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# Instabilities in the Quark-Gluon Plasma \*

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\*“ Dans la vie, rien n'est à craindre, tout est à comprendre.” Marie Curie



# Overview

Quark-gluon  
Plasma

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Hard Expanding  
Loops (HEL)

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Plasma  
Instabilities

## Quark-gluon Plasma

- Early Universe
- Heavy Ion Collision
- QCD Phase diagram
- QGP signatures

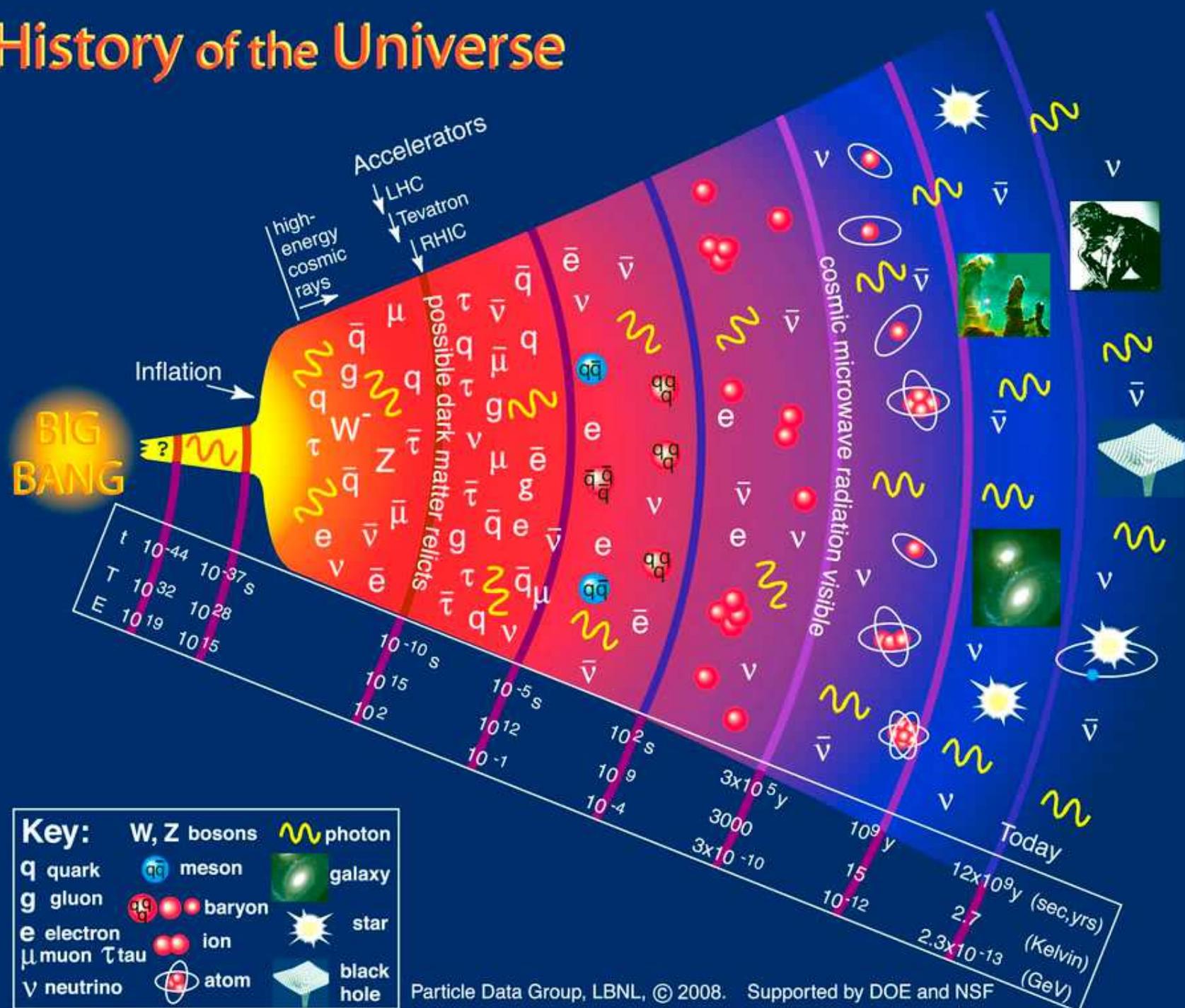
## Hard Expanding Loops (HEL)

- Momentum Anisotropy
- Weibel instabilities
- Scales QGP
- Hard (Thermal) Loops - Boltzmann - Vlasov
- Notations for Bjorken expansion
- Hard-Expanding-Loop formalism

## Plasma Instabilities

- Expanding 1D+3V Abelian plasma
- Expanding 3V plasma
- Conclusions

# History of the Universe



Particle Data Group, LBNL, © 2008. Supported by DOE and NSF



# Relativistic Heavy Ion Collider (RHIC)

Quark-gluon  
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Early Universe

Heavy Ion  
Collision  
QCD Phase  
diagram

QGP signatures

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Au+Au ions  $\sqrt{s_{NN}} = 200\text{GeV}/\text{nucleon pair}$ , p+p, d+A



# Heavy Ion Collision

Quark-gluon

Plasma

Early Universe

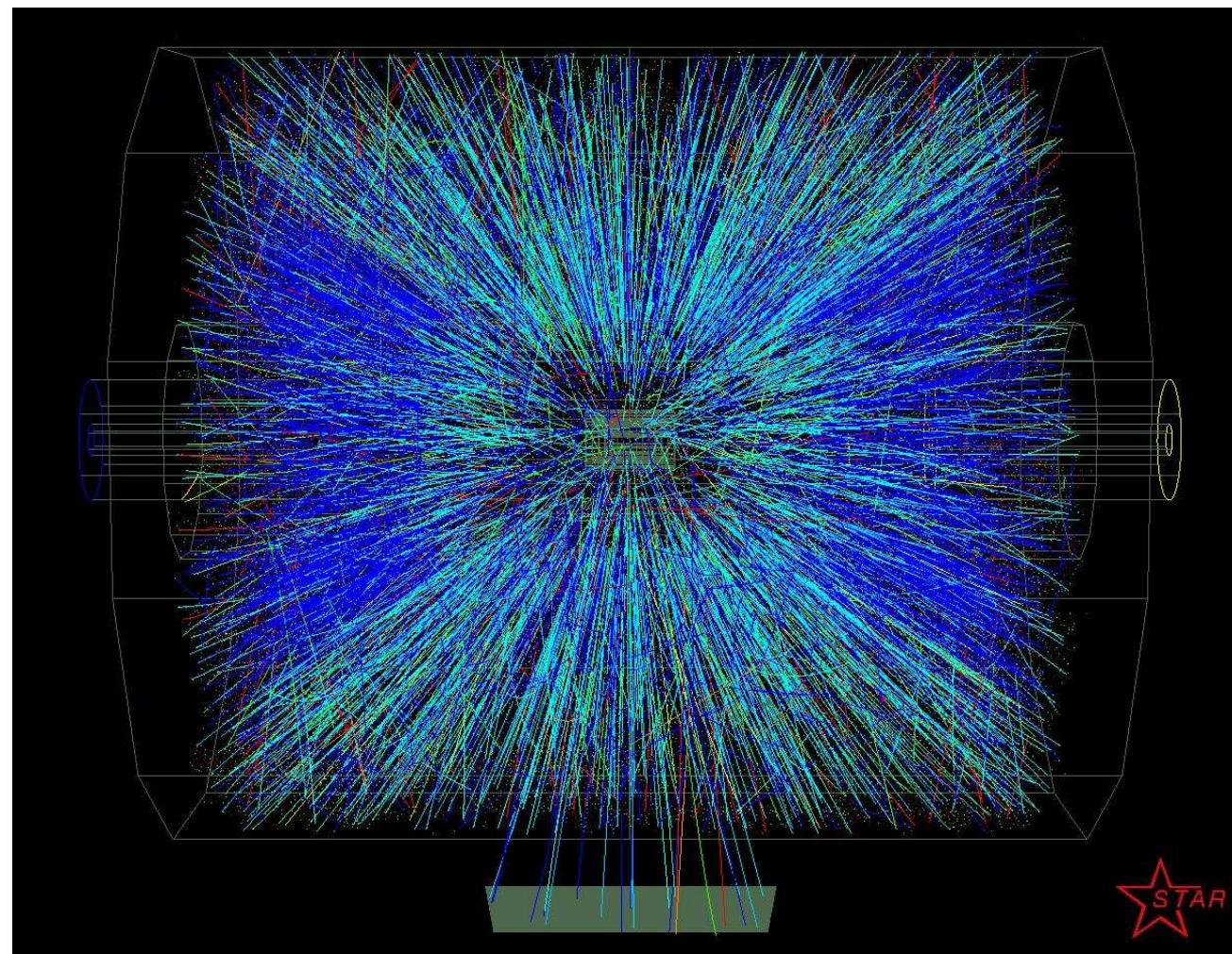
Heavy Ion  
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Side view 2nd STAR event at RHIC, 2001.

# QCD Phase diagram

Quark-gluon  
Plasma

Early Universe

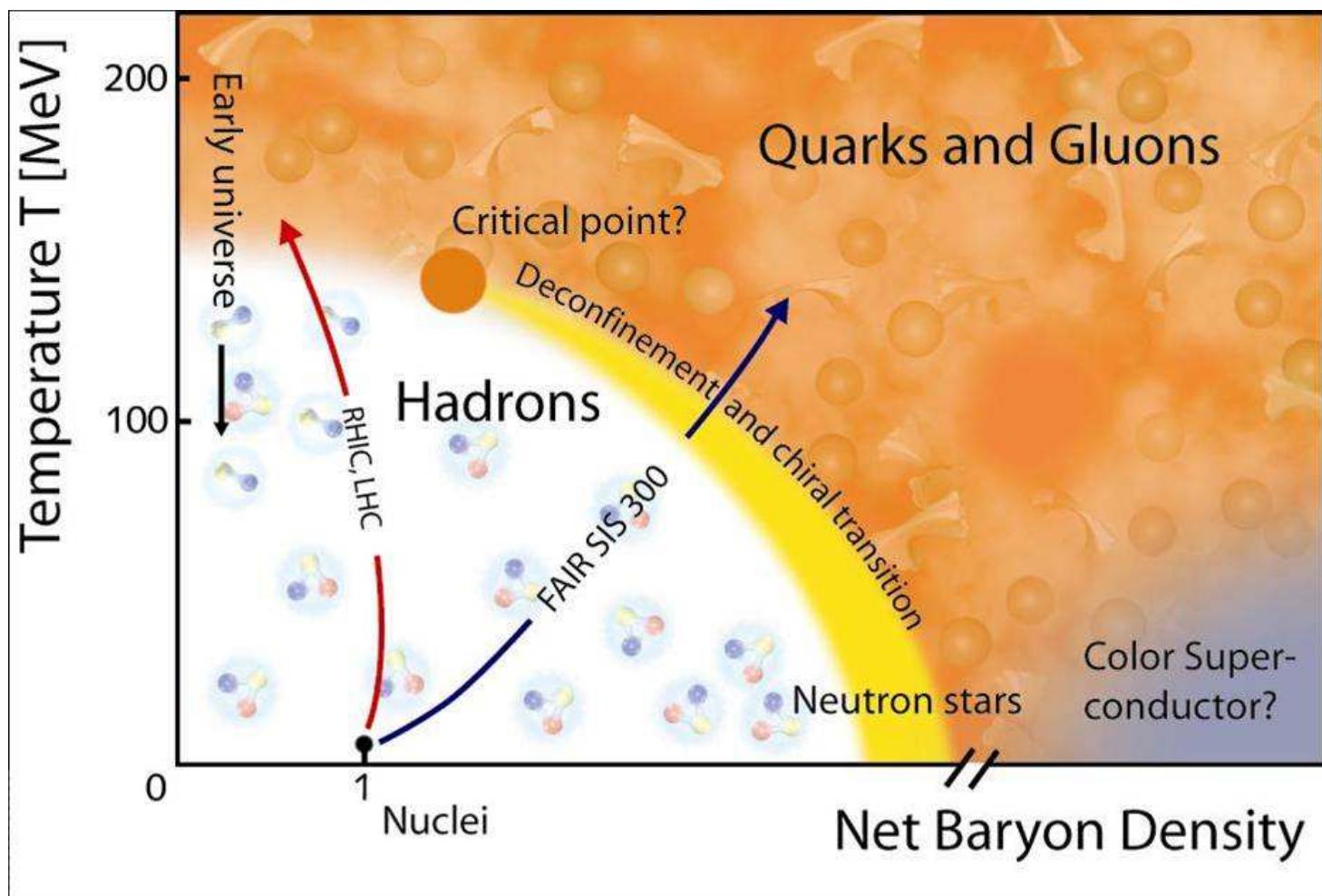
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Schematic QCD phase diagram



# QGP signatures

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- Small viscosity (elliptic flow  $v_2$ )
- Jet quenching
- Experimental observation of  $T > T_c$
- high  $p_T$  suppression of hadrons (for central collisions)
- Rapid thermalization:  
estimates from hydrodynamical computation  $\sim 1 fm/c$



# Elliptic flow

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Pressure gradients generate positive elliptic flow  $v_2$

$$\frac{d^2N}{d\phi dp_T} = N_0 (1 + 2v_2(p_T)\cos(2\phi) + ..) \quad (1)$$

The elliptic flow is quantified by the anisotropy of particle production with respect to the reaction plane  $v_2 = \left\langle \frac{p_x^2 - p_y^2}{p_x^2 + p_y^2} \right\rangle$

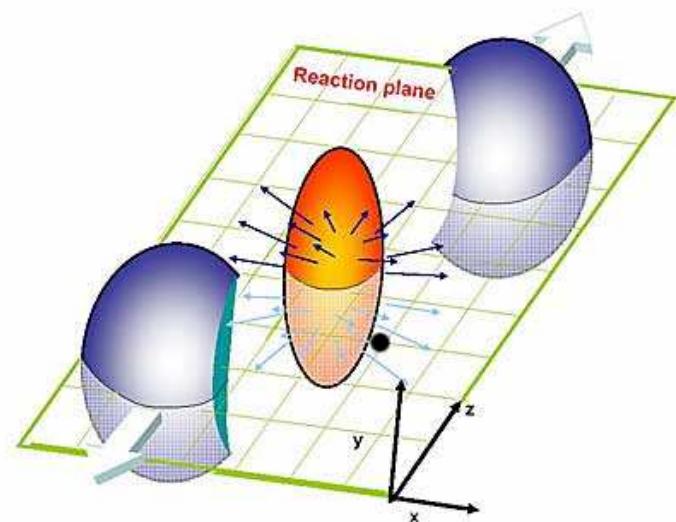


Illustration of the reaction plane definition.



# Hydrodynamic model

Quark-gluon  
Plasma

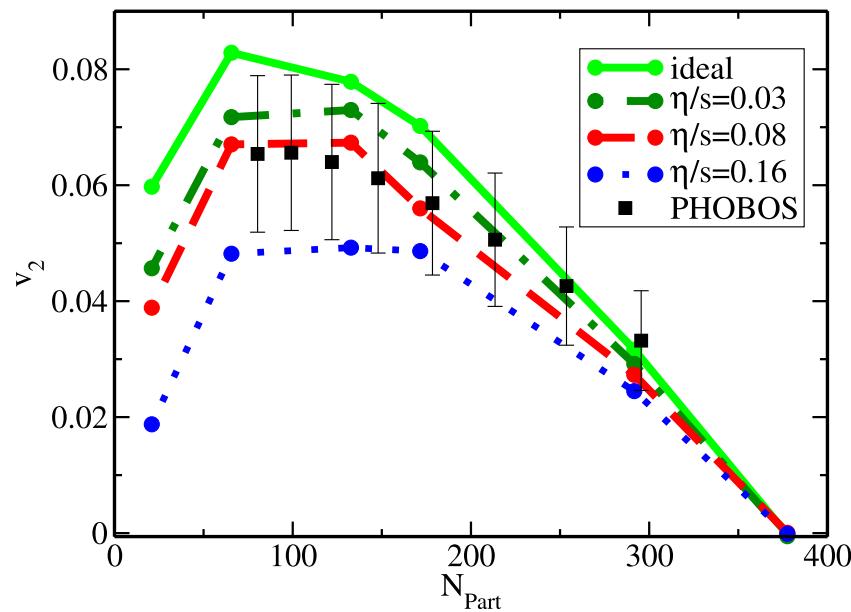
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- PHOBOS data on Au+Au collisions at  $\sqrt{s} = 200$  GeV, compared to hydrodynamic model for various  $\eta/s$  ratios.



P. Romatschke, U. Romatschke hep-th/0706.1522



# Early conditions at RHIC - T

Quark-gluon

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Early Universe

Heavy Ion

Collision

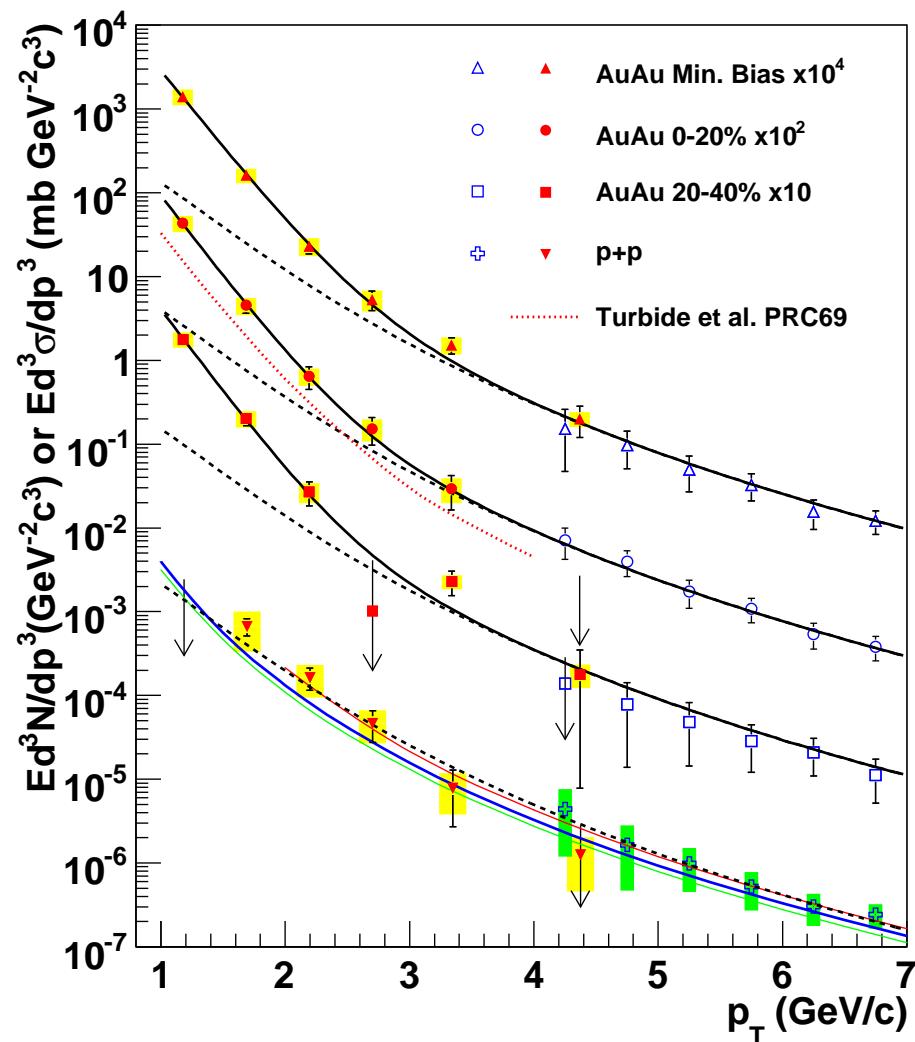
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$$T = 300 - 600 \text{ MeV} > 2 * T_c$$



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Momentum  
Anisotropy

Weibel instabilities

Scales QGP

Hard (Thermal)  
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Boltzmann -

Vlasov  
Notations for  
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# Hard Expanding Loops (HEL)

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# Momentum Anisotropy

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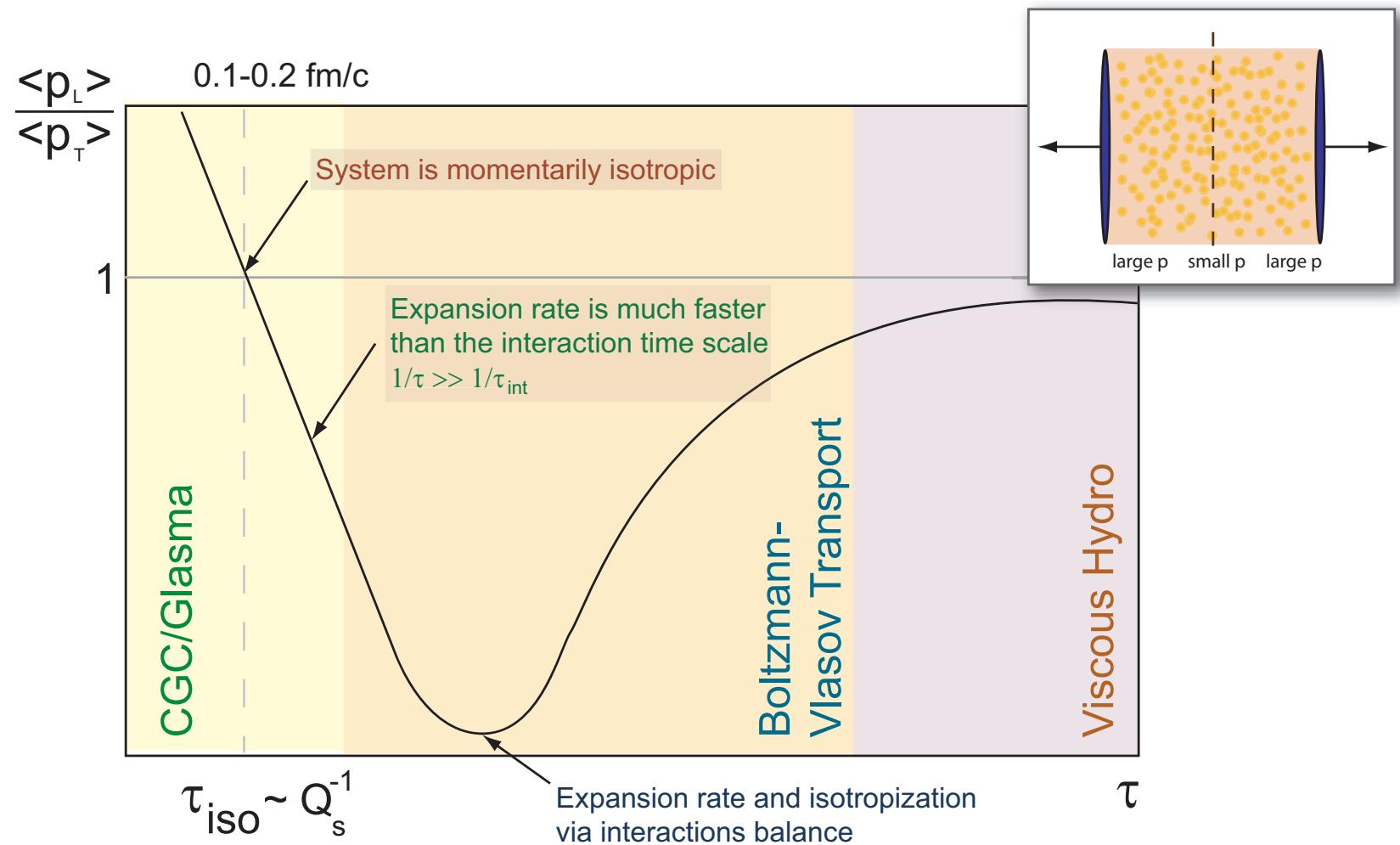
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Momentum space anisotropy time dependence at the early stages of a heavy ion collision



# Weibel instabilities

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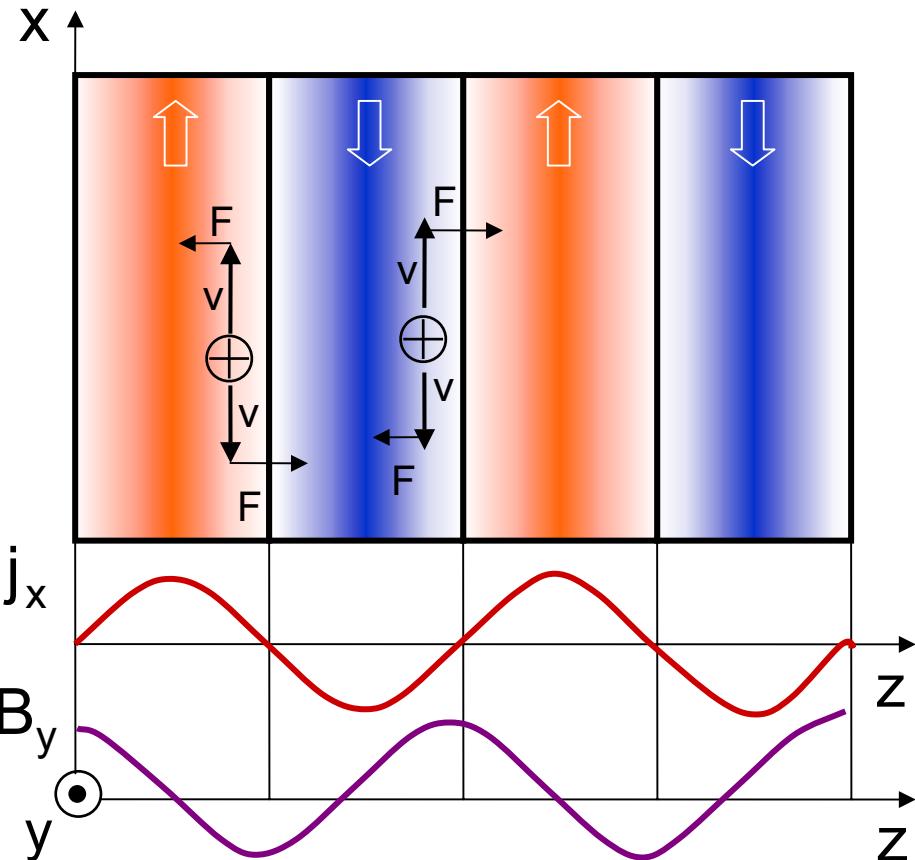
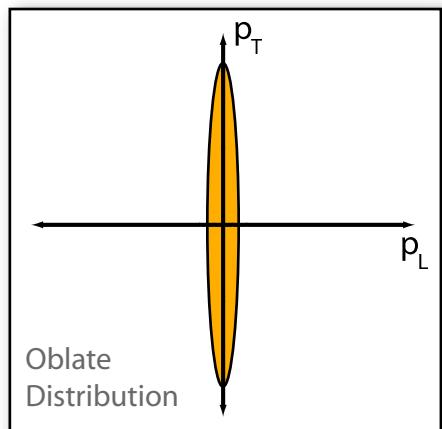


Illustration of the mechanism of filamentation instabilities.



# QED Plasma

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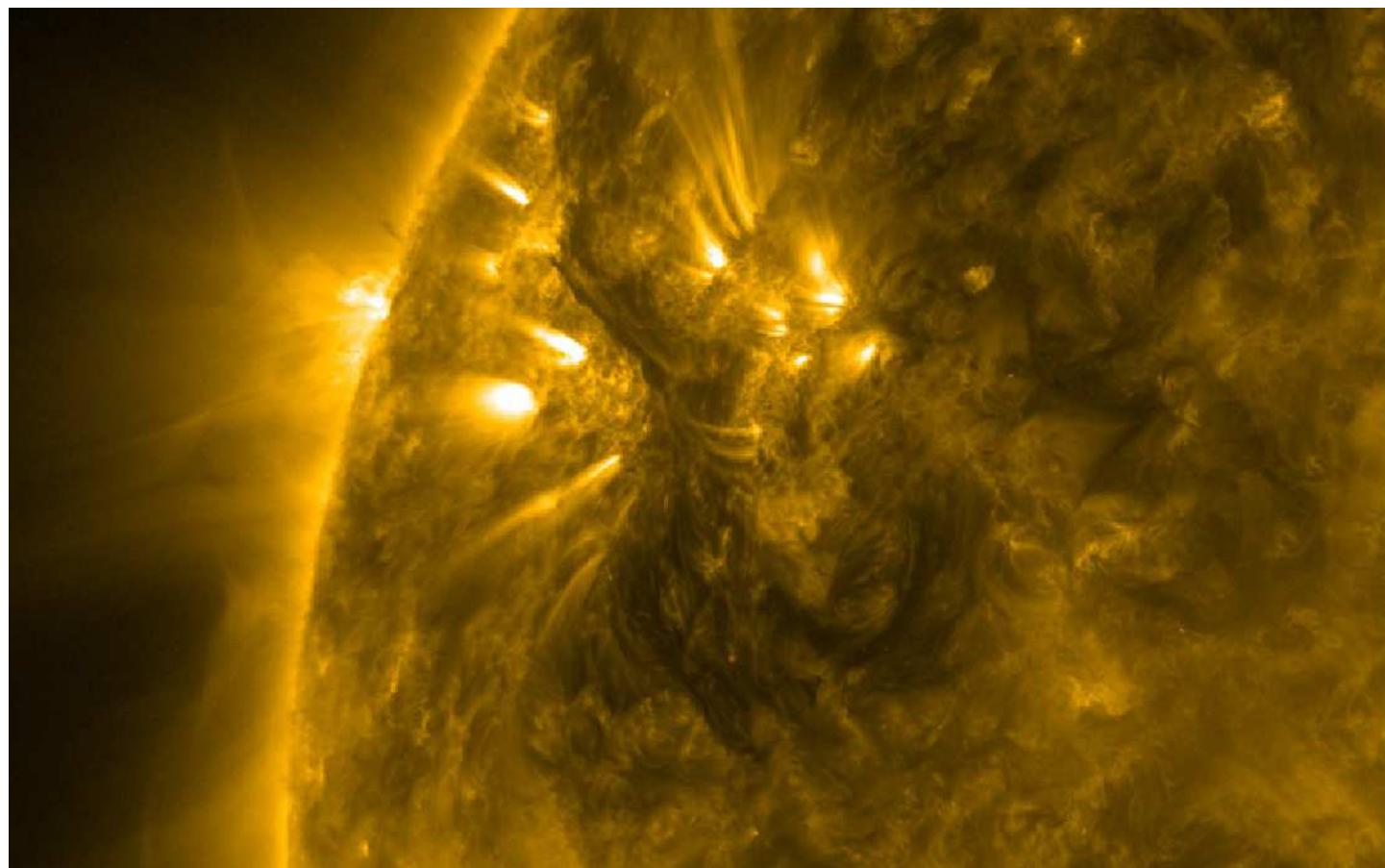
Boltzmann -

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Filaments and active solar region from NASA's Solar Dynamics Observatory



# Scales of weakly coupled QGP

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- $T$ : energy of hard particles
- $gT$ : thermal masses, Debye screening mass,  
Landau damping, **plasma instabilities** [Mrowczynski 1988,  
1993, ...]
- $g^2 T$ : magnetic confinement, color relaxation, rate for small  
angle scattering
- $g^4 T$ : rate for large angle scattering,  $\eta^{-1} T^4$



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## Hard (Thermal) Loops - Boltzmann - Vlasov

With color-neutral background distribution  $v \cdot \partial f_0(\mathbf{p}, \mathbf{x}, t) = 0$ ,  
 $v^\mu = p^\mu/p^0$  gauge covariant Boltzmann-Vlasov:

$$\begin{aligned} v \cdot D\partial f_a(\mathbf{p}, \mathbf{x}, t) &= gv_\mu F_a^{\mu\nu} \partial_\nu^{(p)} f_0(\mathbf{p}, \mathbf{x}, t) = \\ &= -g(\mathbf{E}_a + \mathbf{v} \times \mathbf{B}_a) \cdot \nabla_{\mathbf{p}} f_0, \end{aligned} \quad (2)$$

$$D_\mu F_a^{\mu\nu} = j_a^\nu = g \int \frac{d^3 p}{(2\pi)^3} \frac{p^\mu}{2p^0} \delta f_a(\mathbf{p}, \mathbf{x}, t). \quad (3)$$

So far mostly stationary  $f_0(\mathbf{p})$  with  $\partial_\mu f_0 \equiv 0$

- isotropic:  $f_0(\mathbf{p}) = f_0(|\mathbf{p}|)$ ,  $\nabla_{\mathbf{p}} f_0 \propto \mathbf{v}$

$$v \cdot D\delta f_a(\mathbf{p}, \mathbf{x}, t) = -g\mathbf{E}_a \cdot \nabla_{\mathbf{p}} f_0 \quad (\text{stable}) \quad (4)$$

- anisotropic:  $f_0(\mathbf{p})$ ,  $\nabla_{\mathbf{p}} f_0 \not\propto \mathbf{v}$

$$v \cdot D\delta f_a(\mathbf{p}, \mathbf{x}, t) = -g(\mathbf{E}_a + \mathbf{v} \times \mathbf{B}_a) \cdot \nabla_{\mathbf{p}} f_0 \quad (\text{unstable!}) \quad (5)$$



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# Discretized Hard Loop Effective Theory

Auxiliary field formulation: Mrowczynski, Rebhan & Strickland 2004]

$$\delta f^a(x; p) = -g W_\mu^a(t, \mathbf{x}; \mathbf{v}) \partial^\mu(p) f_0(\mathbf{p}) \quad (6)$$

$$[v \cdot D(A)] W_\mu(x; \mathbf{v}) = F_{\mu\gamma}(A) v^\gamma \quad (7)$$

where  $v^\mu \equiv p^\mu/|\mathbf{p}| = (1, \mathbf{v})$

$$j^\mu(x) = -g^2 \int \frac{d^3 p}{(2\pi)^3} \frac{1}{2|\mathbf{p}|} p^\mu \frac{\partial f(\mathbf{p})}{\partial p^\nu} W^\nu(x; \mathbf{v}), \quad (8)$$

Hard Loop effective theory: (hard) scale  $|\mathbf{p}|$  integrated out  
for real-time lattice simulation: discretize also velocity space in  
"disco balls"

$$D_\sigma(A) F^{\sigma\mu} = j^\mu(x) = \frac{1}{\mathcal{N}} \sum_v v^\mu \mathcal{W}_v(x) \quad (9)$$



# Notations for Bjorken expansion

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It is convenient to switch to comoving coordinates

$$\begin{aligned} t &= \tau \cosh \eta, & \beta &= \tanh \eta, \\ z &= \tau \sinh \eta, & \gamma &= \cosh \eta, \end{aligned} \quad (10)$$

i.e., a coordinate system with metric  $ds^2 = d\tau^2 - d\mathbf{x}_\perp^2 - \tau^2 d\eta^2$ .

We introduce the notation

$$\tilde{x}^\alpha = (x^\tau, x^i, x^\eta) = (\tau, x^1, x^2, \eta) \quad (11)$$

with indices from the beginning of the Greek alphabet for these new coordinates. In addition to space-time rapidity  $\eta$ , we also introduce momentum space rapidity  $y$  for the massless particles according to

$$p^\mu = p_\perp (\cosh y, \cos \phi, \sin \phi, \sinh y). \quad (12)$$



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## Hard-Expanding-Loop formalism

With  $p^\beta \partial_\beta \left[ \partial_{(p)}^\alpha f_0(\mathbf{p}_\perp, p_\eta) \right] \Big|_{p^\mu = \text{const.}} = 0 \quad (13)$

we can commute  $\mathbf{p} \cdot D$  and thus solve gauge-covariant Vlasov equation in comoving coordinates

$$p \cdot D \delta f_a(\mathbf{p}, \mathbf{x}, t) \Big|_{p^\mu = \text{const.}} = g p^\beta F_{\beta\alpha}^a \partial_{(p)}^\alpha f_o(\mathbf{p}, \mathbf{x}, t). \quad (14)$$

Introducing auxiliary fields  $W_\alpha^a(\tau, x^i, \eta; \phi, y)$  similar to the auxiliary field  $W^\nu(x, \mathbf{v})$  of the hard-loop formalism

$$\delta f^a(x; p) = -g W_\alpha^a(\tau, x^i, \eta; \phi, y) \partial_{(p)}^\alpha f_0(p_\perp, p_\eta) \quad (15)$$

that obey  $v \cdot D W_\alpha(\tau, x^i, \eta; \phi, y) \Big|_{\phi, y} = v^\beta F_{\alpha\beta} \quad (16)$

where  $v^\alpha \equiv \frac{p^\alpha}{|\mathbf{p}_\perp|} = \left( \cosh(y - \eta), \cos \phi, \sin \phi, \frac{\sinh(y - \eta)}{\tau} \right)$ .



# Plasma Instabilities

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**Hard-Expanding-Loop formalism**

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# Expanding 1D+3V Abelian plasma

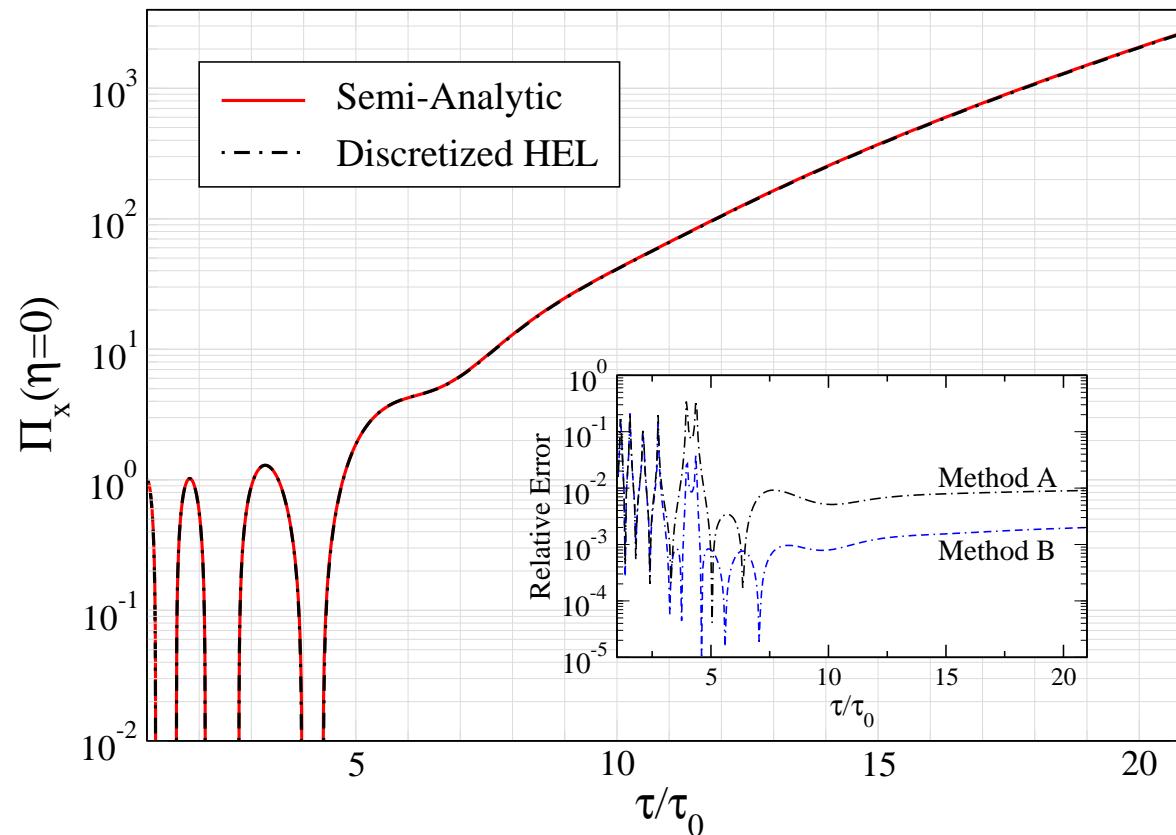
Quark-gluon  
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The proper-time evolution of the canonical field momentum of a single Abelian mode.



# Expanding 1D+3V non-Abelian plasma

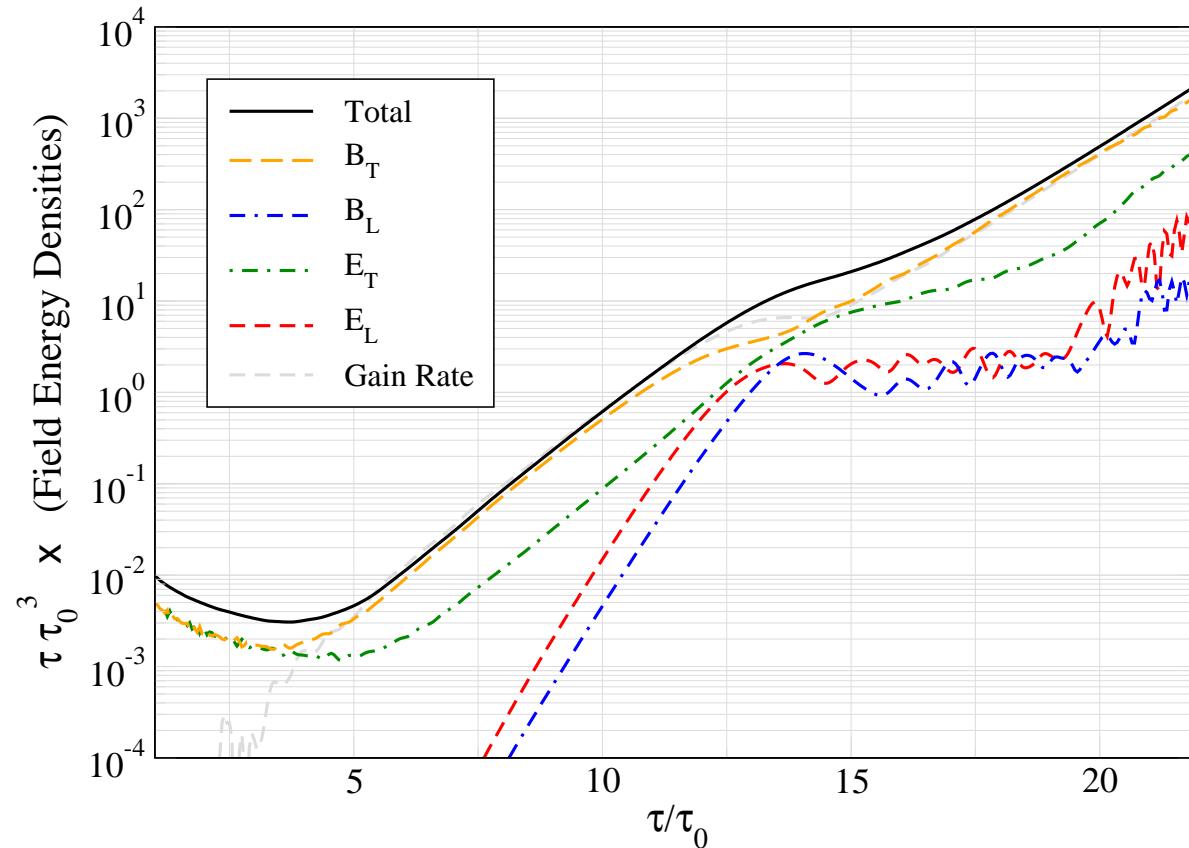
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Plasma

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Conclusions



The proper-time dependence of the chromo-field energy densities and the energy gain rate times an extra factor of  $\tau_0$  resulting from non-Abelian run initialized with Fukushima, Gelis, and McLerran (FGM) initial conditions.



# Expanding 1D+3V non-Abelian plasma

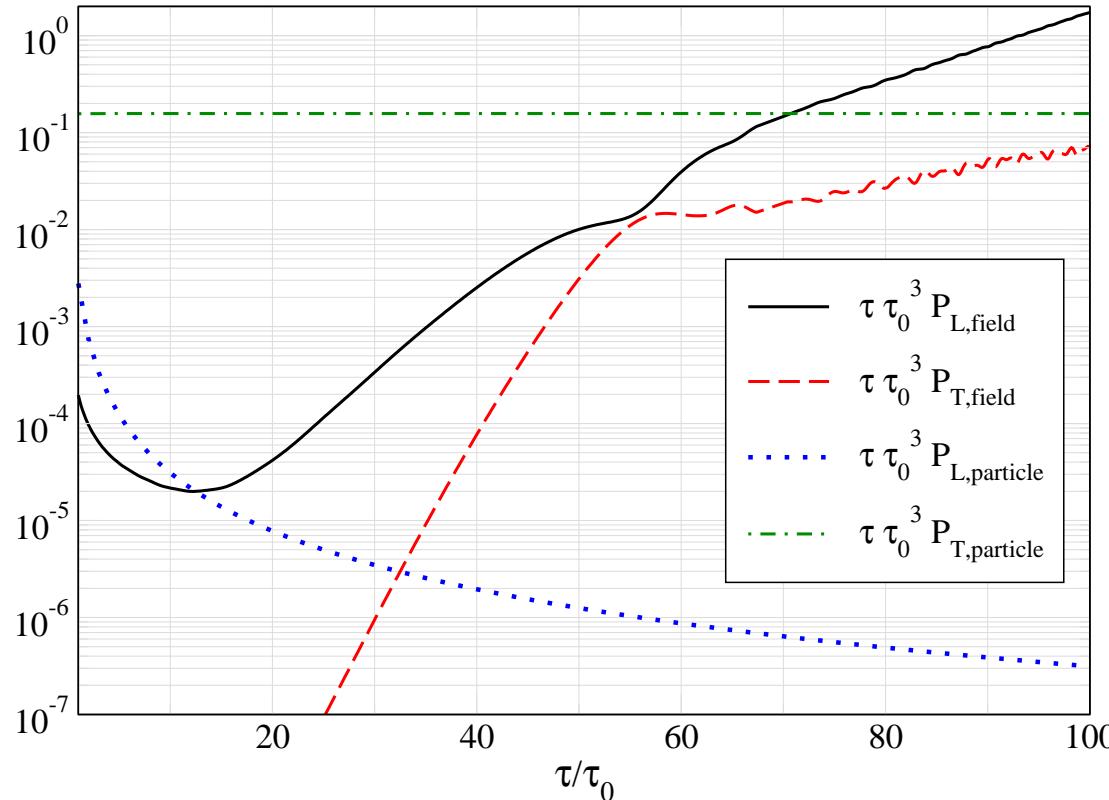
Quark-gluon  
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The comparison of the longitudinal and transverse pressures for the fields and particles resulting from a typical non-Abelian run initialized with FGM (CGC inspired) initial conditions.



# Expanding 1D+3V Abelian plasma

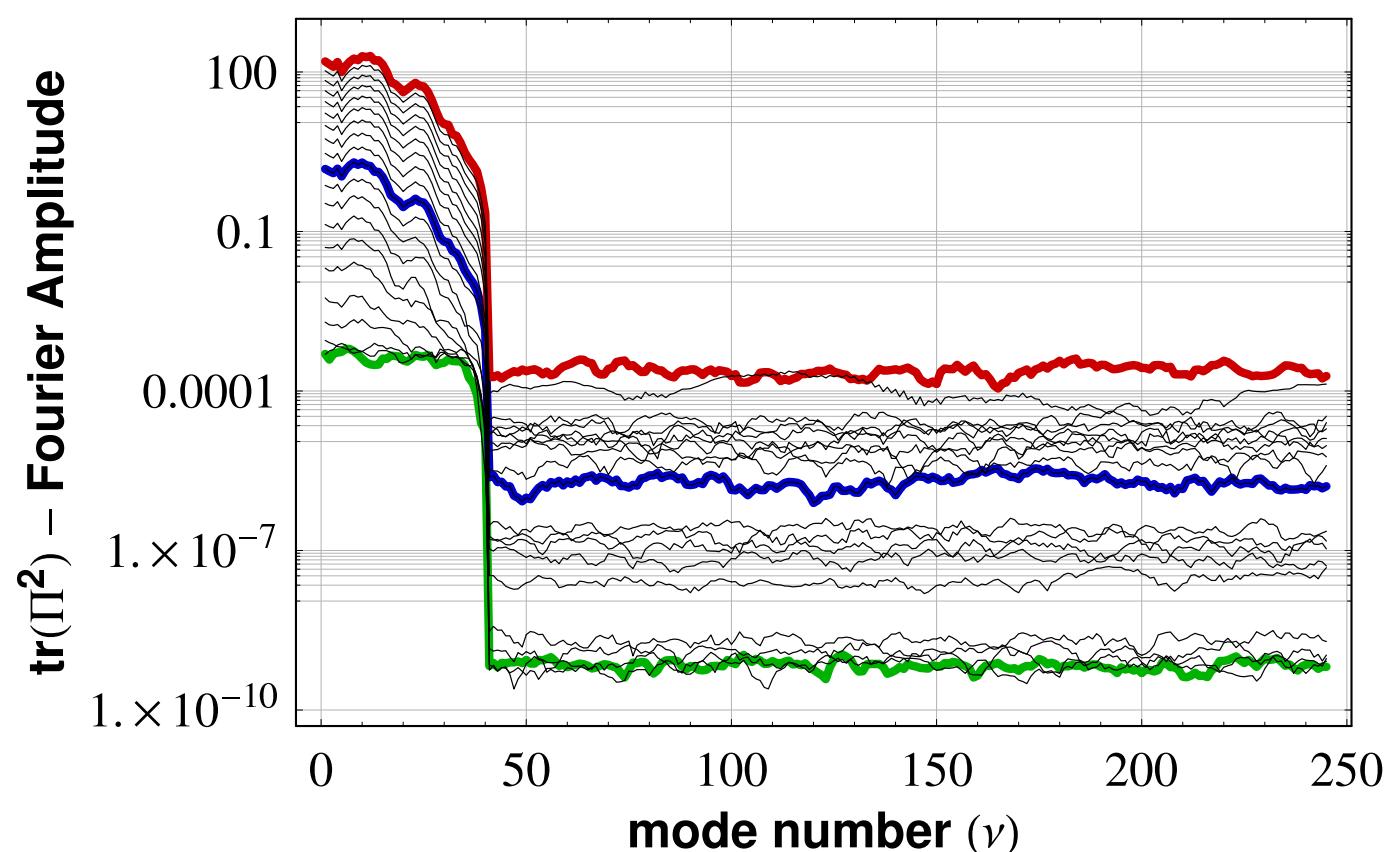
Quark-gluon  
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Fourier spectrum of the color-traced conjugate field momentum obtained from Abelian run with FGM initial conditions.

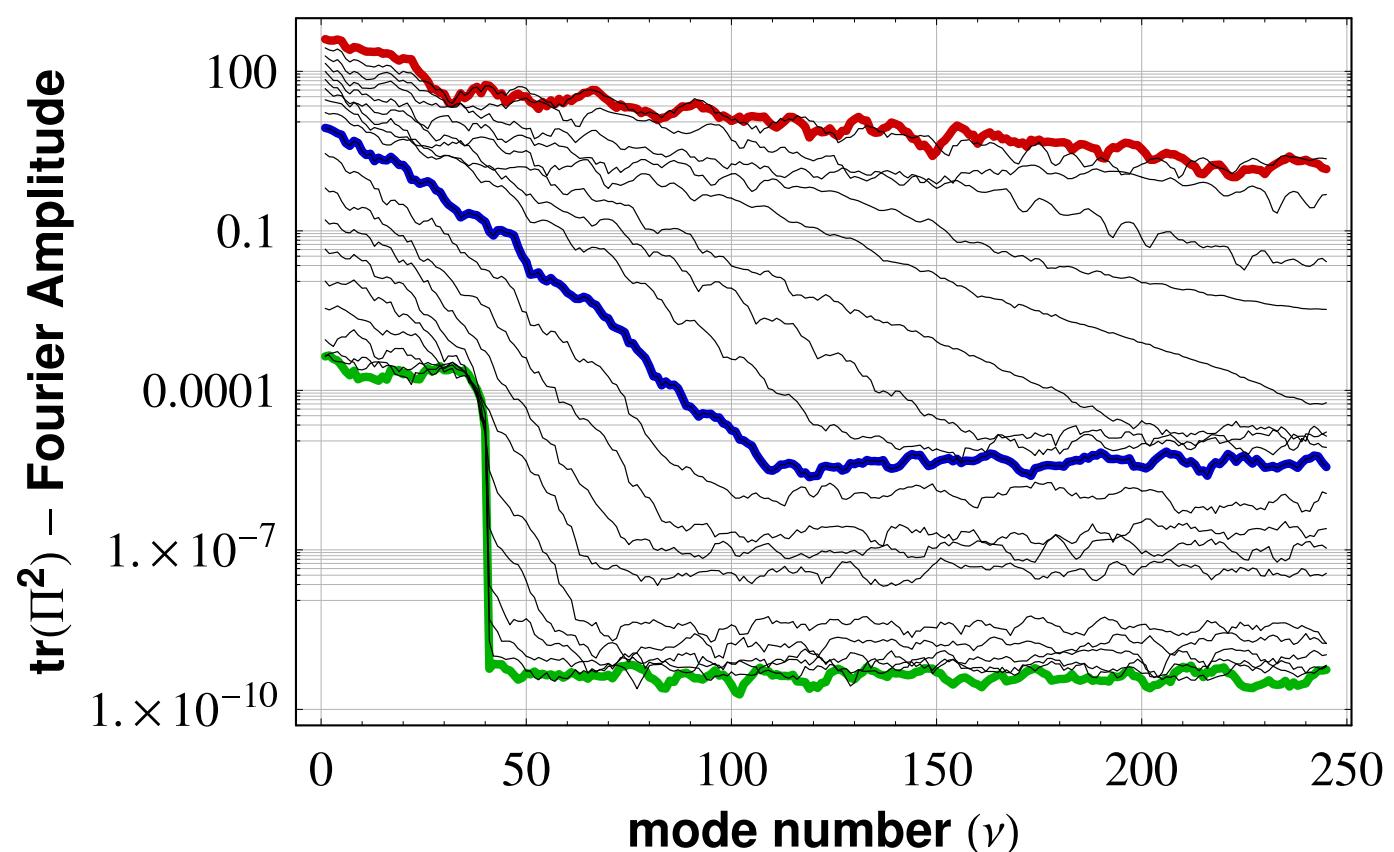


# Expanding 1D+3V non-Abelian plasma

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Fourier spectrum of the color-traced conjugate field momentum obtained from non-Abelian run with FGM initial conditions.



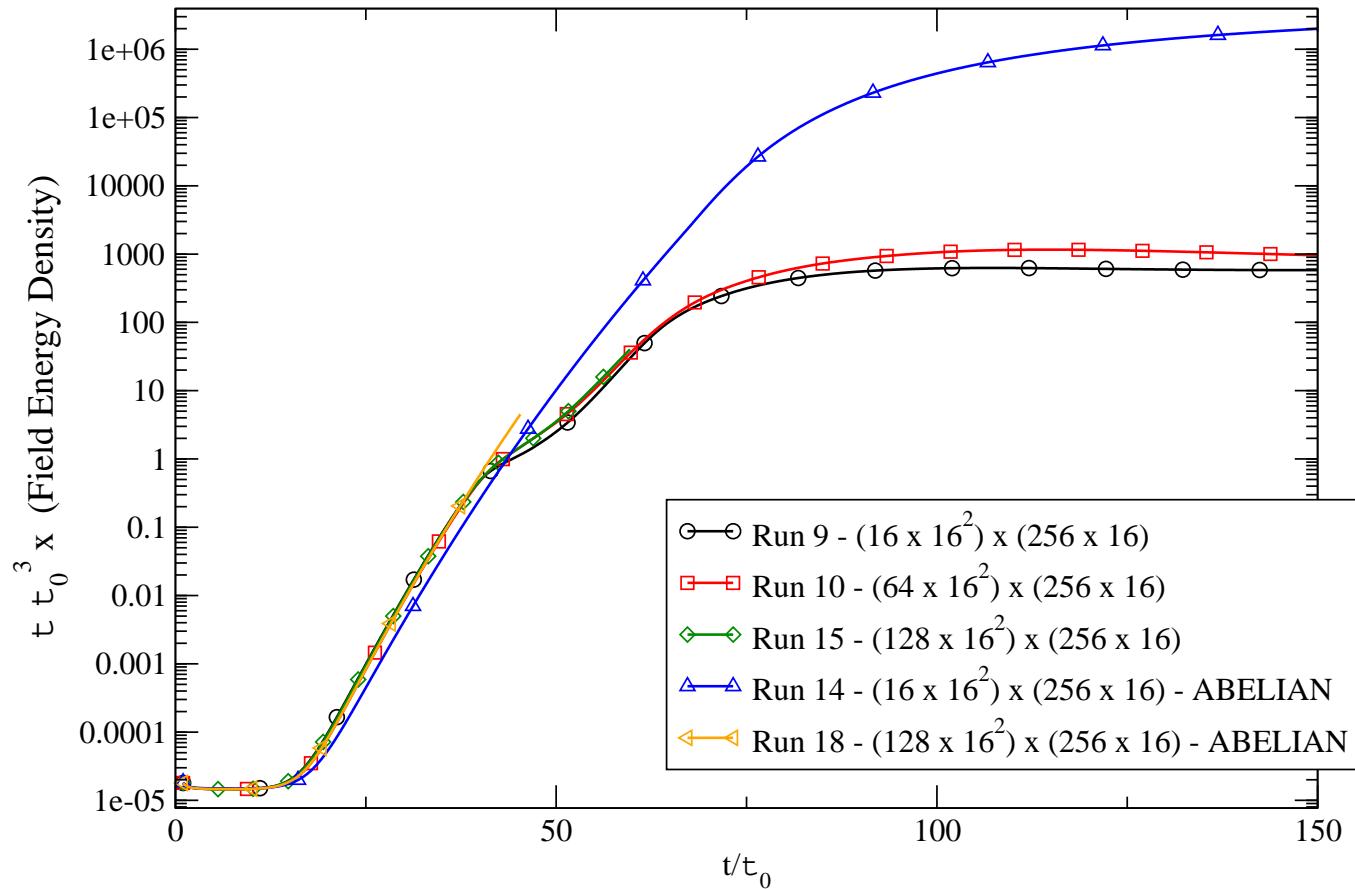
# Expanding 3V plasma

Quark-gluon  
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Conclusions



Preliminary runs from the HEL 3d codes in Abelian and non Abelian setup with different lattice sizing's in the longitudinal  $\eta$  direction, but identical transverse size and  $\mathcal{W}$  auxiliary field numbers.



# Expanding 3V non-Abelian plasma

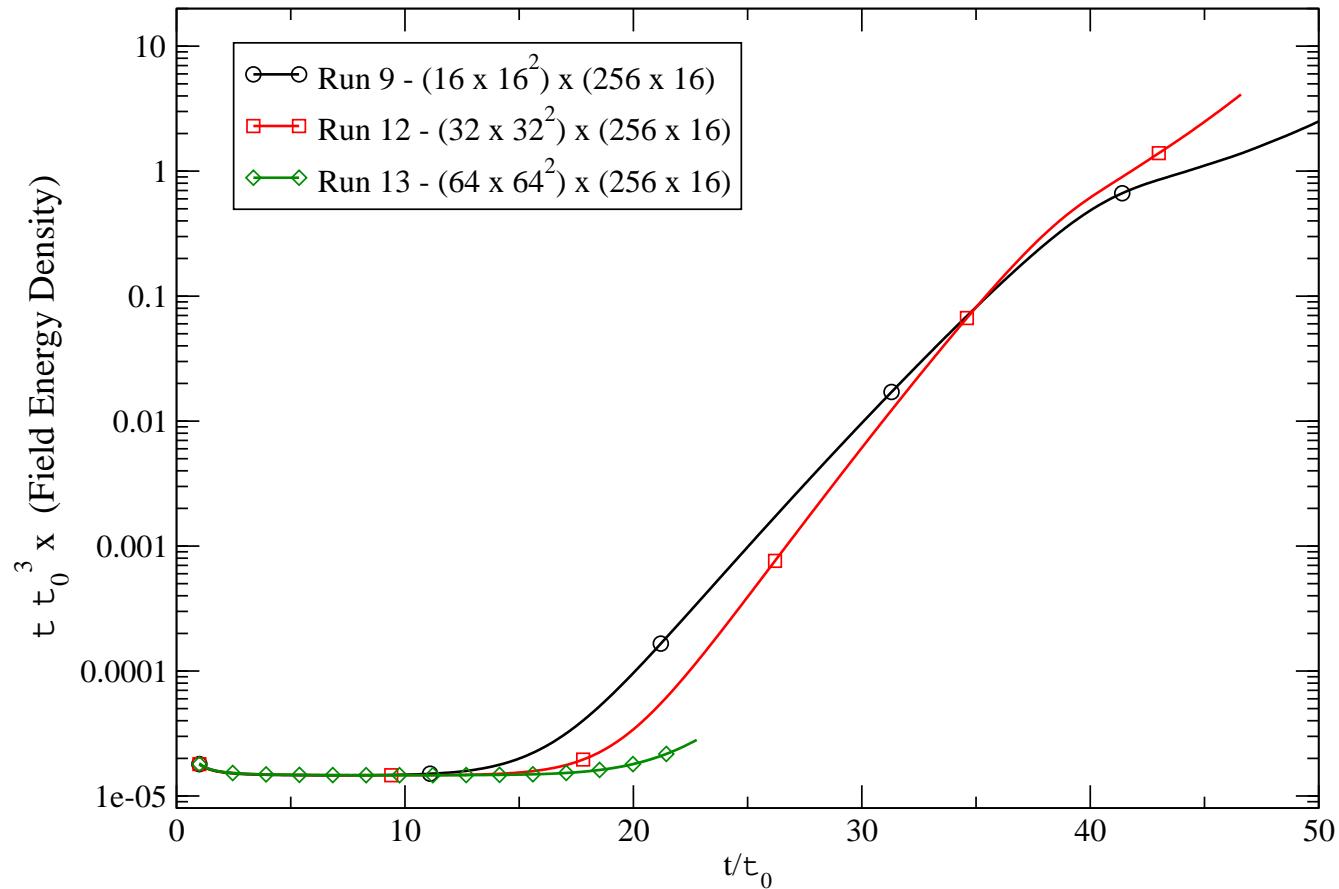
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Preliminary runs from the HEL 3d code with different lattice sizing going from  $16^3$  to  $64^3$  and thus different transverse lattice sizing showing the proper time dependence of the respective total chromo-field energy density.

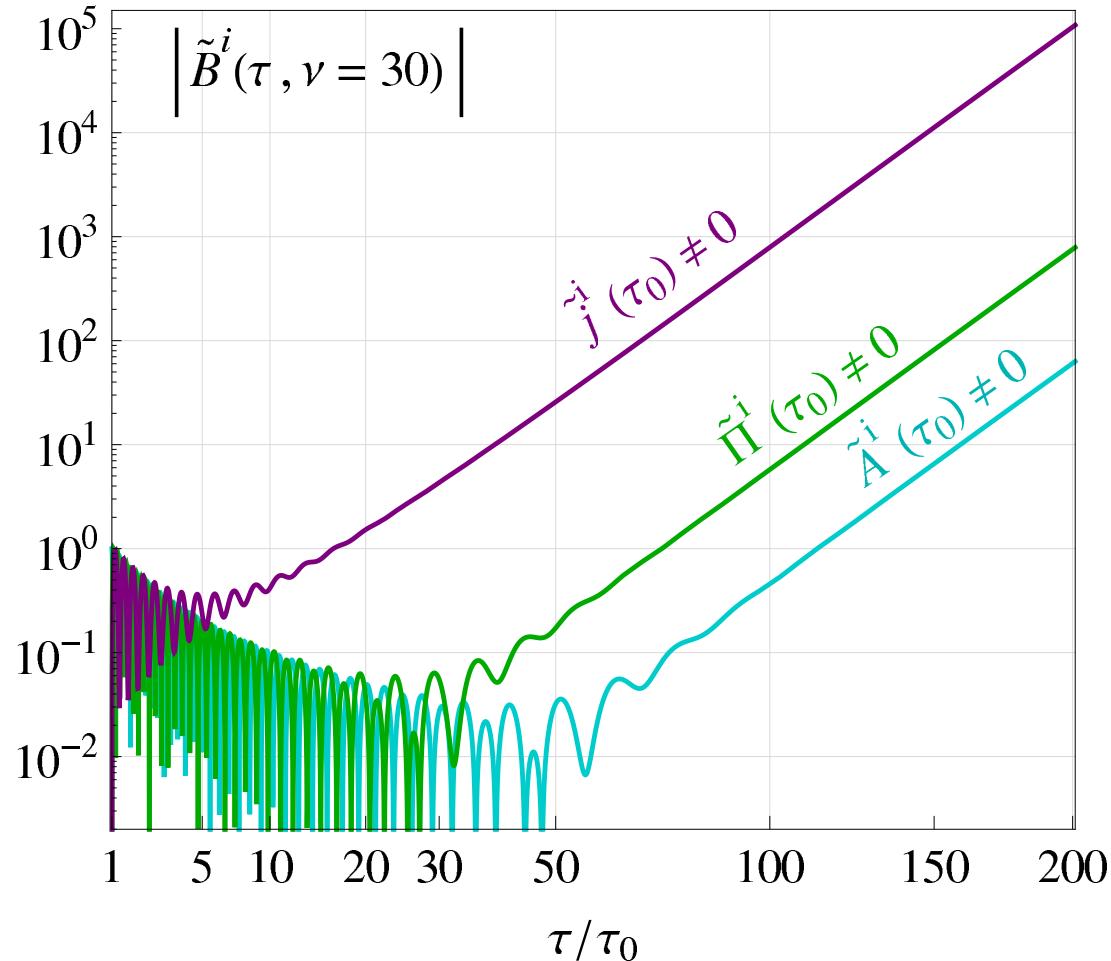


# Unstable transverse modes

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Influence of different initial conditions for a specific mode with  $\nu = 30$



# Expanding 1D+3V non-Abelian plasma

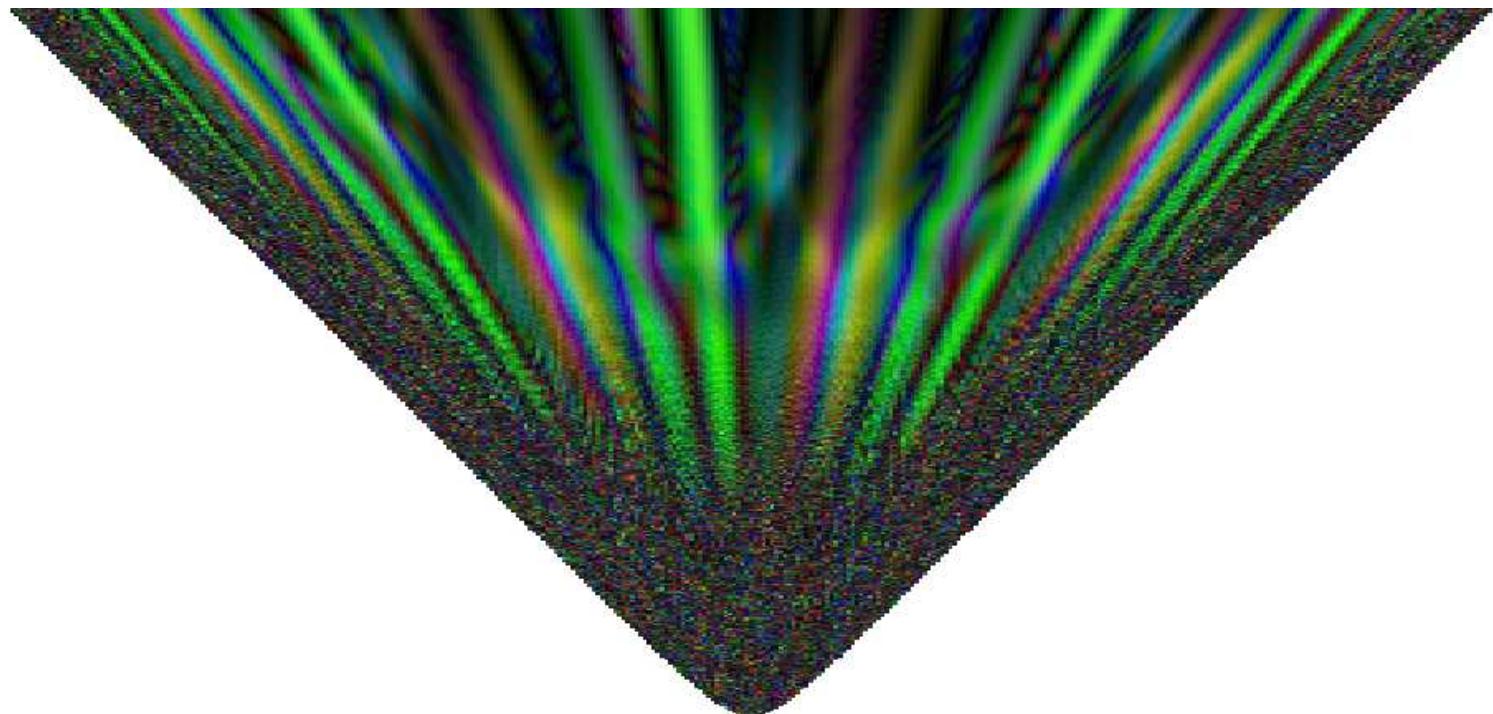
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Visualization of the space-time development of color correlations in a non-Abelian plasma instabilities in Bjorken expansion.



## Conclusions

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Non-abelian plasma instabilities accelerate isotropization and thermalization of the Quark Gluon Plasma.

Large amplitude turbulent field configurations can have an important effect on Quark Gluon Plasma transport such as momentum broadening, energy loss, plasma viscosity, ...

In the 1D+3V Hard Expanding Loop (HEL) 1D we found that the exponential (in  $\sqrt{\tau}$ ) growth in the Abelian (weak-field) phase is only mildly weakened when nonlinearities through non-Abelian self-interactions of the collective fields set in.

The previous 1D HEL code has been extended to full 3D+3V. Final results including different initial conditions are being computed.



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# Thank you.



# Backup - Equation of motions

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## Conjugate Momenta

$$\partial_\tau E_i = +\tau j^i + \frac{1}{\tau} D_\eta^2 A^i + \tau g^2 i[A^{j \neq i}, i[A^{j \neq i}, A^i]] \quad (17)$$

$$\partial_\tau E^\eta = -\tau j^\eta + \frac{ig}{\tau} [A^i, D_\eta A^i] \quad (18)$$

## Gauss law

$$j^\tau = +\frac{1}{\tau} D_\eta E^\eta - \frac{ig}{\tau} [A_i, E^i] \quad (19)$$

with

$$E^i \equiv \tau \partial_\tau A_i, \quad E^\eta \equiv \frac{1}{\tau} \partial_\tau A_\eta \quad (20)$$

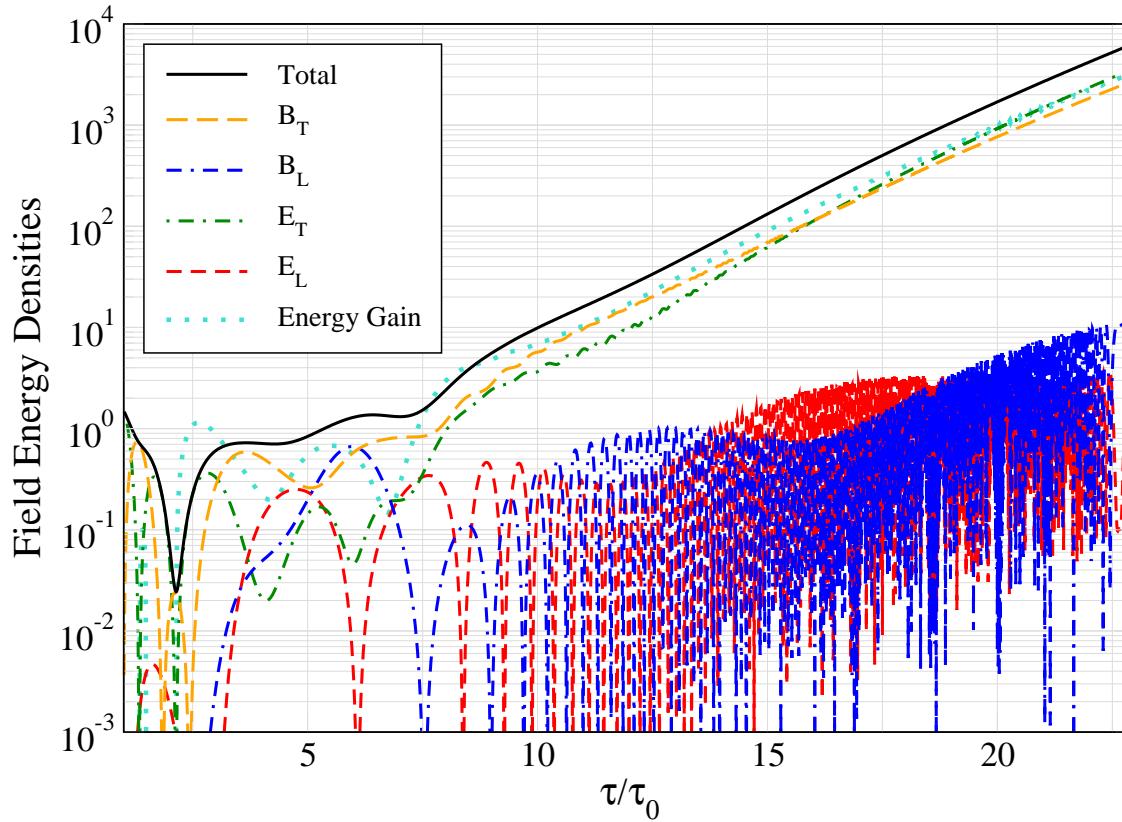


# Backup - Expanding 1D+3V non-Abelian plasma

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The proper-time dependence of the chromo-field energy densities from a run with a single non-Abelian mode seeded with random noise.

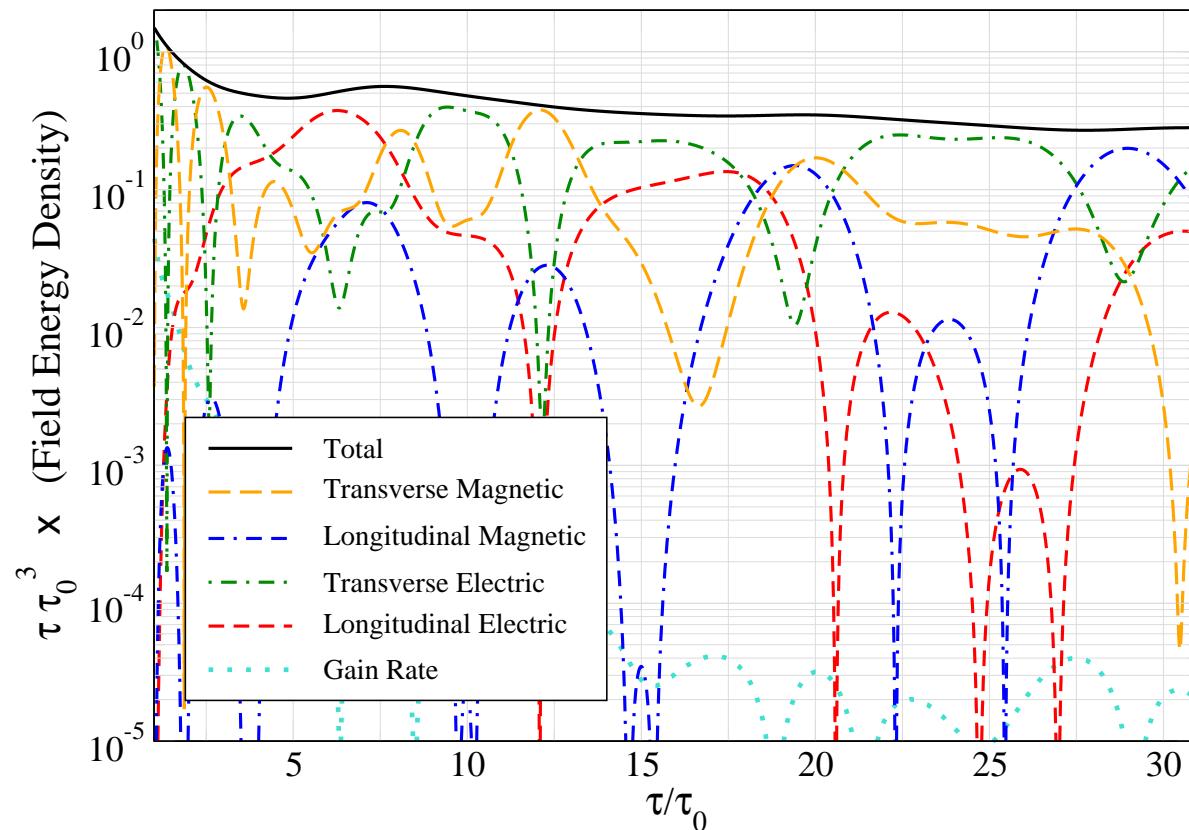


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The proper-time dependence of the chromo-field energy densities with a single non-Abelian mode with decoupled hard particle currents ( $j = 0$ ).

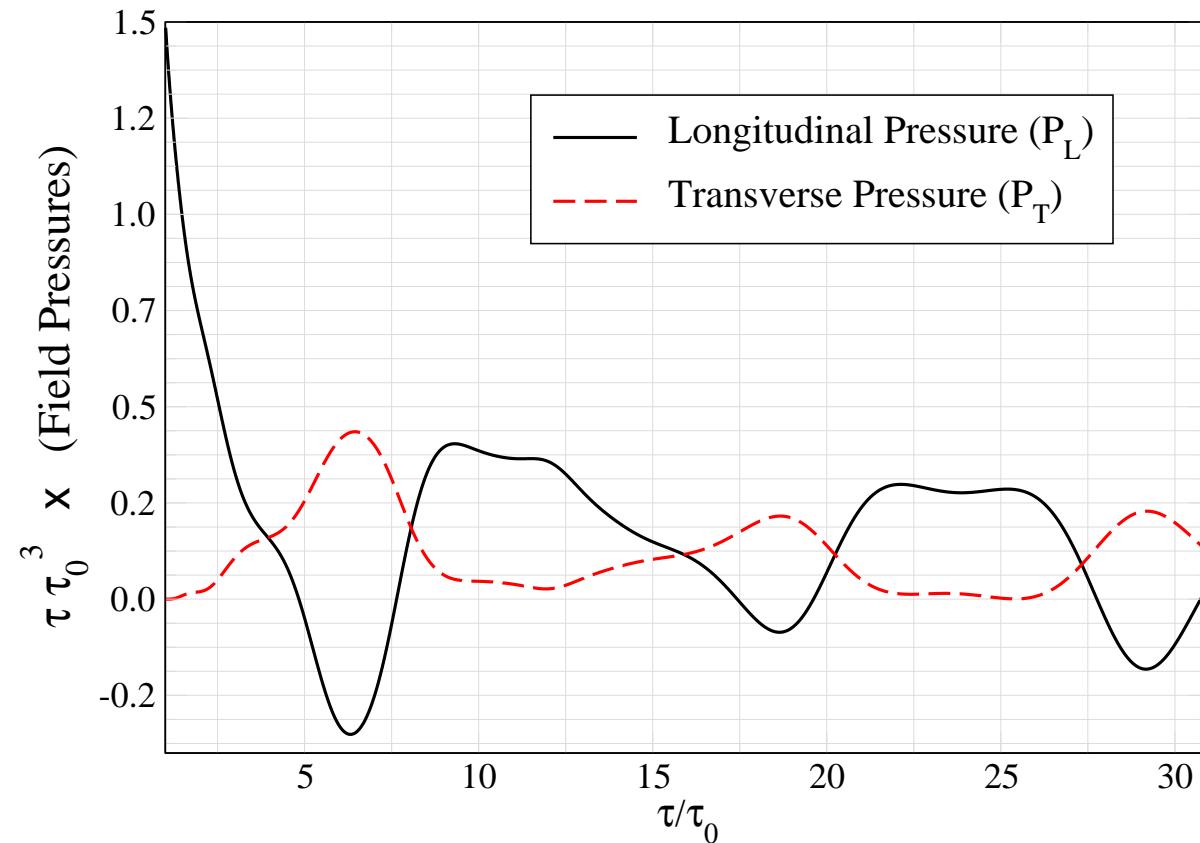


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The proper-time dependence of the longitudinal and transverse pressure with a single non-Abelian mode with decoupled hard particle currents ( $j = 0$ ).