A Uniformly Valid Theory of Turbulent Separation

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Abstract. The contribution deals with recent theoretical results concerning separation of a turbulent boundary layer from a blunt solid object in a uniform stream, accompanied by a numerical study. The investigation is restricted to incompressible nominally steady two-dimensional flow past an impervious obstacle surface. Then the global Reynolds number represents the only parameter entering the description of the Reynolds-averaged flow. It shall be large enough to ensure that laminar–turbulent transition takes place in a correspondingly small region encompassing the stagnation point. Consequently, the concomitant asymptotic hierarchy starts with the external Helmholtz–Kirchhoff potential flow, which detaches at an initially unknown point $\mathscr{D}$ from the body, driving the turbulent boundary layer. It is found that the separation mechanism is inherently reminiscent of the transition process. The local analysis of separation not only fixes the actual scaling of the entire boundary layer but is also expected to eventually predict the position of $\mathscr{D}$ in a rational way.

1 Non-interactive Global and Boundary Layer Flow

Consider a parallel flow just disturbed by the presence of a bluff body and let the Reynolds number *Re* formed with the unperturbed free-stream velocity and a

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typical body dimension take on arbitrarily large values: what is the actual position of flow separation? This question represents a long-standing but nevertheless central challenge in theoretical hydrodynamics. Matched asymptotic expansions prove the adequate means for a rigorous treatment of this problem. Here we present the essential results of this analysis by starting with the overall flow structure and leaving the technical details to [6] and [7]. In the following all quantities are non-dimensional with the aforementioned reference values and the fluid density. Hence x, y, u denote natural coordinates along and normal to the body surface, with $x = y = 0$ indicating the stagnation point $\mathscr{S}$, and the velocity component in x-direction, respectively.

Two findings are decisive. Firstly, laminar–turbulent transition near $\mathscr{S}$ generates a boundary layer that is basically characterised by (i) a main layer exhibiting an asymptotically small relative streamwise velocity deficit and (ii) the classical shear stress equilibrium in the viscous wall layer. Together with the required direct match of both flow regions items (i) and (ii) establish the picture of a classical turbulent boundary layer but with an a priori unknown scaling. This reflects the remarkable property that it never attains a fully developed turbulent state, even for arbitrarily large values of Re. Such an "underdeveloped" turbulent boundary layer contrasts with common reasoning but is reliable as viscous–inviscid flow interaction near separation requires the wall layer thickness to vary predominantly algebraically with the boundary layer thickness δ (rather than exponentially as in the classical case).

Secondly, it has become evident from [3] and numerous subsequent related experimental studies that for increasing values of Re the point of gross separation approaches a position a finite distance remote from its rear stagnation point observed in case of fully attached potential flow. This contradicts the conclusion drawn in the first theoretical investigation of this problem that accounts for the global flow structure [2]. A critical review [5] has finally promoted the experimental observation as a starting point for further analysis. This in turn suggests that in the formal limit $Re^{-1} = 0$ the free-stream flow past the body is to be sought in the class of Helmholtz–Kirchhoff flows, parametrised solely by the position of $\mathscr{D}$ where the well-known Brillouin–Villat (BV) singularity is encountered. Then the surface velocity u_s imposed on the attached boundary layer and the stream function F that accounts for the velocity defect satisfy the fundamental asymptotic relationships

$$u_s(x;k) \sim 1 + 2k\sqrt{-s} + O(-s), \qquad s = x - x_{\mathscr{D}}(k), \qquad s \to 0_-, \tag{1}$$

$$1 - u/u_s(x;k) \sim \varepsilon \partial_\eta F(x,\eta;k) + O(\varepsilon^2), \quad \eta = y/\delta(x;Re), \quad \varepsilon \to 0. \tag{2}$$

Here the positive parameter k measures the strength of the BV singularity and in turn the position $x_{\mathscr{D}}$ of $\mathscr{D}$. Determining both the correct value of k and the dependencies of δ and the defect measure ε on Re represents one crucial goal of this research. Notable previous works (e.g. [1], [11]) unfortunately lacked the discussion of the global flow and thus led to different (large-defect) structures of the boundary layer, where the resulting inconsistencies precluded a uniformly valid flow description.

The canonical situation of the flow around the circular unit cylinder (Re formed with the cylinder radius) is tackled numerically. We first approximate the potential flow by choosing $k = 0.45$ or, by exploitation of Levi–Cività's method,

$x_{\mathscr{D}} \doteq 113.5°$. This seems reliable in view of the experimentally observed separation angles for the largest values of Re available so far ($Re \approx 10^6 - 10^7$) and the basic assumption concerning the asymptotic flow state. Integration of the leading-order small-defect equations subject to $u_s(x;k)$ and supplemented with an (asymptotically consistent) algebraic mixing-length closure starting at stagnation yields the results displayed in Fig. 1. An initially favourable and then adverse pressure gradient (APG) acts on the boundary layer. In contrast to usual reasoning (cf. [8]), it is not in "quasi-equilibrium" as the variation of F with x is fully present. The profiles of $\partial_\eta F$ at various x-stations exhibit the logarithmic near-wall behaviour and admit a wake-type shape near $x = x_{\mathscr{D}}$, well-recognised in other APG turbulent boundary layers (cf. [1], [2], [4]). Furthermore, let $\Delta(x;k)$ denote the appropriately scaled boundary layer thickness. The expansion $[F,\Delta] \sim [(1-4k\sqrt{-s})F_{\mathscr{D}}(\eta),(1-2k\sqrt{-s})\Delta_{\mathscr{D}}] + O(-s)$, $s \to 0_-$, derived from (1) and (2) is revealed by the numerical data. However, due to the associated splitting of $\partial_\eta F$ in an infinite series of logarithmic velocity portions as $\eta \to 0$ a direct match of the small-defect and the wall layer is only accomplished by the introduction of a so-called intermediate layer immediately upstream of $\mathscr{D}$.

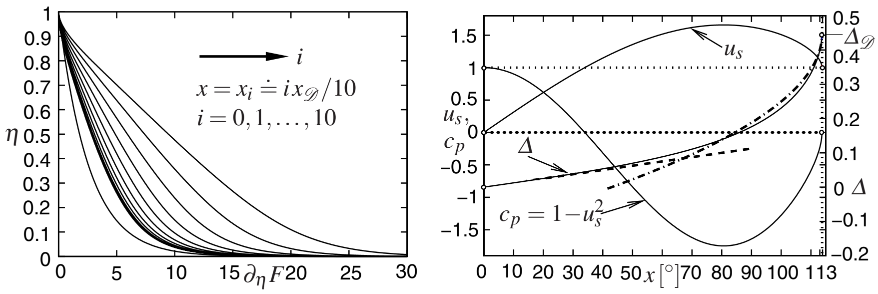

Fig. 1 Defect function $\partial_\eta F$ and key quantities, asymptotes of $\Delta(x;k)$: linear rise in $x = 0$ (*dashed*), square-root behaviour near $x = x_{\mathscr{D}}$ (*dashed-and-dotted*)

Detached Eddy Simulation (DES) underlies the snapshot of instantaneous circular-cylinder flow in Fig. 2 (a). Here $Re = 10^6$ and $x_{\mathscr{D}} \doteq 95°$, which is also promising.

2 "Inner" and "Outer" Flow Interaction

The separation process is vitally governed by the interplay of two locally strong interaction mechanisms, see Fig. 2 (b): an "inner" (viscous–inviscid) one associated with a novel triple-deck structure, and an "outer" (rotational–irrotational) one expressed by a linear problem describing an inviscid vortex flow. The latter is subject to a solvability condition that arises from matching both interactive flow regimes. This is currently expected to finally fix the actual value of k for a given body shape.

In the classical description of the viscous wall layer the shear stress equilibrium is followed by the influence of the pressure and finally the inertia terms on the momentum equation in x-direction. The BV singularity triggers a rather complex multi-stage breakdown of this asymptotic hierarchy a short distance upstream of $\mathscr{D}$, initiated by the emergence of the intermediate layer mentioned above. This eventually leads to the formation of a so-called lower deck adjacent to the surface of streamwise extent δ_{TD}, say, where all those contributions except for the turbulent shear stress are in operation at leading order. Hence, separation takes place within a distance of $O(\delta_{\mathrm{TD}})$ from $\mathscr{D}$ as a self-consistent flow description in terms of viscous–inviscid interaction aims at avoiding the occurrence of both the well-known Goldstein and the BV singularity. This mechanism intrinsically renders the wall layer locally a passive main deck: it transfers the flow displacement exerted by the lower deck to an upper deck formed on its top (together with a passive buffer layer). Here the induced potential flow accounts for the pressure feedback in the lower deck. Herewith the interaction loop is closed and the aforementioned triple-deck structure completed. As a result, the BV singularity is resolved in a manner formally identical to that found in the asymptotic theory of laminar break-away separation [10], [9].

This situation is virtually unaffected by the outer small-defect portion of the boundary layer. On the other hand, expansion (2), applicable to the oncoming boundary layer sufficiently far upstream of $\mathscr{D}$, subject to (1) ceases to be valid in the square region of outer interaction where both s and y are of $O(\delta)$ and which encloses the triple deck. Finally, the triple-deck structure requires that $\delta_{\mathrm{TD}}/\delta = O(\varepsilon)$, with $\varepsilon \propto 1/\ln Re$ (which agrees with the classical boundary layer scaling) and $\delta_{\mathrm{TD}} = Re^{-4/9}$ (which specifies the notion of underdeveloped flow). Activities of current and future research include the aforementioned determination of k and the description of the associated body- and large-scale separated flow.

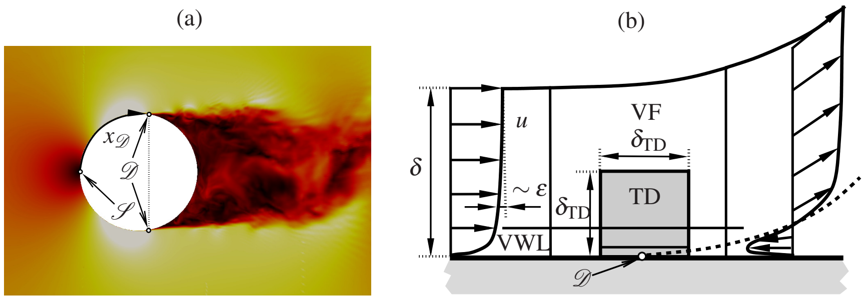

Fig. 2 (a) DES: colours (greytones) distinguish isotachs; (b) sketch of separating flow: viscous wall layer (VWL), triple deck (TD, *shaded*), vortex-flow region (VF), separating streamline (*dashed*)

References

1. Melnik, R.E.: Comput. Fluids 17(1)
2. Neish, A., Smith, F.T.: J. Fluid Mech. 241
3. Roshko, A.: J. Fluid Mech. 10(3)

4. Scheichl, B.: Asymptotic Methods in Fluid Mechanics: Survey and Recent Advances. In: Steinrück, H. (ed.) CISM Courses and Lectures, vol. 523, pp. 221–246. Springer, New York (2010)
5. Scheichl, B., Kluwick, A.: J. Fluids Struct. 24(8)
6. Scheichl, B., Kluwick, A., Alletto, M.: Acta Mech. 201(1-4)
7. Scheichl, B., Kluwick, A., Smith, F.T.: J. Fluid Mech. 670 (2011)
8. Schlichting, H., Gersten, K.: Boundary-layer Theory, 8th edn. Springer, New York (2003)
9. Smith, F.T.: Proc. R. Soc. Lond. A 356 (1687)
10. Sychev, V.V.: Fluid Dyn. 7(3)
11. Sychev, V.V.: Fluid Dyn. 22(3)