Turbulent bluff-body separation: recent advances of a self-consistent flow description for large Reynolds numbers

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The research presented has been carried out in collaboration with

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Institute of Fluid Mechanics and Heat Transfer

TECHNISCHE UNIVERSITÄT WIEN
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Prof Frank T. Smith, FRS

Department of Mathematics

UCL

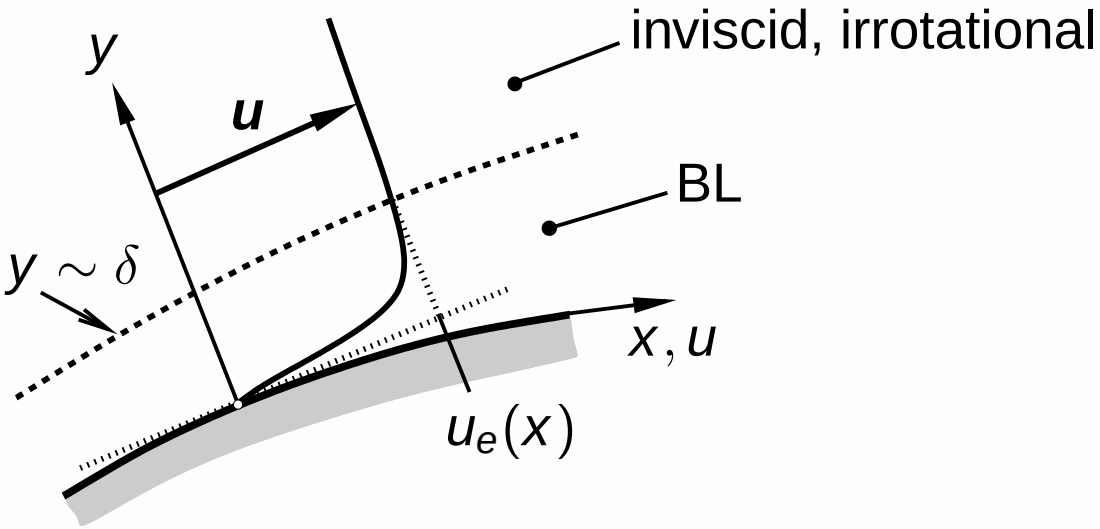
Their contributions are greatly acknowledged.

Sandborn & Liu (JFM, 1968)

"Turbulent boundary-layer separation is normally listed as one of the most important unsolved problems in fluid mechanics..."

Overview

Wall-bounded high-Reynolds-number flows : $Re := \tilde{U}\tilde{L}/\tilde{\nu} \gg 1$



Navier–Stokes eqs ($\rho = const$)

$$\nabla \cdot \boldsymbol{u} = 0\,, \quad \frac{\partial \boldsymbol{u}}{\partial t} + \boldsymbol{u} \cdot \nabla \boldsymbol{u} = -\nabla p + \frac{1}{Re}\nabla^2 \boldsymbol{u}\,, \quad \boldsymbol{u}|_{y=0} = \boldsymbol{0}$$

Reynolds-averaging, ergod hypothesis (nominally steady 2D flow)

$$\boldsymbol{Q}(\boldsymbol{x}, t; Re) = \langle \boldsymbol{Q} \rangle (x, y; Re) + \boldsymbol{Q}'(\boldsymbol{x}, t; Re)$$

$$\langle \boldsymbol{Q} \rangle := \lim_{t_{av} \to \infty} \frac{1}{t_{av}} \int_{-t_{av}/2}^{t_{av}/2} \boldsymbol{Q}(\boldsymbol{x}, t + t'; Re)\, \mathrm{d}t'$$

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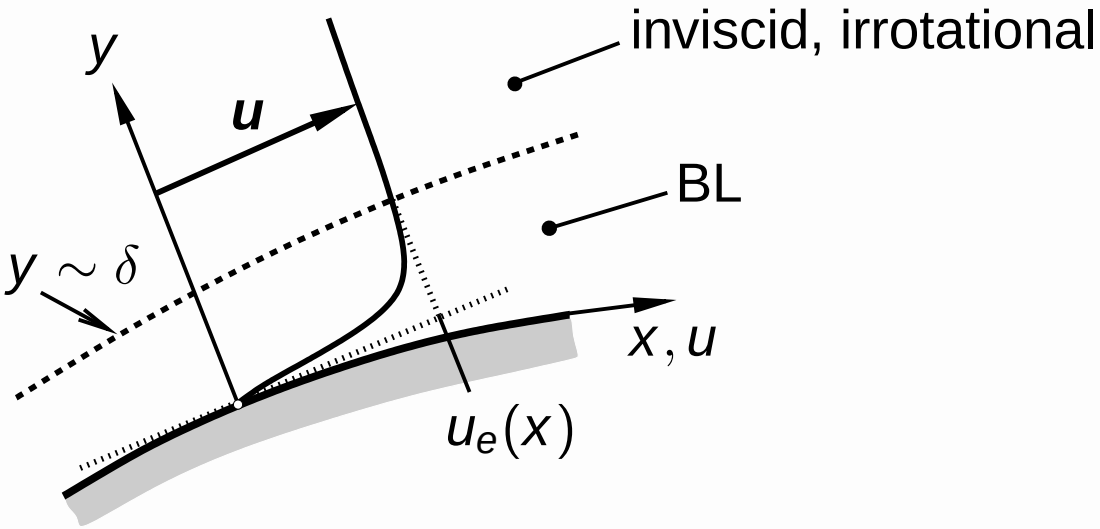

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Outline

Numerically-based methods

- DNS : resolution of all scales $\Rightarrow$ no modelling needed but at present restricted to moderately large values of *Re*
- LES : modelling of short scales $\Rightarrow$ reduction of computational efforts, which, however, are still too massive to be useful for the solution of engineering problems
- RANS : modelling of all scales $\Rightarrow$ engineering problems can be solved with an acceptable amount of computational efforts, which, however, increase with increasing values of *Re*

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Analytically-based methods

- full NS eqs : starting efforts, but still no complete theory exists
  Deriat & Guiraud (J Mec Theor Appl, 1986), Walker (AIAA J, 1989, CISM, 1998), Scheichl & Kluwick (JFS, 2008)

- non-dimensional Reynolds-averaged NS eqs
  (nominally 2D, curvature effects on BL flow of higher order):

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0$$

$$u\frac{\partial u}{\partial x} + v\frac{\partial u}{\partial y} = -\frac{\partial p}{\partial x} - \frac{\partial \langle u'^2 \rangle}{\partial x} - \frac{\partial \overbrace{\langle u'v' \rangle}^{-\tau}}{\partial y} + \frac{1}{Re}\nabla^2 u$$

$$\nabla^2 = \frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2}$$

$$u\frac{\partial v}{\partial x} + v\frac{\partial v}{\partial y} = -\frac{\partial p}{\partial y} - \frac{\partial \langle u'v' \rangle}{\partial x} - \frac{\partial \langle v'^2 \rangle}{\partial y} + \frac{1}{Re}\nabla^2 v$$

$$y = 0: \quad u = v = u' = v' = 0, \quad y \sim \delta(x; Re): \quad u \sim u_e(x), \quad \tau \sim 0$$

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Classical theory of turbulent small-defect BLs: $u_\tau := \sqrt{\tau_w} \to 0$

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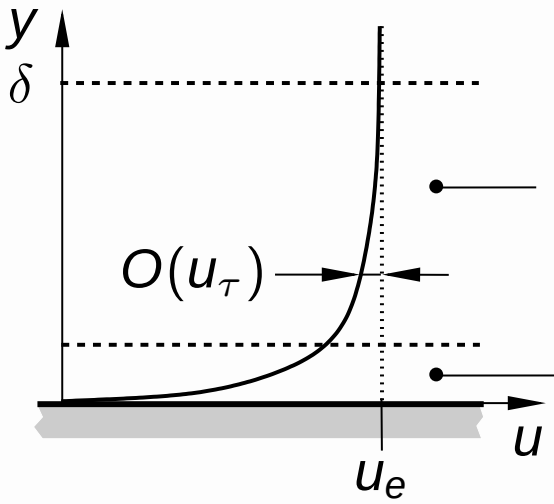

outer predominately inviscid region

overlap region: $y^+ := y u_\tau Re \to \infty$

$$\frac{u}{u_\tau} \sim \frac{1}{\kappa} \ln y^+ + C_i$$

viscous wall layer

$$\tau_w = Re^{-1} (\partial u / \partial y)_{y=0}$$

assumptions — Walker (CISM, 1998), Scheichl, Kluwick & Alletto (Acta Mech, 2008)

(a) locally isotropic turbulence $\Rightarrow$ $\langle u'^2 \rangle$, $\langle u'v' \rangle$, $\langle v'^2 \rangle$ of same magnitude

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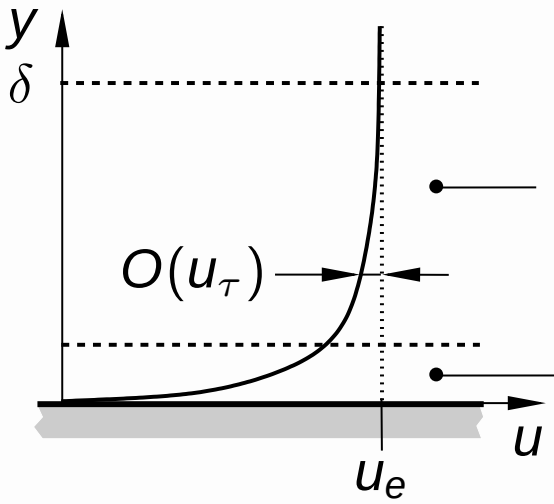

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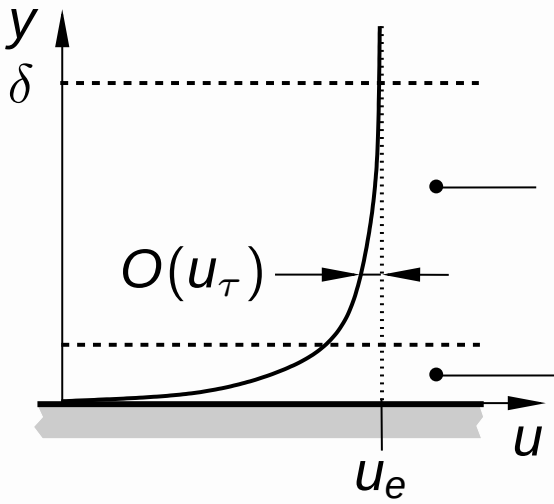

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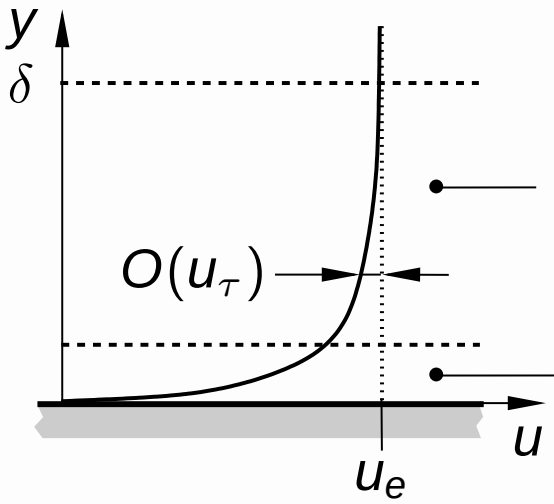

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Distinguished limit $\gamma := u_\tau / u_e \to 0\,,\ \ Re \to \infty$

viscous wall layer: $y^+ = y u_\tau Re$

$$\frac{u}{u_e} \sim \gamma(x, Re)\, u^+(x, y^+) + \cdots$$

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$$p \sim p_0(x) + \cdots$$

outer defect layer: $\eta = y/\delta$

$$\frac{u}{u_e} \sim 1 - \gamma \frac{\partial F_1(x,\eta)}{\partial \eta} + O(\gamma^2)$$

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$$p \sim p_e(x) + O(\gamma^2)$$

experimental observation / result from first principles (?)

Österlund et al. (Phys Fluids, 2000) Walker (AIAA J, 1989, CISM, 1998)

$$u^+(y^+) \sim \kappa^{-1} \ln y^+ + C_i\,, \quad y^+ \to \infty\,, \quad \kappa \approx 0.384\,, \ \ C_i \approx 4.1$$

provides expansion in outer layer and matching with wall layer

$$\eta \to 0\ : \quad \partial F_1/\partial \eta \sim -\kappa^{-1} \ln \eta + C_o(x)\,, \quad T_1 \to 1\,; \qquad p_0(x) = p_e(x)$$

skin-friction law $\kappa/\gamma \sim \ln(Re\, \gamma \delta u_e) + \kappa(C_i + C_o) \ \sim \ln Re$

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Substitution into Reynolds-averaged NS eqs

viscous wall layer: $\mathrm{d}u^+/\mathrm{d}y^+ + t^+ = 1 \quad \ldots \quad$ constant total shear stress

outer defect layer: $\delta \sim \gamma\, \Delta_1(x) + O(\gamma^2)$

leading-order equation

$$(E + 2\beta_0)\eta F_1' - E\, F_1 - \Delta_1 F_{1,e}\, \partial F_1/\partial x = F_{1,e}\,(T_1 - 1)\,, \quad F_{1,e} = F_1(x, 1)$$

$$E := 1 - \Delta_1\, \mathrm{d}F_{1,e}/\mathrm{d}x\,, \quad \beta_0 := -(\Delta_1 F_{1,e}\, \mathrm{d}u_e/\mathrm{d}x)/u_e$$

layer thickness ratio (Kármán number) δ^+ exponentially small

$$\delta^+ = \frac{(u_\tau Re)^{-1}}{\delta} \sim \frac{1}{Re\, \gamma^2 \Delta_1 u_e} \sim \frac{1}{\Delta_1 u_e} \frac{\exp(-\kappa/\gamma)}{\gamma^2}$$

consequences

- classical theory not capable of describing BL separation — since $\mathrm{d}\Delta_1/\mathrm{d}x$ is "manipulated" velocity defect $1 - u/u_e = O(\gamma)$, BL thickness δ

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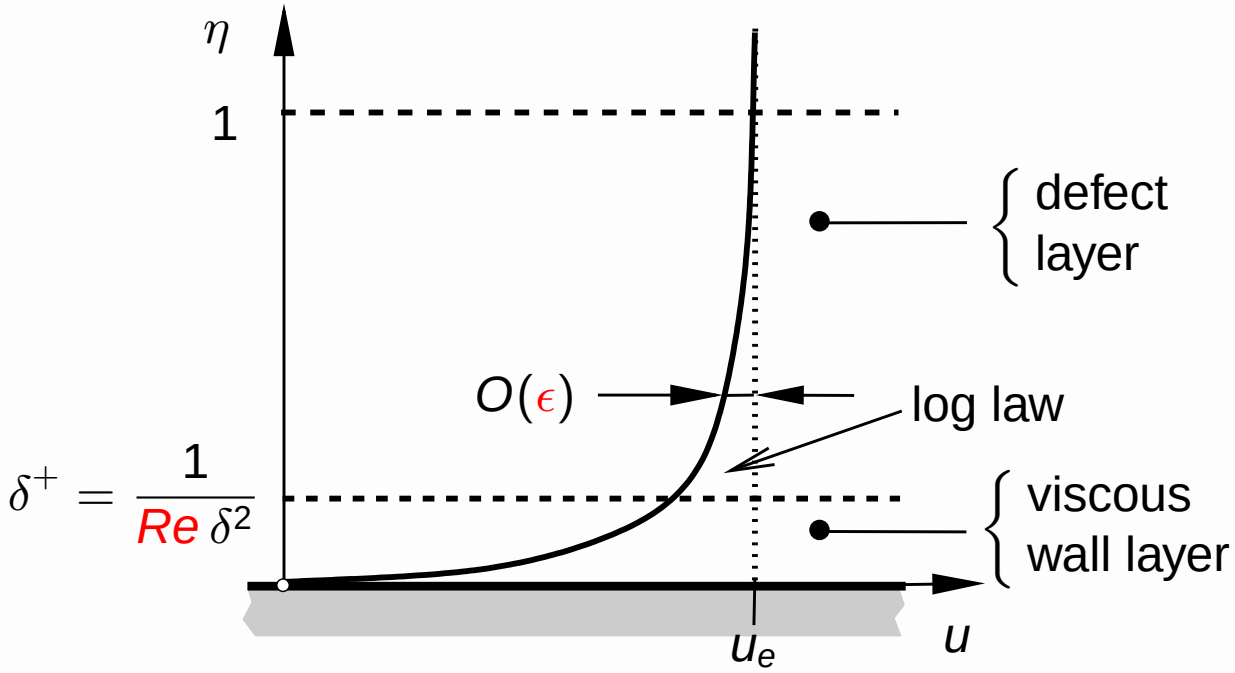

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- $\epsilon \gg \gamma \;\Rightarrow\; \partial p/\partial_x = O(1) \;\Rightarrow$ marginal separation — Scheichl & Kluwick (AIAA J, 2007)
- $\delta^+ = \frac{(\delta Re)^{-1}}{\delta} = O(\delta^4) \;\Rightarrow$ locally $\partial p/\partial x \gg 1 \;\Rightarrow$ massive separation — Scheichl, Kluwick & Smith

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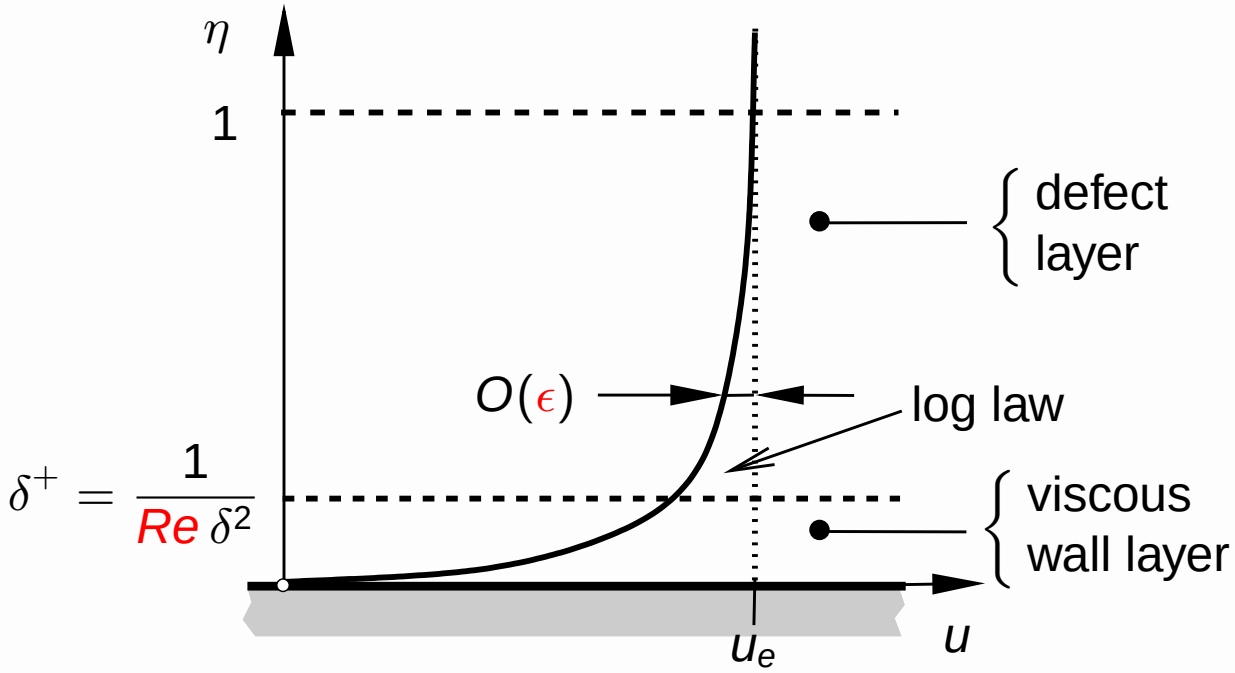

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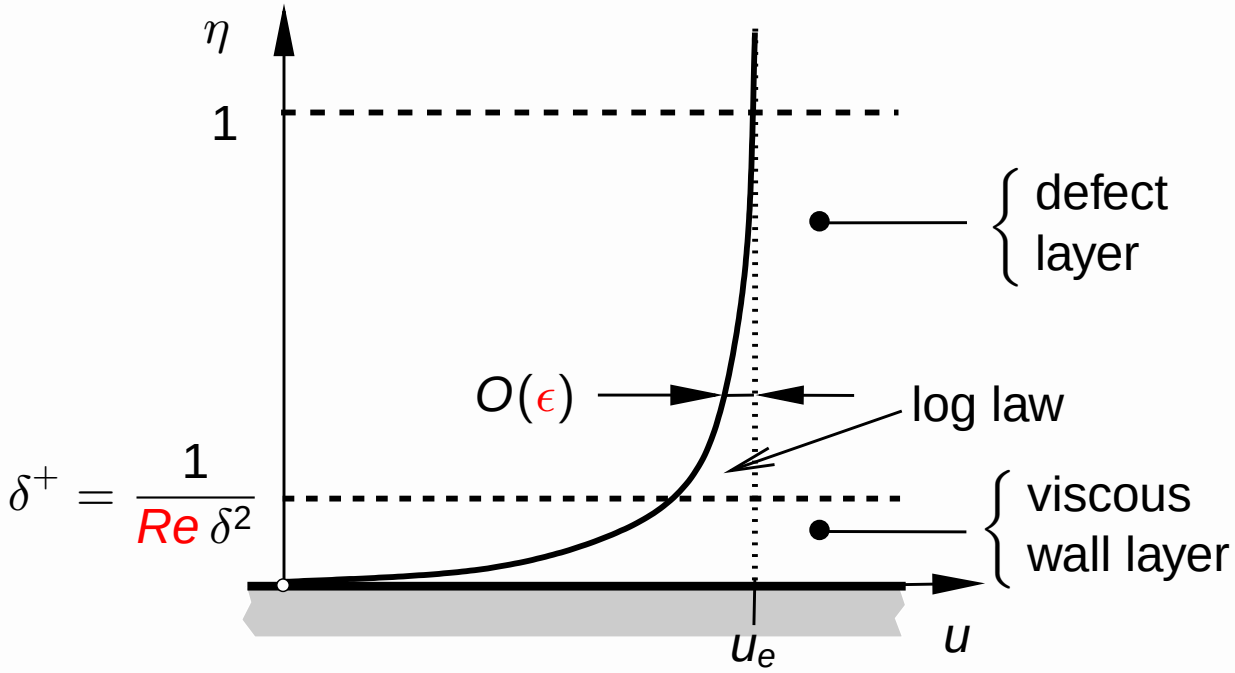

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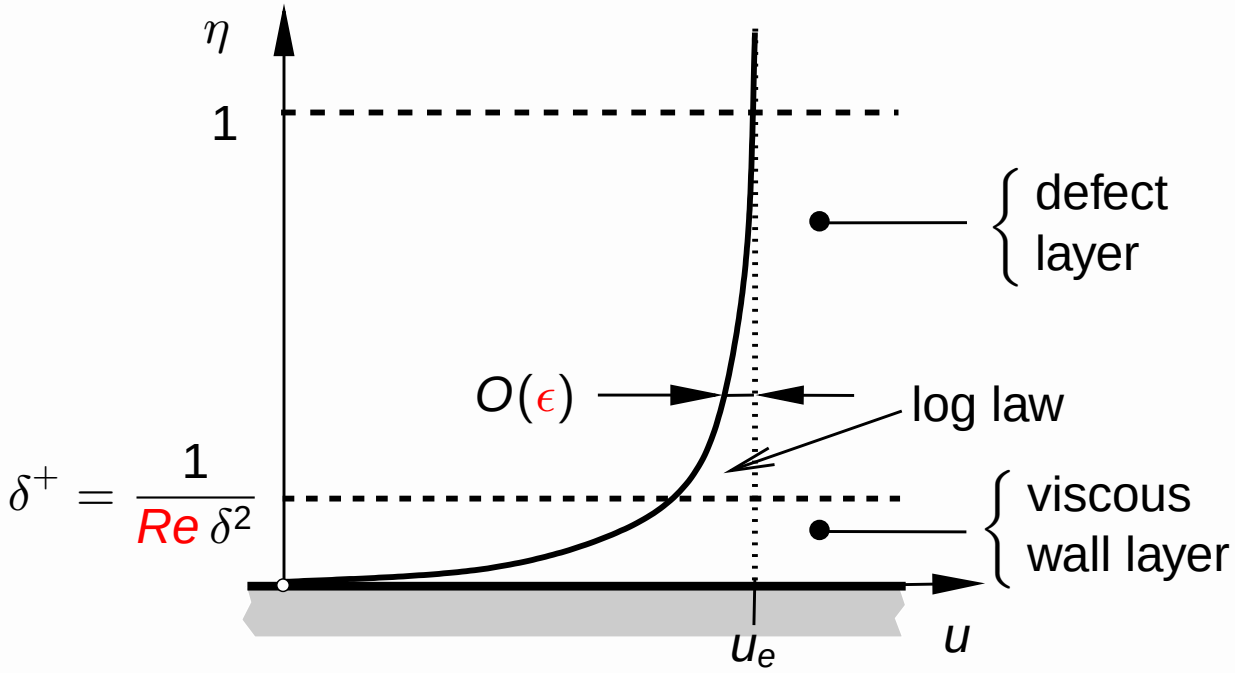

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Problem formulation – motivation

Local description of turbulent break-away separation . . .

. . . requires answers to three basic questions:

(1) Global topology of flow for 'vanishing viscosity'?

(2) Characteristics of incident boundary layer flow?

(3) How do these issues interdepend?

How turbulent are the BL and the separated SL ?

Scheichl, Kluwick & Alletto (Acta Mech, 2008, AIAA J)

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basic assumptions

- flow incompressible, nominally steady, 2D
- free-stream turbulence disregarded
- $Re \gg UL/\nu \to \infty$

canonical example: circular cylinder flow

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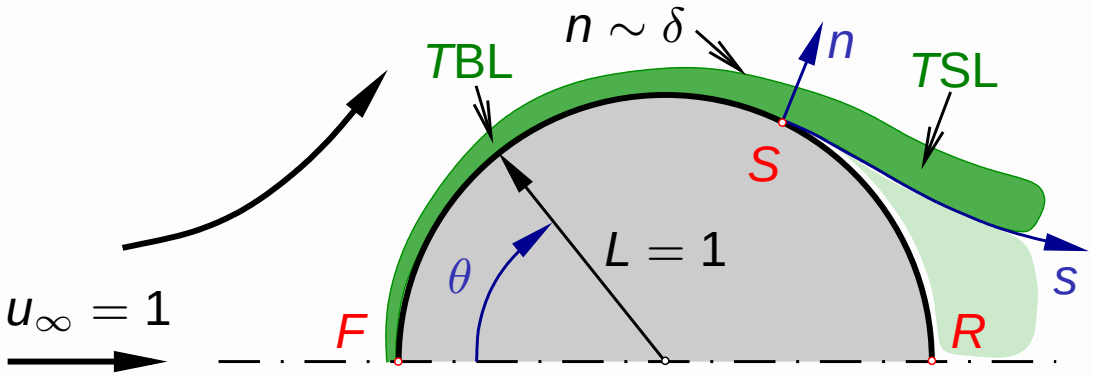

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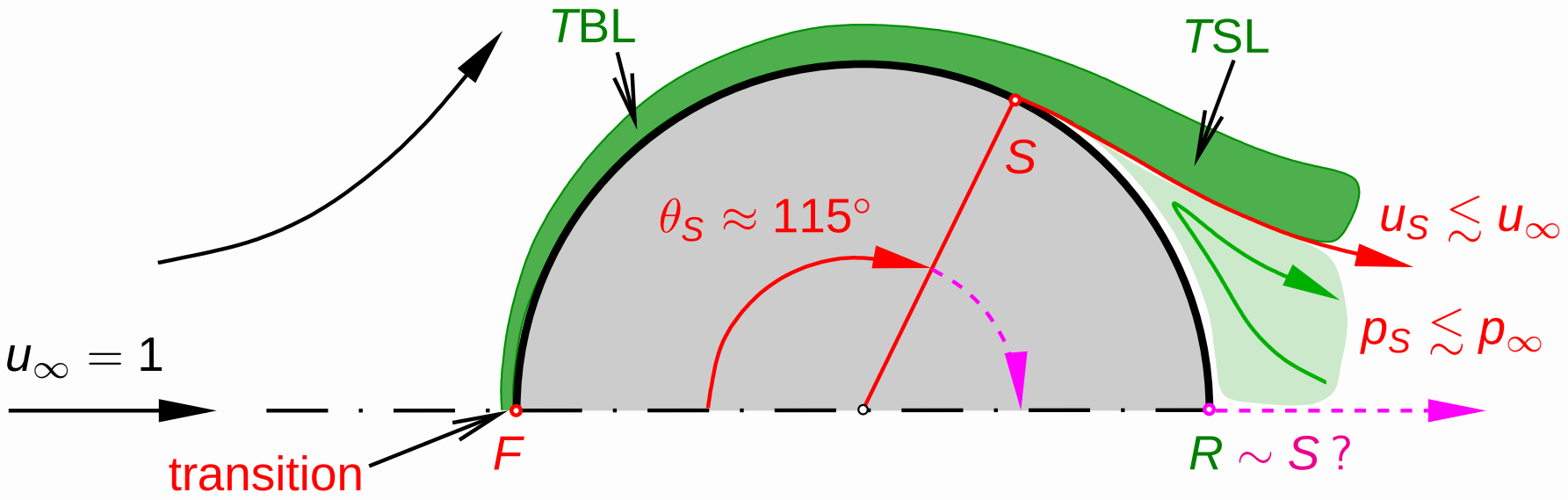
Global flow picture

$Re \gtrapprox 3 \times 10^6$: *postcritical* regime, $Re \to \infty$: transition point approximates F

$Re^{-1} = 0$: *ultimate* or T-state of flow, transition in F



Neish & Smith (JFM, 1992)

unlikely Scheichl & Kluwick (JFS, 2008)

corroborated experimentally up to $Re \approx 4 \times 10^7$

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Global flow picture

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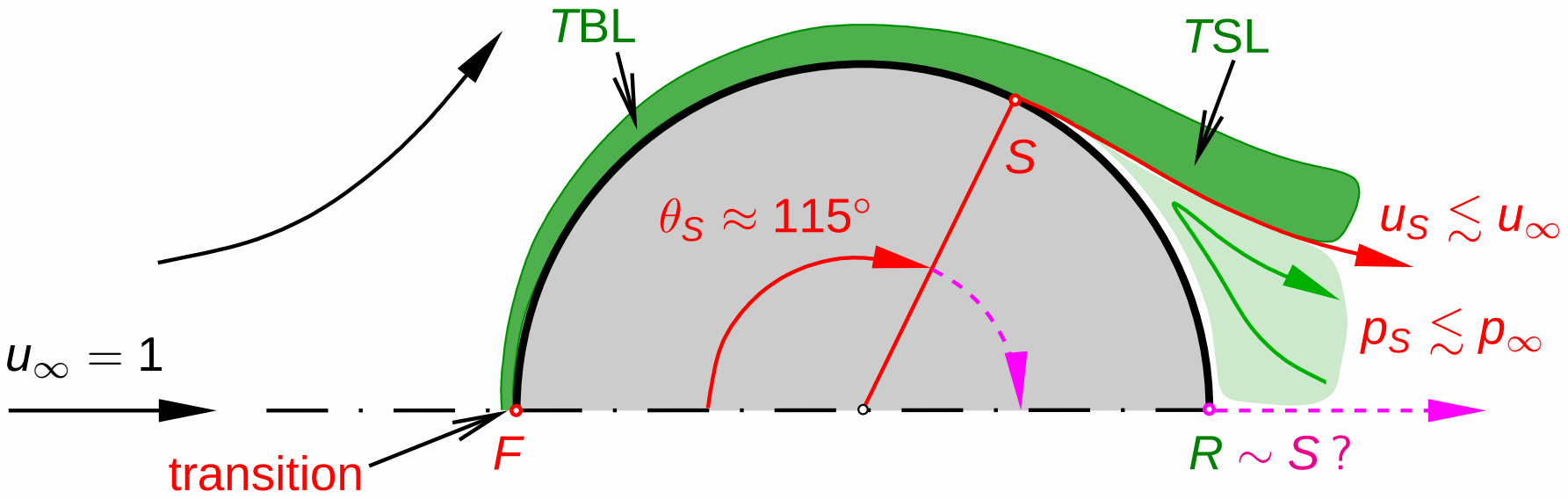

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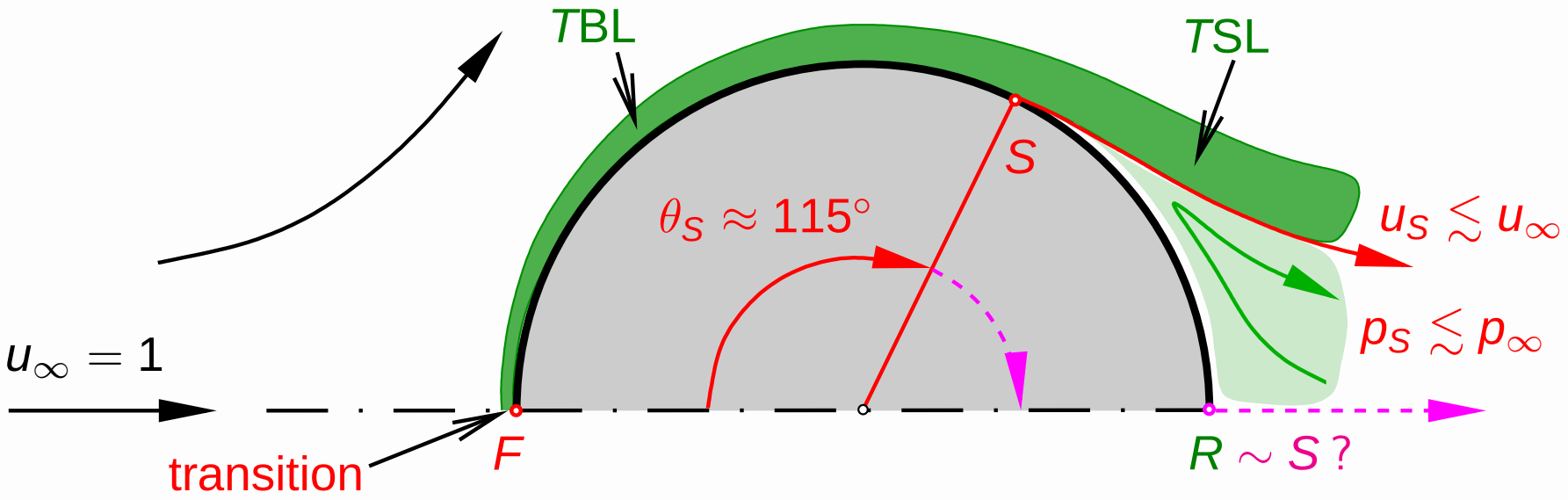

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Outline

Separation

Is there really a fully developed TBL as $Re \to \infty$?

oil-flow measurements : $Re_C := \tilde{U}\tilde{C}/\tilde{\nu}\,, \quad \tilde{C} =$ airfoil chord length

sub- trans- super- post-

by courtesy of G. Schewe (Göttingen, 2001)

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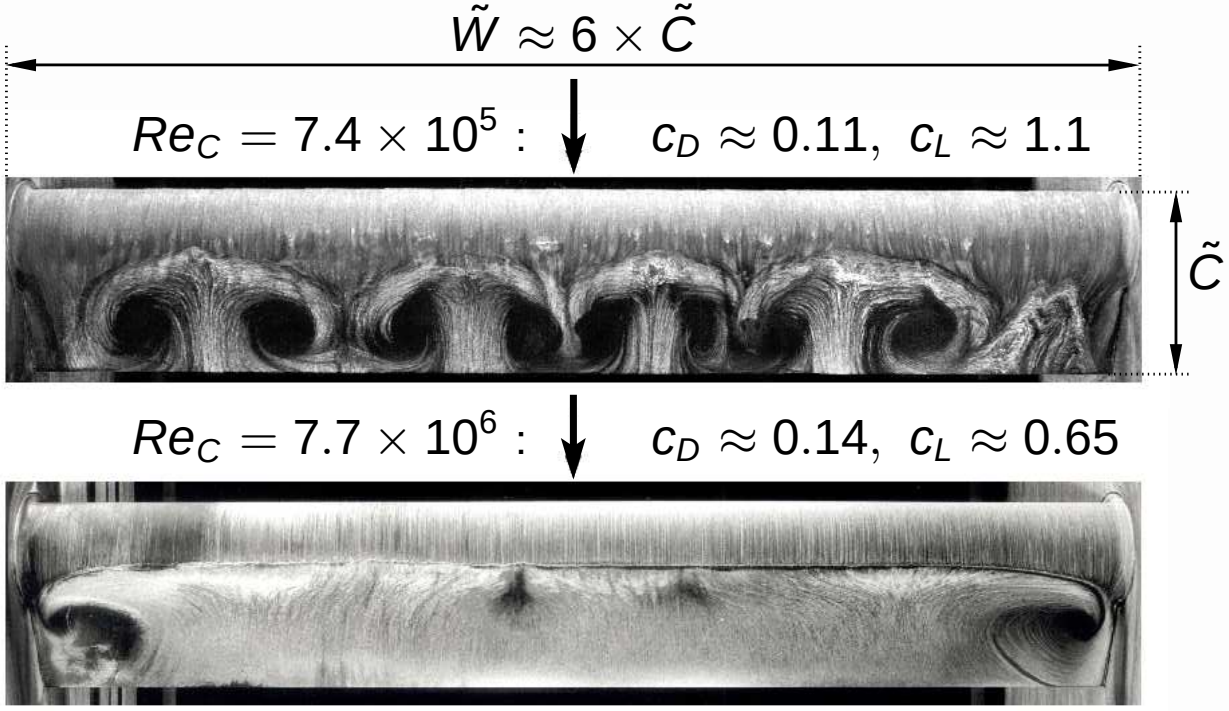

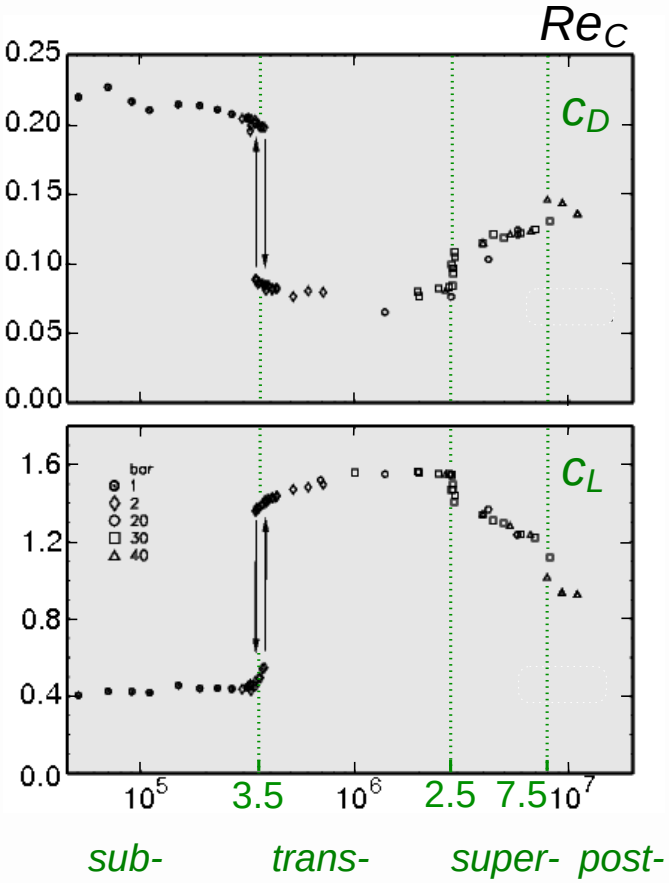

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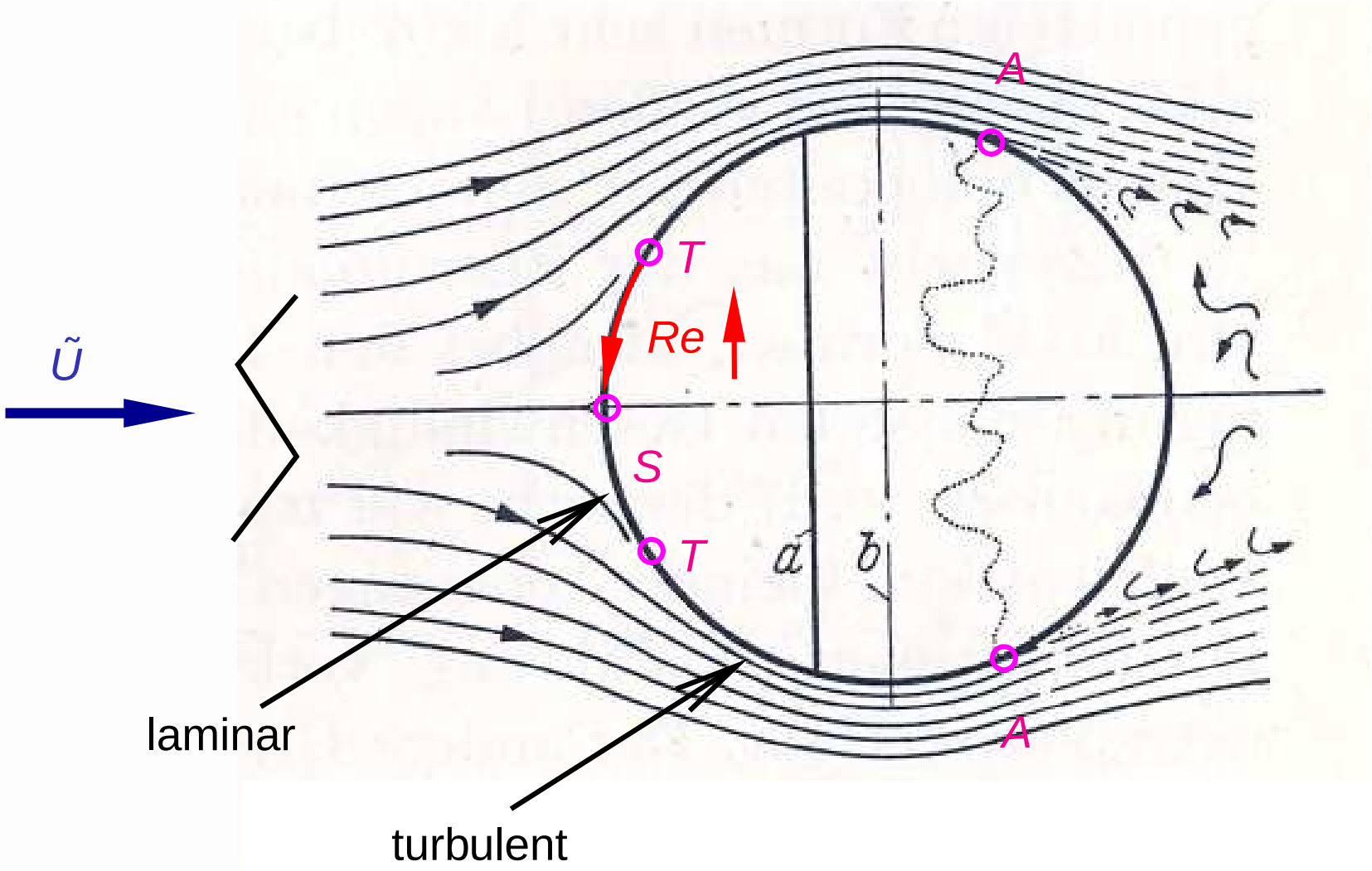

Prandtl (Göttingen, 1914)

Outline

‘Ideal-fluid limit’

Hierarchy of external flow – shear layer: potential flow approached as $Re \to \infty$

Oseen (1927), Lamb (1932), Batchelor (1956), Prandtl (1961), Sychev et al. (1998)

$Re^{-1} = 0$: external potential flows

Imai (J Phys Soc Japan, 1953),
Birkhoff & Zarantonello (1957), Gurevich (1966)

- free streamlines confine (open) dead-water region
- class of flows / θ_S controlled by
  Brillouin–Villat parameter $k \geq 0$, free-stream velocity $u_S \leq u_\infty$

$$u_e \begin{cases} \sim b(k)\theta + O(\theta^2)\,, & \theta \to 0_+ \\ \sim u_S\left[1 + 2k(-s)^{1/2} + O(-s)\right]\,, & s \to 0_- \\ \equiv u_S\,, & s \geq 0 \end{cases}$$

$$\varkappa \begin{cases} = \varkappa_B(s)\,, & s < 0 \\ \sim -k\,s^{-1/2} + \varkappa_B(0) + O(s^{1/2})\,, & s \to 0_+ \end{cases}$$

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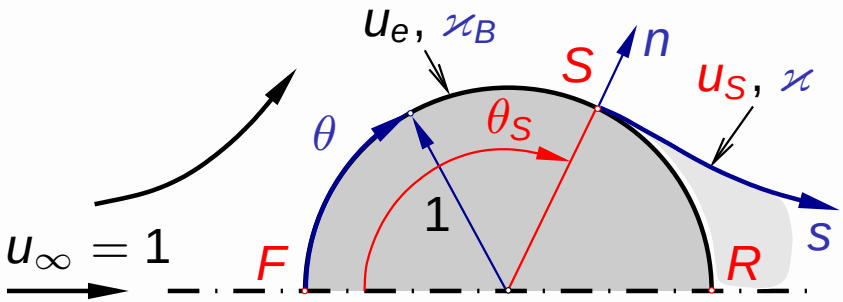

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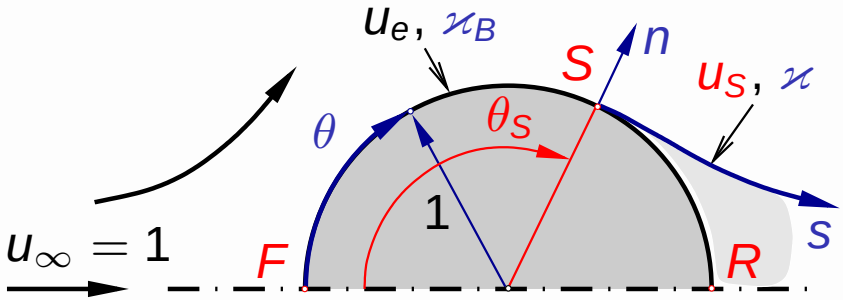

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Kirchhoff-type flows – numerical solutions

Open cavity: $u_S \equiv u_\infty = 1$

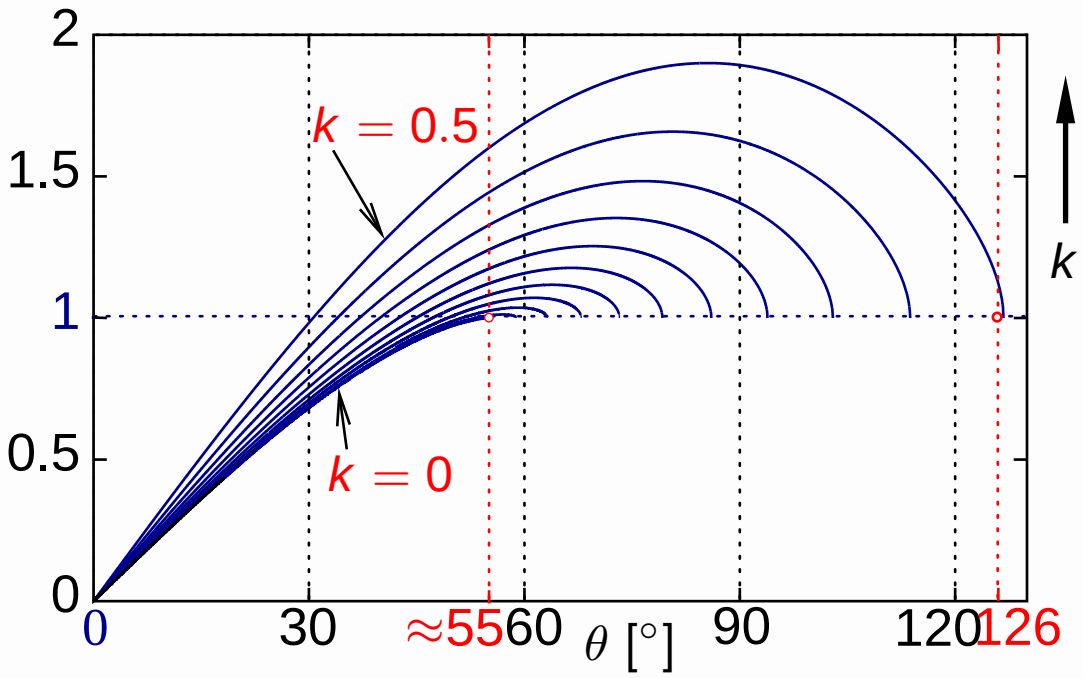
surface velocity $u_e(\theta, k)$

$k = 0,\ 0.05,\ 0.1,\ \ldots,\ 0.5$

$55^\circ\, 2'\, 30'' \ \leq\ \theta_S \ \lessapprox\ 126^\circ$

$k = 0$:

Brillouin–Villat condition



free streamlines

$126^\circ \ \lessapprox\ \theta_S \leq 180^\circ$

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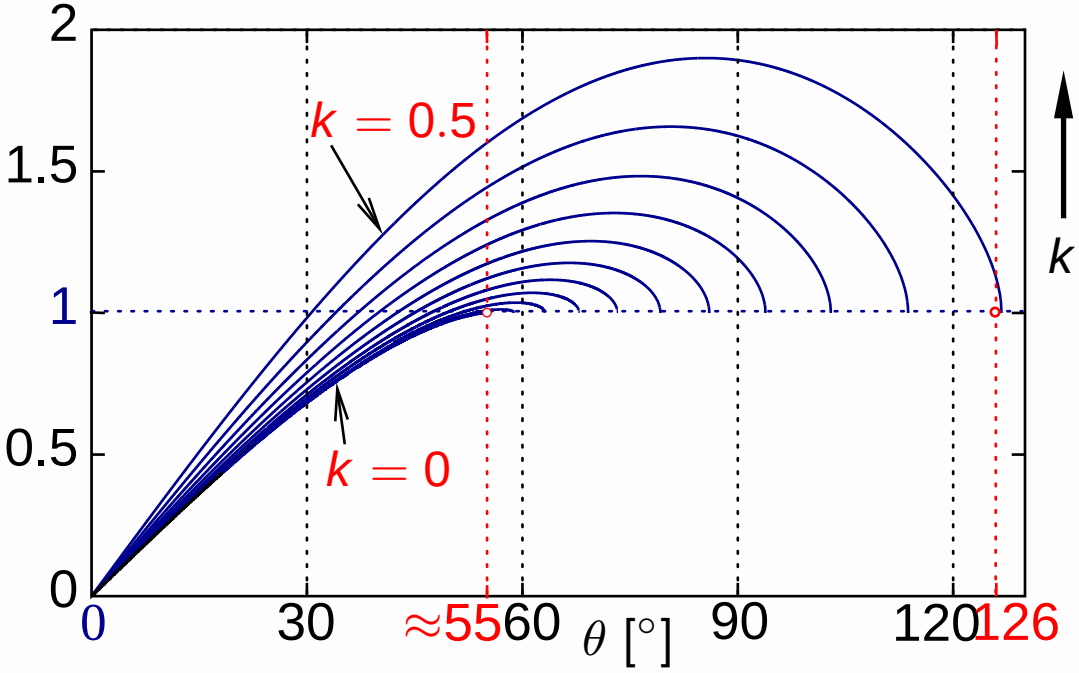

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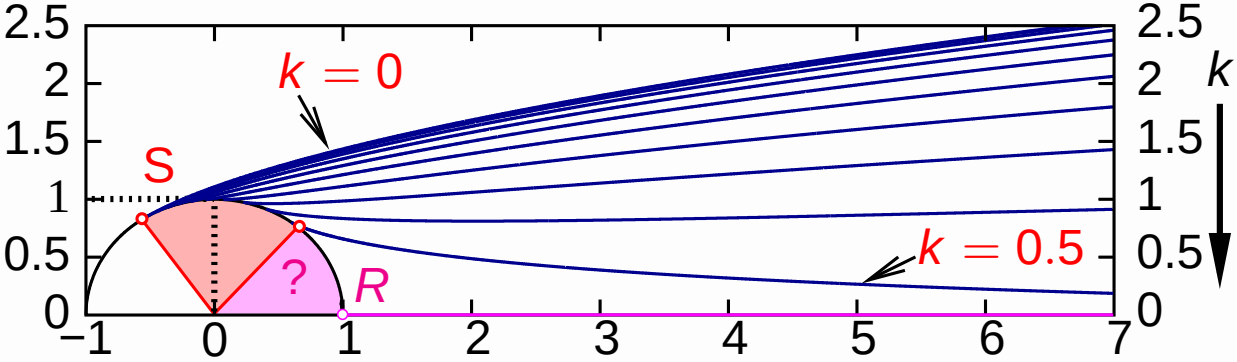
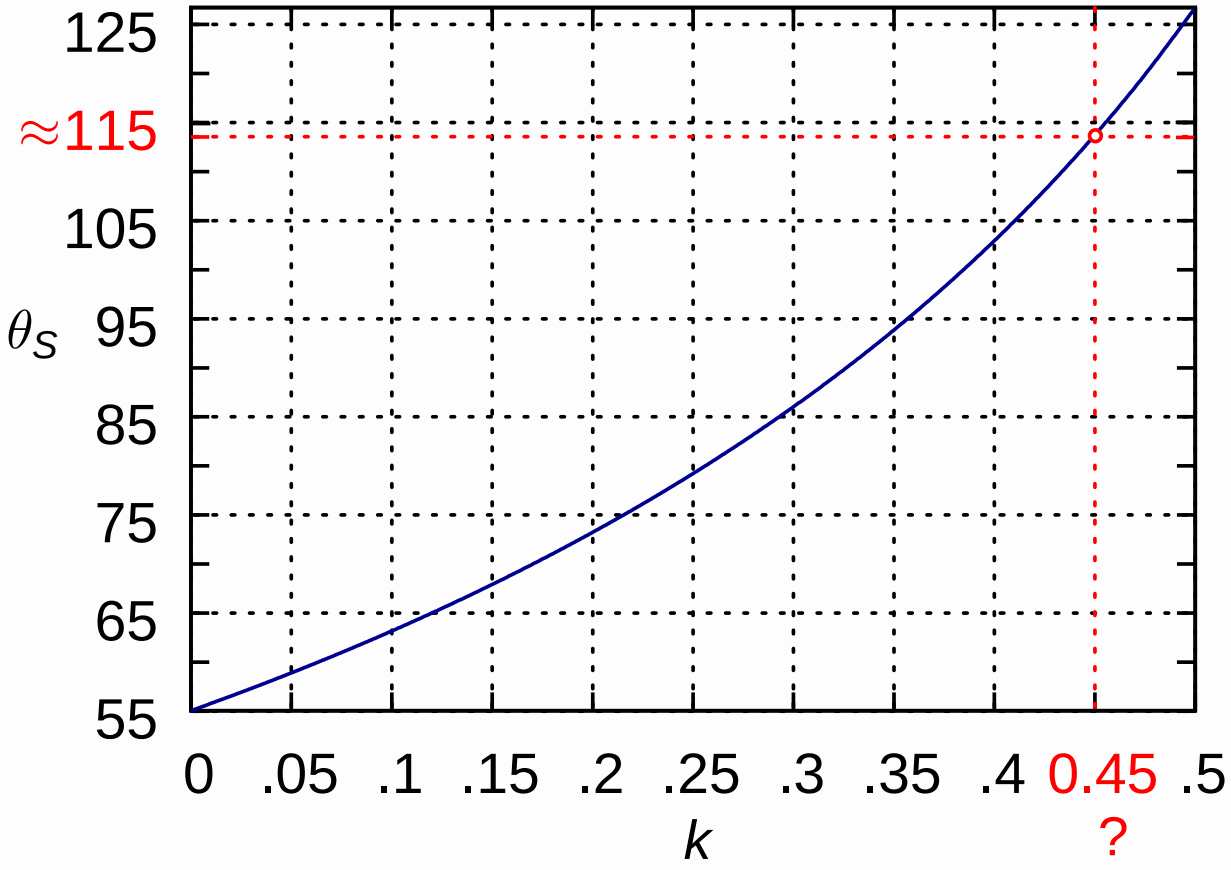


Question: overall flow structure as $Re \to \infty$



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(2) Characteristics of incident boundary layer flow?

(3) How do these issues interdepend?

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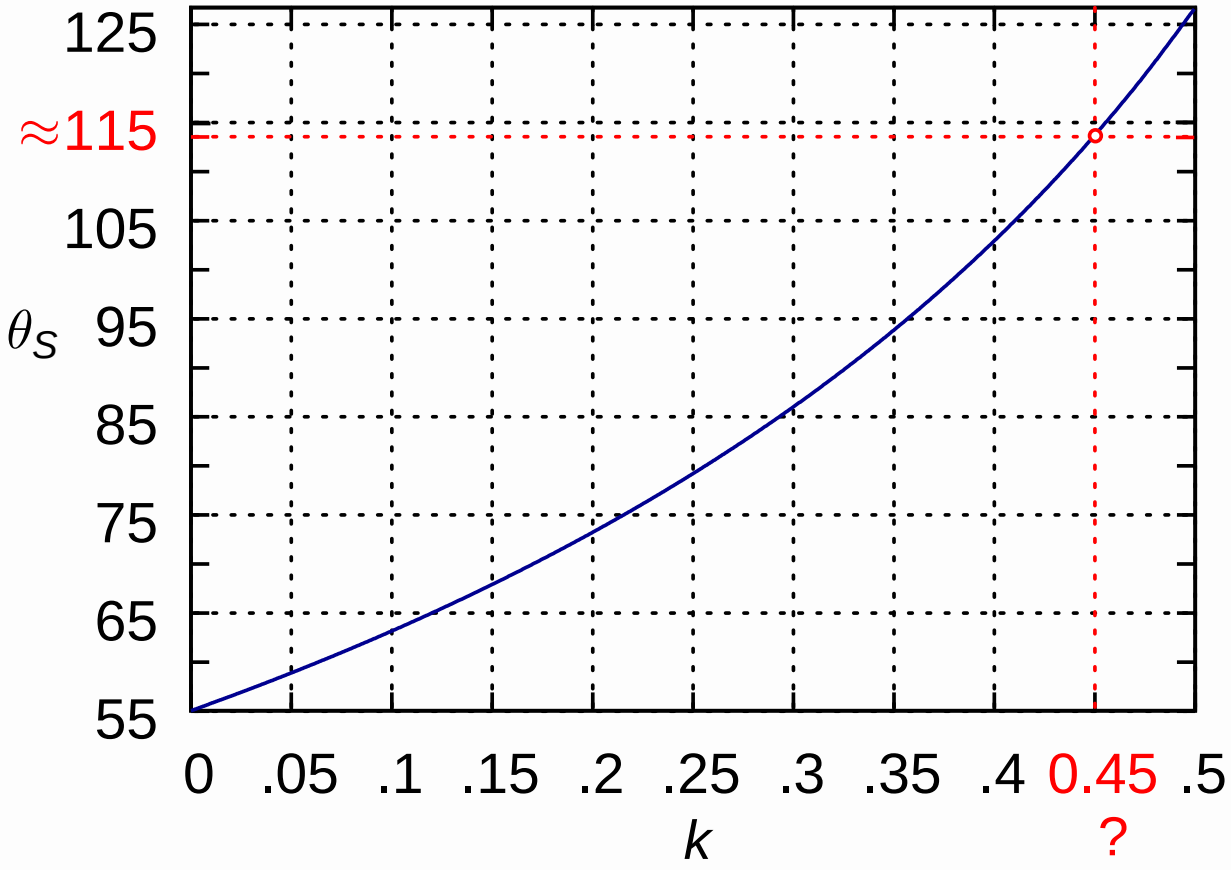

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Asymptotic framework – turbulence intensity level

Neish & Smith (1992), Scheichl, Kluwick & Alletto (Acta Mech, 2008, AIAA J)

Prandtl-type BL: $k, \; T, \; Re \gg 1$

$N = Re^{1/2} n$

$$\left\{ Re^{1/2} \left[\psi, \; -\langle u'v' \rangle \right], \, p \right\} \sim \left\{ \left[\Psi, \, T r \right] (\theta, N; k, T), \, p_F - u_e^2(\theta; k)/2 \right\} + O\left(Re^{-1/2} \right)$$

$$\partial_N \Psi \, \partial_{N\theta} \Psi - \partial_\theta \Psi \, \partial_{NN} \Psi = u_e \, \partial_\theta u_e + \partial_N [T r + \partial_{NN} \Psi], \quad 0 \le T < \infty$$

$$N \to 0: \;\; \Psi \to 0, \;\; \Psi_N \to 0, \;\; r = O(N^3), \quad N \to \infty: \;\; \Psi_N \to u_e, \;\; r \to 0$$

solution as $s = \theta - \theta_S \to 0_-$

$T \gg 1:$

Goldstein singularity at

$s = s_G = O\left[k^6 T^{-8} (\ln T)^{16} \right]$

$\Rightarrow$ interaction length scale δ_{TD}

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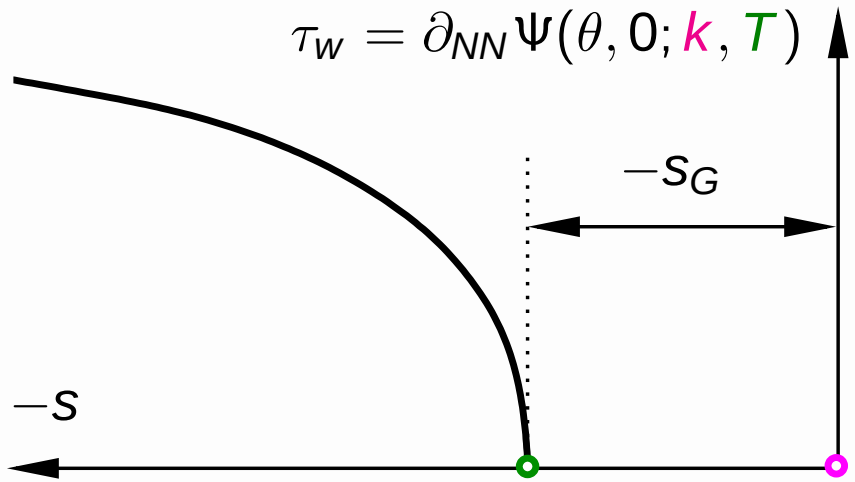

Concept of 'underdeveloped' TBL

BL originating in F parametrised by $T \gg 1$

$$\left.\begin{array}{ll} \text{small defect:} & u_e - u = O(\epsilon), \quad r = O(\epsilon^2) \\ \text{BL eq:} & -\langle u'v' \rangle = Re^{-1/2} T r = O(\sigma\epsilon), \quad \delta = O(\sigma) \end{array}\right\} \Rightarrow T = Re^{1/2}\sigma/\epsilon$$

$$\text{fully developed TBL:} \quad -\langle u'v' \rangle = O(\epsilon^2), \quad \sigma \propto \epsilon \sim \kappa/\ln Re$$

small-defect layer: $\sigma \ll 1, \quad \epsilon = \kappa/(\sigma^2 \ln Re) \ll 1$

$$\left\{ \left[\frac{u_e n - \psi}{u_e\, \delta\epsilon}, \frac{-\langle u'v' \rangle}{u_e^2\, \delta\epsilon} \right], \frac{\delta}{\sigma} \right\} \sim \left\{ [F, \Sigma](\theta, \eta; k), \Delta(\theta; k) \right\} + O(\epsilon), \quad \eta = \frac{n}{\delta}$$

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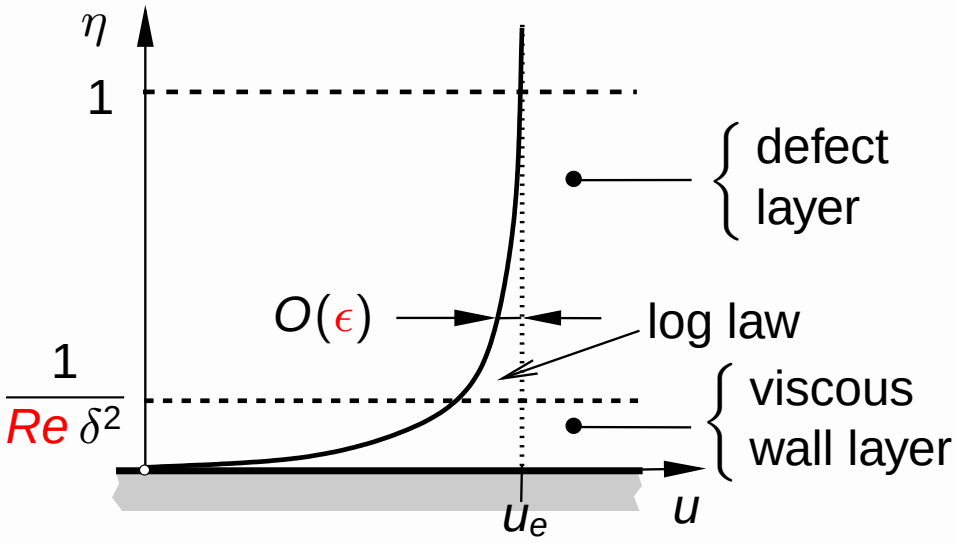

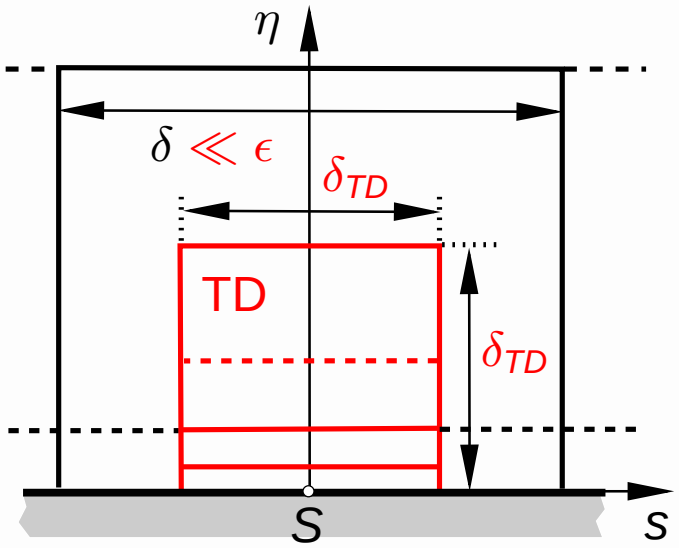

Outline

Solutions of outer-defect eqs for $0 \le \theta < \theta_S \doteq 113.5^\circ \quad k = 0.45$

$$u_e^2\,\partial_\theta(u_e\Delta)\eta F' - \partial_\theta(u_e^3\Delta F) = u_e^3(\Sigma - 1), \quad \Sigma = l^2 F''|F''|, \quad F(\theta,\eta;k), \quad \Delta(\theta;k)$$

$$\eta \to 0: \quad F' \sim -\kappa^{-1}\ln\eta + B(\theta) \;\Leftrightarrow\; \Sigma \sim 1, \quad \eta = 1: \quad F' = F'' = \Sigma = 0$$

$$\theta \to 0: \quad F \sim F_0(\eta), \quad \Delta \sim \Delta_0\,\theta, \quad u_e \sim b(k)\,\theta \qquad \text{stagnant-flow}$$

- algebraic closure for $\ell = \delta l$, Klebanoff's intermittency factor
  Michel, Quémard & Durant (1969), Klebanoff (1955)
- Keller–Box scheme / method of lines, adaptive grid remeshing

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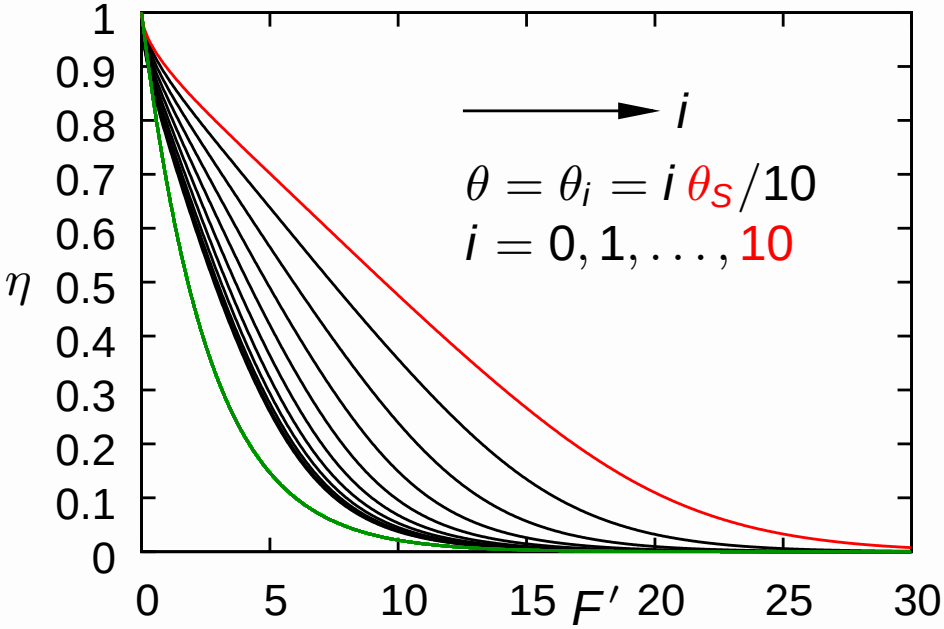
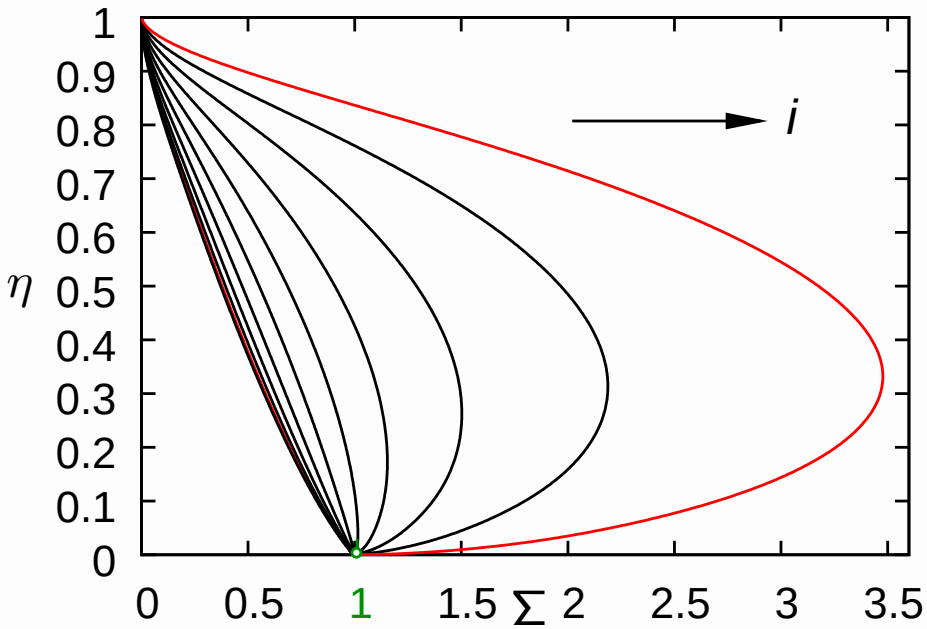
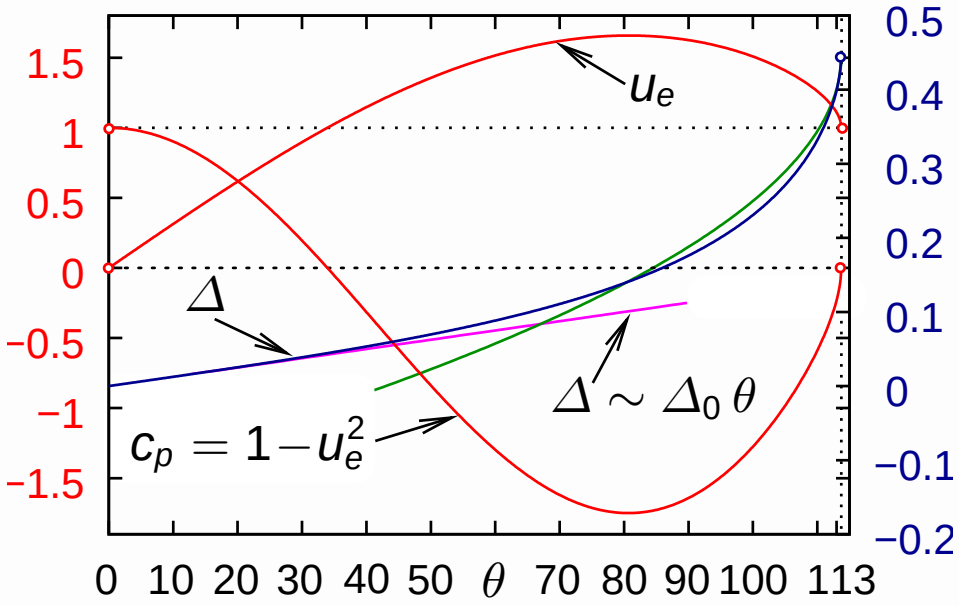




Solutions of outer-defect eqs, cont'd $k = 0.45$

Comparison with leading-order theory as $s = \theta - \theta_S \to 0_-$



$$[F, \Delta] \sim [F_S(\eta; k), \Delta_S(k)] - \sum_{i=1}^{\infty} [f_i(s,k) F_i(\eta; k), g_i(s,k) \Delta_i(k)]$$

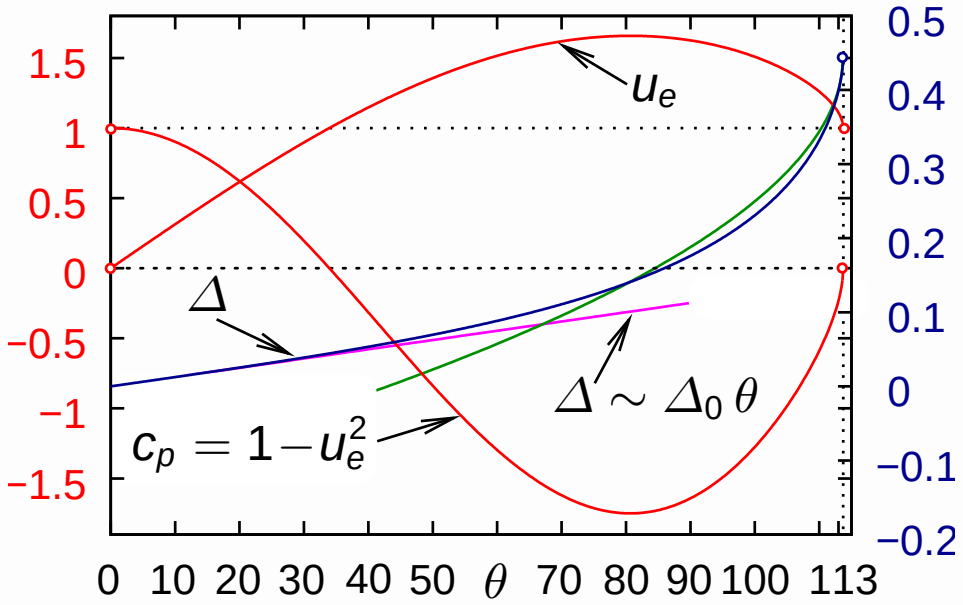
$$[F, \Delta] \sim [F_S(1 - 2g_1), \Delta_S(1 - g_1)] + O(s), \quad g_1 = 2k(-s)^{1/2}$$

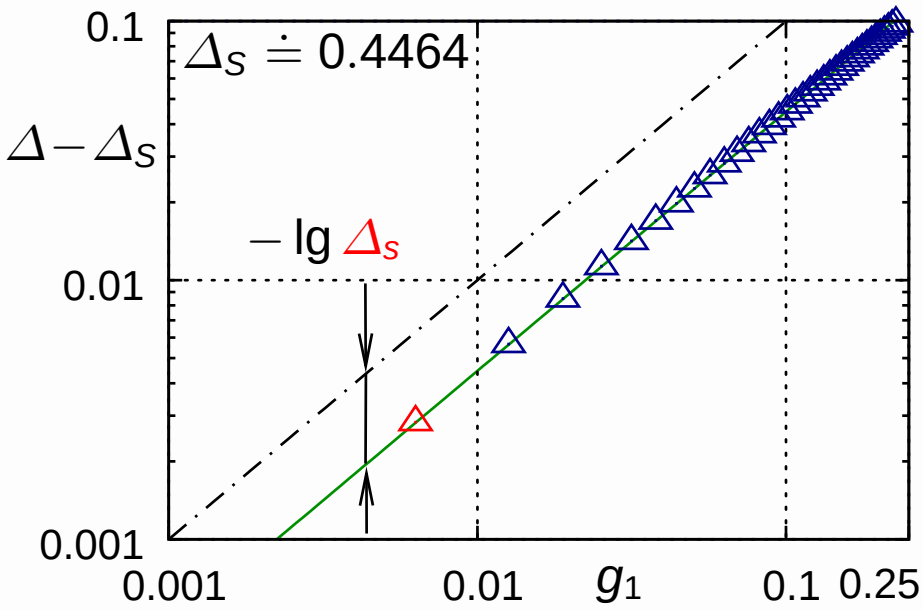
$F_i' = O[\ln(\eta)], \quad \eta \to 0, \quad i = 1, 2, \ldots \quad \Rightarrow \quad$ sublayer for $\eta = O(-s)$

indicates onset of viscous/inviscid interaction

Solutions of outer-defect eqs, cont'd $k = 0.45$

Comparison with leading-order theory as $s = \theta - \theta_S \to 0_-$





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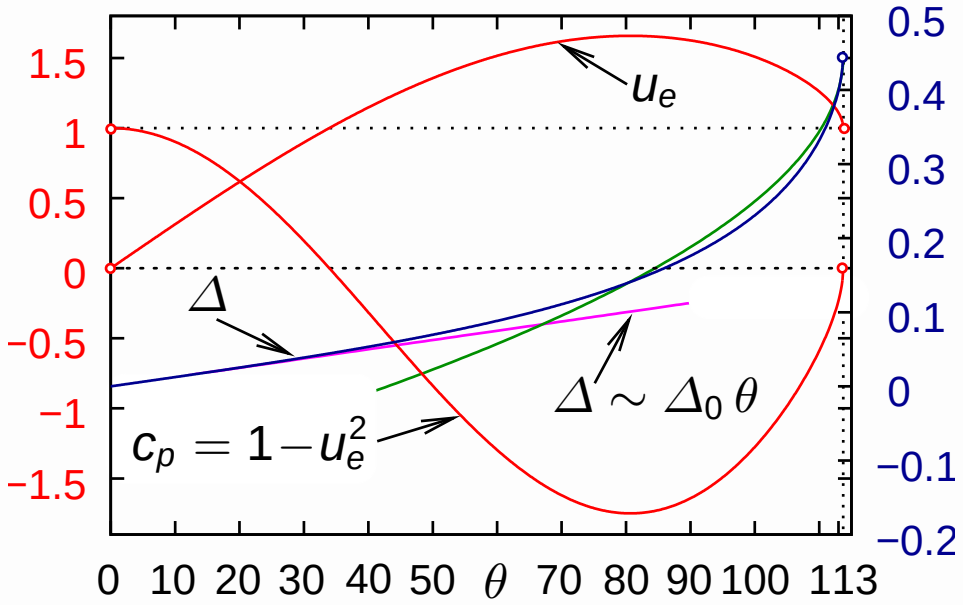
$$[F,\,\Delta] \sim [F_S(1-2g_1),\,\Delta_S(1-g_1)] + O(s), \quad g_1 = 2k(-s)^{1/2}$$

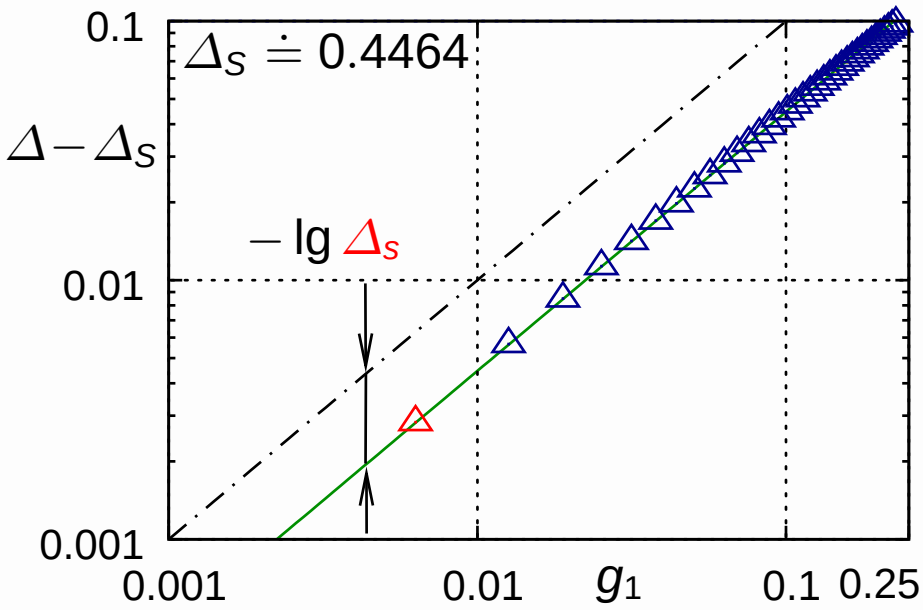
$F_i' = O[\ln(\eta)], \quad \eta \to 0, \quad i = 1, 2, \ldots \quad \Rightarrow \quad$ sublayer for $\eta = O(-s)$

indicates onset of viscous/inviscid interaction

Solutions of outer-defect eqs, cont'd $k = 0.45$

Comparison with leading-order theory as $s = \theta - \theta_S \to 0_-$





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Outline

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- increasing turbulence intensity shifts separation downstream, increases k
- BL never attains fully developed turbulent state

interactive BL $\Rightarrow$ scaling in terms of k, $Re \gg 1$

$$\epsilon \sim \kappa / \ln Re, \quad \delta_{TD} = k^{8/9} Re^{-4/9}, \quad \delta = O(\delta_{TD}/\epsilon), \quad -\langle u_i' u_j' \rangle = O(\epsilon\delta)$$

ongoing research

- TD problem (akin to laminar counterpart)
- self-induced separation $\Leftrightarrow$ (maximum) values of k, $\theta_S(k)$?
- structure of (large-scale) separated flow

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Thank you for your attention !