analysis with numerical examples.

References

[1] DeSimone A.: *Energy minimizers for large ferromagnetic bodies.* Arch. Rational Mech. Anal., 125 (1993), pp. 99-143.