Sensorless Capability of Permanent Magnet Synchronous Machines Due To Saturation- And Reluctance-Based Coupling Effects

Andreas Eilenberger, Erich Schmidt, Manfred Schrödl
Vienna University of Technology
Institute of Electrical Drives and Machines
Gusshausstrasse 25-29, A-1040 Vienna
Email: andreas.eilenberger@tuwien.ac.at

***Abstract*—The paper shows the theoretical derivation of sensorless capability of a permanent magnet synchronous machine due to saturation- and/or reluctance-based coupling effects. Only due to an introduction of coupling terms in the rotor-oriented inductance tensor a dependency of double the electrical rotor position can be calculated.**

I. Introduction

A lot of publications handle with sensorless capability of salient permanent magnet synchronous machines (PMSM) [1] and therefore assume unequal inductances in direct and quadrature direction of a rotor-oriented two-axis inductance tensor. In most cases the coupling terms between the flux-linkage space phasor in direct direction to the current space phasor in quadrature direction and vice versa are neglected. This work gives a theoretical derivation of sensorless capability of PMSM's also due to these coupling terms which consider saturation- and/or reluctance-based effects. These fictive components of the inductance tensor in the rotor-oriented reference frame are represented as rotor position dependent phase inductances which also includes higher harmonics up to second order to sufficiently describe the saliency of a PMSM [2], [3]. Furthermore some simulation results are given for better comprehension.

Andreas Eilenberger
June 10, 2010

II. Fundamentals

A. Phase Inductances

According to the ideas in [2] the self- and mutual phase inductances l, m

$$\underline{\psi}_{\mathrm{s,uvw}} = \mathbf{l}_{\mathrm{s,uvw}} \; \underline{i}_{\mathrm{s,uvw}} \tag{1}$$

$$\begin{pmatrix} \psi_{\mathrm{u}} \\ \psi_{\mathrm{v}} \\ \psi_{\mathrm{w}} \end{pmatrix} = \begin{pmatrix} l_{\mathrm{uu}} & m_{\mathrm{vu}} & m_{\mathrm{wu}} \\ m_{\mathrm{uv}} & l_{\mathrm{vv}} & m_{\mathrm{wv}} \\ m_{\mathrm{uw}} & m_{\mathrm{vw}} & l_{\mathrm{ww}} \end{pmatrix} \begin{pmatrix} i_{\mathrm{u}} \\ i_{\mathrm{v}} \\ i_{\mathrm{w}} \end{pmatrix} \tag{2}$$

are represented by their Fourier series expansion in dependence of the angular rotor position γ up to second order elements with

$$l_{\mathrm{uu}} = l_0 + l_2 \cos(2\gamma)\,, \tag{3}$$
$$l_{\mathrm{vv}} = l_0 + l_2 \cos(2\gamma + 2\pi/3)\,, \tag{4}$$
$$l_{\mathrm{ww}} = l_0 + l_2 \cos(2\gamma + 4\pi/3)\,, \tag{5}$$
$$m_{\mathrm{wv}} = m_{\mathrm{vw}} = m_0 + m_2 \cos(2\gamma)\,, \tag{6}$$
$$m_{\mathrm{uw}} = m_{\mathrm{wu}} = m_0 + m_2 \cos(2\gamma + 2\pi/3)\,, \tag{7}$$
$$m_{\mathrm{vu}} = m_{\mathrm{uv}} = m_0 + m_2 \cos(2\gamma + 4\pi/3)\,. \tag{8}$$

B. Stator-Oriented Inductances

The linear transformation matrix $\mathbf{T}_{\alpha\beta}^{\mathrm{uvw}}$ for transforming three-phase to equivalent two-phase quantities and $\mathbf{T}_{\mathrm{uvw}}^{\alpha\beta}$ for opposite direction

$$\mathbf{T}_{\alpha\beta}^{\mathrm{uvw}} = \frac{2}{3}\begin{pmatrix} 1 & -\frac{1}{2} & -\frac{1}{2} \\ 0 & \frac{\sqrt{3}}{2} & -\frac{\sqrt{3}}{2} \end{pmatrix}, \tag{9}$$

$$\mathbf{T}_{\mathrm{uvw}}^{\alpha\beta} = \begin{pmatrix} 1 & 0 \\ -\frac{1}{2} & \frac{\sqrt{3}}{2} \\ -\frac{1}{2} & -\frac{\sqrt{3}}{2} \end{pmatrix}, \tag{10}$$

yields to the self- and mutual stator-oriented inductances $l_{\alpha\alpha}$, $l_{\beta\beta}$ and $m_{\alpha\beta}$ in the stator-oriented fixed reference frame expressed with the self- and mutual phase inductances l_{uu}, l_{vv}, l_{ww}, m_{wv}, m_{uw} and m_{vu} in a three-phase winding configuration (zero-sequence self and mutual inductances are neclegted)

$$\mathbf{l}_{\mathrm{s},\alpha\beta}(l_{0,2}, m_{0,2}) = \mathbf{T}_{\alpha\beta}^{\mathrm{uvw}} \cdot \mathbf{l}_{\mathrm{s,uvw}}(l_{0,2}, m_{0,2}) \cdot \mathbf{T}_{\mathrm{uvw}}^{\alpha\beta} \tag{11}$$

$$= \begin{pmatrix} \frac{2}{3}(l_{\mathrm{uu}} - m_{\mathrm{uv}} - m_{\mathrm{uw}}) + \frac{1}{3}\left(m_{\mathrm{vw}} + \frac{l_{\mathrm{vv}}+l_{\mathrm{ww}}}{2}\right) & \frac{1}{\sqrt{3}}\left(m_{\mathrm{uv}} - m_{\mathrm{uw}} - \frac{l_{\mathrm{vv}}-l_{\mathrm{ww}}}{2}\right) \\ \frac{1}{\sqrt{3}}\left(m_{\mathrm{uv}} - m_{\mathrm{uw}} - \frac{l_{\mathrm{vv}}-l_{\mathrm{ww}}}{2}\right) & \frac{l_{\mathrm{vv}}+l_{\mathrm{ww}}}{2} - m_{\mathrm{vw}} \end{pmatrix}. \tag{12}$$

Furthermore with above representations eq. 3-8, the self- and mutual stator-oriented inductances follow to

$$\begin{pmatrix} l_{\alpha\alpha} & m_{\alpha\beta} \\ m_{\alpha\beta} & l_{\beta\beta} \end{pmatrix} = \begin{pmatrix} l_0 - m_0 + \left(\frac{1}{2}l_2 + m_2\right)\cos 2\gamma & \left(\frac{1}{2}l_2 + m_2\right)\sin 2\gamma \\ \left(\frac{1}{2}l_2 + m_2\right)\sin 2\gamma & l_0 - m_0 - \left(\frac{1}{2}l_2 + m_2\right)\cos 2\gamma \end{pmatrix}.$$

With the components of the inductance tensor quantity $\mathbf{l}_{\mathrm{s},\alpha\beta}$

$$l_{\alpha\alpha} = l_0 - m_0 + \left(\frac{1}{2}l_2 + m_2\right)\cos 2\gamma, \tag{13}$$

$$l_{\beta\beta} = l_0 - m_0 - \left(\frac{1}{2}l_2 + m_2\right)\cos 2\gamma, \tag{14}$$

$$m_{\alpha\beta} = \left(\frac{1}{2}l_2 + m_2\right)\sin 2\gamma. \tag{15}$$

C. *Rotor-Oriented Inductances*

1) With decoupled stator flux components: With the linear transformation matrix $\mathbf{T}_{\mathrm{dq}}^{\alpha\beta}(\gamma)$ and $\mathbf{T}_{\alpha\beta}^{\mathrm{dq}}(\gamma)$ for linear coordinate transformations between the stator- and rotor-oriented fixed reference frame and vice versa

$$\mathbf{T}_{\mathrm{dq}}^{\alpha\beta}(\gamma) = \begin{pmatrix} \cos\gamma & \sin\gamma \\ -\sin\gamma & \cos\gamma \end{pmatrix}, \tag{16}$$

$$\mathbf{T}_{\alpha\beta}^{\mathrm{dq}}(\gamma) = \begin{pmatrix} \cos\gamma & -\sin\gamma \\ \sin\gamma & \cos\gamma \end{pmatrix}, \tag{17}$$

and the trigonometrical identities

$$\cos^2\gamma = \frac{1+\cos 2\gamma}{2}, \tag{18}$$

$$\sin^2\gamma = \frac{1-\cos 2\gamma}{2}, \tag{19}$$

$$\sin\gamma\cos\gamma = \frac{\sin 2\gamma}{2}, \tag{20}$$

the self- l_{dd}, l_{qq} and mutual l_{dq}, l_{qd} inductances in the rotor-oriented fixed reference frame, the coefficients l_{a}, l_{b}, m_{a} and m_{b} of the Fourier series expansion of the phase inductances (eq. 3-8) and the inductance tensor quantity $\mathbf{l}_{\mathrm{s,dq}}$

$$\mathbf{l}_{\mathrm{s,dq}} = \begin{pmatrix} l_{\mathrm{dd}} & l_{\mathrm{qd}} \\ l_{\mathrm{dq}} & l_{\mathrm{qq}} \end{pmatrix} = \begin{pmatrix} l_{\mathrm{d}} & m \\ m & l_{\mathrm{q}} \end{pmatrix}, \tag{21}$$

follows to

$$\mathbf{l}_{\mathrm{s,dq}}(l_{0,2}, m_{0,2}) = \mathbf{T}_{\mathrm{dq}}^{\alpha\beta}(\gamma)\cdot\mathbf{l}_{\mathrm{s},\alpha\beta}(l_{0,2}, m_{0,2})\cdot\mathbf{T}_{\alpha\beta}^{\mathrm{dq}}(\gamma) \tag{22}$$
$$= \begin{pmatrix} l_0 - m_0 + \left(\frac{1}{2}l_2 + m_2\right) & 0 \\ 0 & l_0 - m_0 - \left(\frac{1}{2}l_2 + m_2\right) \end{pmatrix}.$$

With

$$l_{\mathrm{d}} = l_0 - m_0 + \left(\frac{1}{2}l_2 + m_2\right), \tag{23}$$

$$l_{\mathrm{q}} = l_0 - m_0 - \left(\frac{1}{2}l_2 + m_2\right), \tag{24}$$

$$m = 0. \tag{25}$$

According to the two-axis theory [4], [5] and defining the components of the complex stator flux linkage space phasor $\underline{\psi}_{\mathrm{s,dq}}$, the complex stator current space phasor $\underline{i}_{\mathrm{s,dq}}$ and the complex stator flux linkage space phasor due to the permanent magnets $\underline{\psi}_{\mathrm{pm,dq}}$ in the rotor-oriented fixed reference frame dq and the tensor quantity $\mathbf{l}_{\mathrm{s,dq}}$ as follows

$$\underline{\psi}_{\mathrm{s,dq}} = \begin{pmatrix} \psi_{\mathrm{d}} \\ \psi_{\mathrm{q}} \end{pmatrix} = \psi_{\mathrm{d}} + \jmath\psi_{\mathrm{q}}, \tag{26}$$

$$\underline{\psi}_{\mathrm{pm,dq}} = \begin{pmatrix} \psi_{\mathrm{pm}} \\ 0 \end{pmatrix} = \psi_{\mathrm{pm}}, \tag{27}$$

$$\underline{i}_{\mathrm{s,dq}} = \begin{pmatrix} i_{\mathrm{d}} \\ i_{\mathrm{q}} \end{pmatrix} = i_{\mathrm{d}} + \jmath i_{\mathrm{q}}, \tag{28}$$

$$\underline{\psi}_{\mathrm{s,dq}} = \mathbf{l}_{\mathrm{s,dq}}\,\underline{i}_{\mathrm{s,dq}} + \underline{\psi}_{\mathrm{pm,dq}}, \tag{29}$$

$$\begin{pmatrix} \psi_{\mathrm{d}} \\ \psi_{\mathrm{q}} \end{pmatrix} = \begin{pmatrix} l_{\mathrm{d}} & 0 \\ 0 & l_{\mathrm{q}} \end{pmatrix}\begin{pmatrix} i_{\mathrm{d}} \\ i_{\mathrm{q}} \end{pmatrix} + \begin{pmatrix} \psi_{\mathrm{pm}} \\ 0 \end{pmatrix}, \tag{30}$$

the subsequent flux linkage equations are achieved

$$\psi_{\mathrm{d}} = l_{\mathrm{d}} i_{\mathrm{d}} + \psi_{\mathrm{pm}}, \tag{31}$$

$$\psi_{\mathrm{q}} = l_{\mathrm{q}} i_{\mathrm{q}}. \tag{32}$$

The self- $l_{\alpha\alpha}$, $l_{\beta\beta}$ and mutual $m_{\alpha\beta}$ inductances in the stator-oriented fixed reference frame (eq. 11) follows with the rotor-oriented self inductances l_{d} and l_{q} to

$$\mathbf{l}_{\mathrm{s},\alpha\beta}(l_{\mathrm{d,q}}) = \mathbf{T}_{\alpha\beta}^{\mathrm{dq}}(\gamma)\cdot\mathbf{l}_{\mathrm{s,dq}}\cdot\mathbf{T}_{\mathrm{dq}}^{\alpha\beta}(\gamma) \tag{33}$$
$$= \begin{pmatrix} \frac{l_{\mathrm{d}}+l_{\mathrm{q}}}{2} + \frac{l_{\mathrm{d}}-l_{\mathrm{q}}}{2}\cos 2\gamma & \frac{l_{\mathrm{d}}-l_{\mathrm{q}}}{2}\sin 2\gamma \\ \frac{l_{\mathrm{d}}-l_{\mathrm{q}}}{2}\sin 2\gamma & \frac{l_{\mathrm{d}}+l_{\mathrm{q}}}{2} - \frac{l_{\mathrm{d}}-l_{\mathrm{q}}}{2}\cos 2\gamma \end{pmatrix}.$$

Furthermore subsequent abbreviations

$$l_{\mathrm{m}} = \frac{l_{\mathrm{d}}+l_{\mathrm{q}}}{2}, \quad \mathbf{l}_{\mathrm{m}} = \begin{pmatrix} l_{\mathrm{m}} & 0 \\ 0 & l_{\mathrm{m}} \end{pmatrix}, \tag{34}$$

$$l_{\Delta} = \frac{l_{\mathrm{d}}-l_{\mathrm{q}}}{2}, \quad \mathbf{l}_{\Delta} = \begin{pmatrix} l_{\Delta} & 0 \\ 0 & l_{\Delta} \end{pmatrix}, \tag{35}$$

simplify to

$$\mathbf{l}_{\mathrm{s},\alpha\beta} = \begin{pmatrix} l_{\mathrm{m}} + l_{\Delta}\cos 2\gamma & l_{\Delta}\sin 2\gamma \\ l_{\Delta}\sin 2\gamma & l_{\mathrm{m}} - l_{\Delta}\cos 2\gamma \end{pmatrix}. \tag{36}$$

2) With coupled stator flux components: With the tensor quantity $\mathbf{l}_{\mathrm{s,dq}}$ the components of the stator flux linkage space phasor ψ_{d}, ψ_{q} in the rotor-oriented reference frame and introducing mutual inductances $m = l_{\mathrm{dq}} = l_{\mathrm{qd}}$ according to the rotor-oriented fixed reference frame leads to

$$\psi_{\mathrm{d}} = l_{\mathrm{d}} i_{\mathrm{d}} + m i_{\mathrm{q}} + \psi_{\mathrm{pm}} \tag{37}$$

$$\psi_{\mathrm{q}} = l_{\mathrm{q}} i_{\mathrm{q}} + m i_{\mathrm{d}}. \tag{38}$$

Again with eq. 33 and eq. 18-20 the self- and mutual inductances $l_{\alpha\alpha}$, $l_{\beta\beta}$ and $m_{\alpha\beta}$ in the stator-oriented fixed reference frame (eq. 11) follow with the self inductances l_{d}, l_{q} and the mutual inductance m in the rotor-oriented fixed reference frame to

$$\mathbf{l}_{\mathrm{s},\alpha\beta}(l_{\mathrm{d,q}},m) = \begin{pmatrix} \frac{l_\mathrm{d}+l_\mathrm{q}}{2} + \frac{l_\mathrm{d}-l_\mathrm{q}}{2}\cos 2\gamma - m\sin 2\gamma & \frac{l_\mathrm{d}-l_\mathrm{q}}{2}\sin 2\gamma + m\cos 2\gamma \\ \frac{l_\mathrm{d}-l_\mathrm{q}}{2}\sin 2\gamma + m\cos 2\gamma & \frac{l_\mathrm{d}+l_\mathrm{q}}{2} - \frac{l_\mathrm{d}-l_\mathrm{q}}{2}\cos 2\gamma + m\sin 2\gamma \end{pmatrix} \quad (39)$$

$$= \begin{pmatrix} l_\mathrm{m} + l_\Delta\cos 2\gamma - m\sin 2\gamma & l_\Delta \sin 2\gamma + m\cos 2\gamma \\ l_\Delta \sin 2\gamma + m\cos 2\gamma & l_\mathrm{m} - l_\Delta\cos 2\gamma + m\sin 2\gamma \end{pmatrix} \quad (40)$$

and yields to the components of the tensor quantity $\mathbf{l}_{\mathrm{s},\alpha\beta}$ as stator-oriented self- and mutual inductances

$$l_{\alpha\alpha} = l_\mathrm{m} + l_\Delta\cos 2\gamma - m\sin 2\gamma, \quad (41)$$

$$l_{\beta\beta} = l_\mathrm{m} - l_\Delta\cos 2\gamma + m\sin 2\gamma, \quad (42)$$

$$m_{\alpha\beta} = l_\Delta\sin 2\gamma + m\cos 2\gamma, \quad (43)$$

With comparison of eq. 13 and eq. 41 we get the relation

$$l_{\alpha\alpha} = l_\mathrm{m} + \sqrt{l_\Delta^2 + m^2}\cos(2\gamma+\varepsilon), \quad (44)$$

$$l_{\beta\beta} = l_\mathrm{m} - \sqrt{l_\Delta^2 + m^2}\cos(2\gamma+\varepsilon), \quad (45)$$

$$m_{\alpha\beta} = \sqrt{l_\Delta^2 + m^2}\sin(2\gamma+\varepsilon), \quad (46)$$

with

$$\tan\varepsilon = \frac{m}{l_\Delta}. \quad (47)$$

The self- and mutual phase inductances follow with

$$\mathbf{l}_{\mathrm{s,uvw}}(l_{\alpha\alpha,\beta\beta}, m_{\alpha\beta}) = \mathbf{T}_{\mathrm{uvw}}^{\alpha\beta}\cdot\mathbf{l}_{\mathrm{s},\alpha\beta}(l_{\mathrm{m},\Delta},m)\cdot\mathbf{T}_{\alpha\beta}^{\mathrm{uvw}} \quad (48)$$

$$= \begin{pmatrix} \frac{2}{3}l_{\alpha\alpha} & -\frac{1}{3}l_{\alpha\alpha} + \frac{1}{\sqrt{3}}m_{\alpha\beta} & -\frac{1}{3}l_{\alpha\alpha} - \frac{1}{\sqrt{3}}m_{\alpha\beta} \\ -\frac{1}{3}l_{\alpha\alpha} + \frac{1}{\sqrt{3}}m_{\alpha\beta} & \frac{1}{6}l_{\alpha\alpha} + \frac{1}{2}l_{\beta\beta} - \frac{1}{\sqrt{3}}m_{\alpha\beta} & \frac{1}{6}l_{\alpha\alpha} - \frac{1}{2}l_{\beta\beta} \\ -\frac{1}{3}l_{\alpha\alpha} - \frac{1}{\sqrt{3}}m_{\alpha\beta} & \frac{1}{6}l_{\alpha\alpha} - \frac{1}{2}l_{\beta\beta} & \frac{1}{6}l_{\alpha\alpha} + \frac{1}{2}l_{\beta\beta} + \frac{1}{\sqrt{3}}m_{\alpha\beta} \end{pmatrix}, \quad (49)$$

to a modified approach for the Fourier series expansion of the phase inductances (see eq. 2)

$$l_\mathrm{uu} = l_0 + l_2\cos(2\gamma+\varepsilon), \quad (50)$$

$$= \frac{2}{3}l_\mathrm{m} + \frac{2}{3}\sqrt{l_\Delta^2+m^2}\cos(2\gamma+\varepsilon), \quad (51)$$

$$l_\mathrm{vv} = l_0 + l_2\cos(2\gamma+2\pi/3+\varepsilon), \quad (52)$$

$$= \frac{2}{3}l_\mathrm{m} + \frac{2}{3}\sqrt{l_\Delta^2+m^2}\cos(2\gamma+2\pi/3+\varepsilon), \quad (53)$$

$$l_\mathrm{ww} = l_0 + l_2\cos(2\gamma+4\pi/3+\varepsilon), \quad (54)$$

$$= \frac{2}{3}l_\mathrm{m} + \frac{2}{3}\sqrt{l_\Delta^2+m^2}\cos(2\gamma+4\pi/3+\varepsilon), \quad (55)$$

$$m_\mathrm{wv} = m_\mathrm{vw} = m_0 + m_2\cos(2\gamma+\varepsilon), \quad (56)$$

$$= -\frac{1}{3}l_\mathrm{m} + \frac{2}{3}\sqrt{l_\Delta^2+m^2}\cos(2\gamma+\varepsilon), \quad (57)$$

$$m_\mathrm{uw} = m_\mathrm{wu} = m_0 + m_2\cos(2\gamma+2\pi/3+\varepsilon), \quad (58)$$

$$= -\frac{1}{3}l_\mathrm{m} + \frac{2}{3}\sqrt{l_\Delta^2+m^2}\cos(2\gamma+2\pi/3+\varepsilon) \quad (59)$$

$$m_\mathrm{vu} = m_\mathrm{uv} = m_0 + m_2\cos(2\gamma+4\pi/3+\varepsilon), \quad (60)$$

$$= -\frac{1}{3}l_\mathrm{m} + \frac{2}{3}\sqrt{l_\Delta^2+m^2}\cos(2\gamma+4\pi/3+\varepsilon) \quad (61)$$

D. Stator-Oriented Flux Linkage Space Phasor

The stator flux linkage space phasor $\underline{\psi}_{\mathrm{s},\alpha\beta}$ in the stator-oriented fixed reference frame $\alpha\beta$ follows to

$$\begin{aligned} \underline{\psi}_{\mathrm{s},\alpha\beta} &= \mathbf{T}_{\alpha\beta}^{\mathrm{dq}}(\gamma)\cdot\left[\mathbf{l}_{\mathrm{s,dq}}\cdot\underline{i}_{\mathrm{s,dq}} + \underline{\psi}_{\mathrm{pm,dq}}\right] \\ &= \mathbf{T}_{\alpha\beta}^{\mathrm{dq}}(\gamma)\cdot\mathbf{l}_{\mathrm{s,dq}}\cdot\mathbf{T}_{\mathrm{dq}}^{\alpha\beta}(\gamma)\cdot\underline{i}_{\mathrm{s},\alpha\beta} + \mathbf{T}_{\alpha\beta}^{\mathrm{dq}}(\gamma)\cdot\underline{\psi}_{\mathrm{pm,dq}} \\ &= \mathbf{l}_{\mathrm{s},\alpha\beta}(l_{\mathrm{d,q}},m)\cdot\underline{i}_{\mathrm{s},\alpha\beta} + \mathbf{T}_{\alpha\beta}^{\mathrm{dq}}(\gamma)\cdot\underline{\psi}_{\mathrm{pm,dq}}. \end{aligned}$$

$$\begin{aligned} \begin{pmatrix}\psi_\alpha\\ \psi_\beta\end{pmatrix} &= \begin{pmatrix} l_\mathrm{m}+l_\Delta\cos 2\gamma & l_\Delta\sin 2\gamma \\ l_\Delta\sin 2\gamma & l_\mathrm{m}-l_\Delta\cos 2\gamma\end{pmatrix}\begin{pmatrix}i_\alpha\\ i_\beta\end{pmatrix} \\ &+ \begin{pmatrix} -m\sin 2\gamma & m\cos 2\gamma \\ m\cos 2\gamma & m\sin 2\gamma\end{pmatrix}\begin{pmatrix}i_\alpha\\ i_\beta\end{pmatrix} \\ &+ \begin{pmatrix}\cos\gamma & -\sin\gamma\\ \sin\gamma & \cos\gamma\end{pmatrix}\begin{pmatrix}\psi_\mathrm{pm}\\ 0\end{pmatrix} \\ &= \begin{pmatrix} l_\mathrm{m} & 0\\ 0 & l_\mathrm{m}\end{pmatrix}\begin{pmatrix}i_\alpha\\ i_\beta\end{pmatrix} + \begin{pmatrix} l_\Delta & 0\\ 0 & l_\Delta\end{pmatrix} \\ &\quad \begin{pmatrix}\cos 2\gamma & -\sin 2\gamma\\ \sin 2\gamma & \cos 2\gamma\end{pmatrix}\begin{pmatrix}i_\alpha\\ -i_\beta\end{pmatrix} \\ &+ \begin{pmatrix}0 & m\\ m & 0\end{pmatrix}\begin{pmatrix}\cos 2\gamma & \sin 2\gamma\\ -\sin 2\gamma & \cos 2\gamma\end{pmatrix}\begin{pmatrix}i_\alpha\\ i_\beta\end{pmatrix} \\ &+ \begin{pmatrix}\cos\gamma & -\sin\gamma\\ \sin\gamma & \cos\gamma\end{pmatrix}\begin{pmatrix}\psi_\mathrm{pm}\\ 0\end{pmatrix}. \end{aligned} \quad (62)$$

The expression of the stator-oriented stator flux linkage $\underline{\psi}_{\mathrm{s},\alpha\beta}$ with the *rotor position independent stator-oriented equivalent inductance tensors* $\mathbf{l}_\mathrm{m}$ and $\mathbf{l}_\Delta$ follow to

$$\begin{aligned} \underline{\psi}_{\mathrm{s},\alpha\beta} &= \mathbf{l}_\mathrm{m}\,\underline{i}_{\mathrm{s},\alpha\beta} + \mathbf{l}_\Delta\,\mathbf{T}_{\alpha\beta}^{\mathrm{dq}}(2\gamma)\,\underline{i}_{\alpha\beta}^{*} + \mathbf{m}\,\mathbf{T}_{\mathrm{dq}}^{\alpha\beta}(2\gamma)\,\underline{i}_{\mathrm{s},\alpha\beta} \\ &\quad + \mathbf{T}_{\alpha\beta}^{\mathrm{dq}}(\gamma)\,\underline{\psi}_{\mathrm{pm,dq}} \quad (63) \\ &= \left[\mathbf{l}_\mathrm{m} + (\mathbf{l}_\Delta^{*}+\mathbf{m})\cdot\mathbf{T}_{\mathrm{dq}}^{\alpha\beta}(2\gamma)\right]\cdot\underline{i}_{\mathrm{s},\alpha\beta} + \underline{\psi}_{\mathrm{pm},\alpha\beta}. \end{aligned}$$

with

$$\mathbf{m} = \begin{pmatrix}0 & m\\ m & 0\end{pmatrix}. \quad (64)$$

The expression of the stator-oriented stator flux linkage space phasor $\underline{\psi}_{\mathrm{s},\alpha\beta}$ with the *rotor position dependent stator-oriented equivalent inductance tensor* $\mathbf{l}_{\mathrm{s},\alpha\beta}(2\gamma)$ follows to

$$\underline{\psi}_{\mathrm{s},\alpha\beta} = \mathbf{l}_{\mathrm{s},\alpha\beta}(2\gamma)\cdot\underline{i}_{\mathrm{s},\alpha\beta} + \underline{\psi}_{\mathrm{pm},\alpha\beta}, \quad (65)$$

and yields to the coherence of the rotor position dependent inductance tensor $\mathbf{l}_{\mathrm{s},\alpha\beta}$ and the rotor position independent inductance tensors $\mathbf{l}_\mathrm{m}$ and $\mathbf{l}_\Delta$ with

$$\mathbf{l}_{\mathrm{s},\alpha\beta} = \mathbf{l}_\mathrm{m} + (\mathbf{l}_\Delta^{*}+\mathbf{m})\cdot\mathbf{T}_{\mathrm{dq}}^{\alpha\beta}(2\gamma). \quad (66)$$

III. Simulation and Measurement Results

Fig. 1 depicts the complex solution of the stator-oriented flux linkage space phasor divided by the stator-oriented current space phasor. Fig. 2 depicts the phase shift ε, difference

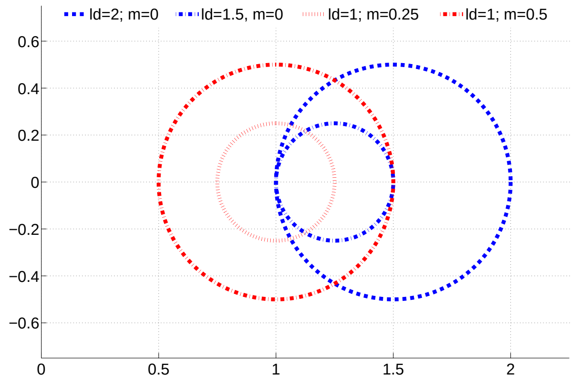

Fig. 1. Simulation of $\Im\left\{\underline{\psi}_{\mathrm{s},\alpha\beta}/\underline{i}_{\mathrm{s},\alpha\beta}\right\}$ ov. $\Re\left\{\underline{\psi}_{\mathrm{s},\alpha\beta}/\underline{i}_{\mathrm{s},\alpha\beta}\right\}$ with $\psi_{\mathrm{pm}} = 0$, $l_{\mathrm{q}} = 1$ and sinusoidal current supply at various direct l_{d}, quadrature l_{q} and mutual m inductances

between measured and sensorless calculated angular rotor position, over the magnitude of the quadrature current space phasor of the PMSM according to fig. 3. The sensorless calculation of the angular rotor position was done with the so called INFORM method which is based on the second order element of the Fourier series expansion.

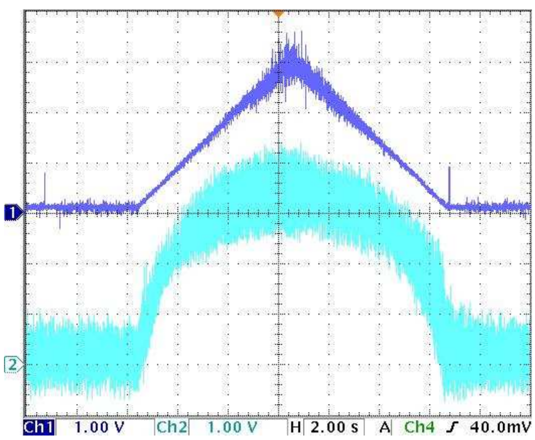

Fig. 2. Angular phase shift ε over the magnitude of the quadrature current load with i_{q} at CH1: 1 p.u./div. and ε at CH2: 22.5 deg./div.

IV. Discussion

Starting with the measureable self- and mutual phase inductances l_{uu}, l_{vv}, l_{ww}, m_{uv}, m_{vw} and m_{wu} we firstly introduce the ideas of [2] with a representation by their Fourier series expansion in dependence of the angular rotor position. Next

Fig. 3. Picture of the used 400 Nm PMSM for measuring ε

step was the well known transformation to the fictive self- and mutual stator-oriented inductances $l_{\alpha\alpha}$, $l_{\beta\beta}$ and $m_{\alpha\beta}$ with neglected self- and mutual zero sequence inductances. Third step was introducing the self inductances in direct and quadrature direction at rotor-oriented reference frame, with the solution of non existing mutual inductances in the rotor-oriented reference frame due to the given approach of the Fourier series expansion up to second order elements. The second order coefficients also show the dependency of the angular rotor position. Next important step is introducing mutual inductances in the rotor-oriented reference frame due to saturation effects in the stator and/or rotor. With this approach we obtain to the same coefficients of the Fourier series expansion but with an additional angular phase shift ε. That means that the sinusoidal spatial distribution of a given phase inductance is shifted by a phase angle ε due to saturation effects. We also get an angular rotor position dependency due to the these effects.

V. Conclusion

The paper discusses an analysis of the rotor-oriented reference frame mutual inductances, which results in coupled stator flux components, with an additional phase shift according to the Fourier series expansion of given self- and mutual phase inductances.

References

[1] M. Schrödl, *Sensorless Control of A.C. Machines*, VDI-Verlag, VII, Fortschritt-Berichte VDI : Reihe 21 Elektrotechnik, Nr. 117, ISBN 3-18-141721-1, January 1992

[2] S. I. Nabeta, *Etude des Regimes Transitoires des Machines Synchrones par la Methode des Elements Finis*, Doctoral thesis (in French), INP Grenoble, 1994

[3] E. Schmidt, *Finite Element Analysis of Electrical Machines, Transformers and Electromagnetic Actuators*, postdoctoral lecture qualification, Vienna University of Technology, 2007.

[4] R.H. Park, *Two-Reaction Theory of Synchronous Machines Generalized Method of Analysis - Part I*, Transactions of the American Institute of Electrical Engineers (AIEE), New York, N.Y., July 1929, Vol. 48, pp. 716-727

[5] J. Štěpina, *Raumzeiger als Grundlage der Theorie der elektrischen Maschinen*, ETZ-A, 1967, Bd.88, pp. 584-588