

# Sudden Short-Circuit Analysis of a Salient Permanent Magnet Synchronous Machine with Buried Magnets for Traction Applications

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**Abstract**—This paper discusses the three-phase sudden short-circuit in a rotor-oriented reference frame and the involved torque of a salient permanent magnet excited synchronous machine with buried magnets (PMSM). The paper will demonstrate the influence of machine parameters like the stator resistance, the direct and quadrature inductance on the resulting currents and torque at short-circuit conditions. It will be shown that the produced drag torque as well as the drag losses could be minimized by varying the machine parameters. The results of the analytical calculation of the non-saturated PMSM are compared with measurements.

## I. INTRODUCTION

Permanent magnet excited synchronous machines with buried magnets becomes more and more standard in the field of traction applications. Such a machine combines high efficiency as well as field weakening capability [1]. However some disadvantages like drag losses or inverter problems at winding errors have to be taken into consideration. The produced torque impulse as well as the drag torque at sudden short-circuit conditions will be discussed and quantified with three-phase sudden short-circuit measurements and also with analytical calculations under non-saturated (direct and quadrature inductances are independent from actual stator current) assumption. The presented PMSM is able to allow short-circuit condition during a faulty traction drive operation without any damaging. So presented machine is short-circuit proofed and induced currents are small enough to fulfill the assumption of a non-saturated machine.

## II. THREE-PHASE SHORT-CIRCUIT CURRENT

Publications like [2]-[4] handle with the transient sudden short-circuit current under various condition. Following derivation firstly discusses the steady-state three-phase short-circuit current and furthermore comes to the mathematical derivation of the transient three-phase sudden short-circuit current without and with initial current conditions. So it is possible to simulate short-circuit conditions during faulty traction drive operation.

### A. Steady-State Condition

In the subsequent derivation for determining the operational behaviour of the three-phase short-circuit, steady-state operation is supposed. All data are given in normalized values.

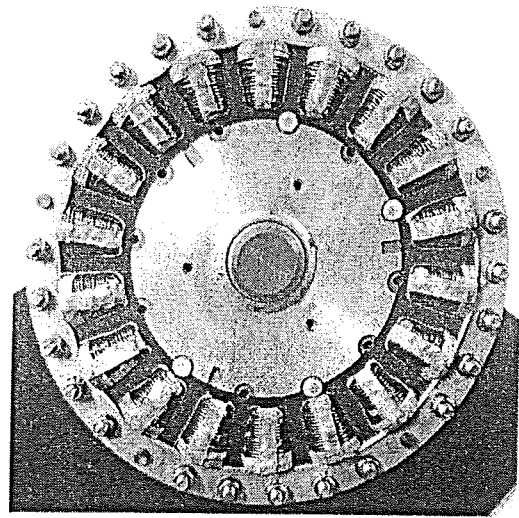


Fig. 1. PMSM with buried magnets - object of investigation

In the  $\alpha\beta$  stator-oriented reference frame the complex stator voltage equation

$$\underline{u}_{\alpha\beta} = r \underline{i}_{\alpha\beta} + \frac{d\psi_{s,\alpha\beta}}{dt}, \quad (1)$$

with the complex stator flux linkage  $\underline{\psi}_s$  and the electrical angular rotor position  $\gamma$

$$\underline{\psi}_{s,\alpha\beta} = \underline{\psi}_{s,dq} e^{j\gamma}, \quad \gamma = \omega t + \gamma_0 \quad (2)$$

follows to

$$\underline{u}_{\alpha\beta} = r \underline{i}_{\alpha\beta} + \frac{d\psi_{s,dq}}{dt} e^{j\gamma} + j\omega \underline{\psi}_{s,dq} e^{j\gamma}. \quad (3)$$

In the  $dq$  rotor oriented reference frame the components of the stator flux linkage are defined by

$$\psi_{s,d} = l_d i_d + \psi_{pm}, \quad (4)$$

$$\psi_{s,q} = l_q i_q, \quad (5)$$

where  $l_d, l_q$  are the inductances in direct and quadrature axis and  $\psi_{pm}$  denotes the flux linkage of the permanent magnets. With the steady-state assumption of a constant stator flux

linkage  $\dot{\psi}_{s,dq} = 0$  the stator voltage equation in the rotor oriented reference frame follows to

$$u_{dq} = r i_{dq} + j\omega \psi_{s,dq}. \quad (6)$$

The components of above equation are

$$u_d = r i_d - \omega l_q i_q, \quad (7)$$

$$u_q = r i_q + \omega l_d i_d + \omega \psi_{pm}. \quad (8)$$

With  $u_d, u_q$  equal to zero the direct and quadrature short-circuit currents  $i_d$  and  $i_q$  can be easily calculated to

$$i_d = \frac{\omega^2 \psi_{pm} l_q}{r^2 + \omega^2 l_d l_q}, \quad (9)$$

$$i_q = \frac{\omega \psi_{pm} r}{r^2 + \omega^2 l_d l_q}. \quad (10)$$

### B. Transient Condition

1) *Without initial currents:* For determining the three-phase sudden short-circuit currents we have to consider the time derivation of the stator flux linkage  $\dot{\psi}_{s,dq} \neq 0$ . Therefore eq. 7 and eq. 8 change to

$$u_d = r i_d - \omega l_q i_q + l_d \dot{i}_d. \quad (11)$$

$$u_q = r i_q + \omega l_d i_d + \omega \psi_{pm} + l_q \dot{i}_q. \quad (12)$$

With the Laplace transformation, the initial currents  $i_d(0_-)$ ,  $i_q(0_-)$  equal to zero, the short-circuit condition ( $u_d = u_q = 0$ ) and the Laplace variable  $s$ , the above two differential equations follow in matrix notation to

$$\begin{pmatrix} r + s l_d & -\omega l_q \\ \omega l_d & r + s l_q \end{pmatrix} \begin{pmatrix} i_d(s) \\ i_q(s) \end{pmatrix} = \begin{pmatrix} 0 \\ -\frac{1}{s} \omega \psi_{pm} \end{pmatrix},$$

which yields, with calculation of the inverse matrix, to the sudden short-circuit currents

$$\begin{pmatrix} i_d(s) \\ i_q(s) \end{pmatrix} = \frac{\frac{1}{l_d l_q}}{s^2 + s \left( \frac{r}{l_d} + \frac{r}{l_q} \right) + \frac{r^2}{l_d l_q} + \omega^2} \cdot \begin{pmatrix} r + s l_q & \omega l_q \\ -\omega l_d & r + s l_d \end{pmatrix} \begin{pmatrix} 0 \\ -\frac{1}{s} \omega \psi_{pm} \end{pmatrix}$$

in direct axis

$$i_d(s) = \frac{-\frac{1}{l_d} \omega^2 \psi_{pm}}{s(s-s_1)(s-s_2)} \quad (13)$$

and quadrature axis

$$i_q(s) = \frac{-\frac{1}{l_q} \left( \frac{r}{l_d} + s \right) \omega \psi_{pm}}{s(s-s_1)(s-s_2)}. \quad (14)$$

with the roots

$$s_{1,2} = \sigma \pm j\nu = -\frac{r}{2} \left( \frac{1}{l_d} + \frac{1}{l_q} \right) \pm \sqrt{\frac{r^2}{4} \left( \frac{1}{l_d} - \frac{1}{l_q} \right)^2 - \omega^2}$$

In most cases the term under the root gets negative which results in conjugate complex roots. Especially at non-salient

assumption  $l_d = l_q = l$  only conjugate complex roots are existing. The partial fraction decomposition with

$$(s-s_1)(s-s_2) = (s-\sigma)^2 + \nu^2$$

and introducing

$$i_d(s) = \frac{A}{s} + \frac{Bs+C}{(s-\sigma)^2 + \nu^2},$$

$$i_q(s) = \frac{X}{s} + \frac{Ys+Z}{(s-\sigma)^2 + \nu^2},$$

yields to following coefficients  $A, B, C, X, Y$  and  $Z$

$$A = -\frac{\omega^2 \psi_{pm} l_q}{r^2 + \omega^2 l_d l_q}, B = -A, C = 2A\sigma \quad (15)$$

$$X = -\frac{\omega \psi_{pm} r}{r^2 + \omega^2 l_d l_q}, Y = -X, Z = -\frac{\omega \psi_{pm}}{l_q} + 2X\sigma \quad (16)$$

and generates with following laplace transformations

$$\frac{s}{(s-\sigma)^2 + \nu^2} \Leftrightarrow e^{\sigma t} \left( \cos(\nu t) + \frac{\sigma}{\nu} \sin(\nu t) \right) \quad (17)$$

$$\frac{1}{(s-\sigma)^2 + \nu^2} \Leftrightarrow \frac{1}{\nu} e^{\sigma t} \sin(\nu t) \quad (18)$$

the solution in the time domain with

$$i_d(\tau) = A e(\tau) \left[ 1 - e^{\sigma \tau} \left( \cos(\nu \tau) - \frac{\sigma}{\nu} \sin(\nu \tau) \right) \right] \quad (19)$$

$$i_q(\tau) = X e(\tau) \left[ 1 - e^{\sigma \tau} \left( \cos(\nu \tau) + \frac{\sigma}{\nu} \sin(\nu \tau) - \frac{Z}{\nu X} \sin(\nu \tau) \right) \right]. \quad (20)$$

2) *With initial currents:* With the initial currents  $i_d(0_-)$  and  $i_q(0_-)$  the direct current (eq. 13) follows to

$$i_d(s) = \frac{-\frac{1}{l_d} \omega^2 \psi_{pm}}{s(s-s_1)(s-s_2)} + \frac{-\frac{1}{l_d} \left( \frac{r}{l_q} + s \right) i_d(0_-)}{(s-s_1)(s-s_2)} + \frac{\frac{\omega}{l_d} i_q(0_-)}{(s-s_1)(s-s_2)}. \quad (21)$$

The quadrature current (eq. 14) follows to

$$i_q(s) = \frac{-\frac{1}{l_q} \left( \frac{r}{l_d} + s \right) \omega \psi_{pm}}{s(s-s_1)(s-s_2)} + \frac{-\frac{1}{l_q} \left( \frac{r}{l_d} + s \right) i_q(0_-)}{(s-s_1)(s-s_2)} - \frac{\frac{\omega}{l_q} i_d(0_-)}{(s-s_1)(s-s_2)}. \quad (22)$$

Again introducing

$$i_d(s) = \frac{A}{s} + \frac{Bs+C}{(s-\sigma)^2 + \nu^2} + \frac{Ds+E}{(s-\sigma)^2 + \nu^2},$$

$$i_q(s) = \frac{X}{s} + \frac{Ys+Z}{(s-\sigma)^2 + \nu^2} + \frac{Vs+W}{(s-\sigma)^2 + \nu^2},$$

yields to the additional coefficients  $D, E, V$  and  $W$

$$D = \frac{1}{l_d} i_d(0_-), E = \frac{r}{l_d l_q} i_d(0_-) + \frac{\omega}{l_d} i_q(0_-). \quad (23)$$

$$V = \frac{1}{l_q} i_q(0_-), W = \frac{r}{l_d l_q} i_q(0_-) - \frac{\omega}{l_q} i_d(0_-) \quad (24)$$

and generates with above laplace transformations (eq. 17-18) the solutions in the time domain with initial currents  $i_d(0_-)$ ,  $i_q(0_-)$

$$i_d(\tau) = A\epsilon(\tau) \left[ 1 - e^{\sigma\tau} \left( \cos(\nu\tau) \left( 1 - \frac{D}{A} \right) - \sin(\nu\tau) \left( \frac{E + D\sigma}{A\nu} + \frac{\sigma}{\nu} \right) \right) \right] \quad (25)$$

$$i_q(\tau) = X\epsilon(\tau) \left[ 1 - e^{\sigma\tau} \left( \cos(\nu\tau) \left( 1 - \frac{V}{X} \right) - \sin(\nu\tau) \left( \frac{Z + W + V\sigma}{X\nu} - \frac{\sigma}{\nu} \right) \right) \right] \quad (26)$$

### III. SIMULATION AND MEASUREMENT RESULTS

Fig. 2 shows the test stand with a torque gauge bar between load-machine and used/measured PMSM with buried magnets. Fig. 3 shows the normalized magnitudes of the measured

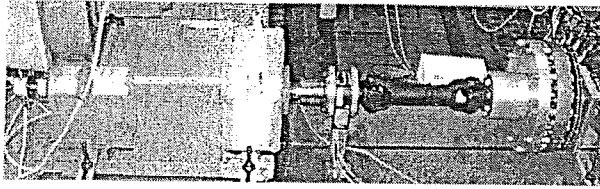


Fig. 2. Teststand with a dc load-machine, a torque measuring shaft or gauge bar and the used PMSM with buried magnets

three-phase steady-state short-circuit currents in direct and quadrature axis, the normalized mechanical rotor speed and normalized short-circuit torque. During measurements the dc load-machine accelerates the mechanical rotor speed slowly enough to fulfill steady-state conditions. Fig. 4 pictures the

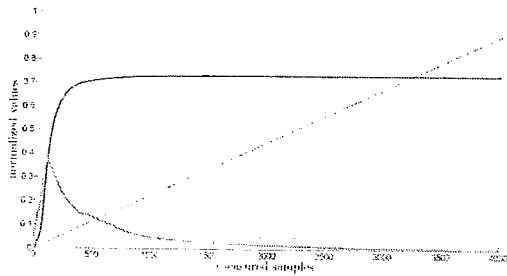


Fig. 3. Normalized magnitude of three-phase steady-state short-circuit currents in direct axis (blue line), quadrature axis (red line), rotor speed (black line) and shaft-torque (green line)

normalized values of the simulated steady-state short-circuit currents in direct and quadrature axis according to eq. 9-10 and torque production (drag torque) according to eq. 27 with a constant rotor flux linkage  $\psi_{pm}$

$$l_i = \psi_{pm} i_q + (l_d - l_q) i_d i_q \quad (27)$$

at two different normalized stator resistances. Fig. 5 compares the magnitude of normalized measured and simulated three-phase sudden short-circuit current space phasor (red lines) with

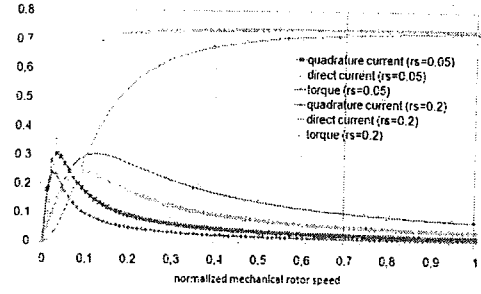


Fig. 4. Normalized magnitude of three-phase steady-state short-circuit currents in direct axis (blue line), quadrature axis (red line), rotor speed (black line) and shaft-torque (green line)

appropriate normalized mechanical rotor speed (blue lines). The simulation was done with a normalized direct inductance of 1.35 and a quadrature inductance of 2. Although the quadrature inductance was calculated with eq. 7 by measuring the applied voltages and belonging rotor oriented currents via a precision power meter. The direct inductance was calculated with eq. 9 by measuring the steady-state short-circuit current by a constant flux linkage due to the permanent magnets at high rotor speed.

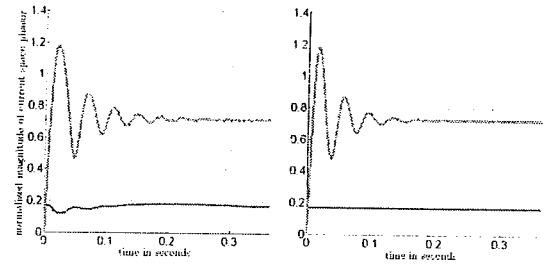


Fig. 5. Normalized magnitude of measured (left) and simulated (right) three-phase sudden short-circuit current space phasor with  $r = 0.05$ ,  $l_d = 1.35$  and  $l_q = 2$  and appropriate normalized rotor speed  $\omega \approx 0.17$  (blue lines)

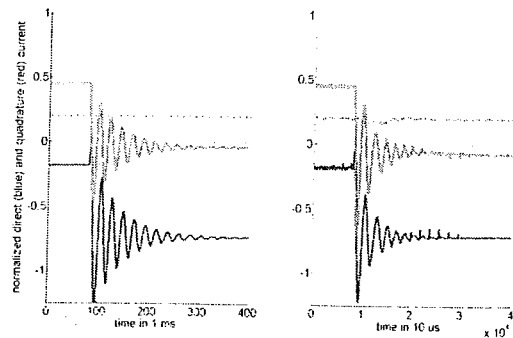


Fig. 6. Simulated (left) and measured (right) normalized three-phase sudden short-circuit current space phasors in direct- (blue line) and quadrature-axis (red line) at a normalized rotor speed (black line) of  $\omega = 0.2$ ,  $r = 0.05$ ,  $l_d = 1.35$  and  $l_q = 2$  with  $i_d(0_-) = -0.18$  and  $i_q(0_-) = 0.45$

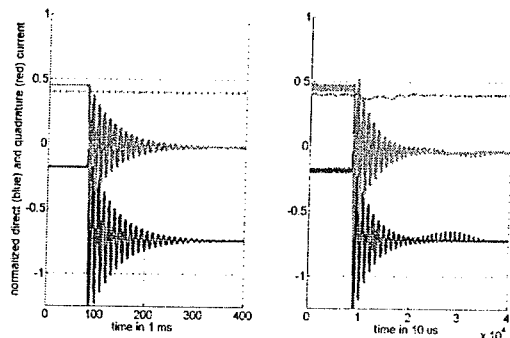


Fig. 7. Simulated (left) and measured (right) normalized three-phase sudden short-circuit current space phasors in direct- (blue line) and quadrature-axis (red line) at a normalized rotor speed (black line) of  $\omega = 0.4$ ,  $r = 0.05$ ,  $l_d = 1.35$  and  $l_q = 2$  with  $i_d(0_-) = -0.18$  and  $i_q(0_-) = 0.45$

#### IV. DISCUSSION OF RESULTS

According to the simulation results in fig. 4 we can see an important dependency between the normalized stator resistance and the produced drag losses over rotor speed. The higher the stator resistance the higher the drag losses over rotor speed. Furthermore, the normalized stator inductance in direct axis determines the short-circuit characteristic of the used/simulated traction PMSM. The comparison between the measured and simulated three-phase short-circuit stator current in fig. 5 shows a good confirmation of the simulation although the rotor speed is not constant at test conditions, because of the dynamic behavior of the dc load-machine. Normally, this imperfection can be neglected. Furthermore fig. 6 and fig. 7 compare the simulated and measured sudden three-phase short-circuit currents in direct and quadrature axis at different rotor speeds and same initial current values. Because of neglected transient inductances the amplitude of the first current impulse is less than the measured value. To improve this analytical calculation of the sudden short-circuit current it is necessary to implement transient inductances in direct and quadrature axis. For estimation of the sudden short-circuit conditions, the presented theoretical calculations are sufficient in most cases.

#### V. CONCLUDING REMARKS

This paper shows the theoretical calculation of the three-phase sudden short-circuit current in direct and quadrature direction with consideration of the direct and quadrature steady-state inductances during constant rotor flux linkage due to the permanent magnets. The simulated results are compared with measurements on a PMSM with buried magnets for traction applications. It is also shown that for improved simulation results transient inductances have to be implemented.

#### VI. DATA OF THE REALIZED DRIVE

description	data	unit
layer configuration	single-layer	
wiring of coils	$\Delta$ -6s1p	
number of pole pairs/slots	15/36	1
reference/rated current	56.6/40	A peak/A rms
reference/rated voltage	325/340	V peak/V rms
reference/rated speed	609/580	rpm
reference (apparent) power	27.6	kVA
reference torque	433	Nm
rated (active) power	16.5	kW
efficiency (at rated power)	92	%
normalized resistance (at 120°C)	0.05	1
max. cont. torque (at 160°C)	346	Nm

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