On Achieving the Shannon Bound in Cellular Systems

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Abstract. *This contribution provides insight into the performance of the currently deployed 3G cellular systems HSDPA and WiMAX. In extensive measurement campaigns we measured the physical layer throughput of these 3G systems in different environments (alpine and urban) and compare the results rigorously to their upper bounds derived from the well known Shannon capacity. By separating the observed losses into a channel state information loss, a design loss, and an implementation loss, we gain more insight into the performance of the different standards which in turn allows us to compare them better and to localize their shortcomings. In general, we find that implementations of the current standards still operate about 10 dB away from the Shannon bound.*

Keywords

MIMO, HSDPA, WiMAX, performance measures, testbed.

1. Introduction

We implemented, measured, and studied the following two standardized cellular systems:

- The WiMAX physical layer as defined in IEEE 802.16–2004 [1] was developed to provide wireless internet access for stationary and low-mobility users. In the standard, three different physical layers for WiMAX are defined, namely the Single Carrier (SC) [1, Sections 8.1 and 8.2], the Orthogonal Frequency Division Multiplexing (OFDM) [1, Section 8.3], and the Orthogonal Frequency Division Multiple Access (OFDMA) [1, Section 8.4] physical layers. The SC physical layer is designed for directional radio links with Line-Of-Sight (LOS), whereas the OFDM and OFDMA physical layers are designed for Non-Line-Of-Sight (NLOS) conditions. The OFDM physical layer utilizes 256 narrow-band sub-carriers to modulate the data symbols to be transmitted. Multiple users are supported by Time-Division Multiple Access (TDMA). The OFDMA physical layer, in contrast, provides in addition to TDMA also multiple access by assigning specific sub-carrier subsets to individual users [2]. In this work, we only consider the OFDM physical layer of WiMAX.
- High Speed Downlink Packet Access (HSDPA) [3] was introduced in Release 5 of the Universal Mobile Telecommunication System (UMTS) to provide high data rates to mobile users [4–7]. This is achieved by several techniques like fast link adaptation [8], fast hybrid automated repeat request [9], and fast scheduling [10]. In contrast to the pure transmit power adaptation performed in UMTS, fast link adaptation in HSDPA adjusts the data rate and the number of spreading codes depending on a so-called Channel Quality Indicator (CQI) feedback. MIMO HSDPA [11], recently standardized in Release 7 of UMTS, further increases the maximum downlink data rate by spatially multiplexing two independently coded and modulated data streams. Additionally, channel-adaptive spatial precoding is implemented at the basestation. The standard defines a set of precoding vectors and one of them is chosen based on a so-called Precoding Control Indicator (PCI) feedback obtained from the user equipment.

This paper is organized as follows: In Section II we define upper bounds for the measured throughput and identify three different throughput losses. Section III presents measurement results and analyzes the different losses. Finally, Section IV provides a conclusion explaining the reasons for the different losses in HSDPA and WiMAX.

2. Performance Metrics

Given the channel impulses $h_k^{(n_r,n_t)}$ of an $N_\mathrm{T}\times N_\mathrm{R}$ MIMO system (for example by measuring them at high SNR) one can compute the so called Shannon *capacity*

$$C(\mathrm{SNR}) = \max_{\mathrm{tr}\{\mathbf{R}\}\leq 1} \log_2 \det\left(\mathbf{I} + \frac{\mathrm{SNR}}{N_\mathrm{T}}\mathbf{H}\mathbf{R}\mathbf{H}^\mathrm{H}\right) \quad (1)$$

where each entry of $\mathbf{H}$ is given by $h_k^{(n_r,n_t)}; 1 \leq n_r \leq N_\mathrm{R}, 1 \leq n_t \leq N_\mathrm{T}$ and $\mathbf{R}$ is the optimal precoding matrix obtained by waterfilling. This measure tells us how much bit/s/Hz we are able to transmit at best. As the calculation of the optimal precoding matrix $\mathbf{R}$ requires precise information about the channel $\mathbf{H}$, typically obtained by feedback from receiver to transmitter, it is of interest to assess the so-called *mutual information* that only requires to know the

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SNR:

$$I(\mathrm{SNR}) = \log_2 \det\left(\mathbf{I} + \frac{\mathrm{SNR}}{N_\mathrm{T}}\mathbf{H}\mathbf{H}^\mathrm{H}\right). \tag{2}$$

Because capacity and mutual information only differ in the knowledge of the channel state information at the transmitter, we define their difference as the ***channel state information loss***

$$L_\mathrm{CSI}(P_\mathrm{Tx}) \triangleq C(P_\mathrm{Tx}) - I(P_\mathrm{Tx}). \tag{3}$$

Note that rather than plotting L_CSI as a function of SNR we have changed the notation to transmit power P_Tx. There are several reasons for this: firstly, we measure over transmit power, secondly, the mean received power in MIMO scenarios is significantly different at each antenna, and thirdly, when employing *adaptive* precoding at the transmitter, the transmit power remains unchanged while the receive SNR varies.

Given a certain wireless standard, we now have to take an overhead into account.

- In HSDPA this overhead is given by the amount of power devoted to the pilots and synchronization sequences. This kind of overhead affects the mutual information by shifting the SNR to lower values. Also the roll-off factor of 0.22, limiting the useful bandwidth, needs to be taken into account. All these factors lower the mutual information.
- In OFDM systems such as WiMAX and LTE, pilots consume bandwidth and the cyclic prefix consumes transmission time. Thus, these losses are proportional to the mutual information and not to the SNR.

Taking all mentioned losses into account, we obtain the so called *achievable mutual information* [12–14]

$$I_\mathrm{a}(\mathrm{SNR}) = \beta \log_2 \det\left(\mathbf{I} + \frac{\alpha\mathrm{SNR}}{N_\mathrm{T}}\mathbf{H}\mathbf{R}_\mathrm{q}\mathbf{H}^\mathrm{H}\right), \tag{4}$$

with $\alpha \leq 1$ and $\beta \leq 1$. In case of precoding (MIMO HSDPA), the matrix $\mathbf{R}_\mathrm{q}$ accounts for the quantization of the precoding matrices. The specific values of α and β depend on the detailed specification of the wireless transmission system. In case of WiMAX transmissions, we obtain $\alpha = 1$ and $\beta = 0.56$, if the parameters are set as in [15]. In case of HSDPA transmissions, we obtain $\alpha = 0.4$ (if -4 dB of the total transmit power is assigned to the data channels) and $\beta = 0.82$ (corresponding to the inverse of the HSDPA root-raised cosine roll-off factor of 1.22).

It is worthwhile noting that all precoding matrices in HSDPA for 2×2 MIMO transmission are unitary ($\mathbf{R}_\mathrm{q}$ is always equal to an identity matrix), thus not altering the achievable mutual information I_a at all. In case of 2×2 MIMO WiMAX, however, Alamouti space-time coding [16] is employed at the transmitter. Thus, by inserting the virtual MIMO channel matrix of the Alamouti space-time code

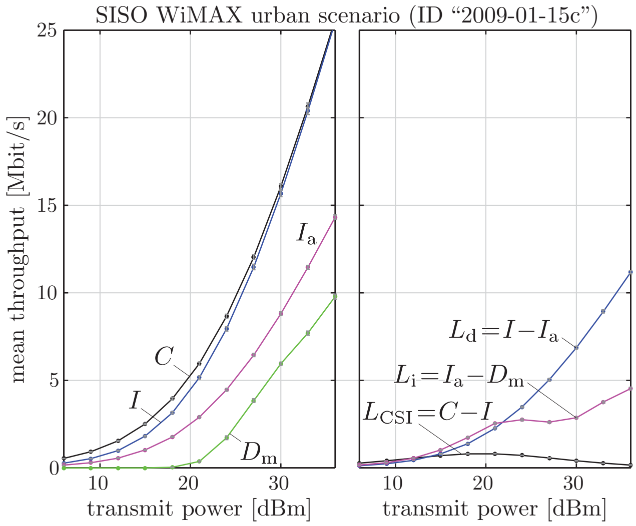

Fig. 1. Throughput and throughput losses of the SISO WiMAX system in the urban environment, obtained from 484 small scale fading channel realizations.

and setting $\mathbf{R}_\mathrm{q} = \mathbf{I}$ in Equation (4), we obtain the achievable mutual information for MIMO WiMAX. Since Alamouti space-time coding is not capacity achieving when two receive antennas are utilized, the consideration of the space-time coding lowers the achievable mutual information.

The achievable mutual information includes the losses purely caused by the design of the system, therefore we define the difference between mutual information and achievable mutual information as the ***design loss***:

$$L_\mathrm{d}(P_\mathrm{Tx}) \triangleq I(P_\mathrm{Tx}) - I_\mathrm{a}(P_\mathrm{Tx}), \tag{5}$$

where we again switched the notation to transmit power P_Tx. Note that the design loss is completely independent of the quality of the implementation.

The achievable mutual information is an upper bound for the physical layer throughput. It can only be achieved, if the implementation of the transmission system is optimal, that is, no further losses occur. However, due to for example imperfect channel knowledge at the receiver or suboptimal receiver design (because of complexity issues), a so-called implementation loss L_i further limits the physical layer throughput. We define the difference between the measured (achieved) throughput $D_\mathrm{m}(\mathrm{SNR})$ and the achievable mutual information I_a as the ***implementation loss***:

$$L_\mathrm{i}(P_\mathrm{Tx}) \triangleq I_\mathrm{a}(P_\mathrm{Tx}) - D_\mathrm{m}(P_\mathrm{Tx}). \tag{6}$$

Note that according to our definition of the implementation loss, also other losses that are not purely caused by the implementation of the transmission system are included in the implementation loss. For example, transmission systems currently employed are not transmitting Gaussian signals of infinite length as it would be required to achieve the Shannon bound. Furthermore, at high transmit power levels (corresponding to high receive SNRs) the throughput of any wireless transmission system saturates because no larger adaptive modulation and coding schemes are available. This fact is currently not reflected in our definition of the achievable throughput definition and causes an increasing implementation loss at high SNR.

3. Measurement Results

As the paper size is quite limited, we cannot provide sufficient details on our measurement setup here. Instead we refer the interested reader to the references [17–21]. In short, the throughput is measured by transmitting and receiving the standard compliant signals. The bounds are calculated from the channel coefficients estimated from the received signals at highest P_{Tx}.

In this paper, we exhibit results obtained in two different environments. The first one is an alpine environment in which the transmitter is placed in a distance of 5.7 km to the receiver. This environment is characterized by an essentially strong LOS component and moderate scattering with an RMS delay spread of 260 ns. The second environment is an urban one with only 430 m distance between transmitter and receiver. In this environment the scattering is much richer with an RMS delay spread of 1.1 µs and no LOS component. In this work, we present only results for SISO and 2×2 MIMO transmissions with cross-polarized transmit antennas although many more situations were investigated.

3.1 SISO Results

In Figure 1 we plot on the left-hand side the capacity C, the mutual information I, the achievable mutual information I_{a}, and the measured throughput D_{m}. On the right-hand side we plot the channel state information loss L_{CSI}, the design loss L_{d}, and the implementation loss L_{i}, as defined in Equations (3), (5), and (6). The left-hand side of Figure 1 shows that the total SNR loss of the measured throughput D_{m} compared to the capacity C amounts to 9 dB at 5 Mbit/s. This loss is further increasing with the SNR and reaches about 11 dB at 10 Mbit/s.

Figure 2 shows the various losses of SISO WiMAX and SISO HDSPA transmissions in the alpine (left-hand side) and the urban environment (right-hand side). We find the following results for alpine as well as urban environments:

- The channel state information loss L_{CSI} is negligibly small compared to the other losses, as it has already been pointed out in [22].

- The design loss L_{d} is monotonically increasing with transmit power. Up to about 30 dBm, the values are very similar for WiMAX and HSDPA but for higher transmit power the design loss of HSDPA starts saturating while it keeps increasing for WiMAX.

- The implementation loss L_{i} is moderately small for

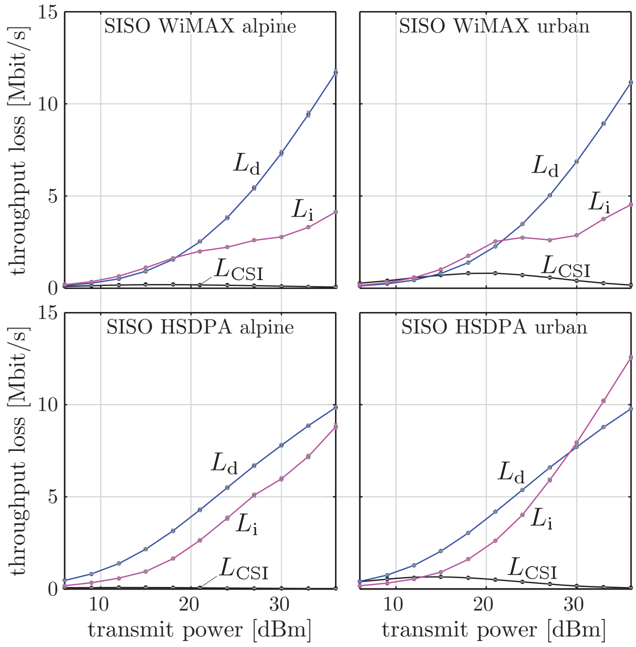

Fig. 2. Throughput losses of the SISO WiMAX and the SISO HSDPA systems in the alpine (ID “2008-09-23”) and the urban environments (ID “2009-01-15c”).

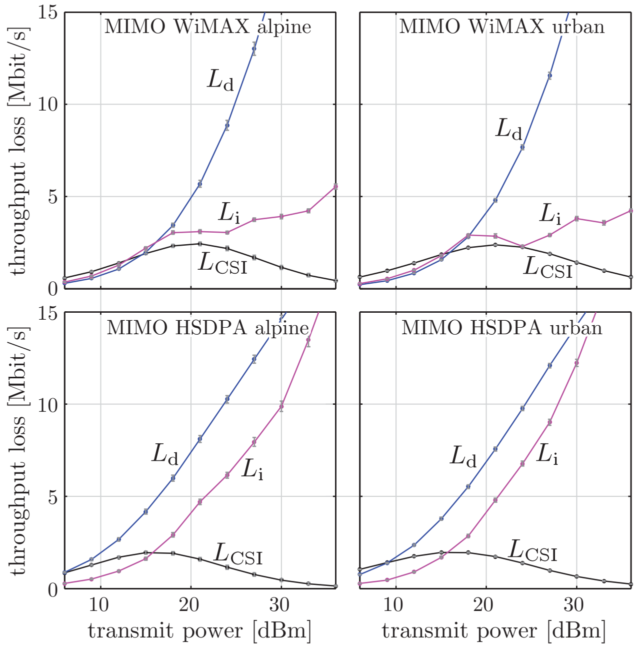

Fig. 3. Throughput losses of the MIMO WiMAX and the MIMO HSDPA systems in the alpine (ID “2008-09-23”) and the urban environments (ID “2009-01-15c”).

WiMAX and only slightly increasing with the transmit power while for HSDPA it is monotonously increasing.

The only relevant difference between alpine and urban environments we encounter in SISO HSDPA, where the implementation loss L_{i} at high SNR is significantly larger in the urban environment. The reason for this is the increased self interference (inter-chip interference) in the richer scattering with larger RMS delay spread. For WiMAX the large RMS delay spread has no influence since the OFDM transmission with cyclic prefix converts the frequency selective channel into several independent frequency flat subchannels.

3.2 MIMO Results

We repeated the measurements with a 2×2 MIMO system and display the results of the various losses of the WiMAX and HSDPA transmissions in Figure 3 for the alpine (left-hand side) and the urban environment (right-hand side).

We find the following results for WiMAX and for HSDPA:

- The channel state information loss L_{CSI} has increased but is still the smallest compared to the other losses.
- The design loss L_{d} is again monotonically increasing with transmit power. At low transmit power, the design loss is larger in case of HSDPA transmission. At large transmit power it is the other way round.
- The implementation loss L_{i} shows a similar behavior as in the case of SISO transmissions

In contrast to the SISO transmissions, MIMO HSDPA shows no significant difference between alpine and urban environments.

4. Conclusion

In this work we presented measurement results for WiMAX and HSDPA in alpine and urban environments. We introduced three different throughput losses in order to analyze the performance. In particular, the introduced losses are:

1. The *channel state information loss* L_{CSI} (capacity minus mutual information): this typically small loss is given by the unknown channel state information at the transmitter. For SISO and cross polarized 2×2 MIMO systems this loss can be neglected compared to the other losses. Only in antenna arrays with at least four antennas this loss is worth considering [22]. In order to combat this loss, a significant amount of feedback information is required.
2. *Design loss* L_{d} (mutual information minus achievable mutual information): In case of WiMAX, this loss increases more dramatically with transmit power than in case of HSDPA.
3. *Implementation loss* L_{i} (achievable mutual information minus measured throughput): The implementation loss behaves very differently for WiMAX and HSDPA. In environments with large RMS delay spread (urban environment with 1.1 µs) the implementation loss of HSDPA is extremely large due to self interference while in small RMS delay spread areas (alpine environment with 260 ns) the behavior is different. In WiMAX the different delay spread has no impact as the cyclic prefix was selected sufficiently large.

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