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Outage Analysis of Cooperative Space-Time Codes with Network Coding

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***Abstract*—With the increasing demand of reliable communications on move, cooperative communications has proved to be an efficient solution to reduce the effects of unreliable nature of wireless channel. In this paper, we study the Space-Time (ST) cooperative communications aided by a fixed relay, forwarding network coded bits to the destination. We present the performance analysis in terms of outage probability. With analytical results, we show that the investigated scheme outperforms the cooperative transmission based on ST codes only. To be specific, the analytical results show that ST codes with the help of dedicated relay forwarding network coding can achieve diversity order of 3 while ST codes based cooperative communications, without use of network coding, only offers diversity order of 2.**

I. Introduction

One of the major challenges of wireless channel is to deal with the fading caused by multipath propagation of the signal transmitted from source to the destination. To reduce the effects of fading, multiple independent copies of the signal are transmitted to average out the channel variations due to multipath propagation. This technique is known as diversity [1]. The most basic ways to provide diversity are by means of frequency, time, and space [2]. The last form of diversity, formally known as spatial diversity, is a way where signal is transmitted or received by multiple, geographically separated, antennas to achieve multiple independently faded copies of the transmitted signal.

Cooperative communications has vastly been investigated to provide the spatial diversity and the capacity improvements to the wireless users [3][4]. It is a communication technique where participating users act as a relay for each other, to forward some information on behalf of the other user, for reliable recovery of the message at the destination. Furthermore, as we move on to high frequencies, coverage problem becomes prominent. Use of relay node is proposed by the Third Generation Partnership Program's Long-Term Evolution Advanced (3GPP-LTE-Advanced) at cell edges to extend the area of reliable reception [5][6]. With the introduction of the relay node, we not only get coverage extension, but improved capacity limits can also be achieved for the edge users. To realize the relay assisted benefits, various relaying protocols have been proposed [7][8].

Decode-and-Forward (DF) relaying strategy or protocol is one of the many investigated relaying protocols where relay decodes and then re-encodes the message received from the transmitting user and forwards it to the destination.

Recently, two ST cooperating users exploiting the availability of free user acting as a fixed relay was proposed and investigated [9][10]. The protocol uses two ST cooperating users transmitting to a common destination and includes a relay node forwarding network coded bits, of the messages received from cooperating mobile stations, to the destination. It was shown that the scheme outperforms cooperative transmission using ST codes only for a range of SNR, and then beyond that range, the protocol starts to degrade its performance.

In this paper, we change the channel model from [9][10] by exploiting the orthogonal channel assignment to the relay node. With analytical results, we show that the proposed scheme improves the performance over cooperative transmission using ST codes only.

The rest of the paper is structured as follows. In Section II, we describe our system model. Section III gives the mutual information and the resulting outage probability of the scheme. We discuss our results in Section IV. Finally, Section V concludes our paper.

II. System Model

As in [9][10], we consider a cellular region having two mobile stations m_1, m_2, and a single base station at the center of the cell. Both of the mobile stations have independent information to be sent to a common destination. Furthermore, these mobile stations are considered to be at the edge of the cellular region with a fixed relay node between them and the base station, as shown in Fig 1. Before explaining our

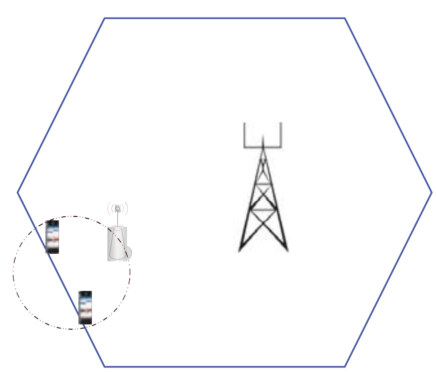
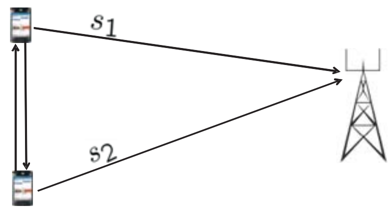


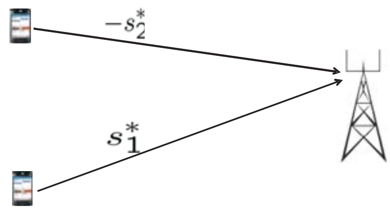
Fig. 1. Cooperative communications

scheme, i.e., cooperative ST codes with network coding, we briefly discuss here the classical, ST coding based, cooperative transmission using Alamouti scheme [11] which works as follows.

Transmission of the message from both of the mobile stations to the destination is accomplished in two phases. In the first Phase, mobile stations m_1 and m_2 broadcast their messages denoted as s_1 and s_2, as illustrated in Fig. 2(a). Along with the base station, both of the mobile stations, acting as cooperative partners, receive these messages from each other. In the next phase, after decoding the message from each other, mobile stations m_1 and m_2, using Alamouti scheme[1], send $-s_2^*$ and s_1^*, respectively, to the destination, as shown in Fig. 2(b).



(a) First Phase



(b) Second Phase

Fig. 2. ST Cooperative communications using Alamouti scheme

Our scheme only differs, from ST coding based cooperative communications using Alamouti scheme, discussed above, in the following sense.

In our scheme, in the first phase of transmission, along with mobile stations m_1, m_2, and the destination, a fixed relay node also receives the messages transmitted by each cooperating user, as illustrated in Fig. 3(a). And, in the next phase, this fixed relay node forwards network coded bits of the messages received from the cooperating users to the destination, as illustrated in Fig. 3(b).

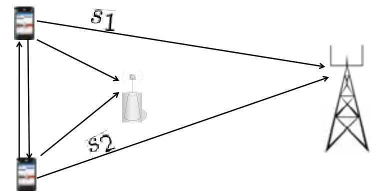
Time slot allocation and the respective transmitted and the received messages by mobile stations, m_1, m_2, relay node, and the base station are shown in Table I. The abbreviation BS is used for base station.

For practical reasons, only half duplex nodes are considered, and the orthogonality is achieved using different time slots used for transmission and the reception of the message from each terminal. Block-fading channel is considered which stays constant for at least four symbols, and the fading coefficients are modeled as independent, circularly symmetric, complex Gaussian random variables with zero mean and $\sigma_{i,j}^2$ variance. Channel gain or the fading coefficient from mobile station m_i to the destination is represented as $h_{i,0}$, and from relay to destination is represented as $h_{r,0}$. It is also assumed that the cooperating nodes are synchronized by some external source. Furthermore, each mobile station satisfies an equal power constraint of Pi, and signal-to-noise ratio (SNR), is defined as

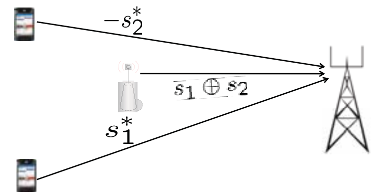
$$\gamma = \frac{P}{N_o} \tag{1}$$

where N_o is the received noise modeled as complex Gaussian random variable distributed as $N_o \sim CN(0, 1)$. We assume that the receivers are provided with the perfect channel state information (CSI), but the receivers are not.

[1]Asterisk $(*)$ represents the complex conjugate operation



(a) First Phase



(b) Second Phase

Fig. 3. ST Cooperative communications using Alamouti scheme with network coding

TABLE I
PROTOCOL DESCRIPTION

Time Slot	Transmitting Node	Transmitted Message	Receiving Node
1	m_1	s_1	m_2, BS, r
2	m_2	s_2	m_1, BS, r
3	m_1	$-s_2^*$	BS
4	m_2	s_1^*	BS
5	r	$s_1 \oplus s_2$	BS

III. Mutual Information and Outage Probaility

In this section, we give the mutual information of the investigated scheme, i.e., cooperative communications based on ST codes aided by a dedicated relay node forwarding network coded bits to the destination. Later, considering the fact that the mutual information, depending on the channel realization between source and the destination, is a random variable, we define outage probability and derive the same for the investigated scheme.

A. Mutual Information

The mutual information, between transmitting mobile station m_i and the destination, can be bounded as follows.[2]

$$I \leq \frac{2}{5}\log\left(1+\frac{\gamma}{2}\sum_{i=1}^{2}|h_{i,0}|^2\right)+\frac{2}{5}\log\left(1+\gamma|h_{r,0}|^2\right) \quad (2)$$

As seen, the resulting expression consists of two parallel mutual information terms. First is due to Alamouti scheme used by cooperating mobile stations [12] and the second is because of the relay node, sending network coded bits to the destination. The factor $\frac{2}{5}$ in front of mutual information expression, indicates the fact that five time slots are used by the two cooperating mobile stations [13].

B. Outage Probability Definition

We consider a quasi-static fading channel model which stays constant for some time and after that it changes from one channel realization to another [1]. As we see, the mutual information expression (2) is dependent on the channel realizations $h_{i,0}$ and $h_{r,0}$, that is, the channel between cooperating users and the destination, and from relay node to the destination; and therefore, is considered to be random. It means that the resulting rate, supported by the channel between nodes, is not constant and is modeled as a random variable.

Information theoretic outage probability defines an event when the channel mutual information can not satisfy a certain target rate. This target rate may be set by some application such as audio, video, or some multimedia application.

Mathematically, the probability of outage can be written as [14]

$$P[R] = P[I < R] \quad (3)$$

where R is used to represent the rate requirement set by some particular application.

C. Outage Probability

For our scheme, using mutual information expression (2) and the outage probability definition (3), we derive the outage probability for the investigated scheme as follows.

$$\begin{aligned}
P[R] &= P[I<R]\\
&\geq P\Big[\frac{2}{5}\log\left(1+\frac{\gamma}{2}\sum_{i=1}^{2}|h_{i,0}|^2\right)+\frac{2}{5}\log\left(1+\gamma|h_{r,0}|^2\right)\\
&\qquad\qquad < R\Big]\\
&\geq P\Big[\frac{2}{5}\log\left(1+\frac{\gamma}{2}\sum_{i=1}^{2}|h_{i,0}|^2\right)\left(1+\gamma|h_{r,0}|^2\right)<R\Big]\\
&\geq P\Big[\Big(|h_{r,0}|^2+\frac{1}{2}\sum_{i=1}^{2}|h_{i,0}|^2+\frac{\gamma}{2}|h_{r,0}|^2\sum_{i=1}^{2}|h_{i,0}|^2\Big)\\
&\qquad\qquad < \frac{2^{\frac{5R}{2}}-1}{\gamma}\Big]
\end{aligned}$$

Let $|h_{r,0}|^2 = y$, $\sum_{i=1}^{2}|h_{i,0}|^2 = x$, and $\frac{2^{\frac{5R}{2}}-1}{\gamma} = \epsilon$, then

$$\begin{aligned}
P[R] &\geq P\left[y+\frac{1}{2}x+\frac{1}{2}\gamma xy<\epsilon\right]\\
&\geq P\left[y\Big(1+\frac{1}{2}\gamma x\Big)<\epsilon-\frac{1}{2}x\right]\\
&\geq P\left[y<\frac{\epsilon-\frac{1}{2}x}{1+\frac{1}{2}\gamma x}\right]\\
&\geq \int_0^{2\epsilon} P\Big[y<\frac{\epsilon-\frac{1}{2}x}{1+\frac{1}{2}\gamma x}\Big|x\Big]P_X(x)\,dx\\
&\geq \int_0^{2\epsilon}\Big[1-\exp\Big(-\lambda\frac{\epsilon-\frac{1}{2}x}{1+\frac{1}{2}\gamma x}\Big)\Big]P_X(x)\,dx \qquad (4)
\end{aligned}$$

In the second step, we used the inequality, showing the lower bound on the resulting outage probability, because the mutual information expression (2) used here shows an upper bound on the mutual information of the investigated scheme.

The second last equation is due to the law of total probability theorem and in the last equation, we used the cumulative distribution function (cdf) of y, that is exponential random variable. $P_X(x)$ is simply the probability distribution function (pdf) of sum of two exponential random variables.

A general form of $P_X(x)$ for the sum of m independent exponential random variables is given by [15]

$$f(x) = \frac{\lambda}{(m-1)!}(\lambda x)^{m-1}\exp(-\lambda x) \quad (5)$$

where $\lambda > 0$ represent the parameter of the exponential distribution.

IV. Results and Discussion

Fig. 4 displays the outage probability curves based on (4), derived in Section III. To achieve these curves, (4) is evaluated at different values of SNR, and the required spectral efficiency R, is set to one.

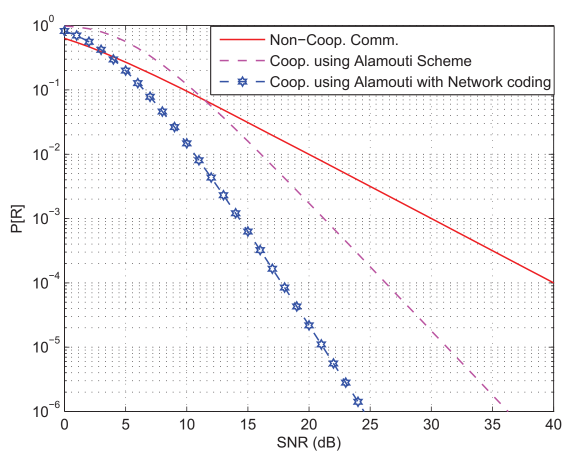


Fig. 4. Outage probability for space-time Codes with network coding based cooperative communications

[2]All logarithms are taken to base 2, unless indicated explicitly

For sake of simplicity, we have taken $\lambda = 1$, but it will not affect our final conclusion about the result. For comparison purpose, we also show the outage results for non-cooperative or direct transmission, and the cooperative communications based on Alamouti scheme without using relay node forwarding network coded bits to the destination. The analytical expression for outage probability for two user cooperative communications, based on ST codes, can be seen in [16] [17].

As clear from the figure, for high SNR, cooperative communications schemes, either based on Alamouti scheme only or Alamouti scheme with network coding, outperform non-cooperative transmission. The resulting gain, due to cooperative communications, can be seen as a slope of corresponding outage probability curve for high SNR. As seen in Fig. 4, when cooperating users use Alamouti scheme only, we can see from outage probability curve that the full order diversity of two is achieved. On the other hand, when we look at the outage curve for our scheme, i.e., cooperation based on ST codes with network coding, the maximum achievable diversity order is three. This shows that with this simple scheme, utilizing availability of free user or a dedicated relay in the network, we can outperform non-cooperative transmission as well as cooperative communications using Alamouti codes only.

While most of the cooperative communications protocols offer gain only for high SNR, the results also show that the investigated scheme has improved performance for low SNR as well. One of the efforts, to improve the performance of these cooperative communication protocols for low SNR, is given in [18][19].

V. Conclusion

We investigated a cooperative transmission scheme where cooperating users exploit a fixed relay node, proposed for future cellular networks. Two cooperating users using ST codes based on Alamouti scheme are considered which are further aided by a dedicated relay which forward network coded bits of the information received from cooperating users to the destination. We used outage probability as a performance metric to evaluate the performance of the investigated scheme. With analytical results, we showed that the investigated scheme outperforms, by offering diversity order 3, both the non-cooperative and cooperative communications using Alamouti scheme only, which offer diversity order of 1 and 2, respectively.

References

[1] David Tse and Pramod Viswanath, "Fundamentals of wireless communication," Cambridge University Press, Jun 2005.

[2] John G. Proakis, *Digital communications*, McGraw-Hill, New York :, 4th ed. edition.

[3] A. Sendonaris, E. Erkip, and B. Aazhang, "User cooperation diversity-Part I: System description," *IEEE Trans. Commun.*, vol. 51, no. 11, pp. 1927–1938, Nov 2003.

[4] A. Sendonaris, E. Erkip, and B. Aazhang, "User cooperation diversity-Part II: Implementation aspects and performance analysis," *IEEE Trans. Commun.*, vol. 51, no. 11, pp. 1939–1948, Nov 2003.

[5] David Martn-Sacristn, Jose F. Monserrat, Jorge Cabrejas-Peuelas, Daniel Calabuig, Salvador Garrigas, and Narcs Cardona, "On the way towards fourth-generation mobile: 3GPP LTE and LTE-Advanced," *EURASIP Journal on Wireless Communications and Networking*, vol. 2009, pp. Article ID 731317, 11 pages, 2009.

[6] Stefan Parkvall and David Astely, "The evolution of LTE towards IMT-Advanced," *Journal of Communications*, vol. 4, no. 3, pp. 146–154, April 2009.

[7] J. N. Laneman, D. N. C. Tse, and G. W. Wornell, "Cooperative diversity in wireless networks: efficient protocols and outage behavior," *IEEE Trans. Inform. Theory*, vol. 50, no. 12, pp. 3062–3080, Dec 2004.

[8] Gerhard Kramer, Michael Gastpar, and Piyush Gupta, "Cooperative strategies and capacity theorems for relay networks," *IEEE Trans. Inform. Theory*, vol. 51, no. 9, 2005.

[9] Gordhan Das Menghwar, Bujar Krasniqi, Asif Ali Shah, and Christoph F. Mecklenbräuker, "Cooperative space-time codes with opportunistic network coding," *Proceedings of IEEE Sarnoff Symposium 2009, Princeton, New Jersey, USA*, April 2009.

[10] Gordhan Das Menghwar, Asif Ali Shah, and Christoph F. Mecklenbräuker, "Cooperative space-time codes with opportunistic network coding with increasing numbers of nodes," *Proc. IEEE Sixth International Symposium on Wireless Communication Systems 2009 (ISWCS-2009)*, Sep 2009.

[11] Siavash M. Alamouti, "A simple transmit diversity technique for wireless communications," *Selected Areas in Communications, IEEE Journal on*, vol. 16, no. 8, pp. 1451–1458, 1998.

[12] Babak Hassibi and Bertrand M. Hochwald, "High-rate codes that are linear in space and time," *IEEE Trans. on Inform. Theory*, vol. 48, no. 7, pp. 1804–1824, 2002.

[13] Xingkai Bao and Jing Li (Tiffany), "Adaptive network coded cooperation (ancc) for wireless relay networks: matching code-on-graph with network-on-graph," *IEEE Trans. on Wireless Communications*, vol. 7, no. 2, pp. 574–583, Feb 2008.

[14] Lawrence H. Ozarow, Shlomo Shamai, and Aaron D. Wyner, "Information theoretic considerations for cellular mobile radio," *IEEE Transcations on Vehicular Technology*, vol. 43, no. 2, pp. 359–378, May 1994.

[15] A. Papoulis, *Probability, Random Variables, and Stochastic Processes*, Mc-Graw Hill, 1984.

[16] J. Nicholas Laneman and Gregory W. Wornel, "Distributed space-time coded protocols for exploiting cooperative diversity in wireless networks," *IEEE Trans. Inform. Theory*, vol. 49, pp. 2415–2525, 2003.

[17] Gordhan Das Menghwar and Christoph F. Mecklenbräuker, "Outage performance of two users cooperative network coding," *Proc. IEEE International Symposium on Communications and Information Technologies (ISCIT-2009)*, Sep 2009.

[18] Charlotte Hucher, Ghaya Rekaya-Ben Othman, and Jean-Claude Belfiore, "How to solve the problem of bad performance of cooperative protocols at low snr," *EURASIP J. Adv. Signal Process*, vol. 2008, pp. 24, 2008.

[19] Charlotte Hucher, Ghaya Rekaya-Ben Othman, and Jean-Claude Belfiore, "AF and DF protocols based on alamouti ST code," in *IEEE International Symposium on Information Theory, 2007 (ISIT 2007)*, Nice, France, June 2007.