PAMM · Proc. Appl. Math. Mech. 11, 591 – 592 (2011) / DOI 10.1002/pamm.201110285

The Mixed Convection Flow past a Horizontal Plate in a Channel: Wake - Potential Flow Interaction

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The mixed convection flow past a horizontal plate in a horizontal channel in a distinguished limit of large Reynolds and Grashof numbers is considered. An interaction problem between the wake and the potential flow is formulated and analyzed. It turns out that stationary solutions exist only if a suitable defined interaction parameter is below a critical value.

1 Introduction

We consider a horizontal plate of length L with a temperature T_p in the mid-plane of a horizontal channel of width b in a uniform flow with velocity velocity U_∞ and temperature T_∞ of an incompressible fluid with kinematic viscosity ν, thermal expansion coefficient β and density ρ. Although the Reynolds number $Re = U_\infty L/\nu$ is assumed to be large we consider the flow to be laminar. Thus the flow field can be decomposed into a boundary-layer flow along the plate and side walls, the wake flow behind the trailing edge and the potential flow in rest of the flow domain.

Due to the temperature perturbation in the wake there will be a hydrostatic pressure difference across the wake such that the potential flow outside of the wake has to adjust to it. However, the adjusted potential flow causes the wake to be inclined. Thus an interaction between the wake and the potential flow will occur. Following [2] and [3] we introduce an appropriate scaling and identify a meaningful interaction parameter κ. Thus we formulate the interaction problem and discuss its solution. In the limit of vanishing Prandtl number and no wake/potential flow interaction ($\kappa = 0$) this problem has been solved analytically in [4] and [1], respectively.

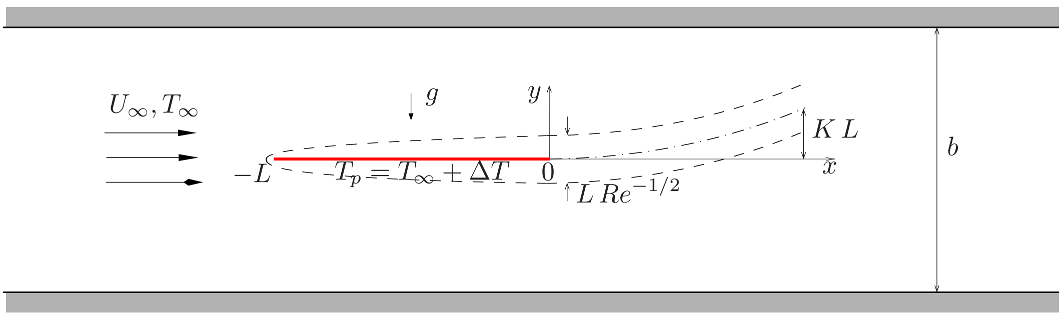

Fig. 1 Mixed convection flow past a horizontal plate in a horizontal channel

2 The interaction mechanism

In order to find a meaningful scaling we estimate the expected interaction mechanism. The vertical hydrostatic pressure gradient is of the order $g\rho\beta\Delta T$ where g is the gravity acceleration and $\Delta T = T_p - T_\infty$ the temperature perturbation. Since the thickness of the wake is of the order $L\,Re^{-1/2}$ the hydrostatic pressure difference across the wake is $\Delta p_h = \rho g\beta\Delta T L\,Re^{-1/2}$. This pressure perturbation induces a velocity perturbation of order $\Delta U = \Delta p_h/\rho U_\infty$ of the potential flow. Thus an inclination of the wake of the order $K = \Delta U/U_\infty = \Delta p_h/\rho U_\infty^2$ is expected. Thus the hydrostatic pressure gradient induced by the temperature perturbation has a component of the order $K\,g\rho\beta\Delta T$ in the main flow direction. Referring this pressure gradient in the wake to its natural reference value $\rho U_\infty^2/L$ we obtain the interaction parameter $\kappa^2 = \rho g\beta\Delta T K \frac{L}{\rho U_\infty^2} = Gr^2 Re^{-9/2}$ which has been expressed in terms of the Reynolds and Grashof number $Gr = g\beta\Delta T L^3/\nu^2$.

The potential flow field is given by $u - iv = 1 + K(u_1 - iv_1) + ...$, where $u_1 - iv_1$ is the first order correction due to the hydrostatic pressure difference across the wake. Note that the pressure difference across the wake can be interpreted as a vortex distribution $\gamma_w(x)$ along the center line of the channel. The scaled inclination $\bar{y}'_w = v_1(x, 0)$ of the wake is equal to

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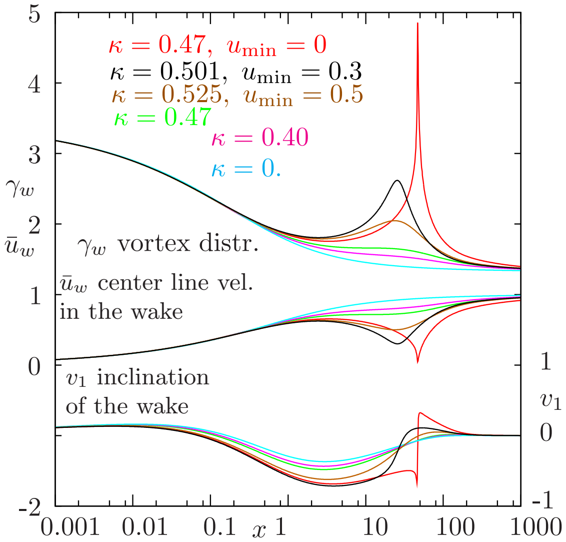

Fig. 2 Vorticity γ_w, center line velocity $\bar{u}_w(x,0)$ of the wake and inclination of the wake v_1 for $b = 100$, $Pr = 1$.

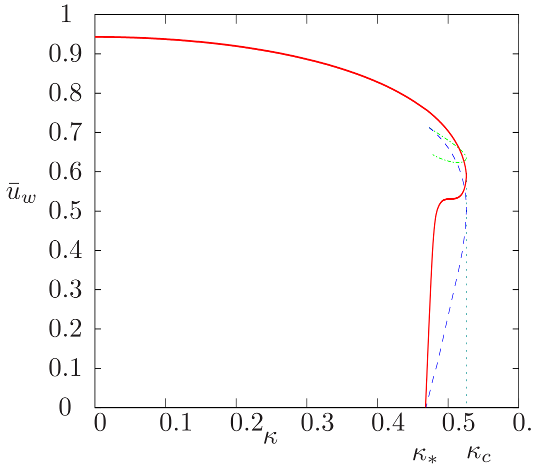

Fig. 3 center line velocity $\bar{u}_c(x_*)$, $x_* = 46.6$, (red solid line), local minimum (blue dashed line) and local maximum (green, dashed dotted line) of the center line velocity $\bar{u}_c$of the wake as a function of the interaction parameter κ for $b = 100$ and $Pr = 1$.

the correction of the potential flow field at the centerline of the channel. Using wake coordinates the equations for the velocity $\bar{u}_w$ and temperature profile $\bar{\theta}_w$ in the wake read

$$\bar{u}_w \bar{u}_{w,x} + \bar{v}_w \bar{u}_{w,\bar{y}} = \kappa^2\, \bar{y}_w' \bar{\theta}_w + \bar{u}_{w,\bar{y}\bar{y}}, \qquad \bar{u}_{w,x} + \bar{v}_{w,\bar{y}} = 0, \qquad \bar{u}_w \bar{\theta}_{w,x} + \bar{v}_w \bar{\theta}_{w,\bar{y}} = \frac{1}{\mathrm{Pr}} \bar{\theta}_{w,\bar{y}\bar{y}}.$$

supplemented by the symmetry conditions $\bar{u}_{w,\bar{y}}(x,0) = \bar{v}_w(x,0) = \bar{\theta}_{w,\bar{y}}(x,0) = 0$, the matching conditions $\bar{u}_w(x,\infty) = 1$, $\bar{\theta}_w(x,\infty) = 0$ and the "initial conditions" $\bar{u}_w(0,\bar{y}) = \bar{u}_B(\bar{y})$, $\bar{\theta}_w(0,\bar{y}) = \bar{\theta}_B(\bar{y})$, where $\bar{u}_B(\bar{y})$ and $\bar{\theta}_B(\bar{y})$ are the velocity and temperature profile in the Blasius boundary-layer.

Following [4] and [2] the potential flow problem in the channel can be solved analytically and we obtain the vertical velocity perturbation at the center line:

$$v_1(x) = \frac{1}{2b}\sqrt{\frac{\sinh\frac{\pi x}{b}}{\sinh\frac{\pi(x+1)}{b}}}\int_0^\infty \frac{\gamma_w(\xi)}{\sinh\frac{\pi(x-\xi)}{b}}\sqrt{\frac{\sinh\frac{\pi(\xi+1)}{b}}{\sinh\frac{\pi\xi}{b}}}\,d\xi, \quad x > 0, \qquad \gamma_w(x) = 2\int_0^\infty \bar{\theta}_w\, \mathrm{d}\bar{y}.$$

These two equations are solved iteratively until convergence is obtained.

3 Results

The wake and potential flow problem has been solved for $Pr = 1$ and $b = 100$ for increasing values of the interaction parameter κ. Starting with $\kappa = 0$, where we have no interaction at all, we obtain first a monotonically decreasing vortex distribution γ_w (see figure 2) . This is due to the fact that due to the preservation of the dimensionless enthalpy flux $\int_0^\infty \bar{u}_w \bar{\theta}_w \mathrm{d}\bar{y} = const$ in the wake and that the flow at the center of the wake is accelerated shortly after the trailing edge, see figure 2. However, we observe that the vertical velocity v_1 is first positive but becomes negative shortly after the trailing edge. Now increasing the interaction parameter κ results in an adverse hydrostatic pressure gradient in the wake. Thus the center line velocity $\bar{u}_w$ decreases with increasing κ. If κ is large enough ($\kappa > 0.47$) a minimum in center line velocity forms. The minimum decreases with increasing κ, However, for $\kappa > \kappa_c = 0.525$ no solutions can be found. Using a different numerical strategy a second solution branch for $\kappa_* = 0.471 < \kappa < \kappa_c$ can be found. This second solution branch terminates in a singularity at $\kappa = \kappa_*$ There the minimum of $\bar{u}_w$ is zero and as a consequence the γ_w has a logarithmic singularity.

In figure 3 the $\bar{u}_w(x_*, 0)$, where x_* is the location of the singularity in the wake flow for $\kappa = \kappa_*$ is shown. Also the the local minimum value and maximum values of the center line velocity $\bar{u}_w$ where they exist are shown.

References

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