

Solid state nuclear clock: interrogation scheme and stability estimation

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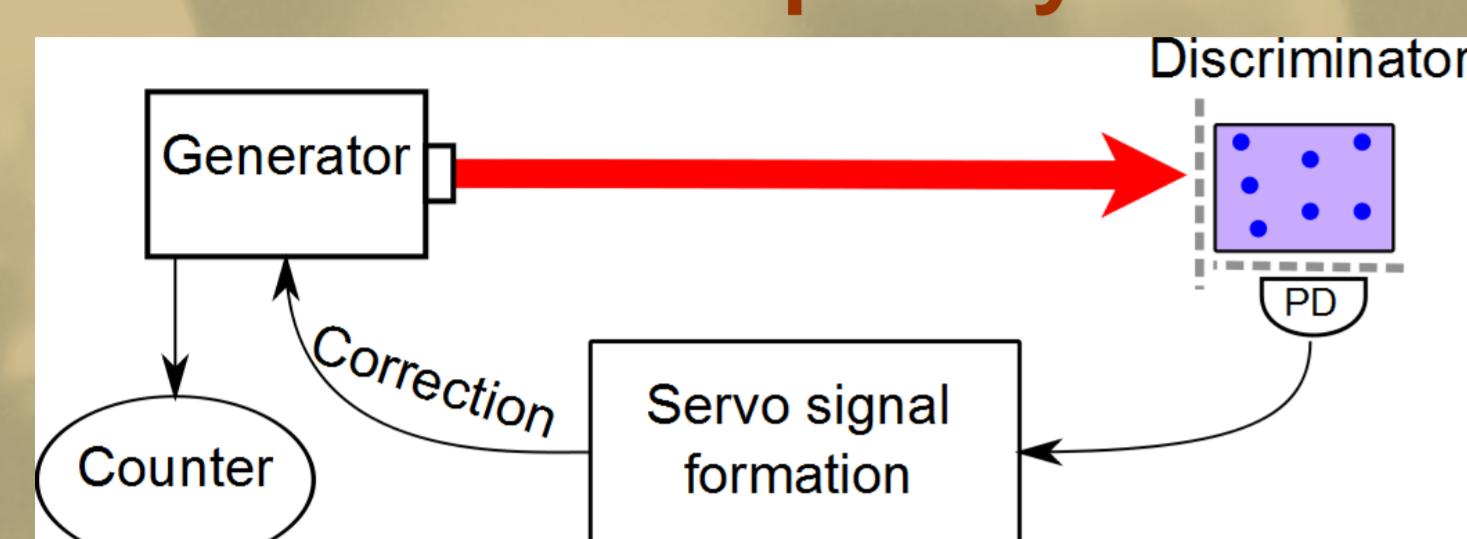
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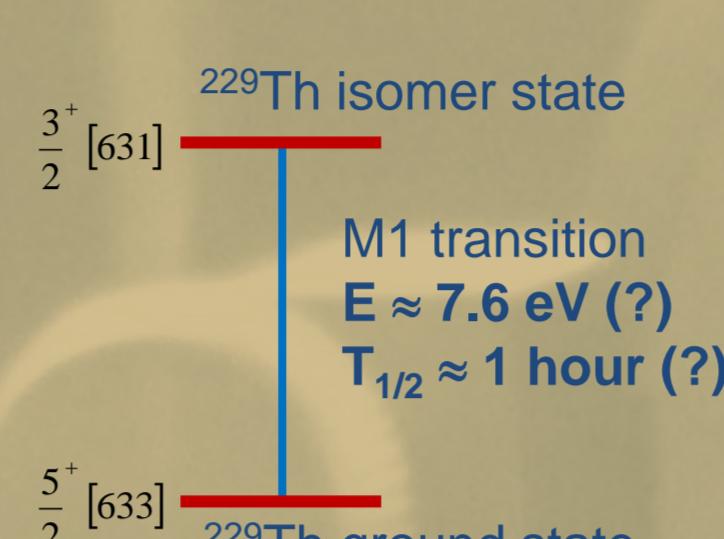
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Frequency stabilization on Th-doped CaF₂

Solid-state frequency standard



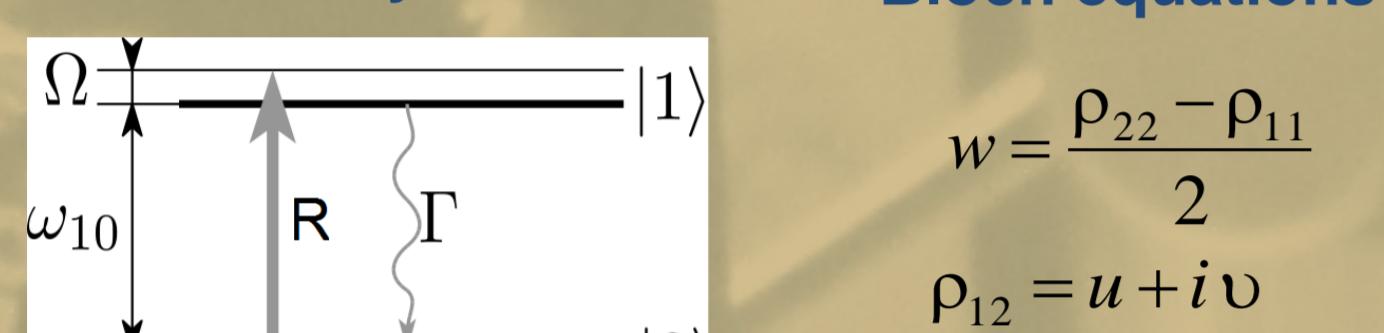
- Huge inhomogeneous broadening
- Huge excited state lifetime
- No way to prepare nuclei in certain state



Frequency stabilization via fluorescence spectroscopy

Quantum discriminator

Two-level system



Bloch equations

$$\begin{aligned} w &= \frac{\rho_{22} - \rho_{11}}{2} \\ \dot{\rho}_{12} &= u + i v \\ \dot{v} &= -\Omega u - \Gamma' v - R w \\ \dot{w} &= R v - \Gamma w - \Gamma/2 \end{aligned}$$

Optical pumping rate $W = W(\Omega) = \frac{R^2 \Gamma'}{2(\Omega^2 + \Gamma'^2)}$

Laser field and Hamiltonian

$$\vec{B} = \vec{B}_0 \cos(\omega t)$$

$$\hat{H} = \hbar \left[2\langle \omega_1 | 2 | + \frac{R}{2} (e^{i\omega t} | 1 \rangle \langle 2 | + e^{-i\omega t} | 2 \rangle \langle 1 |) \right]$$

$$\Gamma' \gg R; \Gamma \Rightarrow \dot{\rho}_{22} = -(2W + \Gamma)\rho_{22} + W$$

$$\omega = 2\pi \cdot v = 2\pi \cdot 1.83 \cdot 10^{15} \text{ Hz}$$

Rabi frequency

$$R^2 = \frac{6\Gamma^2 c^2 \pi}{\omega^3 \hbar} I \left(\frac{2}{9} \right)$$

Coefficient for real multilevel system

Parameters for simulation

$$\text{Laser intensity } I = 1 \text{ mW/cm}^2$$

$$\text{Number of nuclei } N_{th} = 10^{14}$$

$$\text{Rabi frequency } R = 2.12 \text{ s}^{-1}$$

$$\text{Total decay rate } \Gamma \approx 6 \cdot 10^{-3} \text{ s}^{-1}$$

Inamura, Haba, PRC79, 034313 (2009), experiment in a hollow cathode discharge

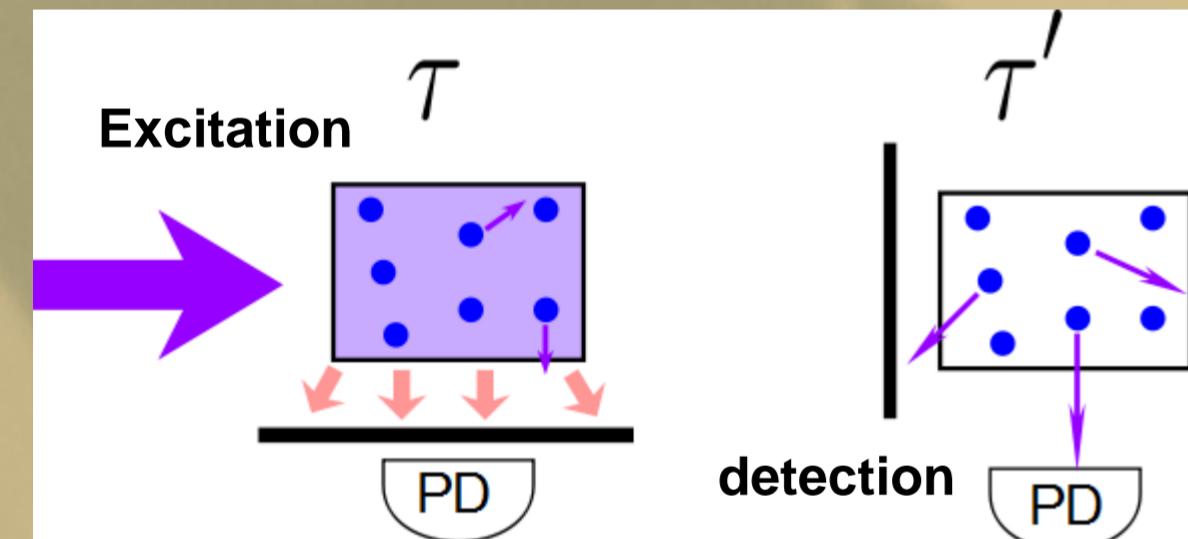
$$\text{Spontaneous decay rate } \Gamma_s [\text{s}^{-1}] = \frac{0.025 \ln(2) E_\gamma^3 [\text{eV}^3]}{10.95 \cdot 3600} = 4.4 \cdot 10^{-7} E_\gamma^3 [\text{eV}^3]$$

Ruchowska et al. PRC 73, 044326 (2006), theory

$$\text{Optical coherence decay rate } \Gamma' = 2\pi \cdot 5 \cdot 10^3 \text{ Hz}$$

Fluorescence spectroscopy

Fluorescence signal



Number of counts in i-th attempt

$$N_n = a(\Omega_n) + b(\Omega_n) \cdot N_{n-1}$$

$$a(\Omega) = \frac{k \Omega_{sp} \gamma N_{th}}{4\pi} \left(1 - e^{-\Omega \tau'} \right) \frac{R^2 \Gamma'}{2(\Gamma^2 + \Gamma(\Gamma^2 + \Omega^2))} \left[1 - e^{-\left(\frac{R^2 \Gamma'}{\Omega^2 + \Gamma^2 + \Gamma^2} + \Gamma \right) \tau} \right]$$

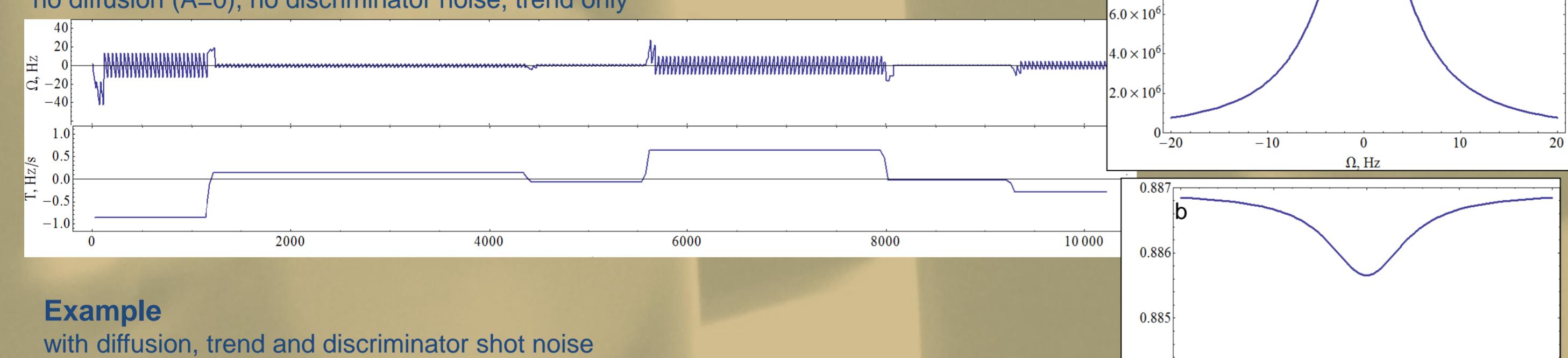
$$b(\Omega) = e^{-\left(\frac{R^2 \Gamma'}{\Omega^2 + \Gamma^2 + \Gamma^2} + \Gamma \right) \tau - \Gamma \tau'}$$

Simulation

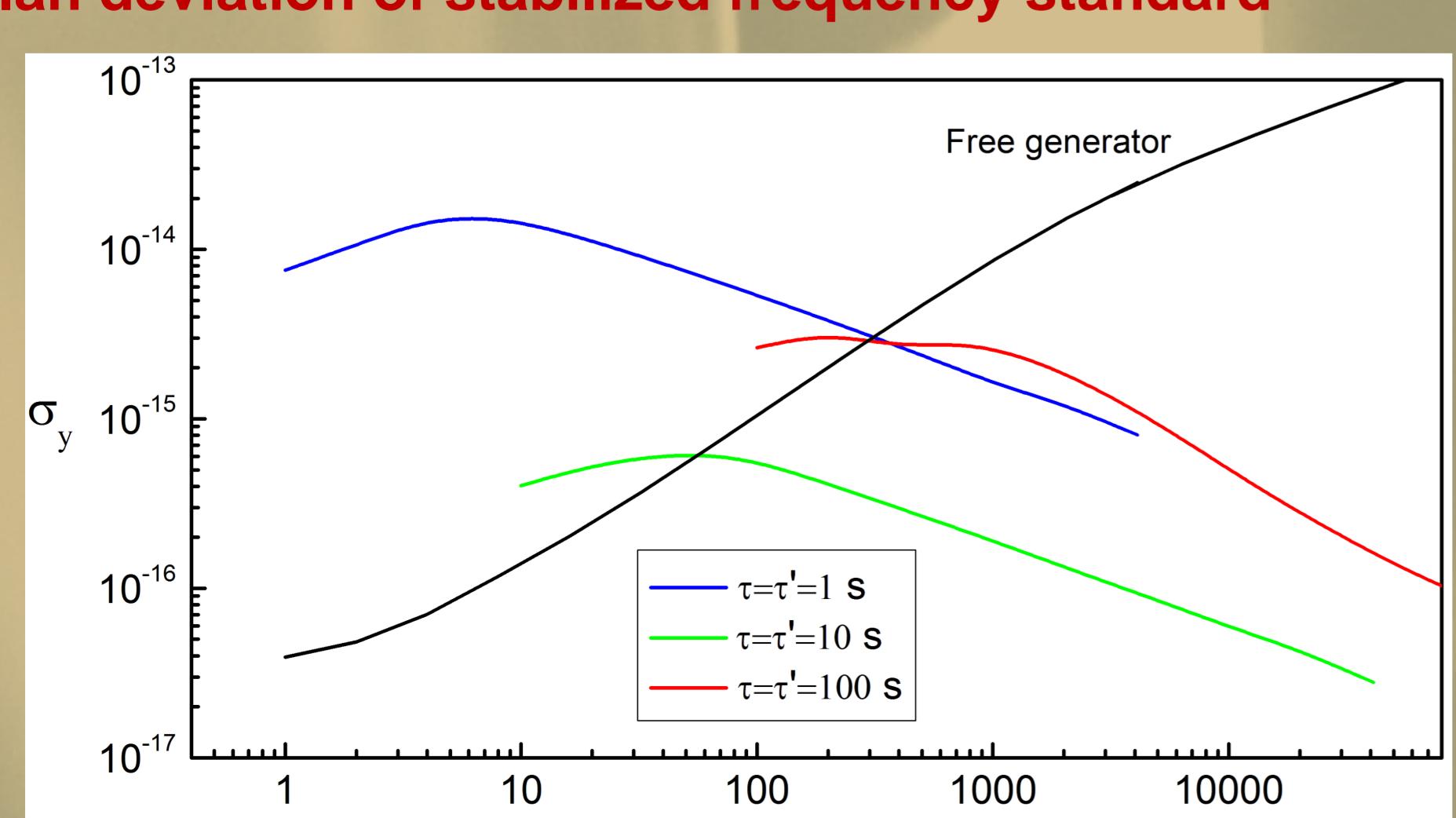
Generator: random walking + random trend;

$$A = 1; B = 0.3; \tau_T = 1000 \text{ s}; \tau = \tau' = 10 \text{ s}$$

Test no diffusion (A=0), no discriminator noise, trend only



Allan deviation of stabilized frequency standard



Interrogation time has an optimum;
Short time stability becomes worse due to discriminator noise

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Crystal shifts and broadenings effects in CaF₂

$$H_{hfs} = H_{E0} + H_{M1} + H_{E2} + \dots$$

electric monopole (contact interaction nucleus – electron cloud, temperature dependent):

- Shift up to 1 GHz if Th at random positions, + 2 kHz/K temperature dependence
- Broadening can be reduced to ≈ 10 Hz for a controlled doping and temperature stabilization

• electric quadrupole (electric gradient at the position of the nucleus)

- Shift up to 5 kHz (worst case) for random positions
- Broadening can be reduced to ≈ 10 mHz

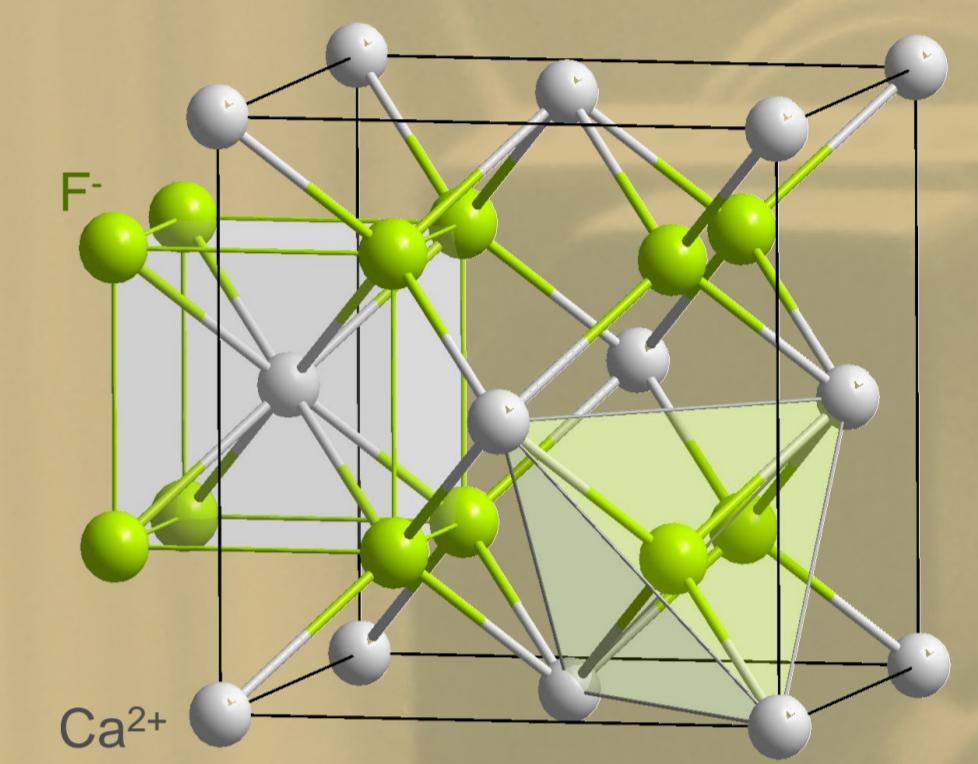
• magnetic dipole (magnetic field at nucleus from neighboring nuclear moments of F)

- Broadening up to 5 kHz for random positions

• 2nd order Doppler (due to vibrations)

- Shift ~ 320 Hz (300 K),
- Broadening ~ 220 Hz (300 K)

Calcium fluoride: CaF₂

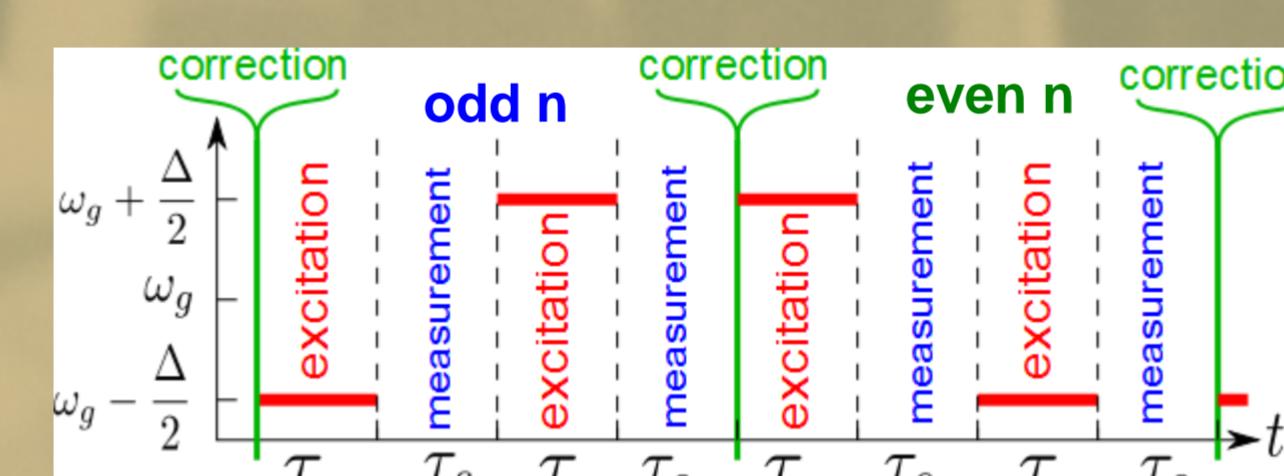


Interaction	Broadening	Shift
electric monopole	< 10 Hz ($\Delta T = 1 \text{ mK}$)	< 1 GHz
electric quadrupole	< 10 mHz ($\Delta T = 1 \text{ mK}$)	~ 5 kHz
magnetic dipole	~ 5 kHz	N/A
2nd order Doppler	~ 220 Hz ($T = 300 \text{ K}$)	~ 320 Hz

Inhomogeneous broadening is about 5 kHz

Error signal

Interrogation cycle

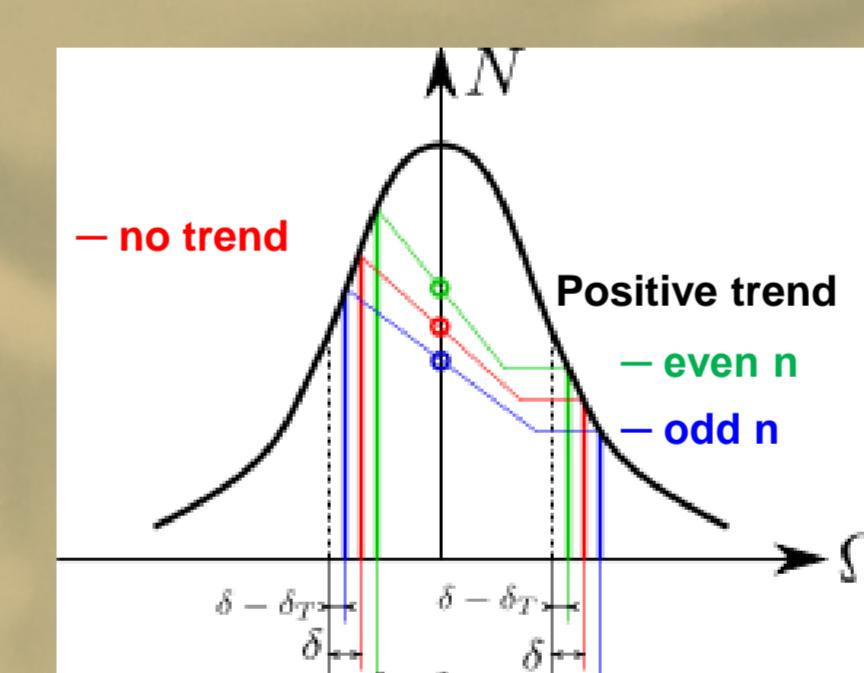


Correction in 1 cycle

$$\delta_n - T \frac{\tau'}{2} = (-1)^n [\alpha(N_{n,2} - N_{n,1}) - \beta(N_{n,1} - N_{n-1,2})]$$

$$\alpha = \frac{1}{2[a'(-\Delta/2) + b'(\Delta/2)\bar{N}]} \quad \beta = \alpha \cdot b(\Delta/2)$$

How to catch the trend?



Correction with trend

$$\delta_n + 2T(\tau + \tau') = (-1)^n [\alpha(N_{n,2} - N_{n,1}) - \beta(N_{n,1} - N_{n-1,2})] + (-1)^n [\gamma(N_{n,2} + N_{n,1} - N_{n-1,2} - N_{n-1,1}) - \epsilon(N_{n-1,2} + N_{n,1} - N_{n-2,2} - N_{n-1,1})]$$

$$\gamma = \alpha \frac{4\tau + 3\tau'}{2[\tau + \tau']} \quad \epsilon = \beta \frac{4\tau + 3\tau'}{2[\tau + \tau']}$$

Idea: odd and even cycles gives different total number of counts for the same trend!

Model of generator

Generator frequency noises

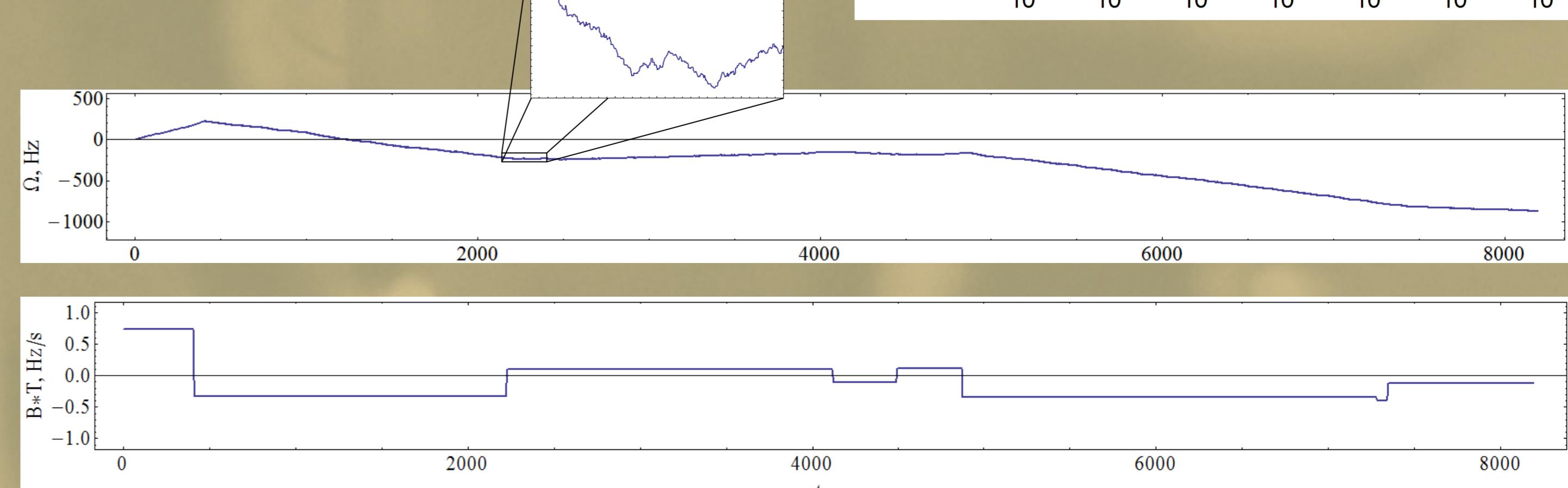
random walking + random trend

$$V_i = V_{i-1} + A \cdot \mathcal{N}(0,1) \cdot \sqrt{\delta t} + B \cdot \delta t \cdot T_i$$

$$T_i = \begin{cases} T_{i-1}, & \mathcal{R}(0,1) < p \\ \mathcal{N}(0,1), & \mathcal{R}(0,1) > p \end{cases}$$

$$p = \frac{\tau_T}{\tau_T + \delta t} \quad \text{--- probability for trend to survive in 1 step} \quad \delta t \\ \tau_T \quad \text{--- average trend lifetime}$$

Example of noised signal



Allan deviation of non-stabilized generator

