Über eine universelle asymptotische Theorie turbulenter Strömungsablösung

On a universal asymptotic theory of turbulent flow separation

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TECHNISCHE UNIVERSITÄT WIEN
Vienna University of Technology

Habilitation colloquium, 7th November 2011

At first: special thanks go to . . .

. . . my mentor and long-term collaborator, Professor Alfred Kluwick,

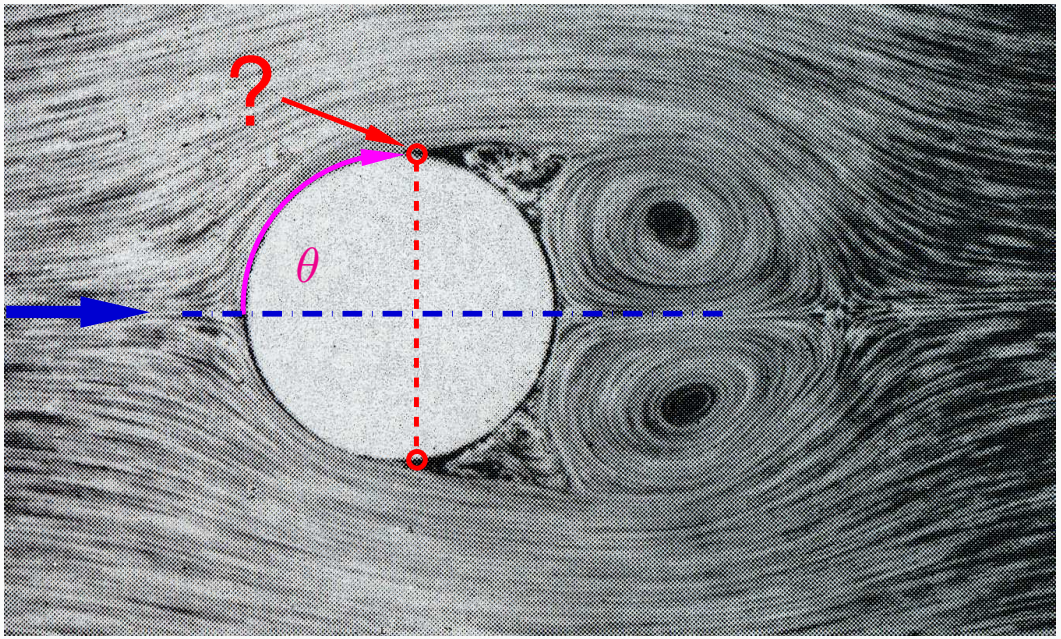
. . . Stefan Braun and Herbert Steinrück for many fruitful discussions,

. . . last not least, all other colleagues at the institute.

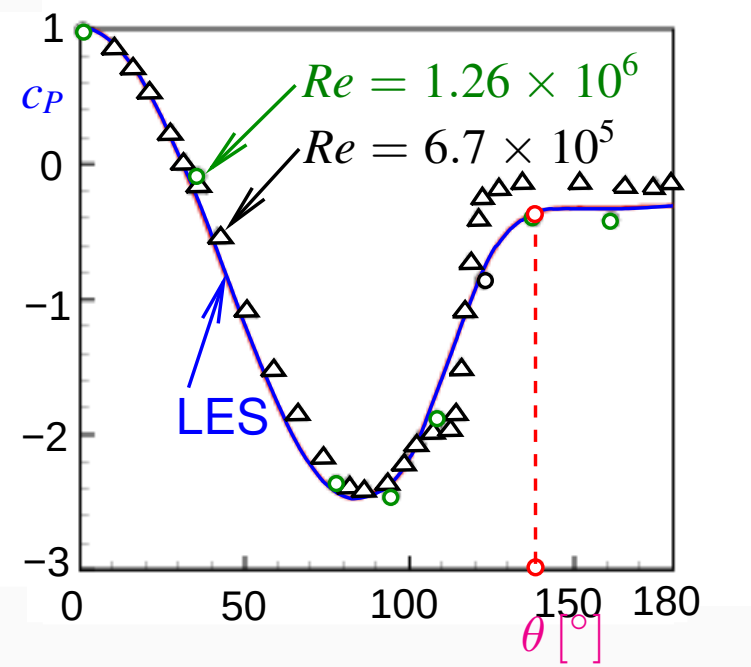
Some introductory remarks on a long-standing problem

Sandborn & Liu (JFM 32/2, 1968)

"Turbulent boundary-layer separation is normally listed as one of the most important unsolved problems in fluid mechanics . . ."



Prandtl (Roy. Aeron. Soc. XXXI, 1927)



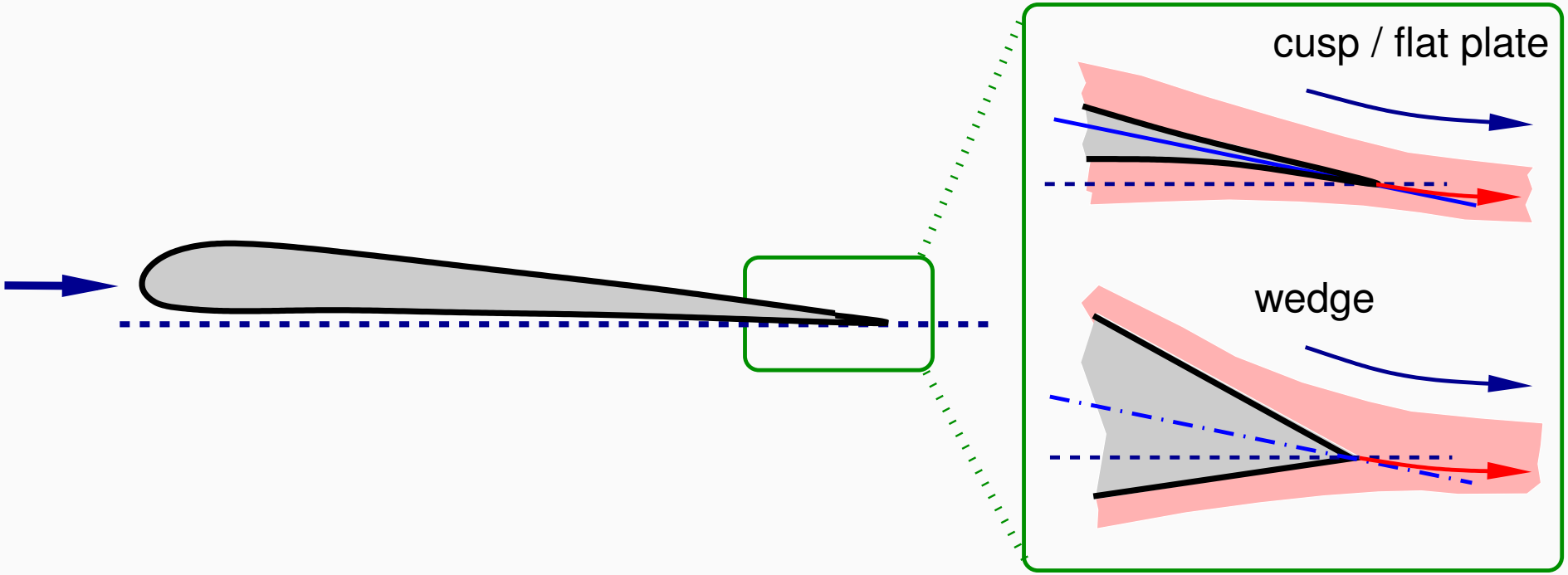
Wang, Catalano & Iaccarino
(*CTR Annual Research Briefs 2001*)

Some introductory remarks on a long-standing problem

Melnik & Grossman (*Numerical and Physical Aspects of Aerodynamic Flows*, 1982)

"Progress . . . will go a long way toward the establishment of rational asymptotic methods for trailing-edge separation."

$Re \to \infty$: Kutta condition



Overview

1. Motivation: flow past blunt body
2. Basic asymptotic concept
3. Marginal (internal, localised) separation
   - Two-parameter asymptotics
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Another long-standing problem: high-Re bluff-body drag

typical: circular cylinder / sphere with radius $\tilde{R}$, drag $\tilde{D} = ?$

$$c_D := \frac{\tilde{D}/(\tilde{R}^2\pi)}{\tilde{\rho}_\infty \tilde{U}_\infty^2/2}\,, \quad Re := \frac{2\tilde{R}\tilde{U}_\infty}{\tilde{\nu}}\,, \quad M = \frac{\tilde{U}_\infty}{\tilde{c}_\infty}$$

dimensional analysis $c_D(Re, M)$

FIFA WC 2010: relatively smooth ball, lowered $\tilde{\rho}_\infty$

cylinder

sphere

Anderson (*Fundamentals of Aerodynamics*, 1991)

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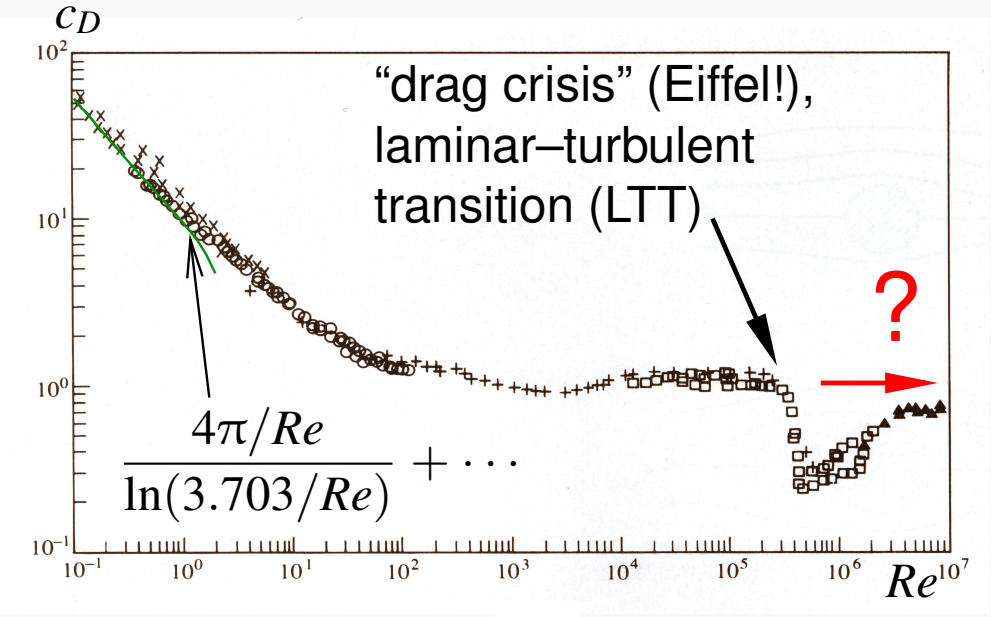

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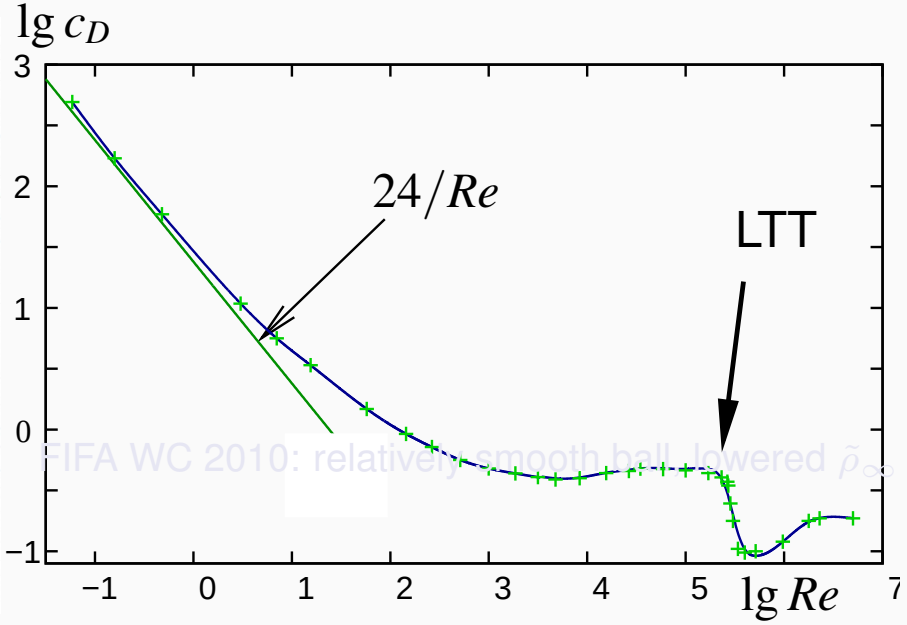

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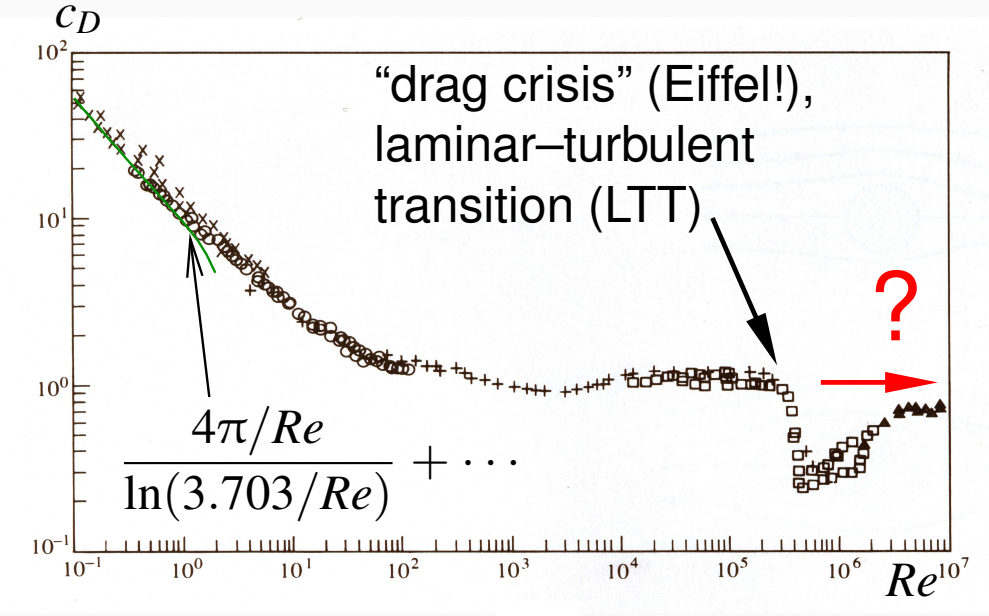

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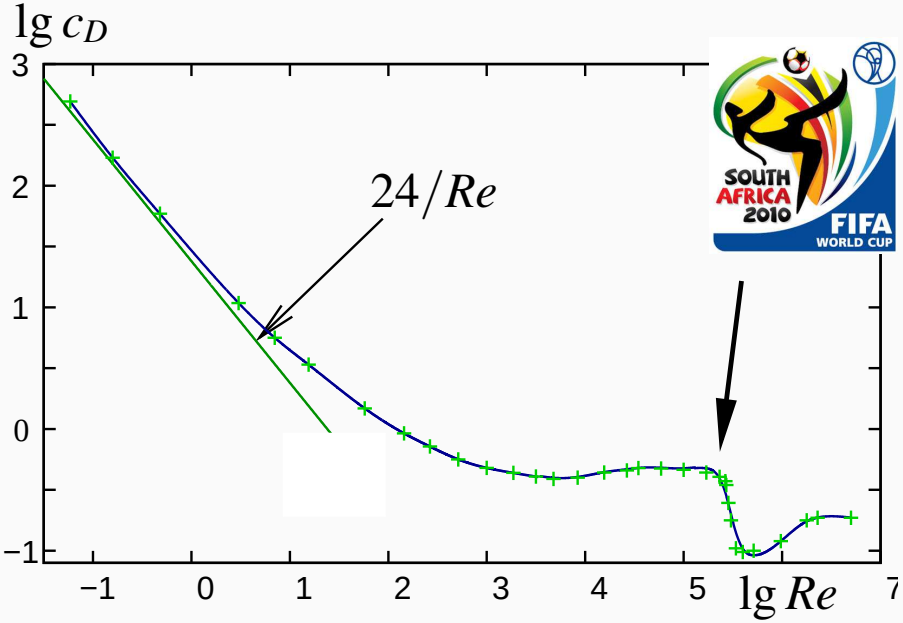

FIFA WC 2010: relatively smooth ball, lowered $\tilde{\rho}_\infty$

. . . and a closely related one: high-Re bluff-body vortex shedding

self-sustained vortex detachment of frequency $\tilde{f}$

Strouhal number $St := \dfrac{2\tilde{R}\tilde{f}}{\tilde{U}_\infty}$

dimensional analysis $St(Re)$

$Re \gg 1$: data scatter, no definite value of St !

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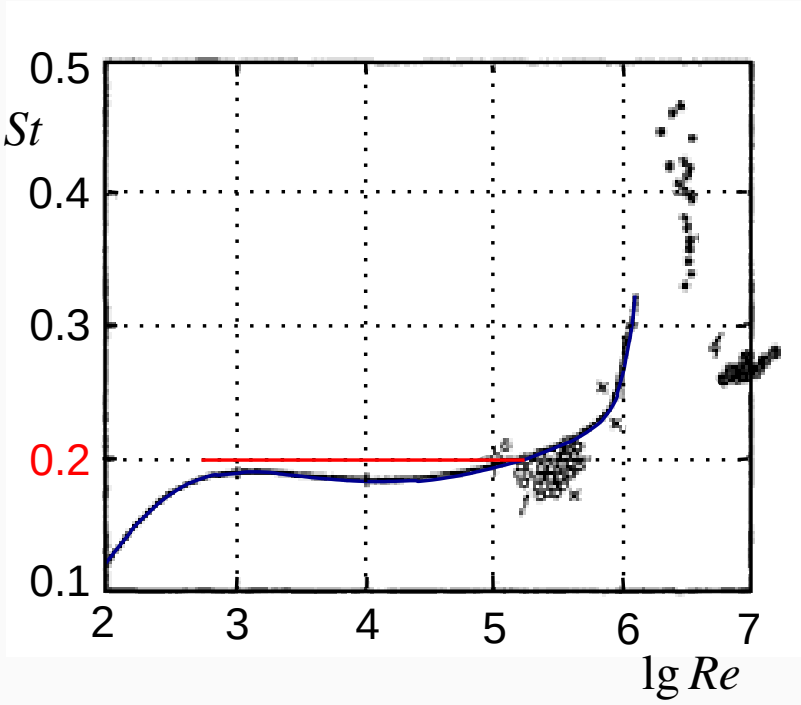

Popov (Fluid Dyn. 1/2, 1966)

$Re \gg 1$: data scatter, no definite value of St !

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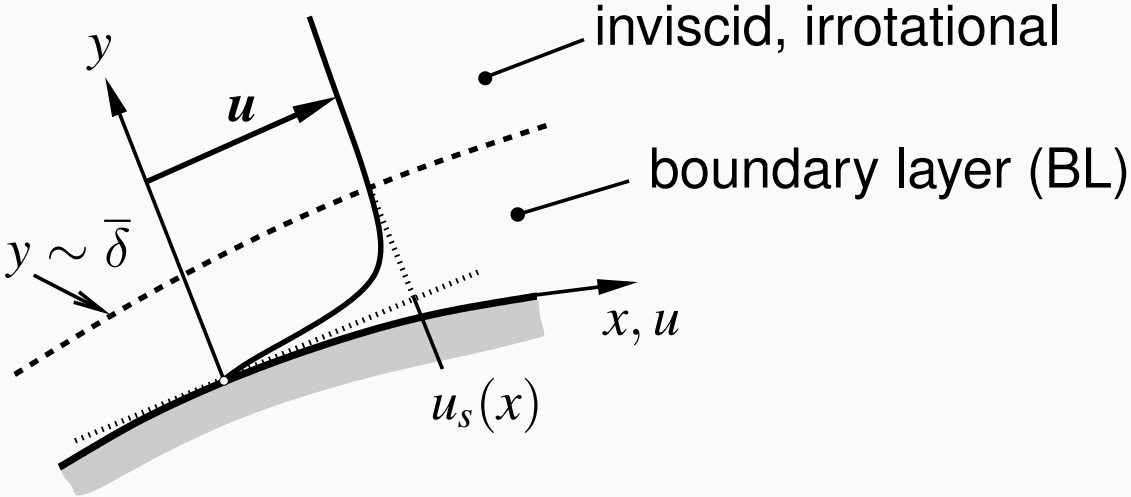
Wall-bounded flows

$$Re := \tilde{L}\tilde{U}/\tilde{\nu} \gg 1$$



Navier–Stokes eqs ($\rho = const$)

$$\nabla \cdot \boldsymbol{u} = 0\,, \quad \partial \boldsymbol{u}/\partial t + \boldsymbol{u} \cdot \nabla \boldsymbol{u} = -\nabla p + Re^{-1} \nabla^2 \boldsymbol{u}\,, \quad \boldsymbol{u}|_{y=0} = \boldsymbol{0}$$

Ensemble-averaging, ergodic hypothesis (nominally steady 2D flow)

$$\boldsymbol{Q}(\boldsymbol{x}, t; Re) = \overline{\boldsymbol{Q}}(x, y; Re) + \boldsymbol{Q}'(\boldsymbol{x}, t; Re)\,, \quad \overline{\boldsymbol{Q}} := \lim_{t_{\mathrm{av}} \to \infty} \frac{1}{t_{\mathrm{av}}} \int_{-t_{\mathrm{av}}/2}^{t_{\mathrm{av}}/2} \boldsymbol{Q}(\boldsymbol{x}, t + t'; Re)\, \mathrm{d}t'$$

Reynolds eqs

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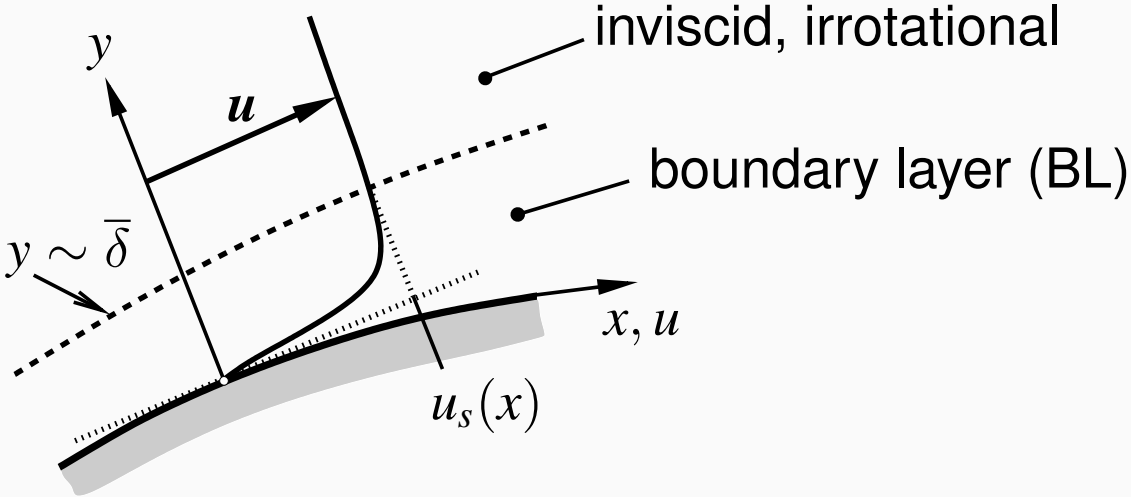

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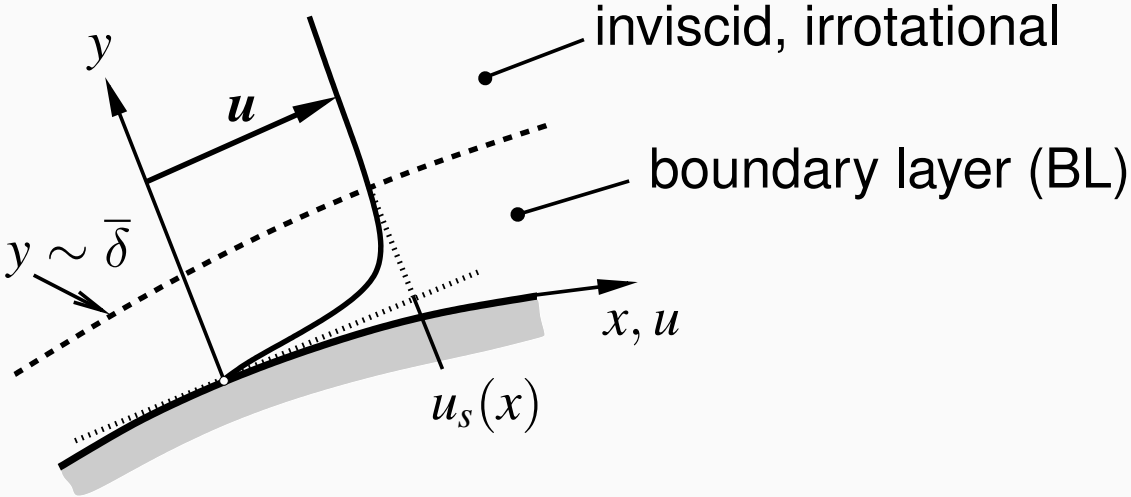

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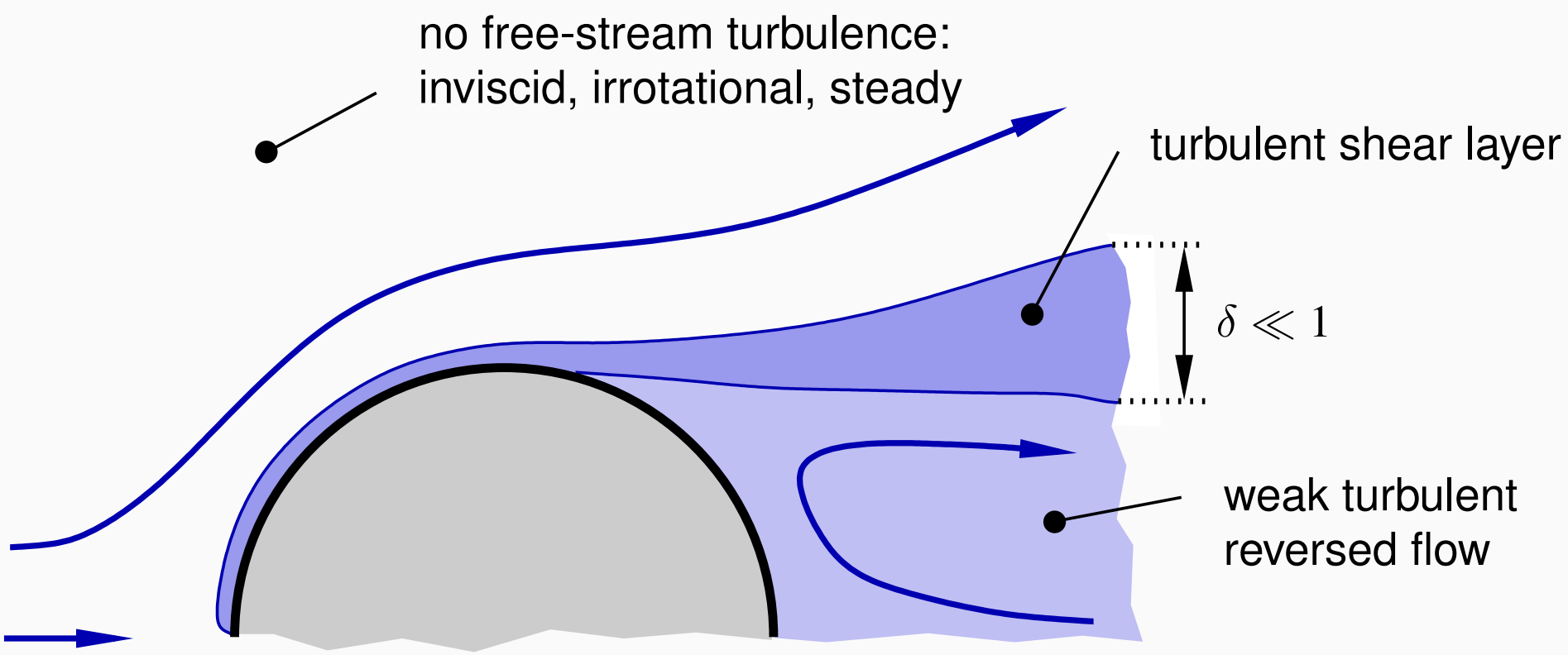
Global flow picture

$Re \to \infty$

Scheichl & Kluwick (J. Fluids Struct. 24/8, 2008),

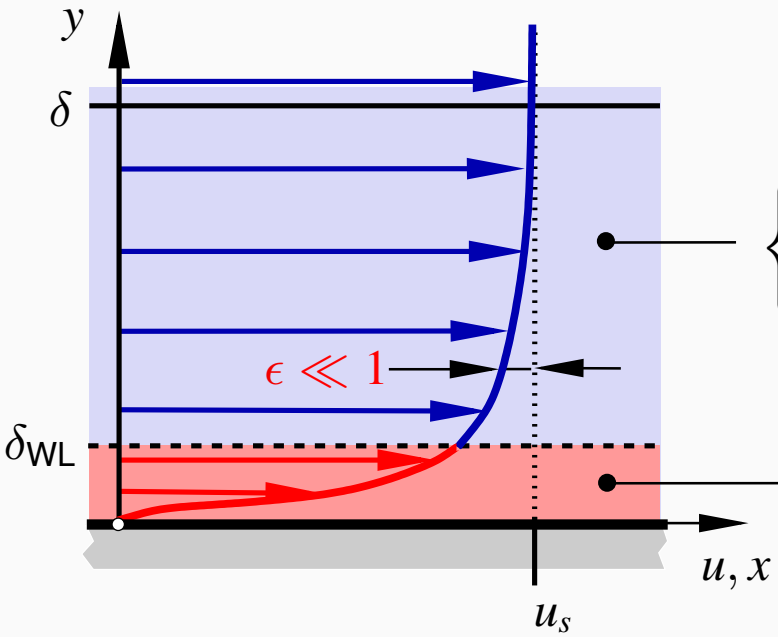
Scheichl, Kluwick & Alletto (Acta Mech. 201/1–4, 2008)



Small-defect BLs

Kluwick & Scheichl (*BAIL 2008*, 2009), Scheichl & Kluwick (Theor. Comp. Fluid Dyn., submitted 2011)

cf. Yajnik (JFM 42/2, 1970), Bush & Fendell (JFM 56/4, 1972), Mellor (Int. J. Eng. Sci. 10/10, 1972)



outer predominantly inviscid region

viscous wall layer

overlap, $y^+ := y u_\tau Re \to \infty$

$$\frac{u}{u_\tau} \sim \frac{1}{\kappa} \ln y^+ + C^+$$

$$u\frac{\partial u}{\partial x} + \cdots \sim -\frac{\partial \overline{u'v'}}{\partial y}$$

hypothesis: $\epsilon \sim 1 - u/u_s \sim u_i'$

$$\Rightarrow \delta \sim \epsilon$$

classical: $\epsilon \sim \frac{\sqrt{\tau_w}}{u_s} \sim \frac{\kappa}{\ln Re}$ (1) $\frac{\delta_{\mathrm{WL}}}{\delta} \ldots$ exponentially small (2)

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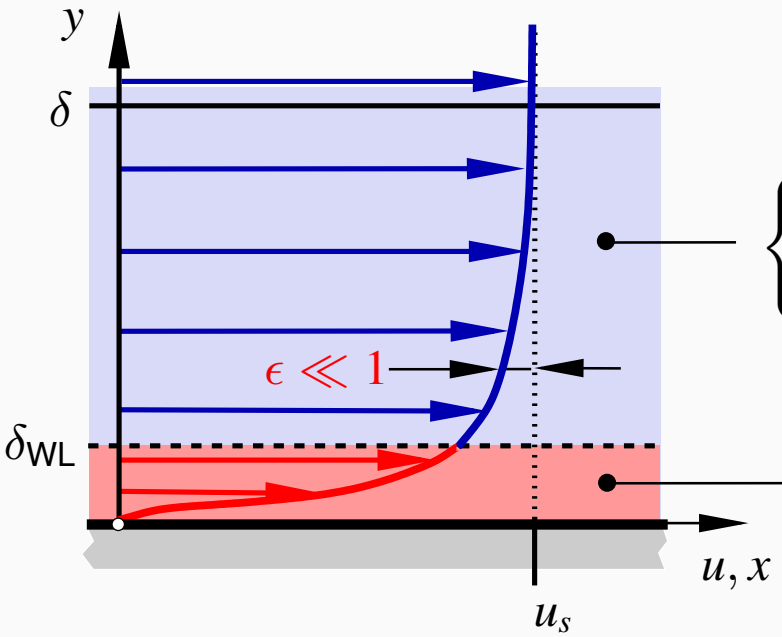

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Large-defect wake-type BLs

Scheichl & Kluwick (AIAA J. 45/1, 2007; IJCSM 1/2–4, 2007),
Smith, Scheichl & Kluwick (J. Eng. Math. 68/3–4, James Lighthill Memorial Paper 2010)
cf. Schneider (J. Flight Sci. Space Res. 15/3, 1991), Melnik (Comput. Fluids 17/1, 1989)

question: $\epsilon \sim 1 - u/u_s \sim 1$ but $\delta \ll 1$?

answer: slenderness empirical !

Goldstein singularity marginal

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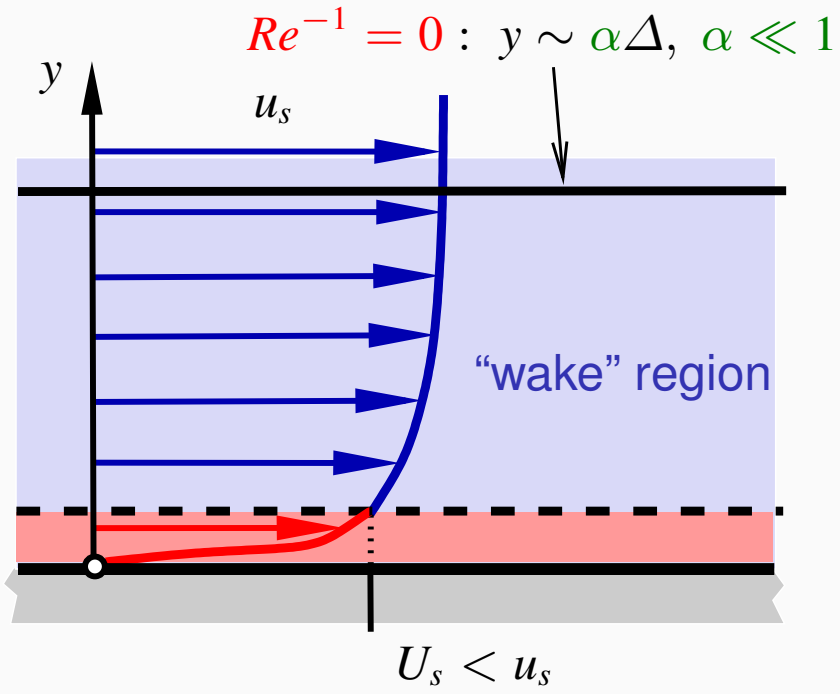

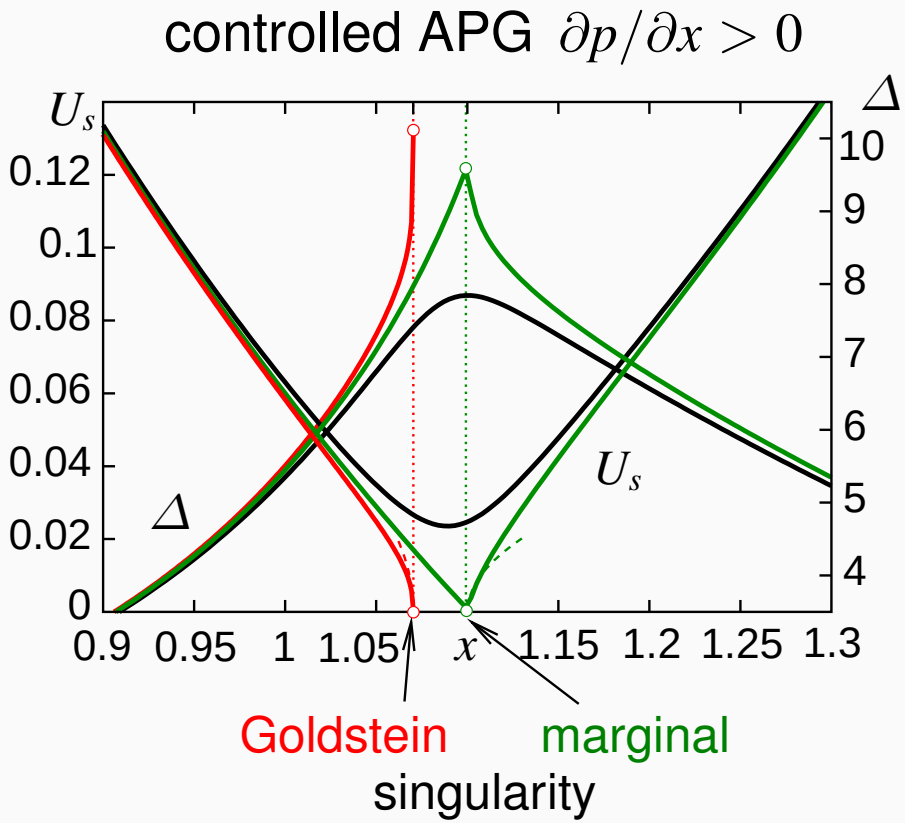

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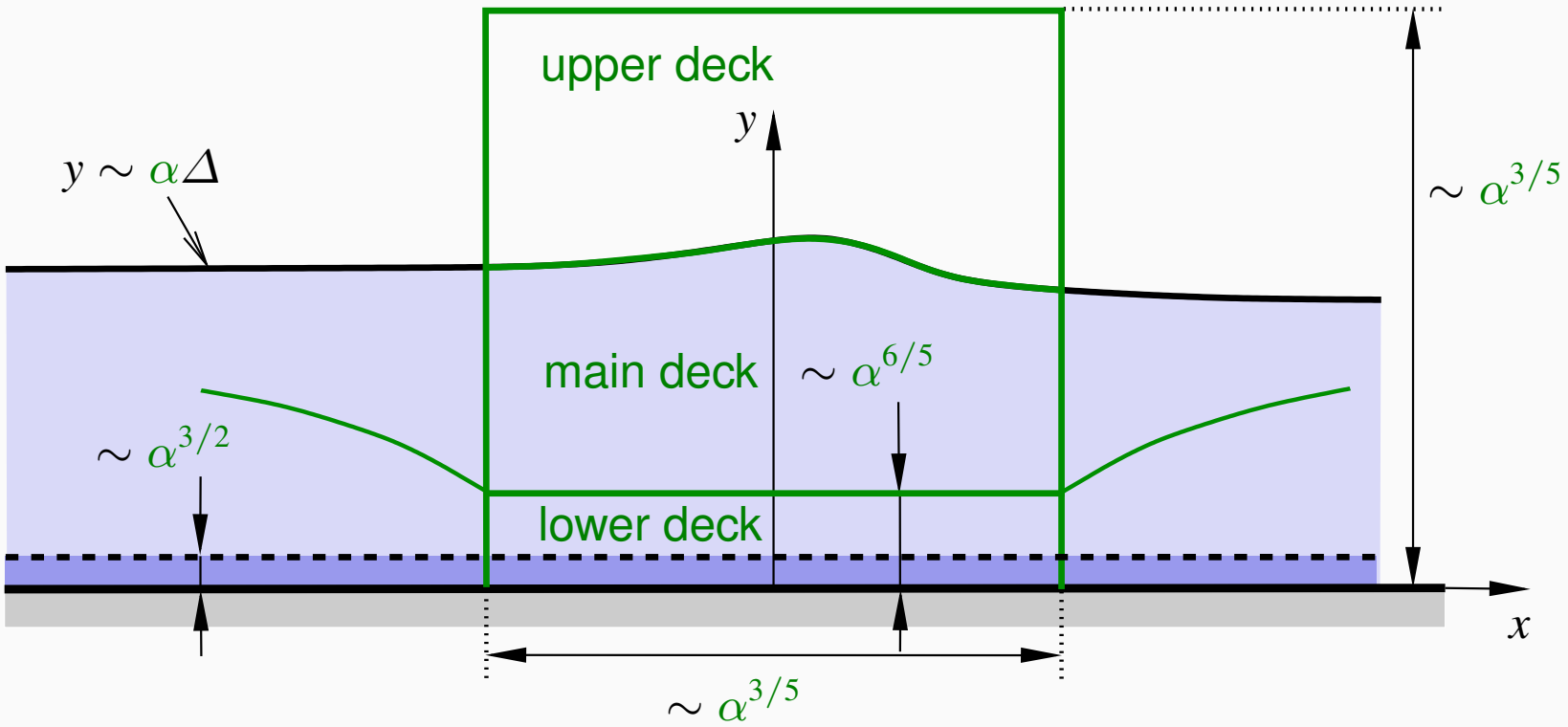
A novel triple deck problem

interaction strategy of marginal separation

BL displacement changes so rapidly that pressure response induced externally is large enough to affect lowest-order BL approximation

$\Rightarrow$ classical asymptotic hierarchy superseded



The condensed separation problem

Lower-deck eqs

lower-deck quantities $\Psi(X,Y)$, $[A,P](X)$, similarity parameter $\Gamma > 0$

$$\frac{\partial\Psi}{\partial Y}\frac{\partial^2\Psi}{\partial Y\partial X} - \frac{\partial\Psi}{\partial X}\frac{\partial^2\Psi}{\partial Y^2} = -1 - \Lambda(\Gamma)\frac{\mathrm{d}P}{\mathrm{d}X} + \frac{\partial\Sigma}{\partial Y}, \quad \Sigma = \frac{\partial^2\Psi}{\partial^2 Y}\left|\frac{\partial^2\Psi}{\partial^2 Y}\right|$$

$$P = \frac{1}{\pi}\fint_{-\infty}^{\infty}\frac{A'(S)}{X-S}\,\mathrm{d}S, \quad A = \lim_{Y\to\infty}(\Sigma - Y)$$

$$Y = 0: \quad \Psi = \partial\Psi/\partial Y = 0$$

$$X \to -\infty: \quad \Psi \sim \Psi_{-\infty} := \Gamma Y + \frac{4}{15}Y^{5/2}$$

eigenvalue A_+

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trivial solution by unperturbed flow

$$\Psi \equiv \Psi_{-\infty}, \quad A \equiv P \equiv 0$$

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non-trivial solution matches exponentially weak upstream perturbations

$$X \to \infty: \quad \lim_{X\to\infty}(A/X^{1/3}) \sim A_+ \doteq 1.038600$$

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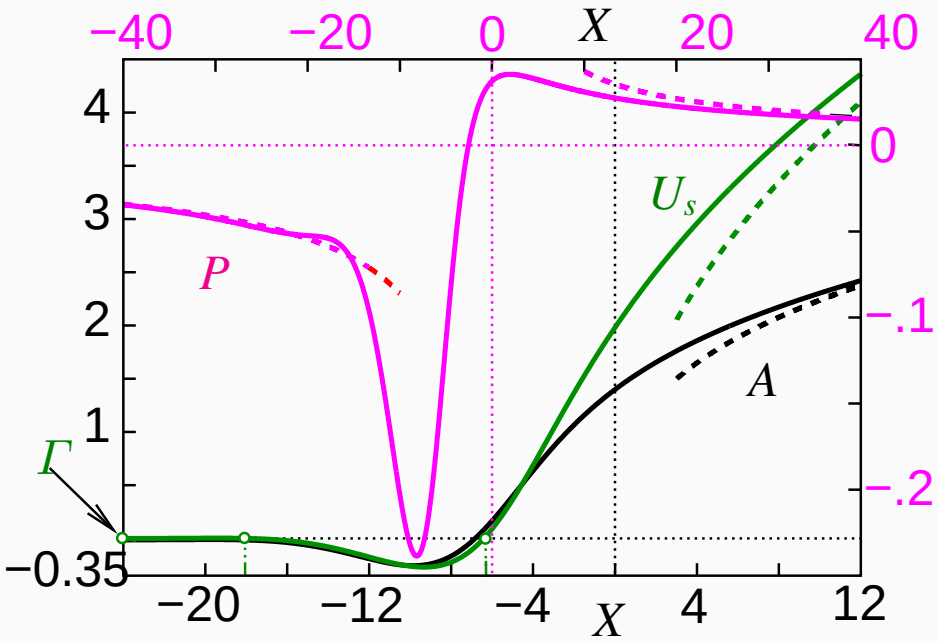
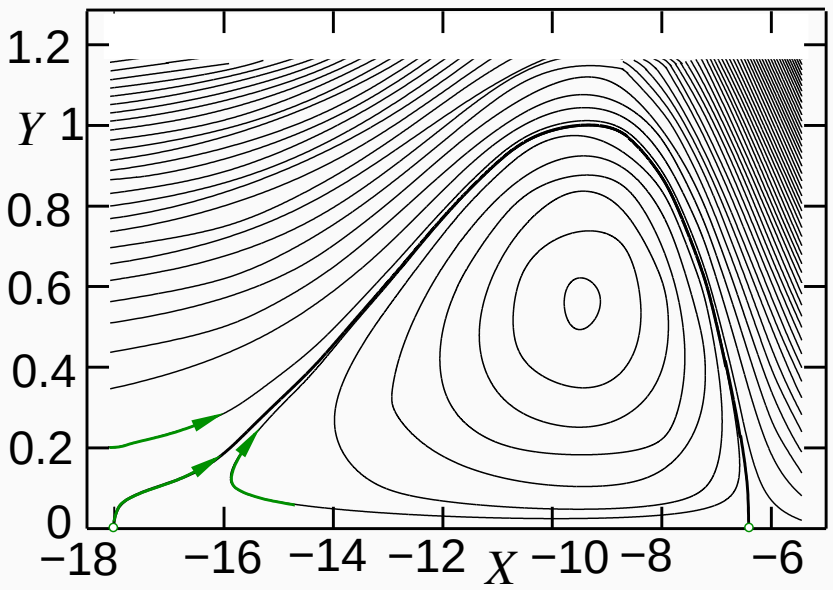
Numerical results
Lower-deck eqs

lower-deck quantities $\Psi(X,Y)$, $[A,P](X)$, similarity parameter $\Gamma > 0$



streamlines $\Psi = const$



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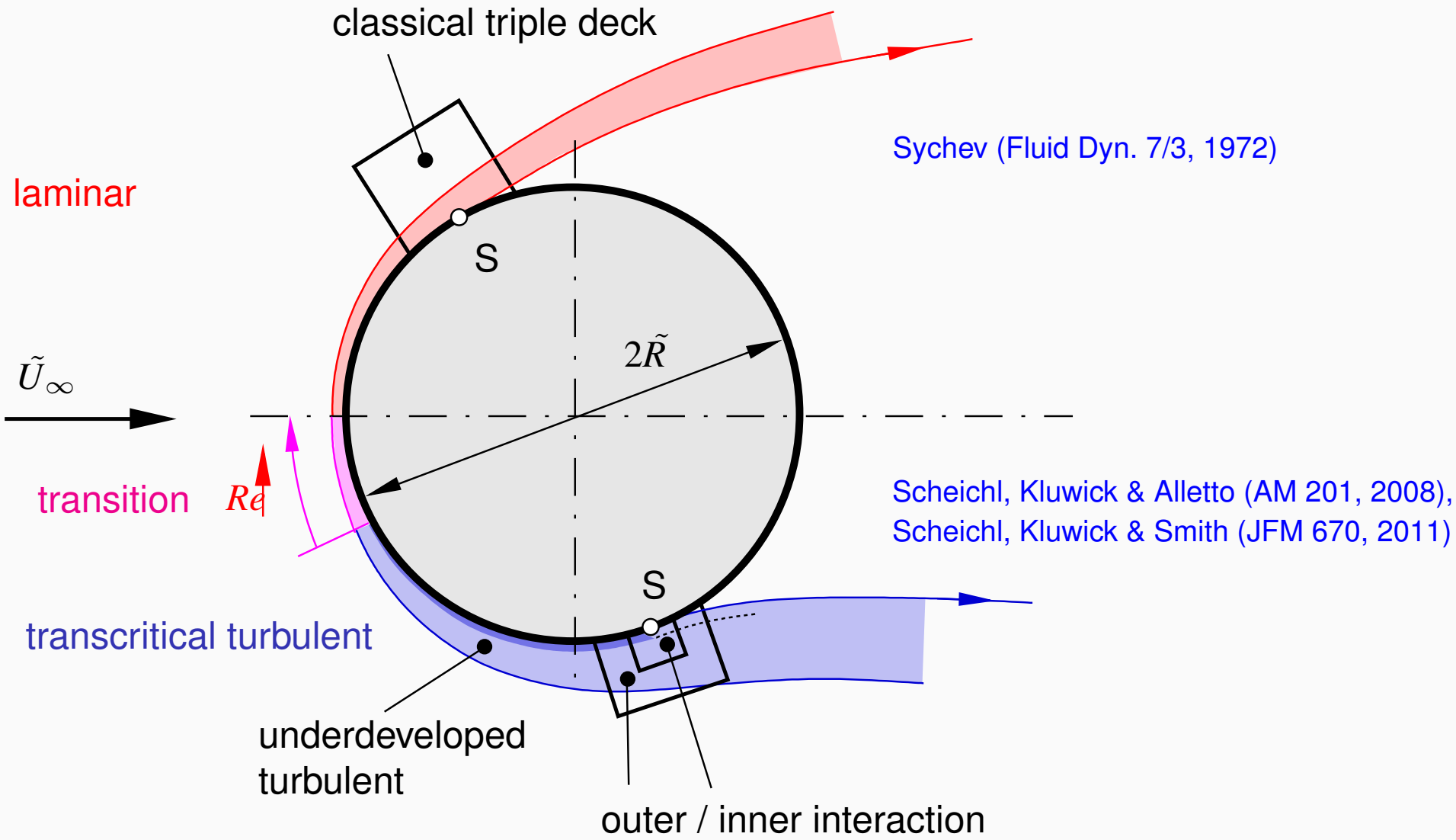
Unravelling the separation riddle

$Re := \tilde{R}\tilde{U}/\tilde{\nu} \to \infty$



cf. Festschrift Alfred Kluwick (ed. S. Braun, 2011)

Helmholtz–Kirchhoff-type flows (open cavity, circulation-free)

surface velocity $u_s(x,k)$

$k = 0,\ 0.05,\ 0.1,\ \ldots,\ 0.5$

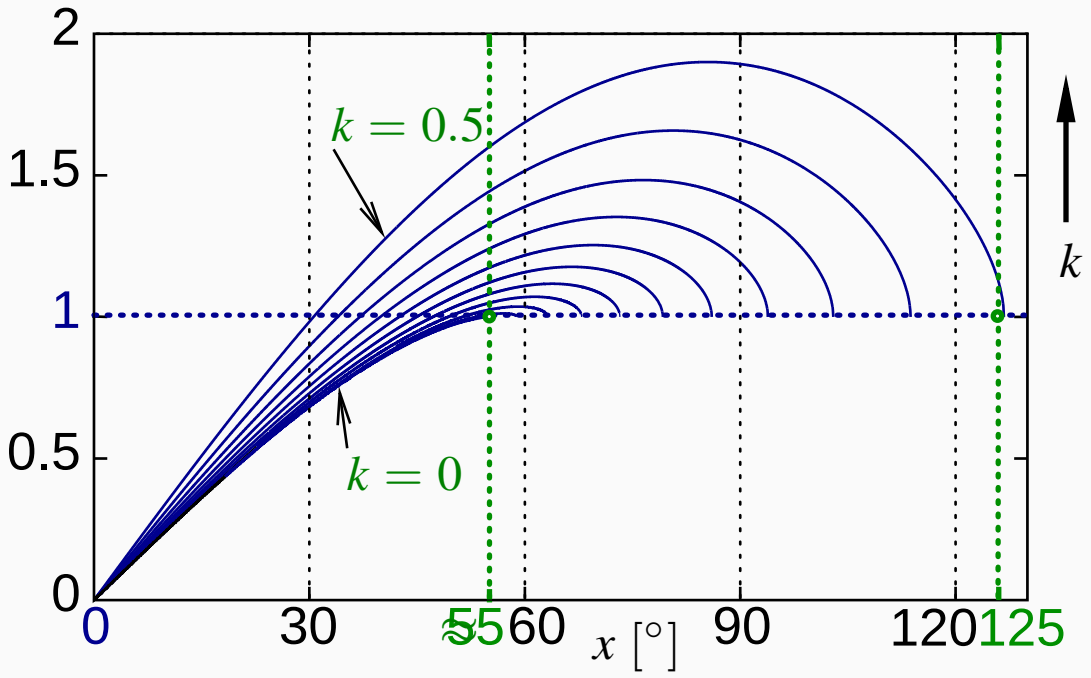
$55^\circ\, 2'\, 30'' \ \leq\ x_{\mathrm{BV}} \ \lessapprox\ 125^\circ$

Brillouin–Villat (BV) singularity

$u_{\mathrm{BV}} \equiv u_\infty = 1$

$u_s/u_{\mathrm{BV}} \sim 1 - 2k\sqrt{-s} + O(-s)\,,$

$s = x - x_{\mathrm{BV}} \to 0_-$



free streamlines

$0.5 \ \lessapprox\ k \ < \ \infty$?

$1 \ \geq\ u_{\mathrm{BV}} \ \geq\ 0$

Helmholtz–Kirchhoff-type flows (open cavity, circulation-free)

surface velocity $u_s(x,k)$

$k = 0,\ 0.05,\ 0.1,\ \ldots,\ 0.5$

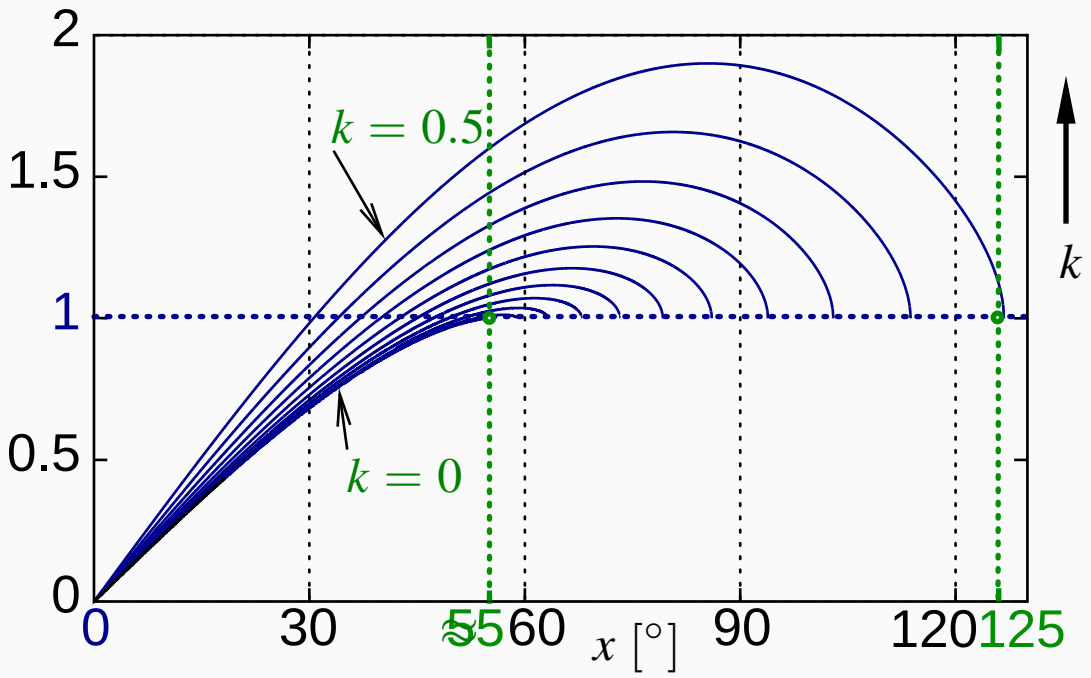
$55^\circ\, 2'\, 30'' \ \leq\ x_{\mathrm{BV}} \ \lessapprox\ 125^\circ$

Brillouin–Villat (BV) singularity

$u_{\mathrm{BV}} \equiv u_\infty = 1$

$u_s/u_{\mathrm{BV}} \sim 1 - 2k\sqrt{-s} + O(-s)\,,$

$s = x - x_{\mathrm{BV}} \to 0_-$



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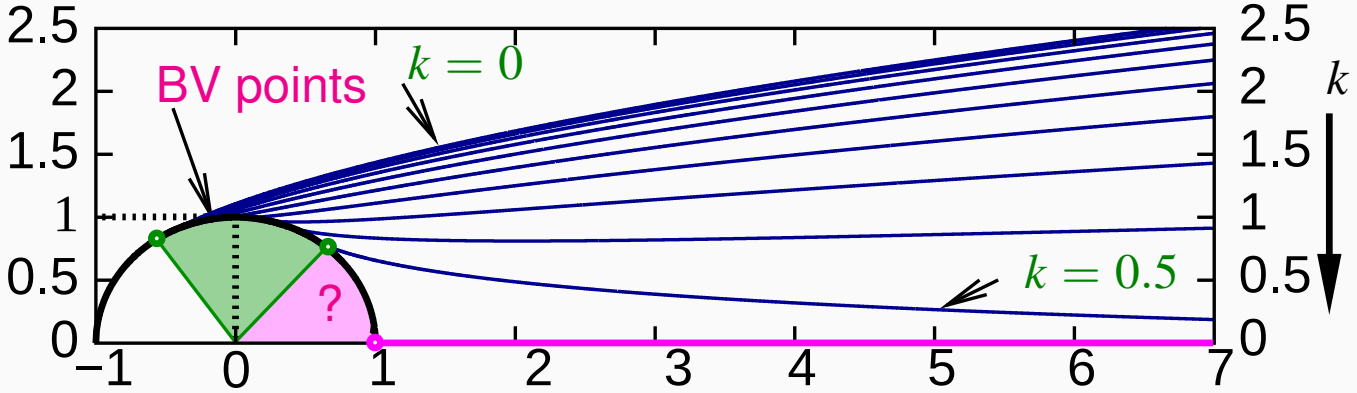

Outline

1. Motivation: flow past blunt body
2. Basic asymptotic concept
3. Marginal (internal, localised) separation
   - Two-parameter asymptotics
   - Local interaction problem
4. Massive (break-away, gross) separation
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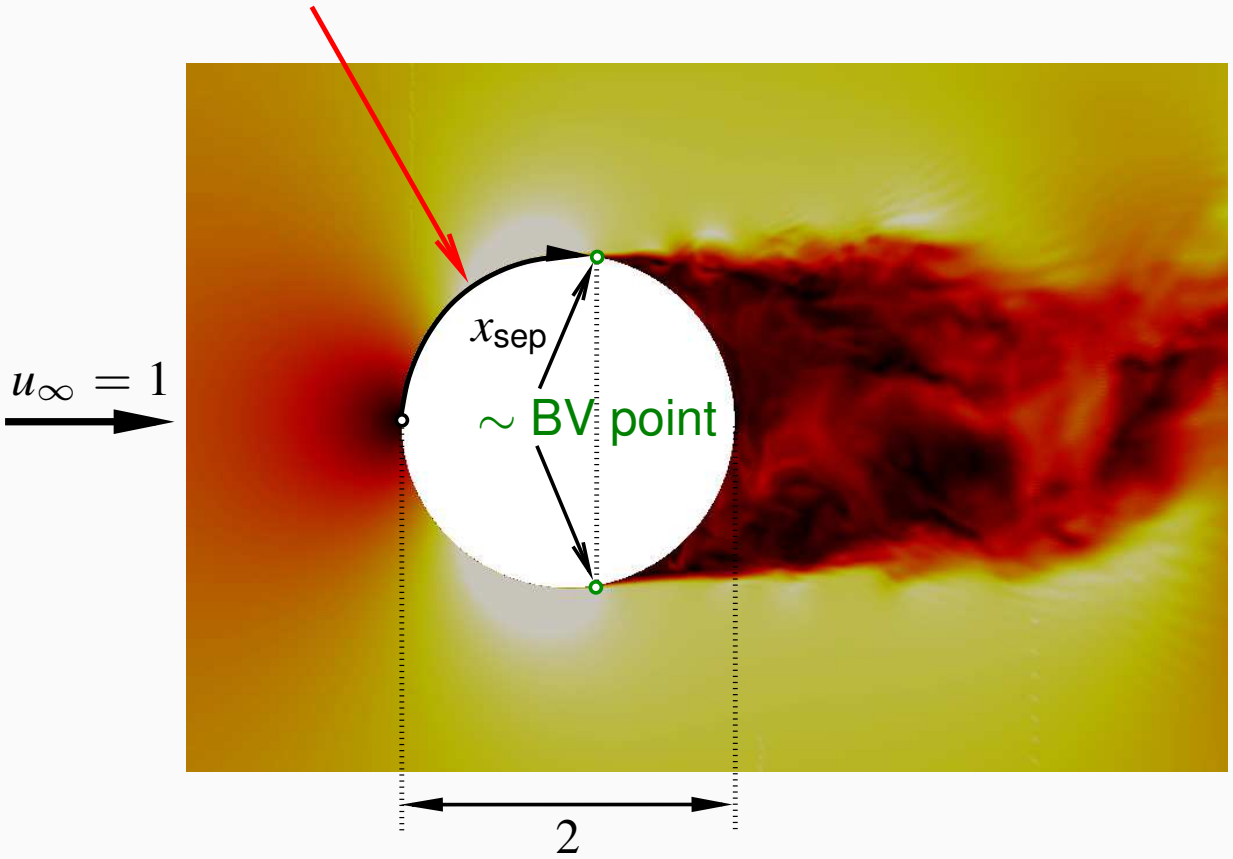
Detached-Eddy Simulation

$Re := \tilde{U}_\infty \tilde{R}/\tilde{\nu} = 10^6$

cf. Roshko (JFM 10/3, 1961), Achenbach (JFM 34/4, 1968), Schewe (DLR, 2001)

fully 3D (aspect ratio $= 1:10$)

viscous sublayer resolved by wall functions



colours distinguish isotachs

Jon Paton (TotalSim Ltd, Brackley, UK, 2010)

$Re \to \infty : \quad x_{\text{sep}} = ?$

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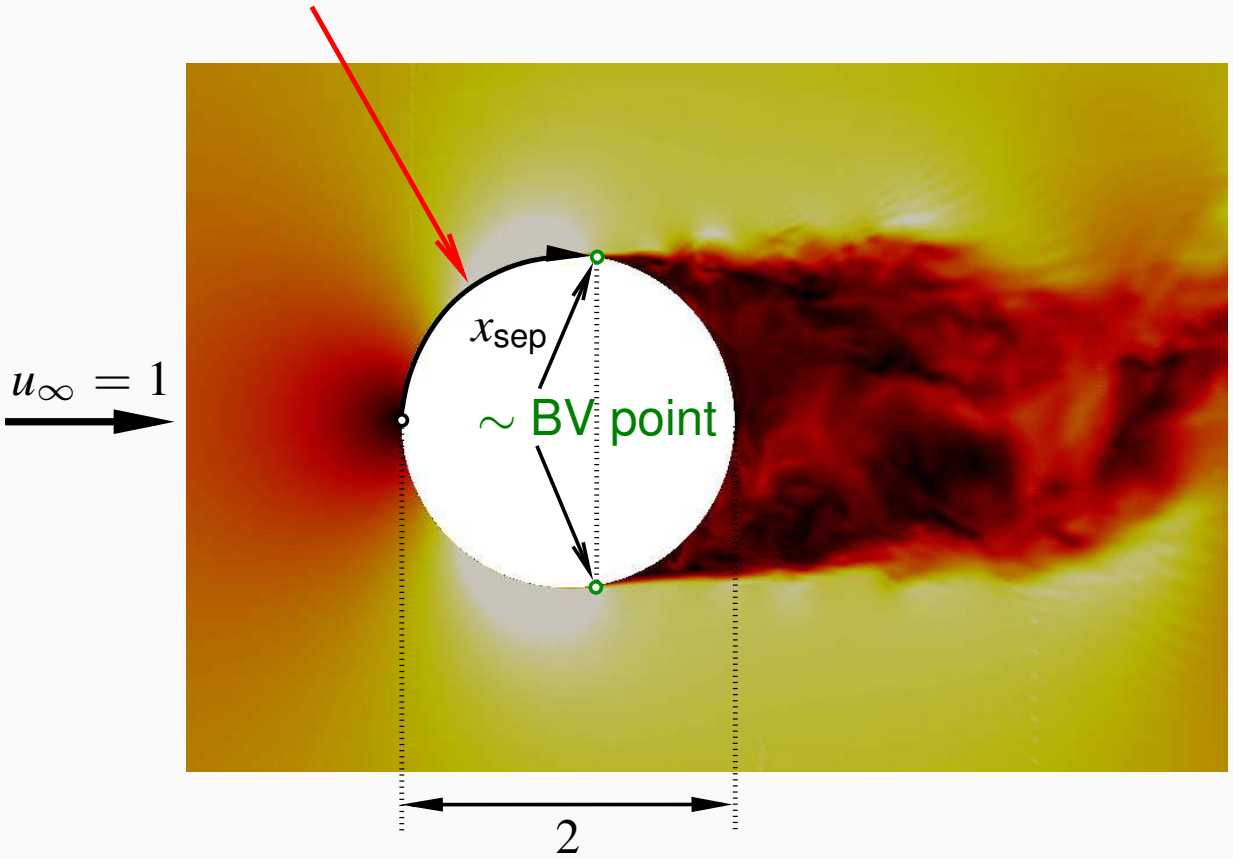

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An “upstream-dilemma” in classical boundary layer theory . . .

- pressure rise due to BV singularity triggers
- Goldstein singularity upstream
- but both singularities to be avoided

local interaction strategy of massive separation

pressure induced by BL displacement alters the rapid pressure variation such that it affects lowest-order BL approximation adjacent to the wall

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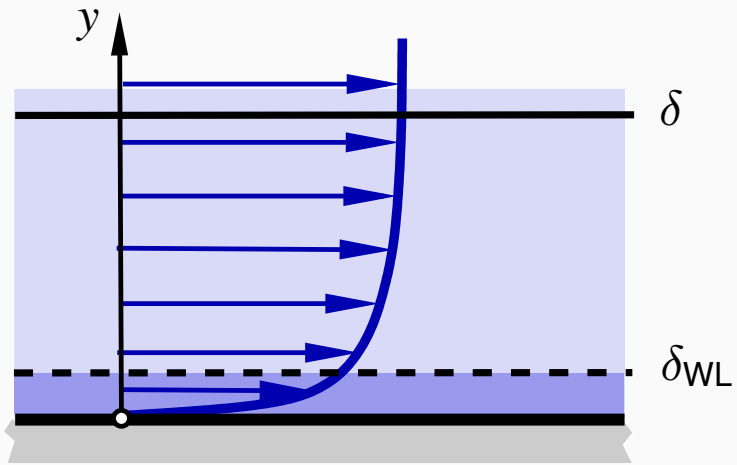

underdeveloped turbulence:

$\dfrac{\delta_{\mathrm{WL}}}{\delta}$ . . . algebraically small

Outer inviscid/irrotational interaction I

Scheichl, Kluwick & Smith (JFM 670, 2011)

local expansion on BL scale

$$[X,Y] = [x - x_{\mathsf{BV}}(k), y]/\delta_{\mathsf{BV}}\,, \quad Z = X + \mathrm{i}Y$$

$$\psi/\delta_{\mathsf{BV}} \sim \underbrace{Y + \delta_{\mathsf{BV}}^{1/2}\,\psi_{\mathsf{BV}} + \cdots}_{\text{irrotational}} - \epsilon[F_{\mathsf{BV}}(Y;k) - \underbrace{\delta_{\mathsf{BV}}^{1/2}\,\psi_{\mathsf{v}}(X,Y;k) + \cdots}_{\text{vorticity-induced}}] + O(\epsilon^2)$$

potential of BV singularity

$$\psi_{\mathsf{BV}} = -\frac{4k}{3}\,\Re\big(Z^{3/2}\big)$$

Log law and Cole's law of the wake

$$F'_{\mathsf{BV}} = -\kappa^{-1}\ln Y + F'_{\mathsf{w}}(Y;k)$$

$$Y \to 0: \quad F'_{\mathsf{w}} \sim A + \underbrace{B\,Y^{1/2}}_{\text{separation}} + O(Y\ln Y)$$

Outer inviscid/irrotational interaction II

Scheichl, Kluwick & Smith (JFM 670, 2011)

Wiener–Hopf-type vortex flow problem

$$\left(\partial_X^2 + \partial_Y^2\right)\psi_{\mathsf{v}} = -F_{\mathsf{BV}}'''(Y;k)\,\psi_{\mathsf{BV}}$$

$Y \to 0:\quad \psi_{\mathsf{v}}|_{X<0} \to 0\,,$ induced pressure $\to 0$ for $X > 0$

$Y \to \infty:\quad \psi_{\mathsf{v}} \sim -4kF_{\mathsf{BV}}(1;k)\,\Im\sqrt{Z}$ (induced potential flow)

solution

Scheichl & Kluwick (JFM, in prepration)

$$\psi_{\mathsf{v}} = \frac{4k}{3}\Re\underbrace{\left[Z^{3/2}\,F_{\mathsf{BV}}'(Y;k) - 3\mathrm{i}\int_0^Y \sqrt{Z - 2\mathrm{i}(Y-\eta)}\,F_{\mathsf{BV}}'(\eta;k)\,\mathrm{d}\eta\right]}_{O(Z^{3/2}\ln Z)\,,\quad Z\to 0}$$

cf. flow past inclined flat plate

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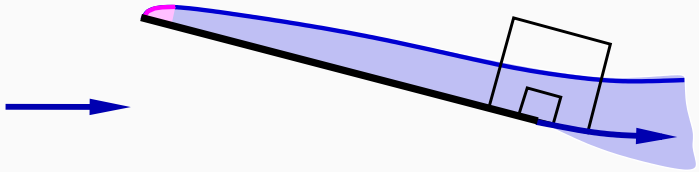

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Inner viscous/inviscid interaction

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Triple deck problem

lower-deck quantities $\Psi(\hat{X},\hat{Y})$, $[A,P](\hat{X})$, similarity parameter $\Gamma > 0$

$$\frac{\partial\Psi}{\partial\hat{Y}}\frac{\partial^2\Psi}{\partial\hat{Y}\partial\hat{X}} - \frac{\partial\Psi}{\partial\hat{X}}\frac{\partial^2\Psi}{\partial\hat{Y}^2} = -\frac{\mathrm{d}P}{\mathrm{d}\hat{X}} + \frac{\partial^3\Psi}{\partial\hat{Y}^3}, \quad \frac{\mathrm{d}P}{\mathrm{d}\hat{X}} = \frac{1}{\pi\Gamma}\fint_{-\infty}^{\infty}\frac{A''(S)}{\hat{X}-S}\,\mathrm{d}S$$

$$\hat{Y}=0: \quad \Psi = \partial\Psi/\partial\hat{Y} = 0, \quad \hat{Y}\to\infty: \quad \Psi \sim (\hat{Y}+A)^2/2 + O\big[(\hat{Y}+A)^{3/2}\big]$$

$\hat{X}\to\infty: \quad P\to 0$ (normalised)

$$\hat{X}\to\infty: \quad P \neq O(\ln X) \quad \Rightarrow \quad 2\Gamma^{1/8} \doteq 0.415 \mp 0.005$$

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trivial solution by unperturbed flow

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Matching inner $\sim$ outer interaction

$$\Psi|_{\hat{X}\to\infty} \quad \sim \quad \psi_{\mathsf{v}}|_{X\to 0_+}$$

separating streamline

$$\hat{X}\to\infty: \quad \hat{Y}_{\mathsf{sep}} \sim -A > 0\,, \quad A + (4\Gamma/3)\,\hat{X}^{3/2} = O(\hat{X}^{1/2})$$

arbitrary origin shift

$$\Psi(\hat{X}-\Lambda,\hat{Y})\,,\ [A,P](\hat{X}-\Lambda)\,, \quad \tau_w(\hat{X}-\Lambda) = \frac{\partial^2\Psi}{\partial\hat{Y}^2}(\hat{X}-\Lambda,0)\,, \quad \tau_w(0) := 0$$

match fixes Λ

$$x_{\mathsf{sep}} - x_{\mathsf{BV}}(k) \propto Re^{-4/9}\Lambda\,, \quad |\Lambda| \ll 1$$

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A recent wall layer analysis . . .

Scheichl & Kluwick (JFM, in preparation)

. . . shows that this intricate interaction mechanism is

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- triggered only if k is chosen properly

Chances are high that the effective value of k is fixed by a rational separation criterion – a sound combination of asymptotics and numerics !

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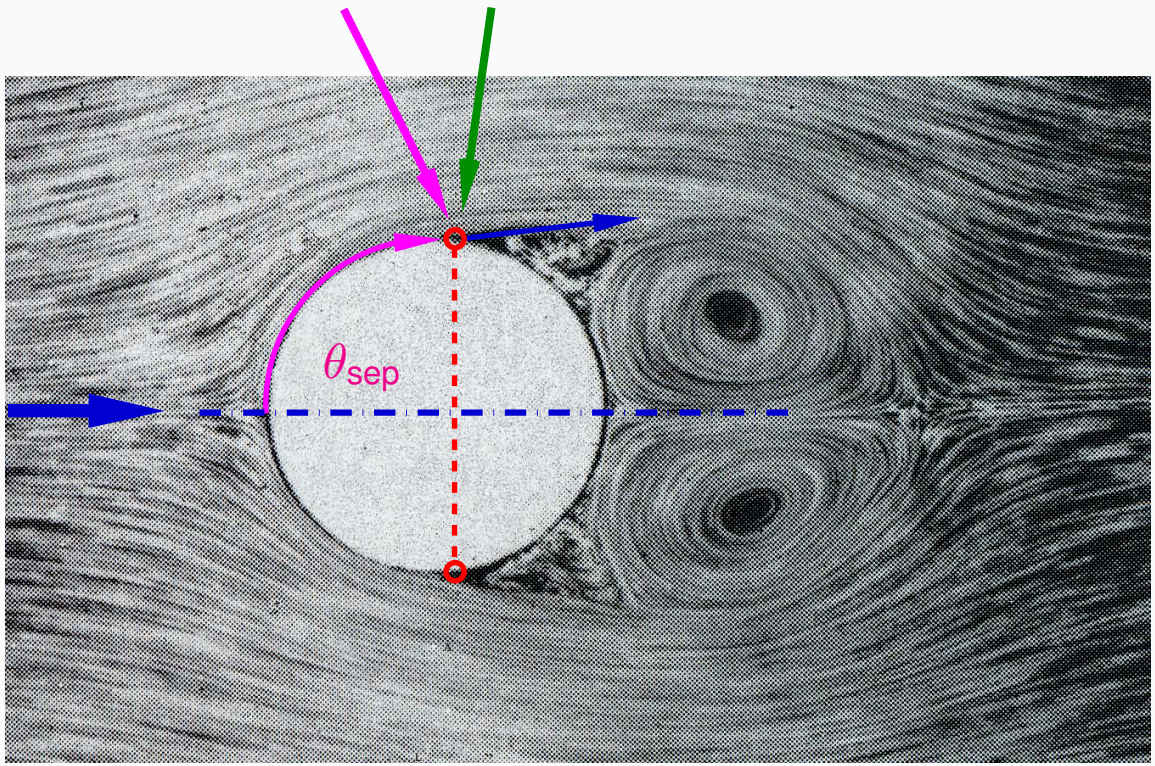

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- hypothesis: each layer distinguished by specific turbulent velocity scale
- velocity deficit of $\mathcal{O}(\epsilon) \ldots \mathcal{O}(1)$

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further ongoing research

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- finite or zero c_D as $Re \to \infty$?

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Thank you for your attention !