# Finite Element Analysis of Electrical Machines and Transformers – State of the Art and Future Trends

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*Abstract* – **The paper discusses state of the art of finite element analyses of electrical machines and transformers. In particular, various optimized methods of modelling and analysis concerning the repetitive structure of electrical machines for electromagnetic analyses are compared with their advantages and drawbacks. Further, various methods of coupling the different domains of multi-field analyses are described. Some examples of coupled analyses in the mentioned sector are presented finally.**

*Index Terms* – **Electrical machines, Transformers, Multi-field analysis, Finite element analysis**

## I. Introduction

The design process of an electrical machine can be commenced by defining certain basic characteristics such as machine type, type of construction, number of poles, number of phases, rated as well as overload operational data and intended duty cycles [1]. Most additional data cover efficiency, thermal, protection and enclosure classes, design standards and last but not least manufacturability.

The basic design of electrical machines and transformers, in particular the dimensioning of magnetic and electric circuits, is mostly carried out by applying analytical calculations. The permitted loading levels are defined on the basis of the design of the insulation and the cooling of the machine or transformer. Typically, the initial dimensions of these devices are first selected by including predetermined geometrical, electrical, thermal as well as mechanical constraints. Next, the devices are designed electrically in order to cover the operational parameters. Afterwards the cooling and finally the mechanical construction of these devices are calculated. Nowadays, in particular with transformers as well as in more and more cases of electrical machines, acoustic investigations are performed additionally.

If one or more of the obtained parameters in this first design step are not sufficient enough, the design will start from the beginning again by typically increasing the dimensions of the machine, by using better materials or by selecting a more efficient cooling method. With this classical approach for a design of electrical machines and transformers, the interaction between the various physical domains are only included with simplifications or even neglected.

However, an accurate operational performance with different electrical and mechanical load situations is more and more evaluated using various numerical analysis methods. With these numerical methods, in particular tests which are not feasible in laboratory circumstances can be virtually performed. On the other hand, the couplings between the various physical domains can now be considered in an appropriate way [2]–[4]. Therefore, numerical analysis methods are increasingly utilized not only for the verification of contractual values of existing machines, but also for the initial design process and for the design optimization of new machines.

## II. Electromagnetic Analyses

In most cases, electrical machines possess a higher number of poles in circumferential direction. In accordance with the winding arrangement, there is a repetitive section of the machine which fully represents the entire machine. By using periodical boundary conditions for the utilized electromagnetic potentials, the overall size of the finite model can be reduced to cover only the complete or even the half repetitive section [5].

On the other hand, with the electromagnetic analyses of various positions of stator and rotor, there are mostly invariant geometries of all parts. In such cases, the separation of the entire model into distinct stator and rotor parts significantly influences both modelling and solving efforts [6]–[8]. For the coupling of the model parts with various rotor positions, there are three basic approaches as moving band, locked-step or interpolated sliding surface and a semi-analytical airgap macro technique as a special application of finite and boundary element coupling [2], [5], [24].

### A. Domain decomposition

The construction of fully independent stator and rotor model parts denoted as $\Omega_{st}$ and $\Omega_{rt}$ allows for a separate setup of the governing finite element equations,

$$\mathbf{K}_m \, \mathbf{U}_m + \mathbf{G}_m = \mathbf{P}_m \ , \quad m = \{st, rt\} \ . \qquad (1)$$

The elimination of the degrees of freedom inside both subdomains $\Omega_{st}$ and $\Omega_{rt}$ as well as the subsequent pro-

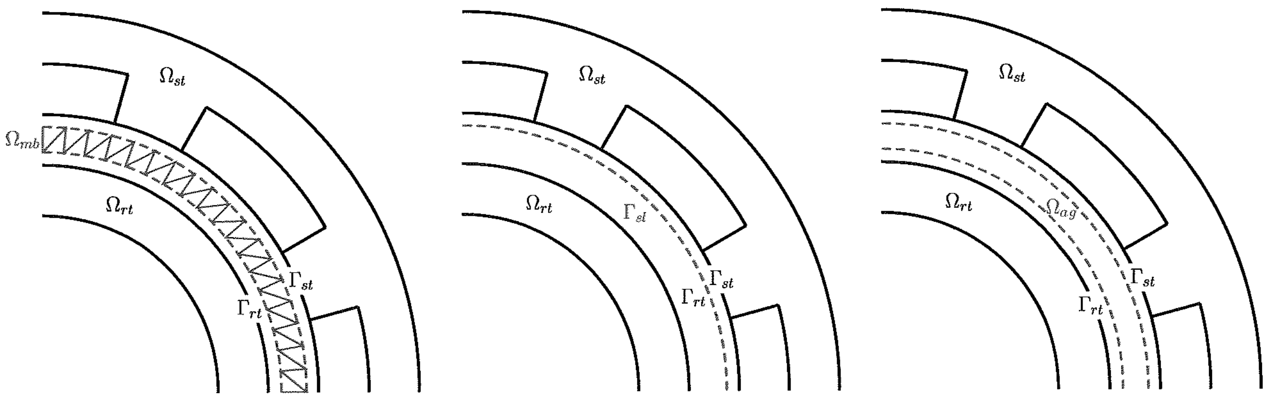


Fig. 1: Domain parts with moving band approach (left), sliding surface approach (middle) and air-gap macro approach (right)

cessing of only the degrees of freedom at the boundaries $\Gamma_{st}$ and $\Gamma_{rt}$ yields

$$\begin{bmatrix} \mathbf{K}_{m,ii} & \mathbf{K}_{m,ie} \\ \mathbf{K}_{m,ie}^T & \mathbf{K}_{m,ee} \end{bmatrix} \begin{bmatrix} \mathbf{U}_{m,i} \\ \mathbf{U}_{m,e} \end{bmatrix} + \begin{bmatrix} \mathbf{0} \\ \mathbf{G}_{m,e} \end{bmatrix} = \begin{bmatrix} \mathbf{P}_{m,i} \\ \mathbf{P}_{m,e} \end{bmatrix} \quad (2)$$

where the subscripts (i) and (e) denote the degrees of freedom allocated at the domains $\Omega_m \setminus \Gamma_m$ and $\Gamma_m$, respectively.

For the subsequent processing, only the exterior degrees of freedom $\mathbf{U}_{st,e}$ and $\mathbf{U}_{rt,e}$ at the boundaries $\Gamma_{st}$ and $\Gamma_{rt}$ are retained. The corresponding matrix equation of the residual structure reads as

$$\begin{bmatrix} \overline{\mathbf{K}}_{st} & 0 \\ 0 & \overline{\mathbf{K}}_{rt} \end{bmatrix} \begin{bmatrix} \mathbf{U}_{st,e} \\ \mathbf{U}_{rt,e} \end{bmatrix} + \begin{bmatrix} \mathbf{G}_{st,e} \\ \mathbf{G}_{rt,e} \end{bmatrix} = \begin{bmatrix} \overline{\mathbf{P}}_{st} \\ \overline{\mathbf{P}}_{rt} \end{bmatrix} . \quad (3)$$

The three basic methods discussed afterwards provide different coupling algorithms between the exterior degrees of freedom of the subdomains. With all methods concerned, significantly reduced calculation efforts for successive rotor positions are established even in a nonlinear analysis due to the smaller stiffness matrices and the very small matrix equation of the residual structure [9], [10].

### B. Moving band modelling

In case of the moving band approach, the stator and rotor parts should have an equidistant discretization in moving direction and a coincident discretization in axial direction. As shown in Fig. 1, the coupling of the model parts is provided by using the non-empty domain $\Omega_{mb}$ within the air-gap which usually consists of a single element layer.

With various angular rotor positions, the moving band is remeshed mainly in dependence on the distortion of the finite elements inside this layer [8]. Thus, a different quality of numerical results regarding the various angular rotor positions is established. Additionally, the simultaneous allocation of periodical boundary conditions needs a special treatment within the remeshing procedure [11], [12].

The single layer mesh of the moving band domain $\Omega_{mb}$ provides a direct coupling of the domains $\Omega_{st}$ and $\Omega_{rt}$. Consequently, the coupled system of the residual structure reads as

$$\begin{bmatrix} \overline{\mathbf{K}}_{st} + \mathbf{K}_{mb,st,st} & \mathbf{K}_{mb,st,rt} \\ \mathbf{K}_{mb,rt,st} & \overline{\mathbf{K}}_{rt} + \mathbf{K}_{mb,rt,rt} \end{bmatrix} \begin{bmatrix} \mathbf{U}_{st,e} \\ \mathbf{U}_{rt,e} \end{bmatrix} = \begin{bmatrix} \overline{\mathbf{P}}_{st} \\ \overline{\mathbf{P}}_{rt} \end{bmatrix} . \quad (4)$$

The most important advantage is given by the fact that there is almost no constraint on the angular displacement. In case of 3D models however, the implementation can become very complex if irregular discretizations exist at the boundaries $\Gamma_{st}$ and $\Gamma_{rt}$.

### C. Sliding surface modelling

In case of the sliding surface approach, the stator and rotor parts must have an equidistant discretization in moving direction and a coincident discretization in axial direction. As shown in Fig. 1, the coupling of the model parts is provided by using floating boundary conditions on the common boundary $\Gamma_{sl} = \Gamma_{st} = \Gamma_{rt} = \Omega_{st} \cap \Omega_{rt}$ in dependence on the rotor position.

With all angular rotor positions, there is a completely unchanged finite element discretization without any remeshing. Thus, an identical quality of numerical results regarding the various angular rotor positions is obtained. Additionally, the simultaneous allocation of periodical boundary conditions can be easily done without any precautions.

The permutation of the boundary degrees of freedom and the boundary terms according to the rotor position can be written as

$$\mathbf{U}_{rt,e} = \mathbf{E}_k \, \mathbf{U}_{st,e} \ , \quad \mathbf{G}_{st,e} = -\mathbf{E}_k^T \, \mathbf{G}_{rt,e} \ . \quad (5)$$

Consequently, the coupled system of the residual structure can be combined into the single matrix equation

$$\left( \overline{\mathbf{K}}_{st} + \mathbf{E}_k^T \, \overline{\mathbf{K}}_{rt} \, \mathbf{E}_k \right) \mathbf{U}_{st,e} = \overline{\mathbf{P}}_{st} + \mathbf{E}_k^T \, \overline{\mathbf{P}}_{rt} \ . \quad (6)$$

Two different coupling strategies are established [5], [13]–[16]. The locked-step coupling provides a direct mapping of the degrees of freedom between stator and rotor parts. The various angular rotor positions are directly determined by the angular discretization of the sliding surface. The interpolation coupling provides an interpolation of the stator degrees of freedom onto the rotor degrees of freedom. Thus, a full flexibility regarding the angular rotor positions is obtained.

### D. Application of an air-gap macro element

In case of the air-gap macro approach, the stator and rotor parts must have an equidistant discretization in moving direction and a coincident discretization in axial direction. As shown in Fig. 1, the coupling of the model parts is provided by using an analytical approximation of the air-gap field within the non-empty domain $\Omega_{ag}$, which does not necessarily coincide with the physical air-gap [17]–[20].

With all angular rotor positions, there is a completely unchanged finite element discretization without any remeshing. Therefore, an identical quality of numerical results regarding the various angular rotor positions is obtained. Additionally, the simultaneous allocation of periodical boundary conditions can be easily done without any precautions.

The air-gap macro element provides an additional symmetric stiffness matrix $\mathbf{K}_{ag}$ which combines Fourier expansions of the boundaries $\Gamma_{st}$ and $\Gamma_{rt}$. Consequently, the coupled system of the residual structure reads as

$$\begin{bmatrix} \overline{\mathbf{K}}_{st} & 0 \\ 0 & \overline{\mathbf{K}}_{rt} \end{bmatrix} \begin{bmatrix} \mathbf{U}_{st,e} \\ \mathbf{U}_{rt,e} \end{bmatrix} + \mathbf{K}_{ag} \begin{bmatrix} \mathbf{U}_{st,e} \\ \mathbf{U}_{rt,e} \end{bmatrix} = \begin{bmatrix} \overline{\mathbf{P}}_{st} \\ \overline{\mathbf{P}}_{rt} \end{bmatrix} . \quad (7)$$

The operator $\mathbf{K}_{ag}$ representing an air-gap reluctance will not be assembled into an algebraic matrix. Instead, the coupled system is always solved iteratively [18], [19].

Finally, the electromagnetic quantities such as force and torque values can be directly evaluated from the potential results without any need of calculating the field vectors resulting in the best possible accuracy [5], [21], [22].

## III. Multi-Field Simulations

The most widely used numerical method is the finite element method which can be used in the analysis of electromagnetic, mechanical, thermal, fluid and acoustic field problems [2], [4], [23], [24]. Fig. 2 depicts the general coupling of these different physical domains in case of electrical machines and transformers while Fig. 3 depicts challenges and couplings with finite element analyses of these devices.

As described above, with the electromagnetic analyses it is state of the art to use invariant stator and rotor model parts while considering the periodicity of the electrical machine additionally. But for the other mentioned physical domains, especially with cooling and mechanical construction, there is almost no periodicity of a repetitive section along the circumferential direction. Nevertheless, the separation of the entire model into distinct stator and rotor parts is again applicable in many situations.

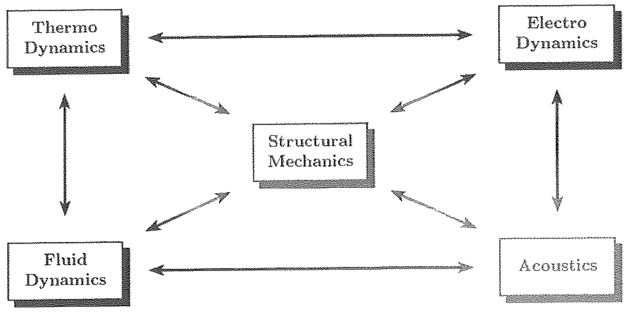


Fig. 2: Physical domains of multi-field simulations

In case of both electrical and magnetic loadability, particularly the conductivities of the conductive regions such as windings and permanent magnets are affected by the temperature distribution within the device. On the other hand, the thermal behaviour shows significant couplings with the cooling media and their fluid field. In case of the mechanical loadability, mechanical stresses caused by centrifugal forces, natural frequencies, by the highest permissible electrical as well as magnetic loadings and the additional temperature rise restrict output power and maximum speed of an electrical machine.

Finally, the sound emission of electrical machines and transformers has growing importance due to more and more tightened low emission standards. Thus, an accurate prediction and consequently reduction of sound emissions are of increasing interest for the electric power industry [4]. The noise is mainly caused by cooling armatures as well as forces of electromagnetic and magnetostrictive origin.

### A. Overview of coupling procedures

In general [2], [4], [23], [24], there are two different strategies for the coupling of the various physical domains arising with multi-field simulations. Table I lists a brief comparison of the strong coupling approach as well as the weak coupling approach.

The requirements of the strong coupling technique are very restrictive. There are always identical finite element discretizations and the same time-stepping procedures with all physical domains concerned. Thus, the best possible convergence is obtained when dealing with strongly coupled physical phenomena. On the other hand, the direct assembly of different physical domains into one set of equations results in a very huge number of unknown degrees of freedom with mostly ill-conditioned matrices.

The strategies of the weak coupling technique are completely different. In terms of finite element discretizations and time-stepping procedures, each physical domain will be considered separately. Thus, the

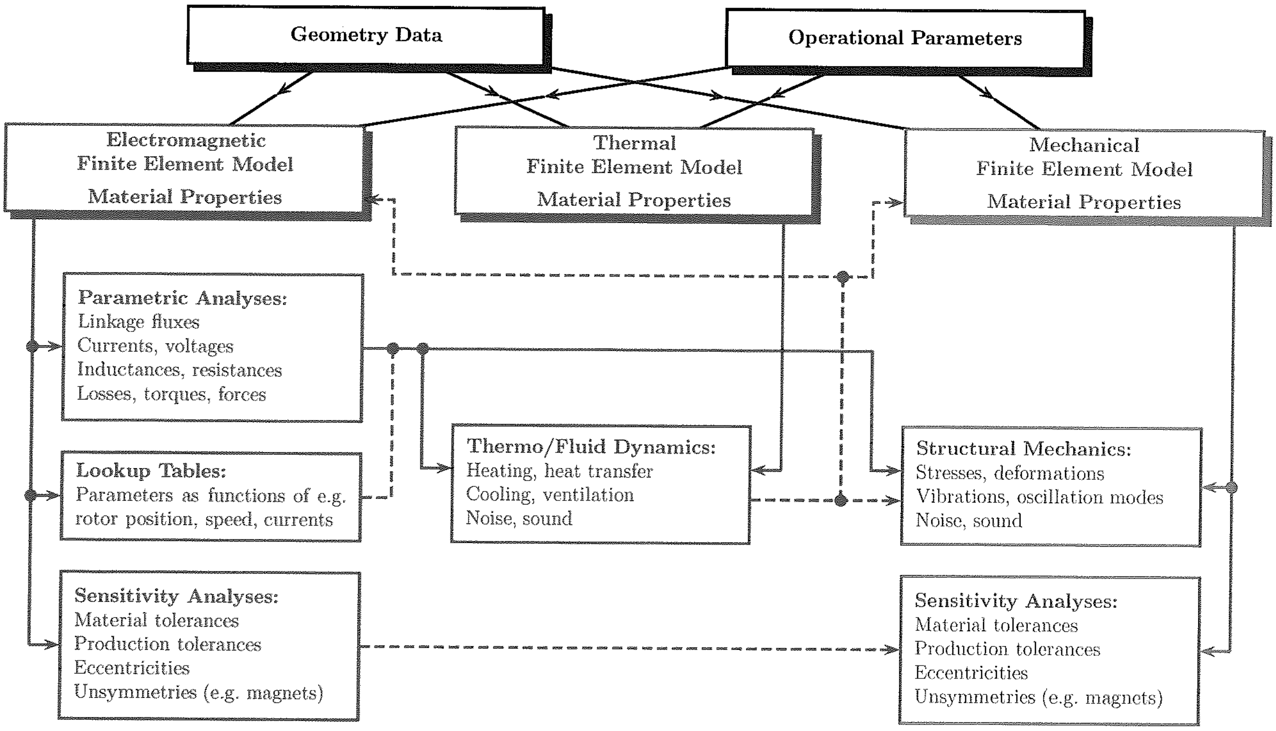


Fig. 3: Typical challenges and couplings with finite element analyses of electrical machines and transformers

TABLE I
OVERVIEW OF COUPLINGS WITH MULTI-PHYSICAL ANALYSES

| | Strong coupling | Weak coupling |
|---|---|---|
| **Approach** | Simultaneous setup and solution of the governing equations | Separated setup and solution of the governing equations, coupling by data transfer between the domains |
| **Requirements** | Identical discretization and solution strategy with all domains | Interpolation of the results between the domains |
| **Advantages** | Robust approach with strongly nonlinear interactions | Distinct discretization and solution strategy with all domains |
| **Drawbacks** | Ill-conditioned matrices, expensive calculation times | Convergence problems, sparsely defined update strategies |
| **Applications** | E.g. induction heating, continuous steel casting, magnetostrictive and piezoelectric phenomena | E.g. electrical machines and transformers |

number of unknown degrees of freedom will be preserved within each of the domains. The coupling between the various domains is accomplished by introducing the interaction between the different physical domains as excitations on the actual domain generated from the other physical domains. Consequently, special attention has to be paid to accurate formulations and evaluations of the coupling terms.

This separate treatment of the physical domains has its advantages by the possibility of using different finite element discretizations and time-stepping algorithms, too. The utilization of different finite element meshes with the different domains requires an inherent necessity of an interpolation between the numerical results of the different domains. Due to the consecutive processing of the various physical domains, convergence and update strategies for the data transfer between the physical domains gain in significance.

### *B. Weak coupling procedures*

Usually, the characteristic thermal and mechanical time constants of electrical machines and transformers are several orders larger than the characteristic electromagnetic time constants [2]. In many cases, the electromagnetic quantities generate only a low impact on the thermal and mechanical properties of the mentioned devices. Coevally, the mechanical deformations have mostly no influence on the electromagnetic behaviour. But there are strong interactions between the temperature distribution and in particular the material properties for both electromagnetic

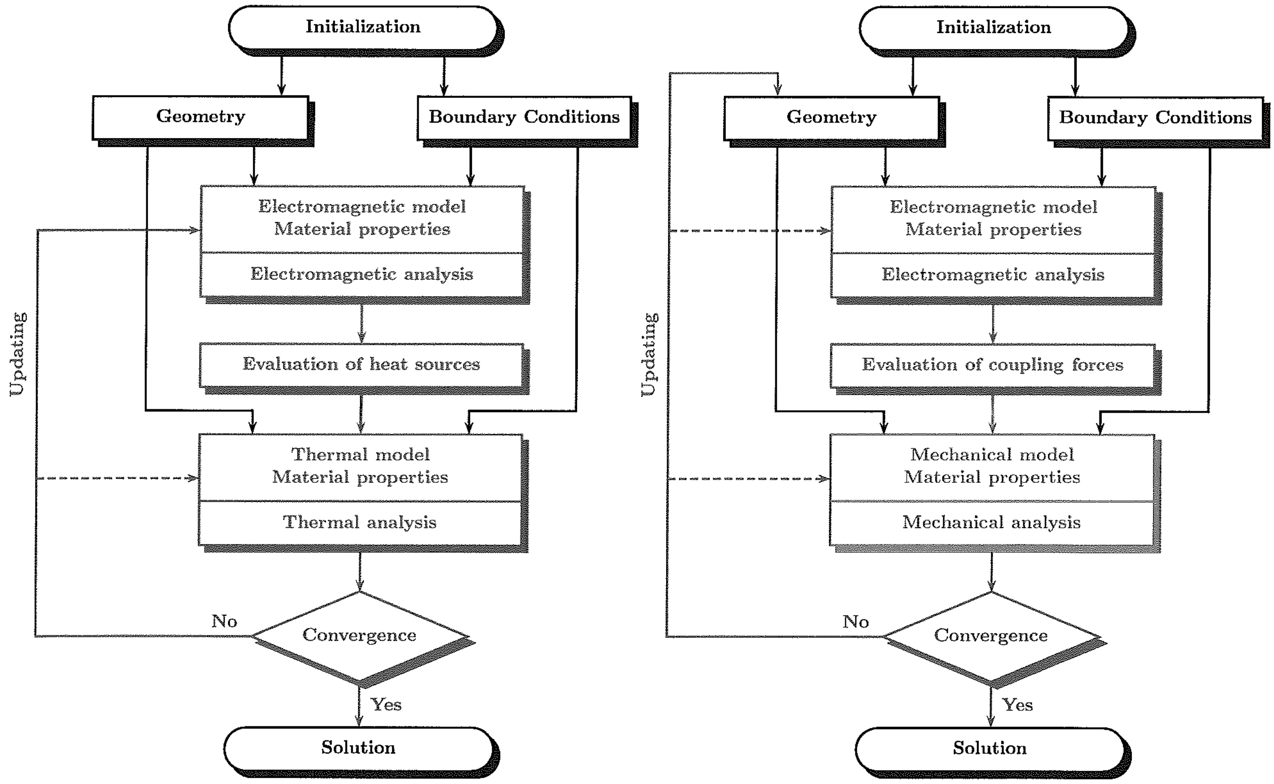


Fig. 4: Weak-coupled electromagnetic-thermal (left) and electromagnetic-mechanical (right) finite element analyses of electrical machines and transformers

and mechanical analyses. Consequently, electrical machines and transformers can be analyzed efficiently by using weakly coupled solution strategies.

Fig. 4 depicts the solution processes of weak-coupled electromagnetic-thermal and electromagnetic-mechanical finite element analyses of electrical machines and transformers as the most important coupled problems arising with these devices.

With an coupled electromagnetic-thermal problem, the heat sources of electromagnetic nature such as eddy current, power and iron losses result in coupling terms with the right-hand side of the thermal equations. The material data are often strongly dependent on the temperature. Thus, they have to be updated during the coupled solution process.

With an coupled electromagnetic-mechanical problem, the local as well as global forces and torques of electromagnetic origin result in coupling terms with the right-hand side of the mechanical equations. The material data can be affected by deformations or situations of transitions between usually fixed and occasionally loose parts. The most important impact on the coupled solution process now arises from changed or even distorted geometries.

## IV. Application example

The long-time operational reliability of large synchronous generators is nowadays one of the main objectives of power system management. One of the most typical failure situation will be caused by loose Roebel bars inside the stator slots of such machines. With short circuit operating conditions, the high electromagnetic forces arising additionally to the mechanical forces will cause a coupled electromagnetic-mechanical problem that is discussed in detail.

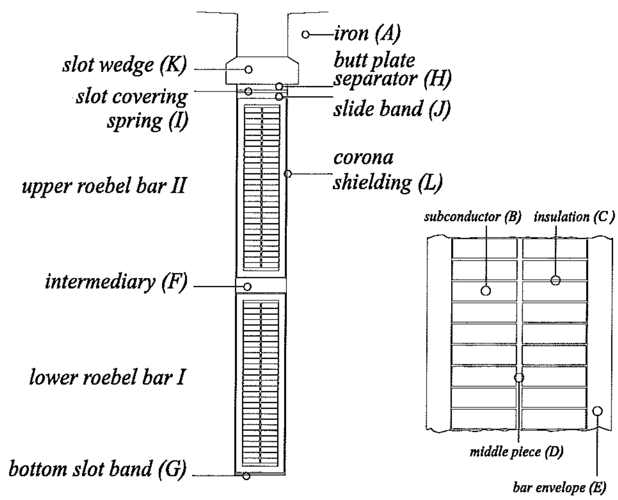


Fig. 5: Arrangement of the Roebel bars inside the slot

The cross section of a typical Roebel bar structure with filling materials inside the slot is shown in Fig. 5. The upper and lower bar itself consists of several subconductors, insulation, middle filler piece and enclosing envelope. A slot wedge fixes all parts inside the slot and should avoid any free space in radial direction. The geometry data used with the following analyses are listed in Table II.

TABLE II
GEOMETRY DATA OF SLOT AND ROEBEL BARS

| | | | |
|---|---|---|---|
| Rated bar current | $I_N$ | 2750 | A |
| Width of slot | $a$ | 22.0 | mm |
| Width of conducting region | $b$ | 15.8 | mm |
| Thickness of envelope | $c$ | 3.1 | mm |
| Height of conducting region | $h$ | 70.9 | mm |
| Height of bottom inlet | $h_0$ | 5.3 | mm |
| Height of intermediary | $h_1$ | 15.5 | mm |
| Height of slot wedge and filler | $h_2$ | 21.5 | mm |
| Height of tooth top | $h_3$ | 19.9 | mm |

During regular operational states, the mechanical preload applied by the slot wedge mainly determines the mechanical behaviour. This is due to the fact that stress contributions caused by rated currents are not rarely observed in the total mechanical stress distribution. On the other hand during electric failures, the electromagnetic forces rise up in an enormous way. They cause significant stress contributions accompanied by an according deformation of the complete Roebel bar structure.

Under any circumstances, a slackening of the normally fixed Roebel bars due to acting electromagnetic and mechanical forces has to be avoided. Therefore, requirements for secure Roebel bars are calculated for various current loads with several material compositions typically used in the industrial manufacturing process [25].

With this fully coupled electromagnetic-mechanical problem, the behaviour is completely different for the two cases of mechanical loose and fixed bars. A weakly coupled cascade algorithm with nonlinear electromagnetic and linear mechanical finite element calculations as described in [25], [26] allows to describe even the transitional state between both fixed and loose bars in dependence of the electrical bar currents.

By using an analytical approach, an equivalent common elastic modulus $E'_{xx}$ of the complete arrangement as listed in Table III as well as the necessary preload of the slot wedge

$$u_0 \geq \frac{8}{3}\frac{\mu_0}{E'_{xx}}\frac{h}{ab}I^2 \qquad (8)$$

with inphase bar currents can be calculated in advance.

In comparison of the various materials, Fig. 6 depicts the displacement of the Roebel bar structure in dependence of inphase bar currents. According to the utilized material sets, various mechanical preloads of the slot wedge $u_0$ are applied. Thus, fixed Roebel bars can be obtained with currents up to 20pu for all material sets depending on the preload applied with the slot wedge.

Fig. 7, Fig. 8 and Fig. 9 depict the stress distribution obtained from rated inphase currents within half height of the lower bar (II), intermediary (F), bottom

TABLE III
EQUIVALENT COMMON ELASTIC MODULUS

| Material set | Numerical approach $E'_{xx}$ (GN/m$^2$) | Analytical approach $E'_{xx}$ (GN/m$^2$) |
|---|---|---|
| Weak | 28.6 | 24.2 |
| Standard | 68.7 | 66.1 |
| Strong | 84.3 | 82.9 |

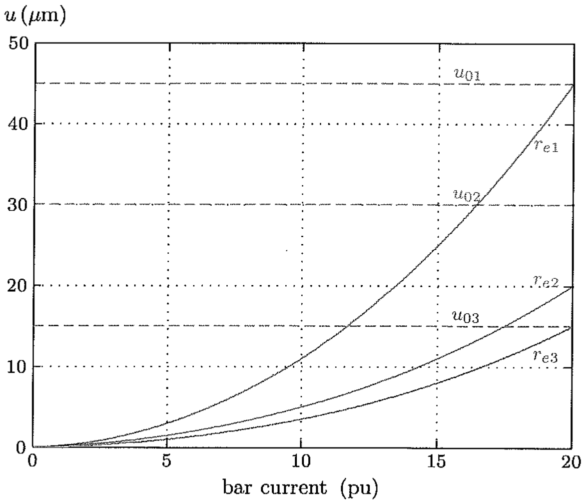


Fig. 6: Displacement of the Roebel bar structure with various mechanical preloads of the slot wedge in dependence on inphase bar currents for three material sets

slot band (G) and butt plate separator (H) for the selected material compositions. Fig. 10, Fig. 11 and Fig. 12 depict the stress distribution along the slot height within conductor region (B), middle filler piece (D) and bar envelope (E) for the selected material compositions.

For the standard material set, bar envelope and conducting regions are stressed with nearly the same mechanical stress values. For the weak material set, the copper regions shows higher stress values compared to the insulation. However, for the strong material set, an increase of high stress values inside the strong insulation is obtained. For all investigated arrangements, local rising stress values can be observed at the slot bottom within the width of the envelope.

## V. CONCLUSION

Electrical machines and transformers are prime examples of multi-physical systems involving electromagnetics, thermal issues, fluid dynamics, structural mechanics as well as acoustic phenomena. The finite element method is the most powerful numerical analysis method for such multi-physical devices.

Since optimizations with respect to the overall performance and also the total manufacturing costs will become more important, the utilization of coupled multi-physical analyses is of growing interest. For the fast and powerful application of this numerical analysis method, special attention should be given to the requirements of these electromagnetic devices.

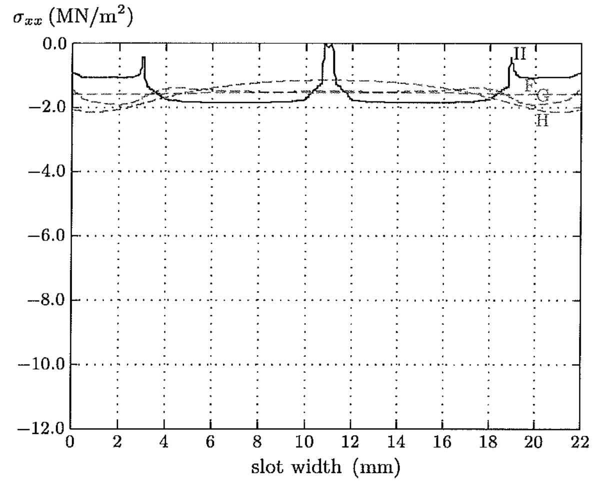


Fig. 7: Stress distribution $\sigma_{xx}$ along slot width in different slot heights with rated inphase currents, weak insulation material

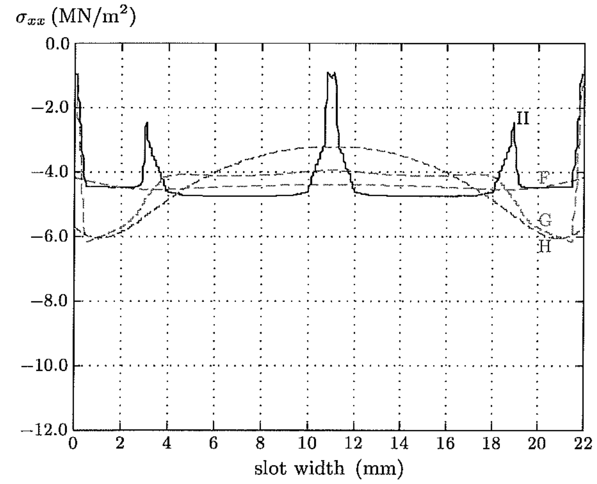


Fig. 8: Stress distribution $\sigma_{xx}$ along slot width in different slot heights with rated inphase currents, standard insulation material

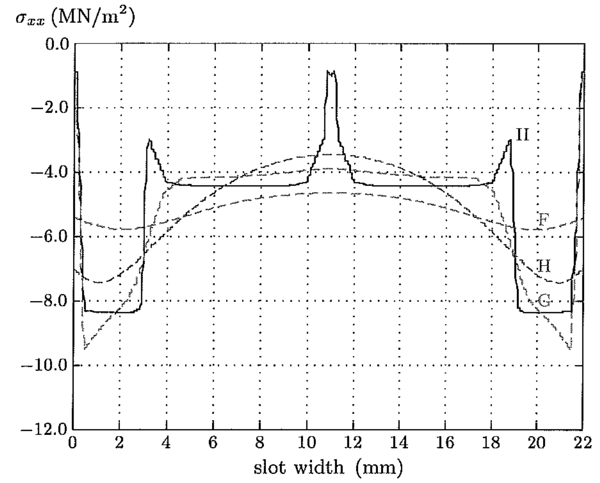


Fig. 9: Stress distribution $\sigma_{xx}$ along slot width in different slot heights with rated inphase currents, strong insulation material

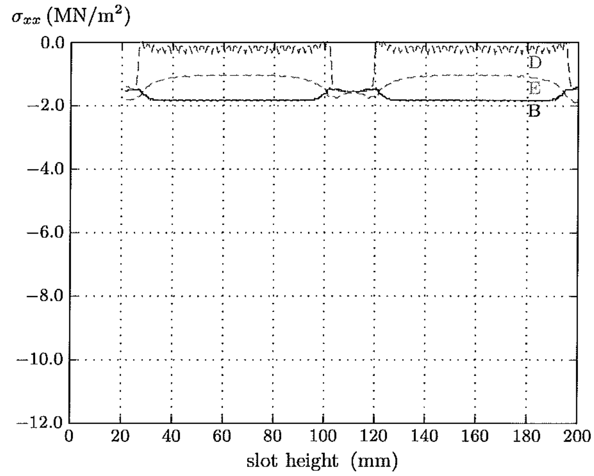


Fig. 10: Stress distribution $\sigma_{xx}$ along slot height in different slot widths with rated inphase currents, weak insulation material

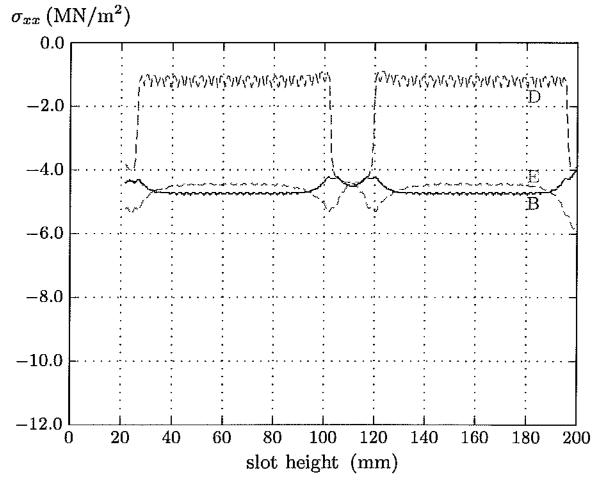


Fig. 11: Stress distribution $\sigma_{xx}$ along slot height in different slot widths with rated inphase currents, standard insulation material

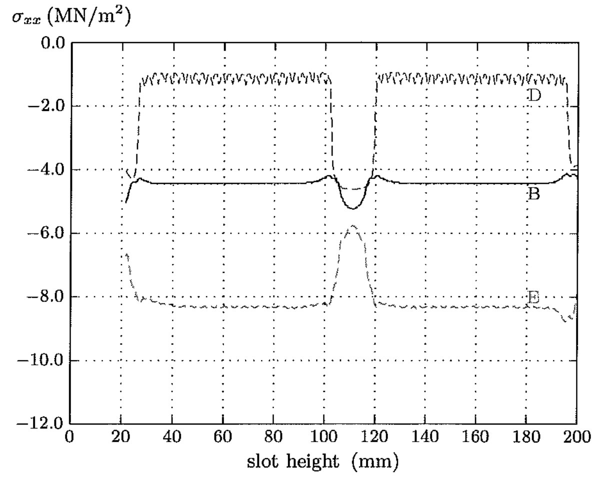


Fig. 12: Stress distribution $\sigma_{xx}$ along slot height in different slot widths with rated inphase currents, strong insulation material

First, the paper presents electromagnetic finite element modelling and analysis methods for electrical machines concerning the various rotor positions with fully invariant stator and rotor model parts. The basic methods are compared against their advantages and drawbacks with modelling and analysis.

In addition to electromagnetic analyses, in particular highly utilized or large-scaled electrical machines and transformers have the necessity of thermal, fluidal and mechanical analyses. An introduction to typical challenges of coupled multi-physics analyses of electrical machines and transformers is given. Various methods of coupling the different physical domains of multifield finite element analyses are described. Thereby, weakly coupled cascade algorithms can be used with most problems in the field of electrical machines and transformers.

The development of robust and reliable computer aided tools for an optimal design of multi-physical devices such electrical machines and transformers as has to argue about the best possible coupling of various simulation methods. Special consideration shall be paid more and more to a treatment of uncertainties and tolerances by means of statistical and probabilistic approaches. Consequently, a prime objective is to derive comprehensive, multi-physical simulation models which can be easily incorporated into design tools used by engineering professionals.

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