Fast Fading Channel Estimation for UMTS Long Term Evolution

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Abstract — *In this paper, we present an Approximate Linear Minimum Mean Square Error (ALMMSE) fast fading channel estimator for Orthogonal Frequency Division Multiplexing (OFDM). The ALMMSE channel estimator utilizes the knowledge of the structure of the autocorrelation matrix given by the Kronecker product between the time correlation matrix and the frequency correlation matrix. Simulation results show that the proposed ALMMSE channel estimator allows to move with about 25 km/h higher velocity than the Least Squares (LS) channel estimator in realistic scenarios.*

I. INTRODUCTION

An essential part of modern wireless communications receivers is the channel estimator whose quality has a direct impact on the data throughput. Today's wireless communication systems are designed to provide data rates also to high velocity users. The channel of a high velocity user is changing rapidly during a subframe. In order to estimate fast fading channel estimation sophisticated techniques have to be employed.

II. SYSTEM MODEL

The n-th received Orthogonal Frequency Division Multiplexing (OFDM) symbol $\mathbf{y}_n$ at one receive antenna port can be written as

$$\mathbf{y}_n = \mathbf{X}_n \mathbf{h}_n + \mathbf{w}_n, \tag{1}$$

where the vector $\mathbf{h}_n$ contains the channel coefficients in the frequency domain and $\mathbf{w}_n$ is additive white zero mean Gaussian noise with variance σ_w^2. The diagonal matrix $\mathbf{X}_n = \mathrm{diag}(\mathbf{x}_n)$ comprises the data symbols $\mathbf{x}_{\mathrm{d},n}$ and the pilot symbols $\mathbf{x}_{\mathrm{p},n}$ permuted by a permutation matrix $\mathbf{P}$ on the main diagonal

$$\mathbf{x}_n = \mathbf{P} \left[\mathbf{x}_{\mathrm{p},n}^{\mathrm{T}} \, \mathbf{x}_{\mathrm{d},n}^{\mathrm{T}}\right]^{\mathrm{T}}. \tag{2}$$

The length of the vector $\mathbf{x}_n$ is K corresponding to the number of subcarriers. Note that according to Equation (2), also the vectors $\mathbf{y}_n$, $\mathbf{h}_n$ and $\mathbf{w}_n$ can be divided into two parts corresponding to the pilot symbol positions and to the data symbol positions.

III. STATE-OF-THE-ART CHANNEL ESTIMATION

In this section, we present two typical state-of-the-art channel estimators that are used as a benchmark for our Approximate Linear Minimum Mean Square Error (ALMMSE) channel estimator [1].

A. LS CHANNEL ESTIMATION

The Least Squares (LS) channel estimator [2] for the pilot symbol positions is given as the solution to the minimization problem

$$\hat{\mathbf{h}}_{\mathrm{p}}^{\mathrm{LS}} = \arg\min_{\hat{\mathbf{h}}_{\mathrm{p}}} \left\| \mathbf{y}_{\mathrm{p}} - \mathbf{X}_{\mathrm{p}}\, \hat{\mathbf{h}}_{\mathrm{p}} \right\|_2^2 = \mathbf{X}_{\mathrm{p}}^{-1}\, \mathbf{y}_{\mathrm{p}}. \tag{3}$$

At the non-pilot symbol positions, the remaining channel coefficients are obtained by two dimensional linear interpolation. The LS channel estimator does not require knowledge about the channel and noise statistics and can be implemented with low complexity.

B. LMMSE CHANNEL ESTIMATION

The Linear Minimum Mean Square Error (LMMSE) channel estimator requires the second order statistics of the channel and the noise. It can be shown that the LMMSE channel estimate is obtained by multiplying the LS estimate with a filtering matrix $\mathbf{A}_{\mathrm{LMMSE}}$ [3]

$$\hat{\mathbf{h}}_{\mathrm{LMMSE}} = \mathbf{A}_{\mathrm{LMMSE}} \hat{\mathbf{h}}_{\mathrm{p}}^{\mathrm{LS}}. \tag{4}$$

The LMMSE filtering matrix is given as

$$\mathbf{A}_{\mathrm{LMMSE}} = \mathbf{R}_{\mathbf{h},\mathbf{h}_{\mathrm{p}}} \left(\mathbf{R}_{\mathbf{h}_{\mathrm{p}},\mathbf{h}_{\mathrm{p}}} + \sigma_w^2 \mathbf{I}\right)^{-1}, \tag{5}$$

where the matrix $\mathbf{R}_{\mathbf{h}_{\mathrm{p}},\mathbf{h}_{\mathrm{p}}} = \mathbb{E}\left\{\mathbf{h}_{\mathrm{p}}\mathbf{h}_{\mathrm{p}}^{\mathrm{H}}\right\}$ is the channel autocorrelation matrix at the pilot symbols, and the matrix $\mathbf{R}_{\mathbf{h},\mathbf{h}_{\mathrm{p}}} = \mathbb{E}\left\{\mathbf{h}\mathbf{h}_{\mathrm{p}}^{\mathrm{H}}\right\}$ is the channel crosscorrelation matrix.

IV. ALMMSE CHANNEL ESTIMATION

In the following section, we briefly discuss the ALMMSE fast fading channel estimator, which approximates the LMMSE channel estimator. The main idea is to make use of the structure of the channel autocorrelation matrix, assumed to be given by

$$\mathbf{R}_{\mathbf{h}} \triangleq \mathbf{R}_{\mathrm{time}} \otimes \mathbf{R}_{\mathrm{freq}}, \tag{6}$$

where $\mathbf{R}_{\text{time}}$ is the time correlation matrix and $\mathbf{R}_{\text{freq}}$ is the frequency correlation matrix. The Kronecker structure assumption of the channel autocorrelation matrix $\mathbf{R}_{\mathbf{h}}$ corresponds to independent time- and frequency-correlation.

Let us consider the following problem

$$\min_{\mathbf{B}_{\text{freq}},\mathbf{C}_{\text{time}}} \mathbb{E}\left\{\|\mathbf{H}-\mathbf{B}_{\text{freq}}\hat{\mathbf{H}}_{\text{LS}}\mathbf{C}_{\text{time}}^{\text{T}}\|_{\text{F}}^2\right\}, \tag{7}$$

with the channel $\mathbf{H} = [\mathbf{h}_0, \cdots, \mathbf{h}_{N_s-1}]$ and the LS channel estimate $\hat{\mathbf{H}}_{\text{LS}} = [\hat{\mathbf{h}}_0^{\text{LS}}, \cdots, \hat{\mathbf{h}}_{N_s-1}^{\text{LS}}]$. Here, N_{s} denotes the number of OFDM symbols, $\mathbf{B}_{\text{freq}}$ and $\mathbf{C}_{\text{time}}$ are matrices of dimension $K \times K$ and $N_{\text{s}} \times N_{\text{s}}$, respectively. $\|\cdot\|_{\text{F}}$ refers to the Frobenius norm.

We assume, that the eigenvectors of matrices $\mathbf{B}_{\text{freq}}$ and $\mathbf{C}_{\text{time}}$ are the same as of the frequency and time correlation matrices, respectively. Therefore, just the eigenvalues of the matrices $\mathbf{B}_{\text{freq}}$ and $\mathbf{C}_{\text{time}}$ have to be found. Using the vectors $\underline{\lambda}_{\text{time}}$, $\underline{\lambda}_{\text{freq}}$, $\underline{\lambda}_{\mathbf{C}_{\text{time}}}$ and $\underline{\lambda}_{\mathbf{B}_{\text{freq}}}$, that store the eigenvalues of the corresponding matrices, solution to this problem can be reformulated as

$$\underline{\lambda}_{\text{time}}\underline{\lambda}_{\text{freq}}^{\text{T}}./\left(\underline{\lambda}_{\text{time}}\underline{\lambda}_{\text{freq}}^{\text{T}}+\sigma_w^2\mathbf{1}\mathbf{1}^{\text{T}}\right) \approx \underline{\lambda}_{\mathbf{C}_{\text{time}}}\underline{\lambda}_{\mathbf{B}_{\text{freq}}}^{\text{T}}, \tag{8}$$

where $\mathbf{1}$ is the all ones vector and $./$ denotes elementwise division. This is a so-called rank-one approximation, where the best approximation is achieved when taking the left and right eigenvectors corresponding to the largest singular value, and having one of them scaled by it

$$\underline{\lambda}_{\mathbf{C}_{\text{time}}} = \sigma_{\max}\mathbf{u}_{\max}, \tag{9}$$

$$\underline{\lambda}_{\mathbf{B}_{\text{freq}}} = \mathbf{v}_{\max}. \tag{10}$$

The ALMMSE channel estimate utilizing the rank-one approximation of Equation (8) is given by

$$\hat{\mathbf{H}}_{\text{ALMMSE}} = \mathbf{B}_{\text{freq}}\hat{\mathbf{H}}_{\text{LS}}\mathbf{C}_{\text{time}}^{\text{T}}, \tag{11}$$

where the matrices $\mathbf{B}_{\text{freq}}$ and $\mathbf{C}_{\text{time}}$ are given by

$$\mathbf{B}_{\text{freq}} = \mathbf{U}_{\text{freq}}\text{diag}\left(\underline{\lambda}_{\mathbf{B}_{\text{freq}}}\right)\mathbf{U}_{\text{freq}}^{\text{H}}, \tag{12}$$

$$\mathbf{C}_{\text{time}} = \mathbf{U}_{\text{time}}\text{diag}\left(\underline{\lambda}_{\mathbf{C}_{\text{time}}}\right)\mathbf{U}_{\text{time}}^{\text{H}}. \tag{13}$$

V. Simulation Results

In this section, we present simulation results and discuss the performance of the different channel estimation techniques. All results are obtained with an Long Term Evolution (LTE) Link Level Simulator, developed at the Vienna University of Technology [4].

In Figure 1 we present the data throughput of the LTE physical layer at SNR=20 dB using CQI=10 over user velocity. It can be observed that with increasing velocity the throughput is decreasing. The LMMSE estimator outperforms the remaining channel estimator. Up to a certain velocity, about $v = 175\,\text{km/h}$ the performance of the proposed ALMMSE channel estimator is close to the LMMSE estimator. A user is able to move 25 km/h faster while achieving the same throughput, if the ALMMSE estimator is utilized instead of the LS estimator.

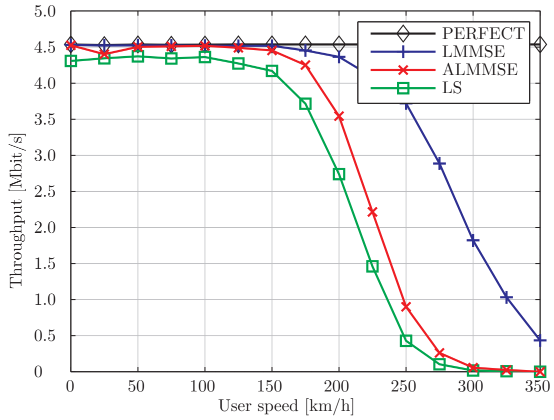

Figure 1: Throughput of the LTE system at a fixed CQI=14 using different channel estimators at SNR=20 dB

VI. Conclusion

In this paper we investigated the performance of some known fast fading channel estimators and our ALMMSE channel estimator. The ALMMSE channel estimator is able to outperform the LS estimator and up to a velocity of about 175 km/h it achieves a throughput performance close to the LMMSE estimator. The next step is to analyze the complexity of the ALMMSE estimator and the huge performance loss compared to LMMSE at high user velocities.

References

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