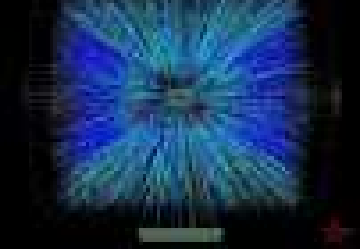


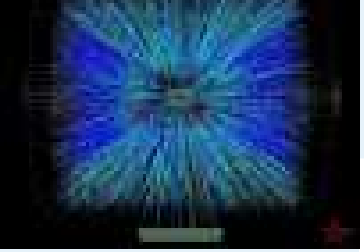
Instabilities in the Quark-Gluon Plasma *

Maximilian Attems

Institute for Theoretical Physics, TU Vienna

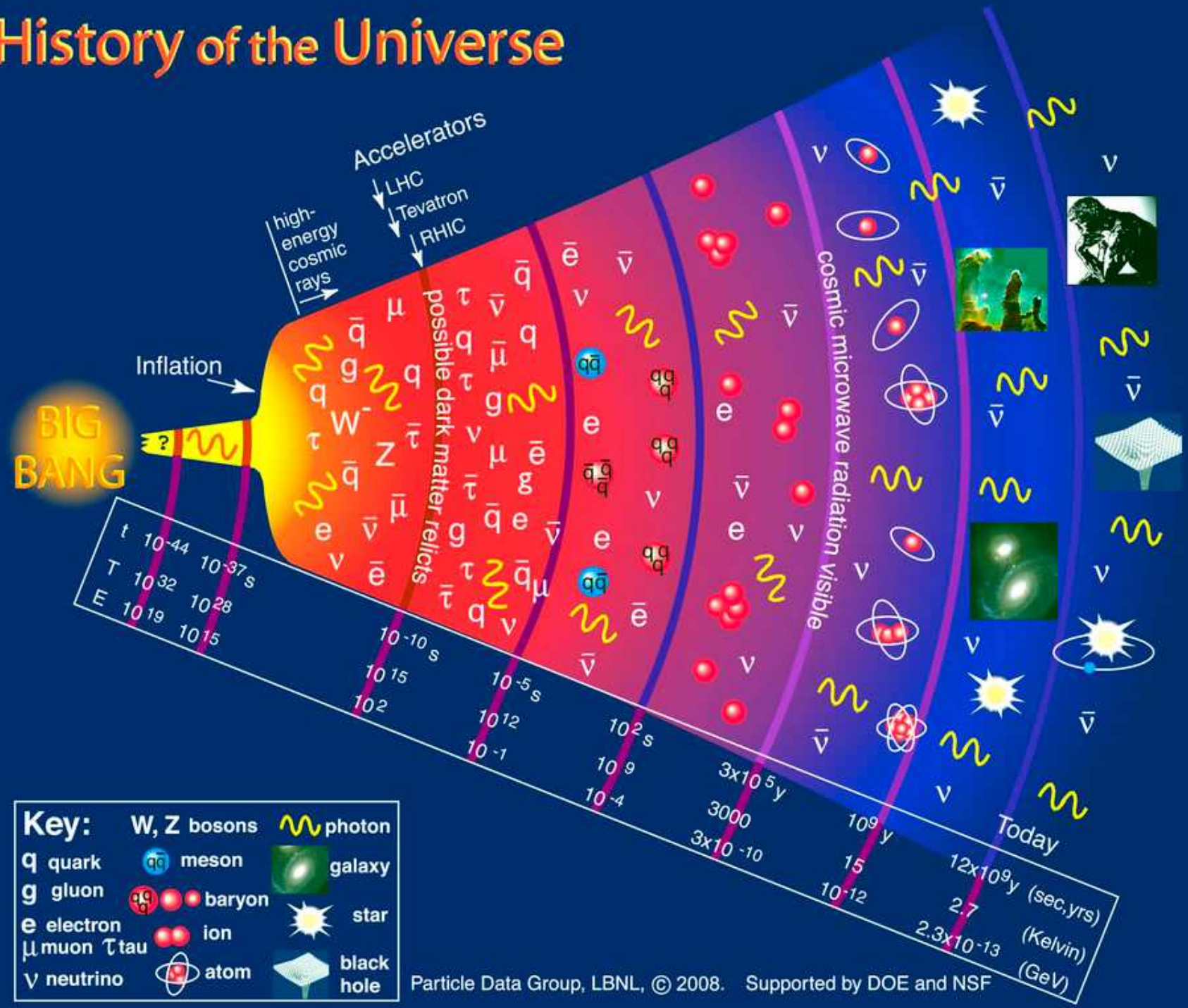
January 19, 2011

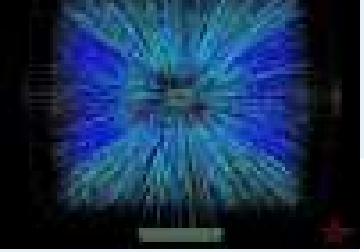
*“ Dans la vie, rien n’est à craindre, tout est à comprendre.” Marie Curie



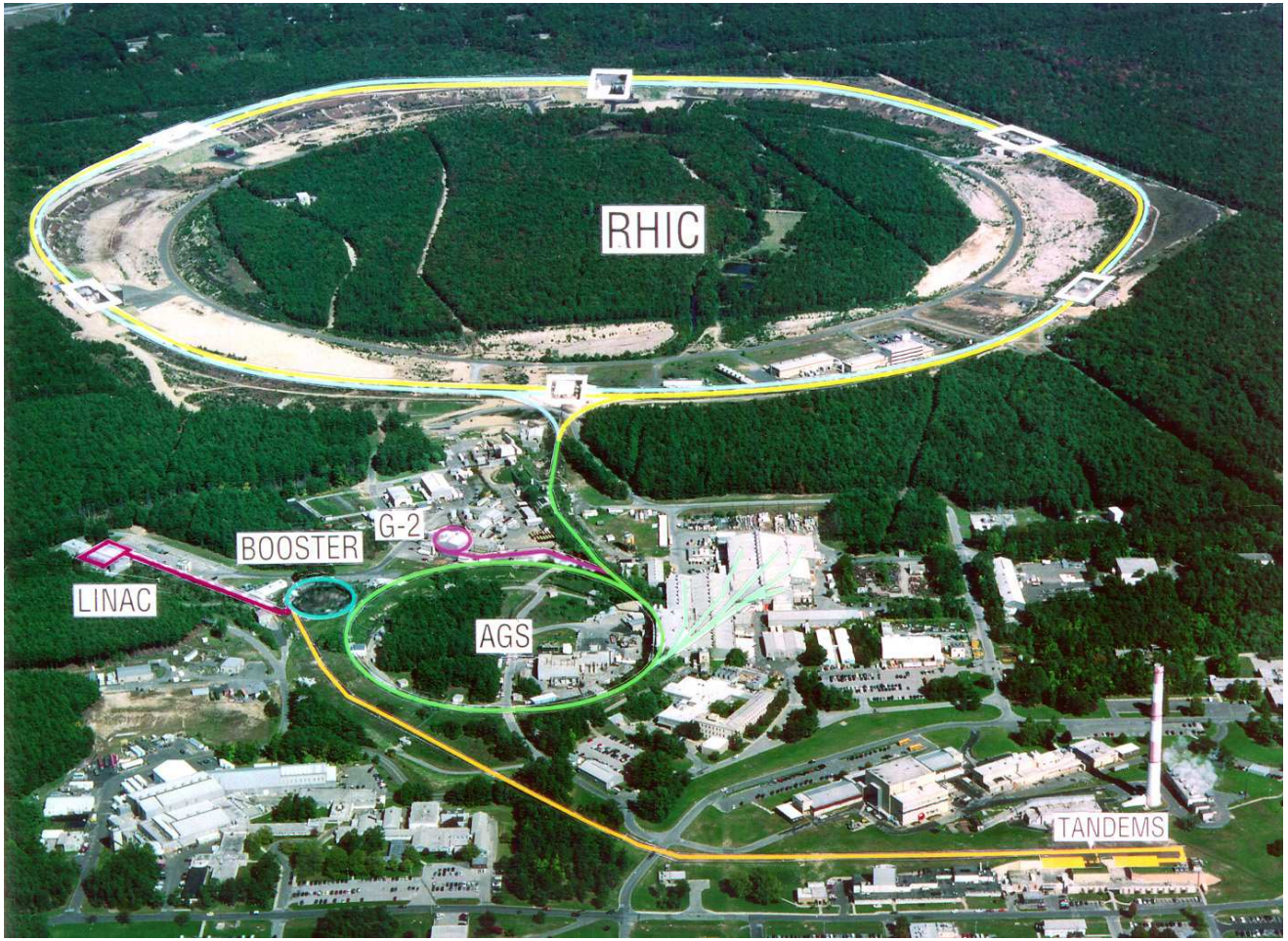
Overview

History of the Universe

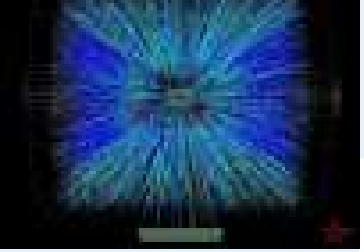




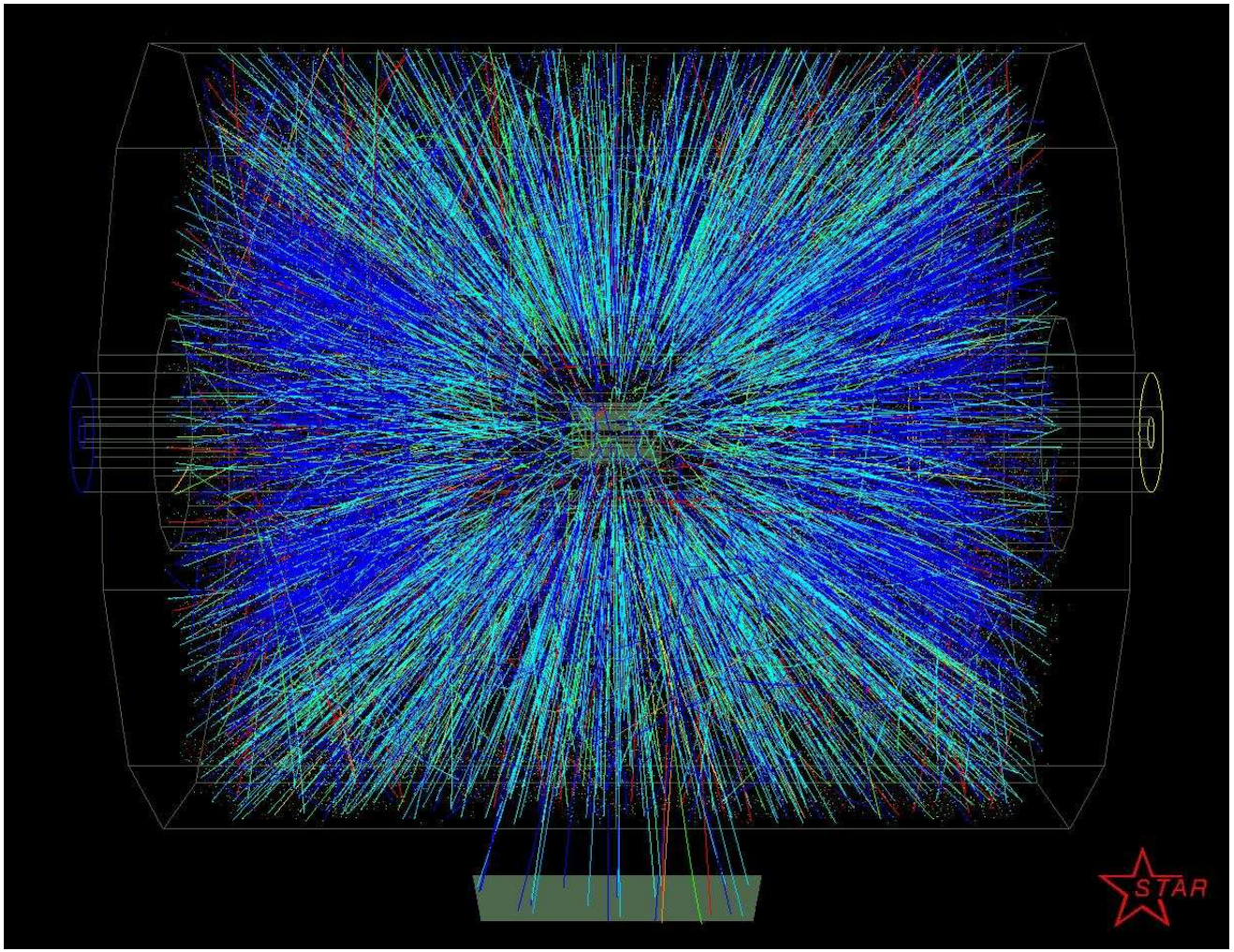
Relativistic Heavy Ion Collider (RHIC)



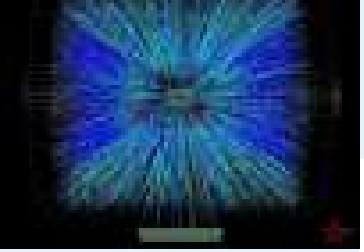
Au+Au ions $\sqrt{s_{NN}} = 200\text{GeV}/\text{nucleon pair}$, p+p, d+A



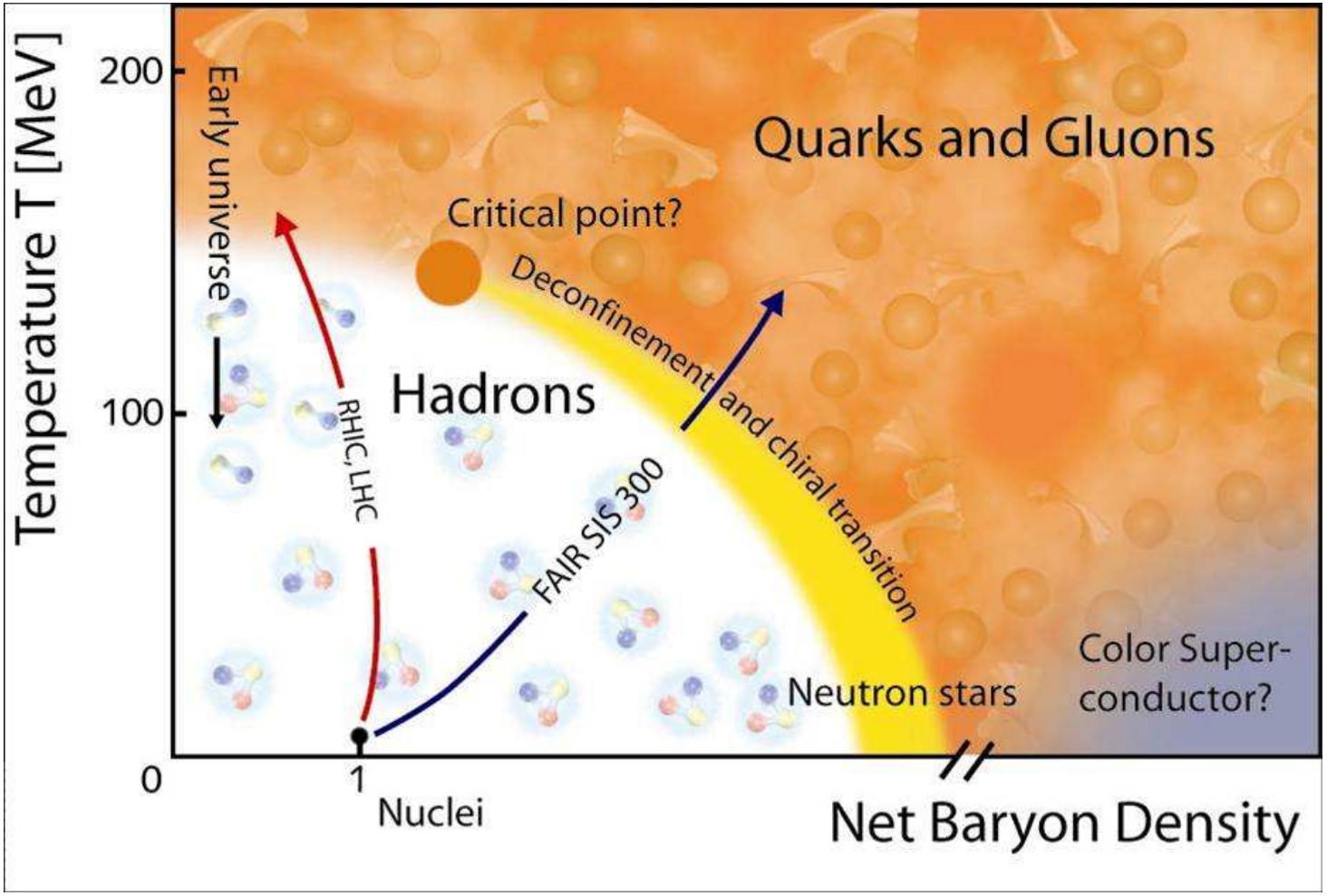
Heavy Ion Collision



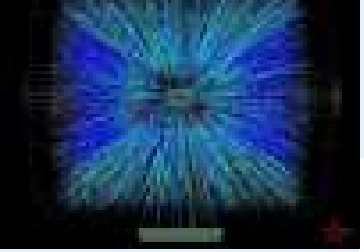
Side view 2nd STAR event at RHIC, 2001.



QCD Phase diagram

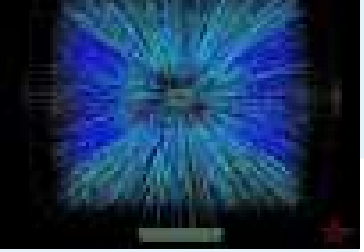


Schematic QCD phase diagram



QGP signatures

- Small viscosity (elliptic flow v_2)
- Jet quenching
- Experimental observation of $T > T_c$
- high p_T suppression of hadrons (for central collisions)
- Rapid thermalization:
  estimates from hydrodynamical computation $\sim 1fm/c$



Elliptic flow

Pressure gradients generate positive elliptic flow v_2

$$\frac{d^2N}{d\phi dp_T} = N_0\left(1 + 2v_2(p_T)cos(2\phi) + ..\right) \tag{1}$$

The elliptic flow is quantified by the anisotropy of particle production with respect to the reaction plane $v_2 = \left\langle \frac{p_x^2 - p_y^2}{p_x^2 + p_y^2} \right\rangle$

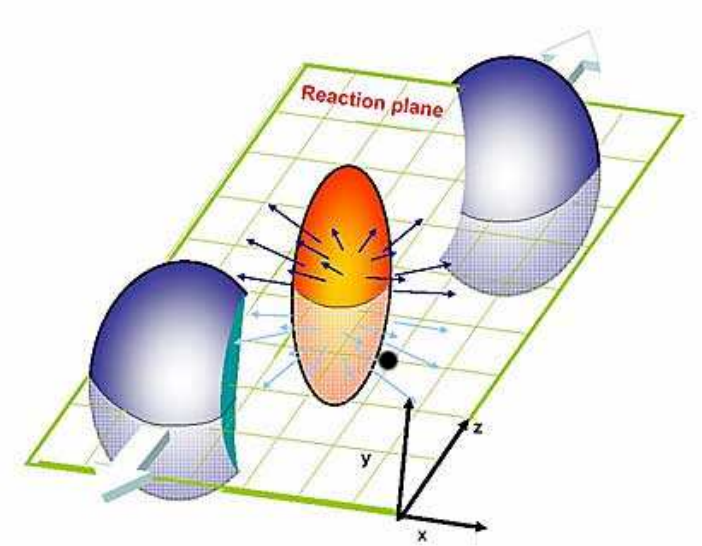
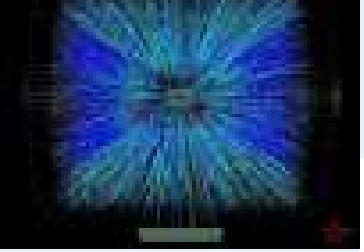
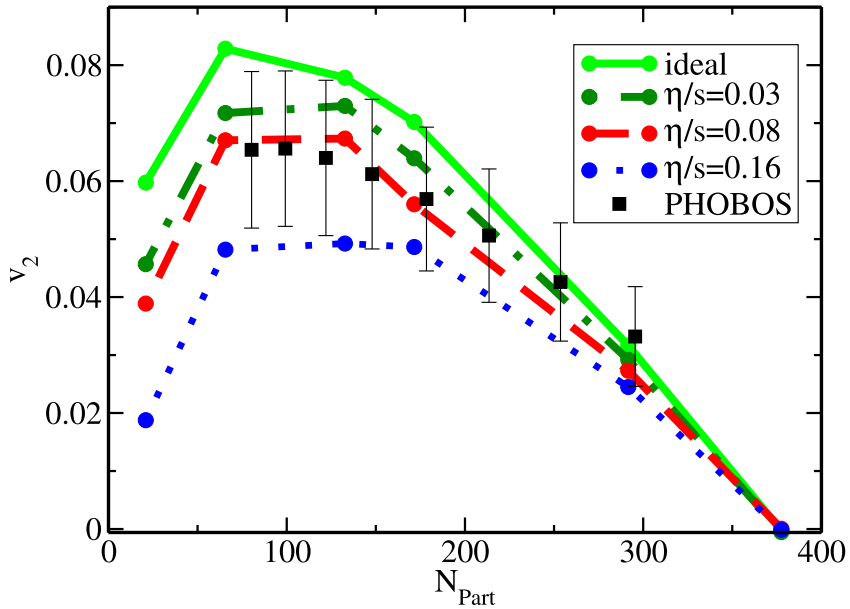


Illustration of the reaction plane definition.

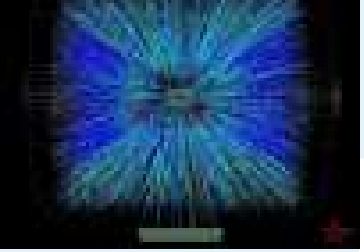


Hydrodynamic model

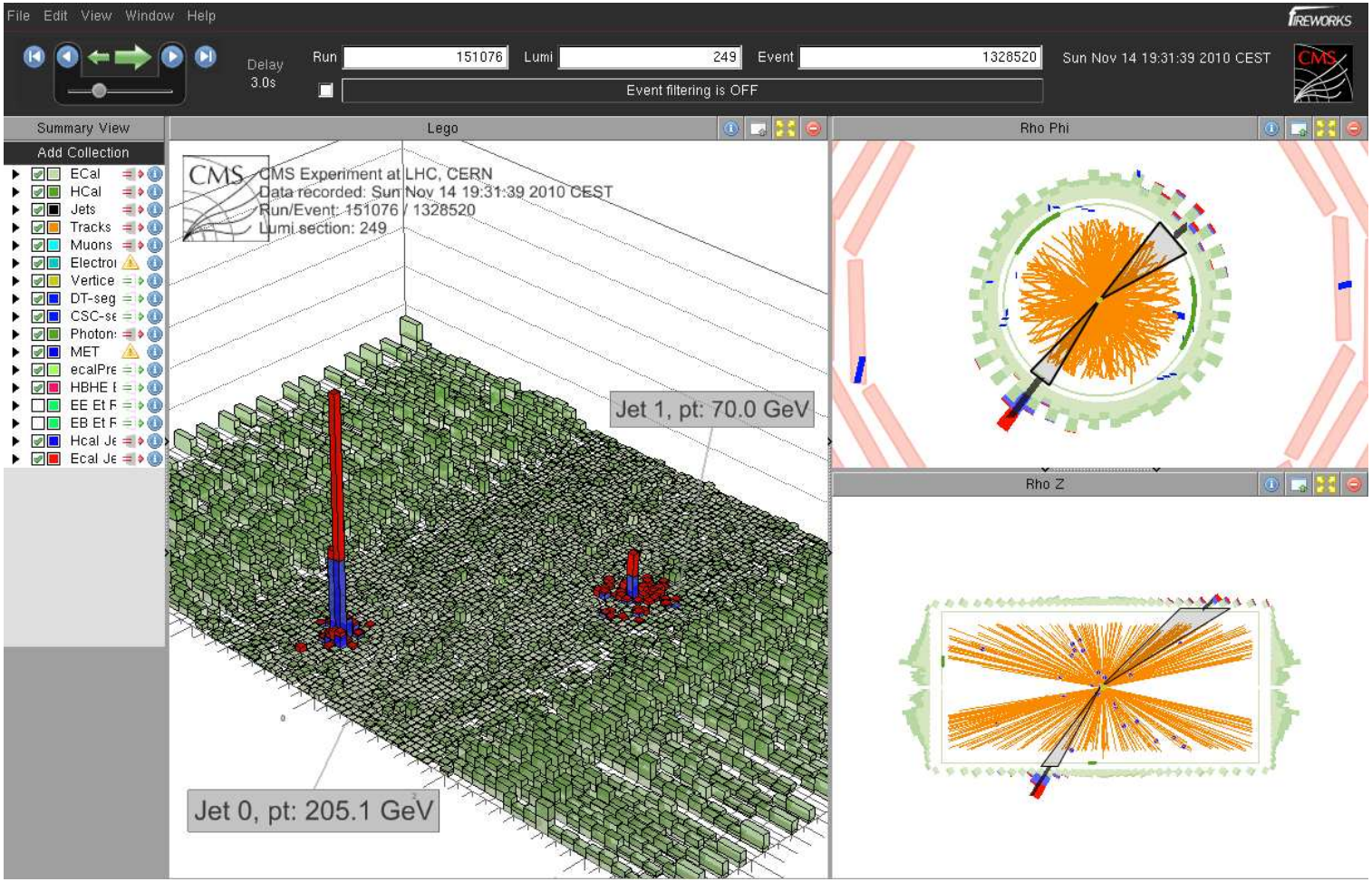
- PHOBOS data on Au+Au collisions at $\sqrt{s_{NN}} = 200$ GeV, compared to hydrodynamic model for various η/s ratios.



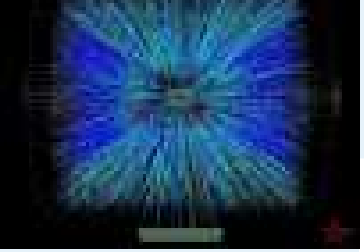
P. Romatschke, U. Romatschke hep-th/0706.1522



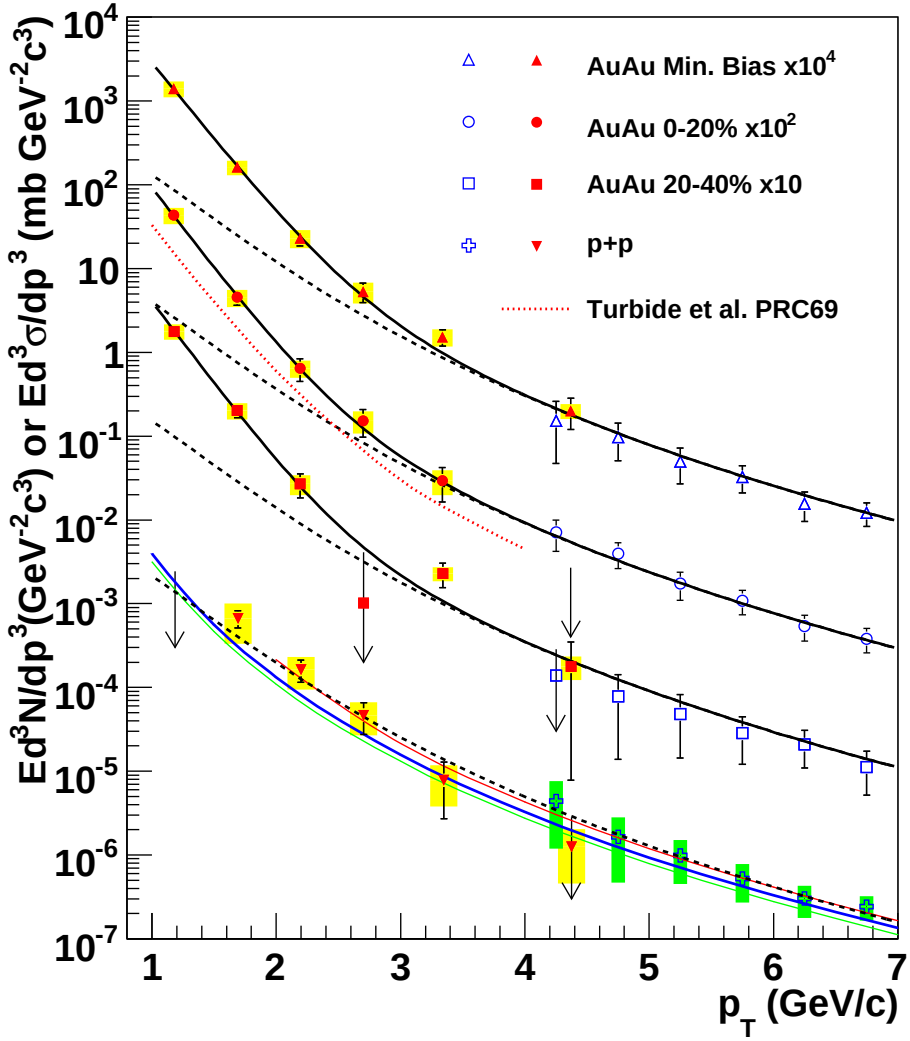
Jet Quenching



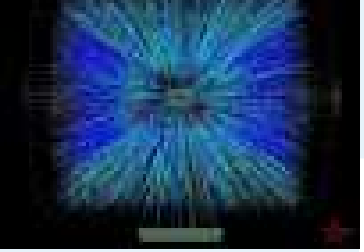
Single CMS event Pb-Pb collision at $\sqrt{s_{NN}} = 2.76$TeV/nucleon pair



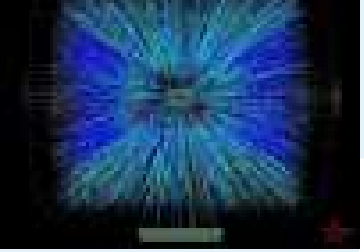
Early conditions at RHIC - T



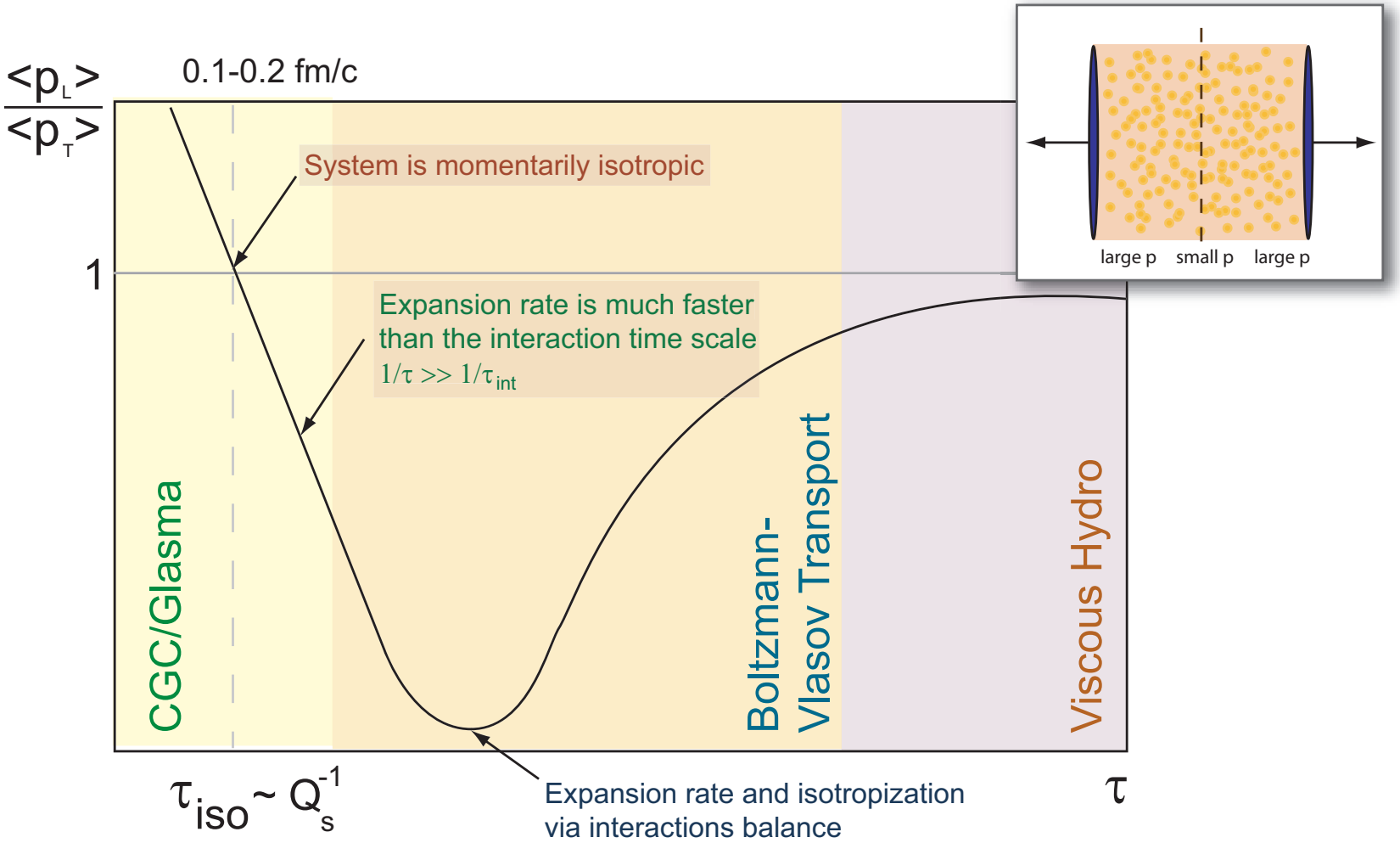
$$T = 300 - 600\text{MeV} > 2 * T_c$$



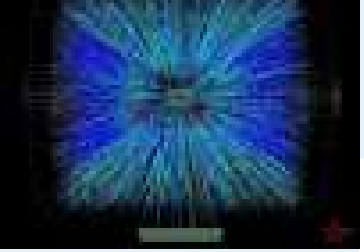
Hard Expanding Loops (HEL)



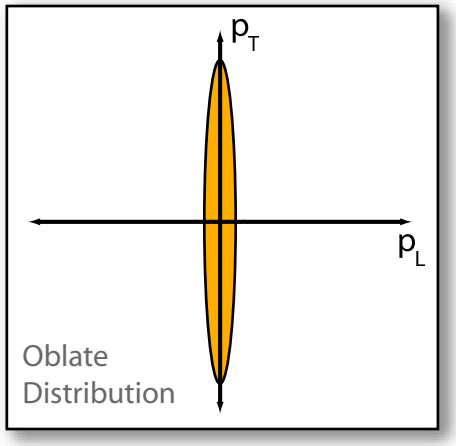
Momentum Anisotropy



Momentum space anisotropy time dependence at the early stages of a heavy ion collision



Weibel instabilities



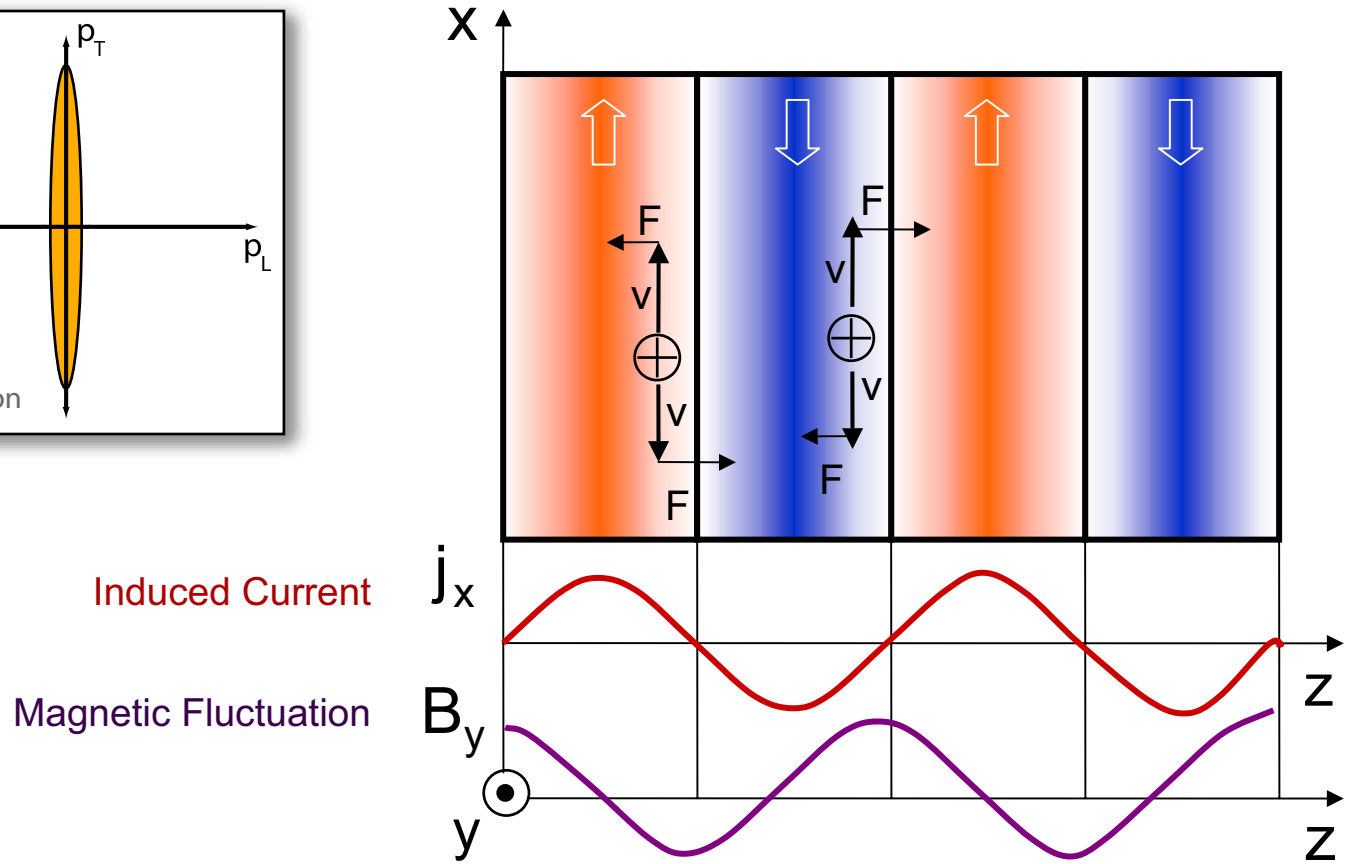
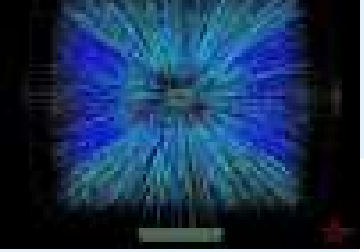
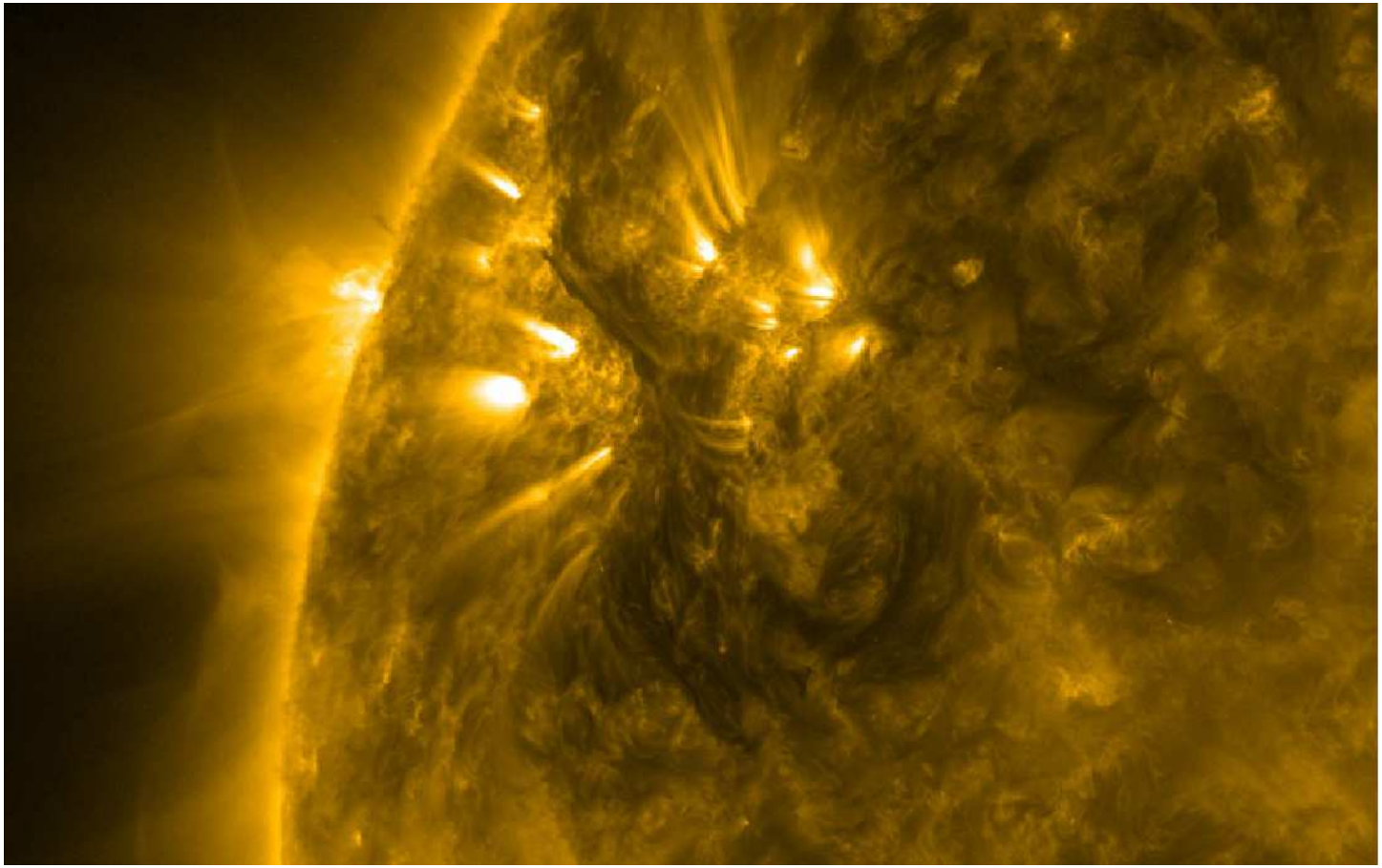


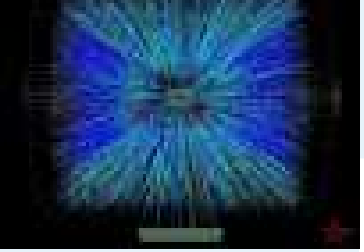
Illustration of the mechanism of filamentation instabilities.



QED Plasma

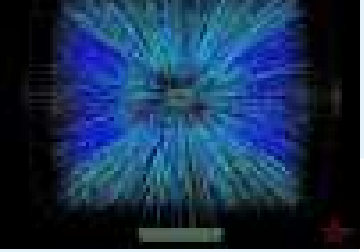


Filaments and active solar region from NASA's Solar Dynamics Observatory



Scales of weakly coupled QGP

- T: energy of hard particles
- gT: thermal masses, Debey screening mass, Landau damping, plasma instabilities [Mrowczynski 1988, 1993, ..]
- g^2T: magnetic confinement, color relaxation, rate for small angle scattering
- g^4T: rate for large angle scattering, $\eta^{-1}T^4$



Hard (Thermal) Loops - Boltzmann - Vlasov

With color-neutral background distribution $v \cdot \partial f_0(\mathbf{p}, \mathbf{x}, t) = 0$, $v^\mu = p^\mu / p^0$ gauge covariant Boltzmann-Vlasov:

$$v \cdot D\partial f_a(\mathbf{p}, \mathbf{x}, t) = g v_\mu F_a^{\mu\nu} \partial_\nu^{(p)} f_0(\mathbf{p}, \mathbf{x}, t) = \\ = -g(\mathbf{E}_a + \mathbf{v} \times \mathbf{B}_a) \cdot \nabla_\mathbf{p} f_0, \tag{2}$$

$$D_\mu F_a^{\mu\nu} = j_a^\nu = g \int \frac{d^3p}{(2\pi)^3} \frac{p^\mu}{2p^0} \delta f_a(\mathbf{p}, \mathbf{x}, t). \tag{3}$$

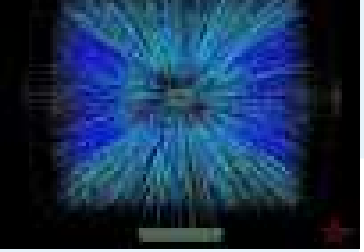
So far mostly stationary $f_0(\mathbf{p})$ with $\partial_\mu f_0 \equiv 0$

- isotropic: $f_0(\mathbf{p}) = f_0(|\mathbf{p}|), \nabla_\mathbf{p} f_0 \propto \mathbf{v}$

$$v \cdot D\delta f_a(\mathbf{p}, \mathbf{x}, t) = -g\mathbf{E}_a \cdot \nabla_\mathbf{p} f_0 \quad (stable) \tag{4}$$

- anisotropic: $f_0(\mathbf{p}), \nabla_\mathbf{p} f_0 \not\propto \mathbf{v}$

$$v \cdot D\delta f_a(\mathbf{p}, \mathbf{x}, t) = -g(\mathbf{E}_a + \mathbf{v} \times \mathbf{B}_a) \cdot \nabla_\mathbf{p} f_0 \quad (unstable!) \tag{5}$$



Discretized Hard Loop Effective Theory

Auxiliary field formulation: Mrowczynski, Rebhan & Strickland 2004]

$$\delta f^a(x;p) = -gW^a_\mu(t,\mathbf{x};\mathbf{v})\partial^\mu(p)f_0(\mathbf{p}) \tag{6}$$

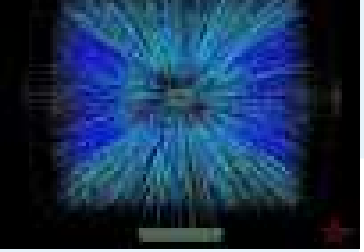
$$[v \cdot D(A)]W_\mu(x;\mathbf{v}) = F_{\mu\gamma}(A)v^\gamma \tag{7}$$

where $v^\mu \equiv p^\mu/|\mathbf{p}| = (1,\mathbf{v})$

$$j^\mu(x) = -g^2 \int \frac{d^3p}{(2\pi)^3}\frac{1}{2|\mathbf{p}|}p^\mu \frac{\partial f(\mathbf{p})}{\partial p^\nu} W^\nu(x;\mathbf{v}), \tag{8}$$

Hard Loop effective theory: (hard) scale $|\mathbf{p}|$ integrated out
for real-time lattice simulation: discretize also velocity space in "disco balls"

$$D_\sigma(A)F^{\sigma\mu} = j^\mu(x) = \frac{1}{\mathcal{N}}\sum_v v^\mu \mathcal{W}_v(x) \tag{9}$$



Notations for Bjorken expansion

It is convenient to switch to comoving coordinates

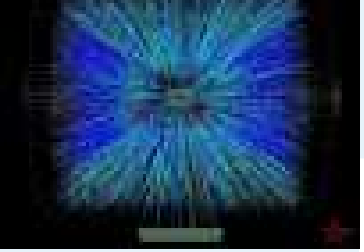
$$t = \tau \cosh \eta, \qquad \beta = \tanh \eta,$$
$$z = \tau \sinh \eta, \qquad \gamma = \cosh \eta, \tag{10}$$

i.e, a coordinate system with metric $ds^2 = d\tau^2 - d\mathbf{x}_\perp^2 - \tau^2 d\eta^2$. We introduce the notation

$$\tilde{x}^\alpha = (x^\tau, x^i, x^\eta) = (\tau, x^1, x^2, \eta) \tag{11}$$

with indices from the beginning of the Greek alphabet for these new coordinates. In addition to space-time rapidity η, we also introduce momentum space rapidity y for the massless particles according to

$$p^\mu = p_\perp (\cosh y, \cos \phi, \sin \phi, \sinh y). \tag{12}$$



Hard-Expanding-Loop formalism

With $$p^\beta \partial_\beta \left[\partial^\alpha_{(p)} f_0(\mathbf{p}_\perp, p_\eta)\right]\Big|_{p^\mu = const.} = 0 \tag{13}$$

we can commute $\mathbf{p}\cdot D$ and thus solve gauge-covariant Vlasov equation in comoving coordinates

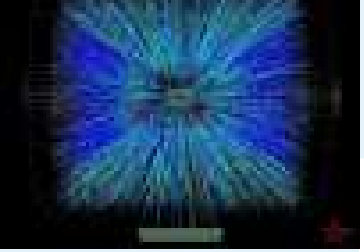
$$p.D\delta f_a(\mathbf{p},\mathbf{x},t)\Big|_{p^\mu = const.} = g p^\beta F^a_{\beta\alpha}\partial^\alpha_{(p)} f_o(\mathbf{p},\mathbf{x},t). \tag{14}$$

Introducing auxiliary fields $W^a_\alpha(\tau, x^i, \eta; \phi, y)$ similar to the auxiliary field $W^\nu(x,\mathbf{v})$ of the hard-loop formalism

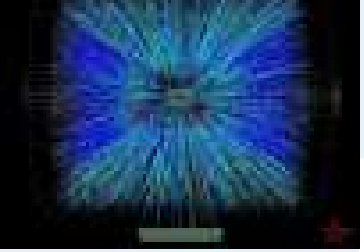
$$\delta f^a(x;p) = -g W^a_\alpha(\tau, x^i, \eta; \phi, y)\partial^\alpha_{(p)} f_0(p_\perp, p_\eta) \tag{15}$$

that obey $$v\cdot D W_\alpha(\tau, x^i, \eta; \phi, y)\Big|_{\phi,y} = v^\beta F_{\alpha\beta} \tag{16}$$

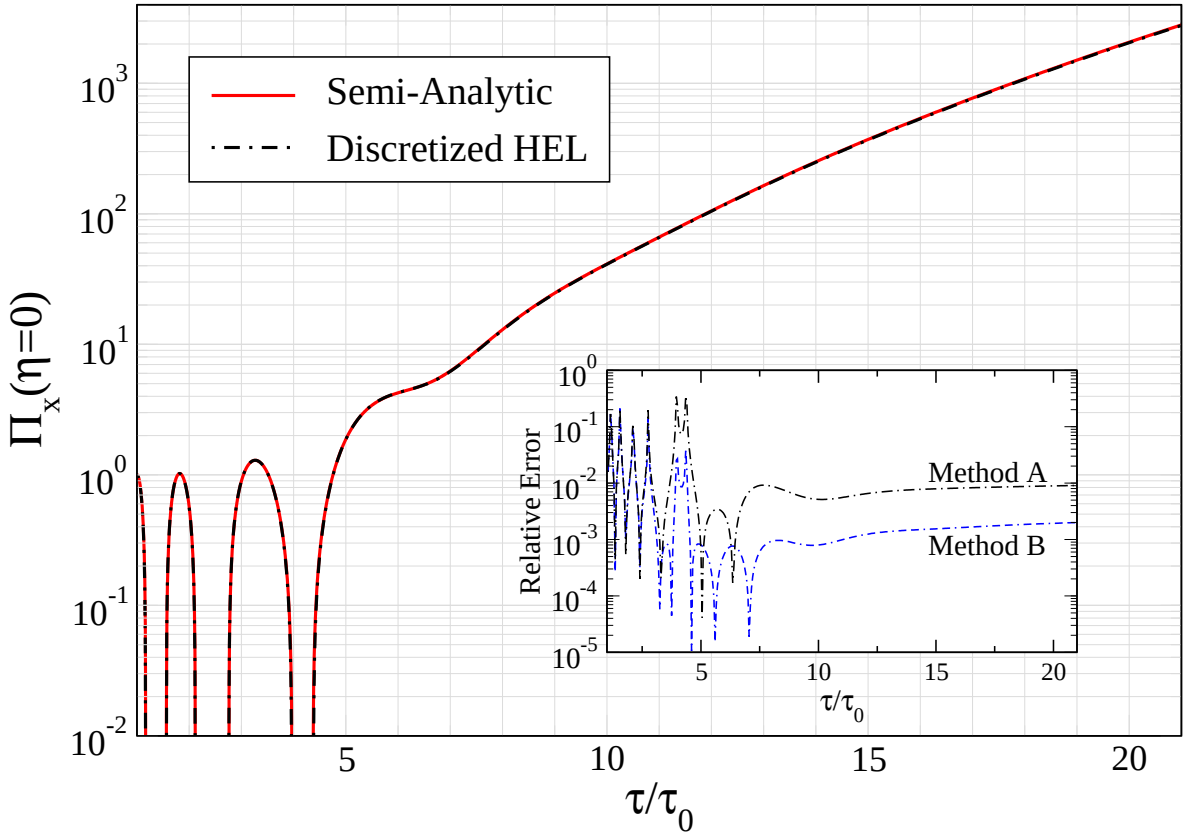
where $v^\alpha \equiv \frac{p^\alpha}{|\mathbf{p}_\perp|} = \left(cosh(y-\eta), \cos\phi, \sin\phi, \frac{sinh(y-\eta)}{\tau}\right)$.



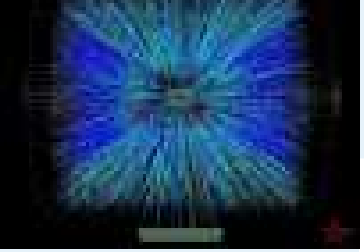
Plasma Instabilities



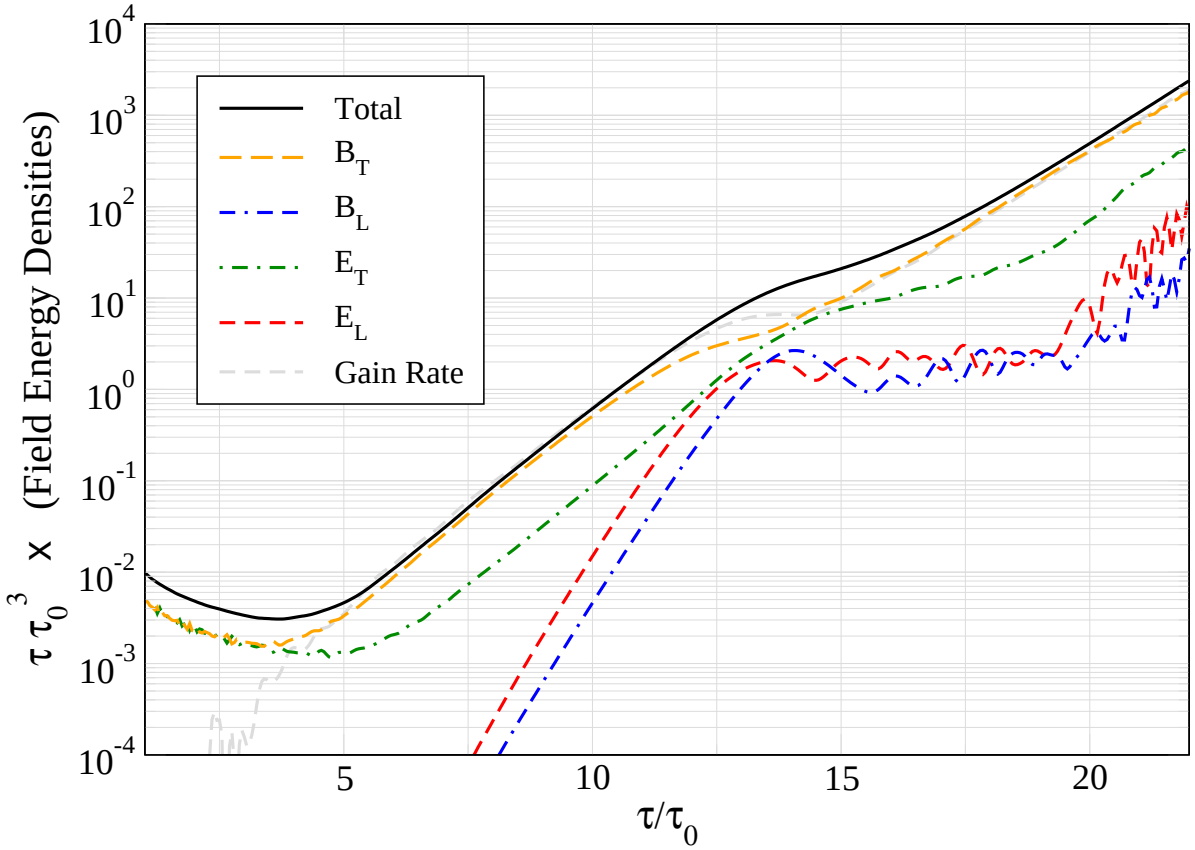
Expanding 1D+3V Abelian plasma



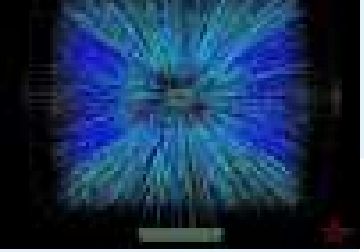
The proper-time evolution of the canonical field momentum of a single Abelian mode.



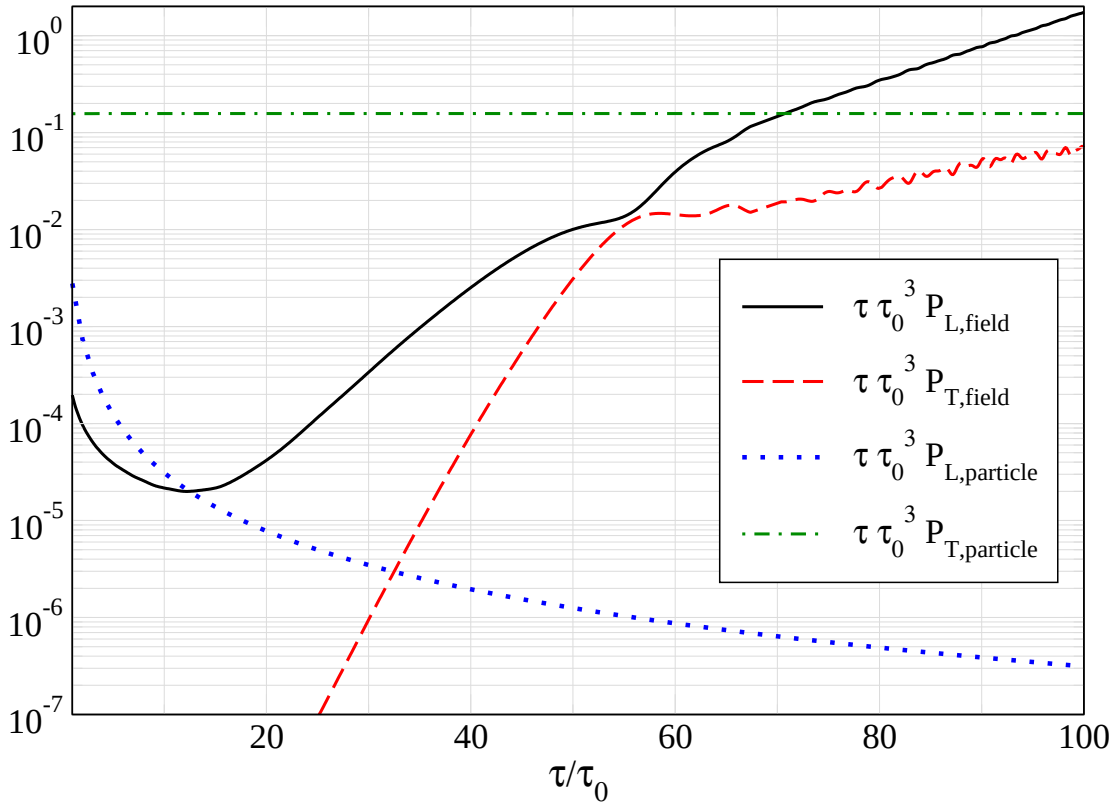
Expanding 1D+3V non-Abelian plasma



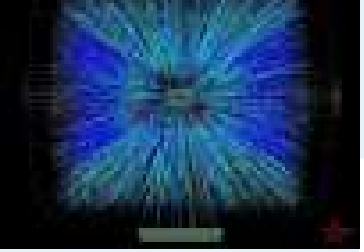
The proper-time dependence of the chromo-field energy densities and the energy gain rate times an extra factor of τ_0 resulting from non-Abelian run initialized with Fukushima, Gelis, and McLerran (FGM) initial conditions.



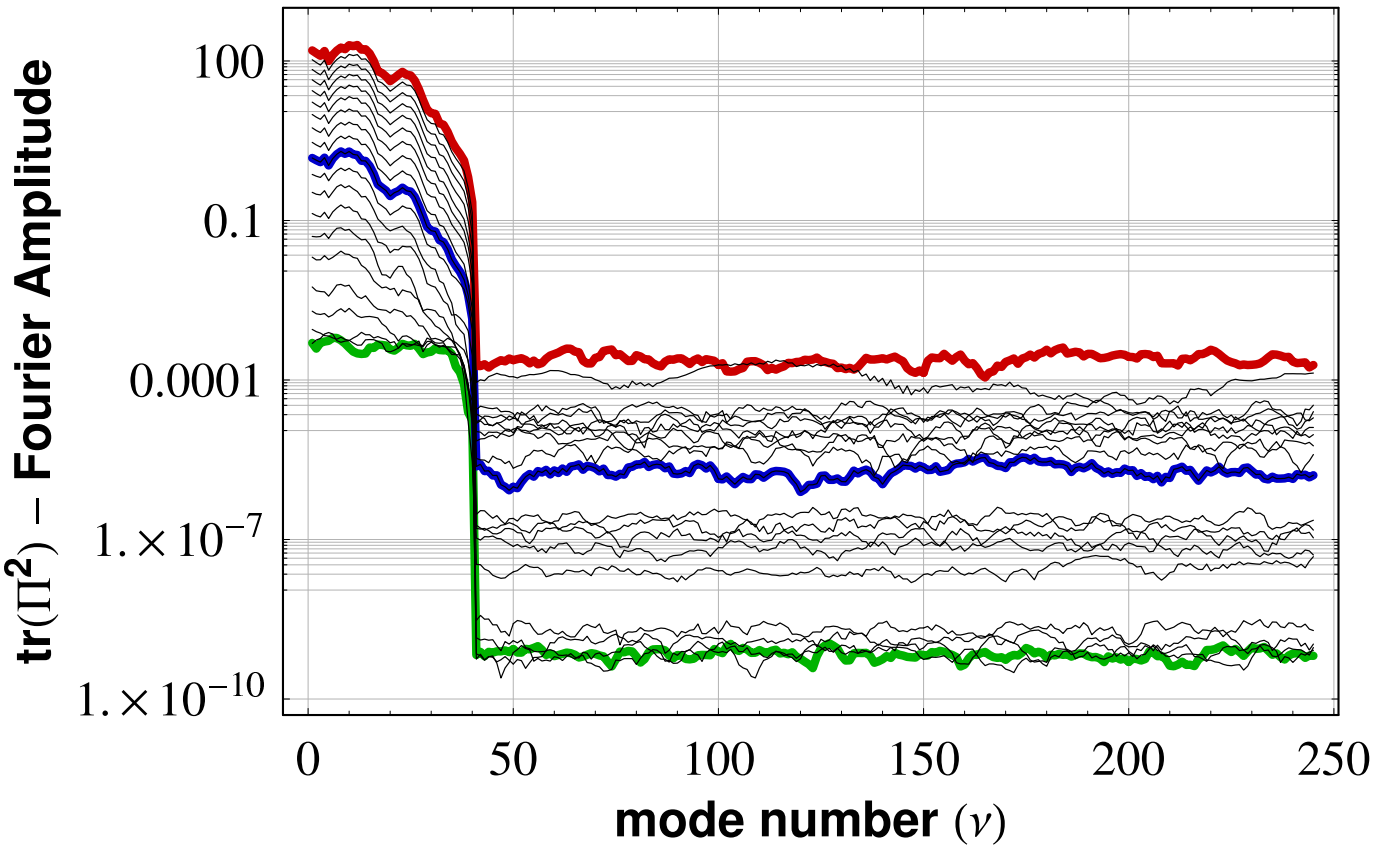
Expanding 1D+3V non-Abelian plasma



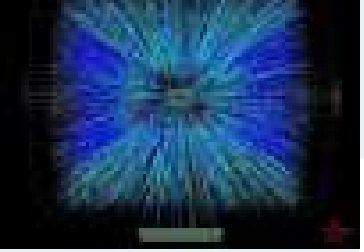
The comparison of the longitudinal and transverse pressures for the fields and particles resulting from a typical non-Abelian run initialized with FGM (CGC inspired) initial conditions.



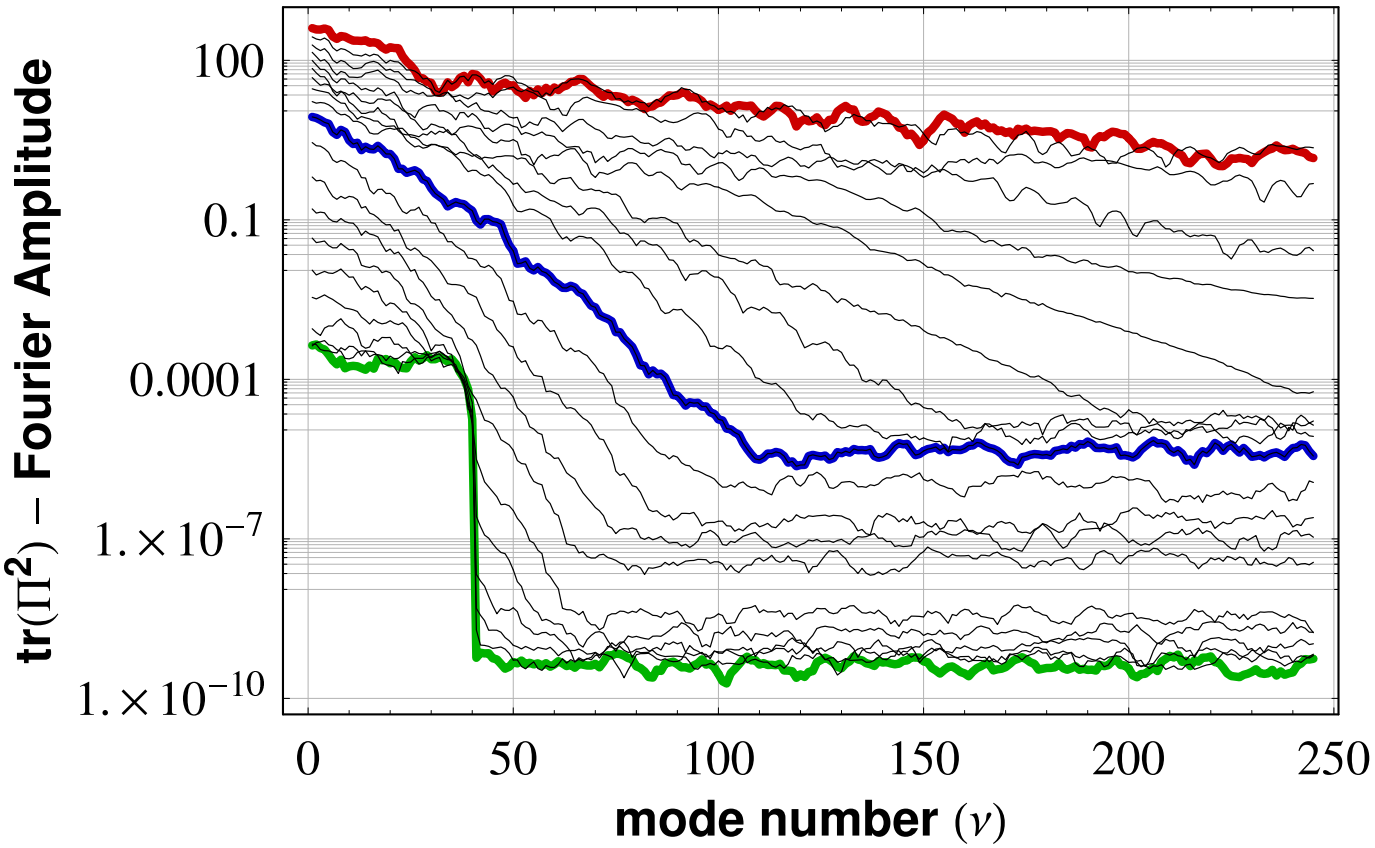
Expanding 1D+3V Abelian plasma



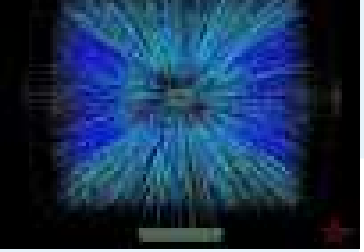
Fourier spectrum of the color-traced conjugate field momentum obtained from Abelian run with FGM initial conditions.



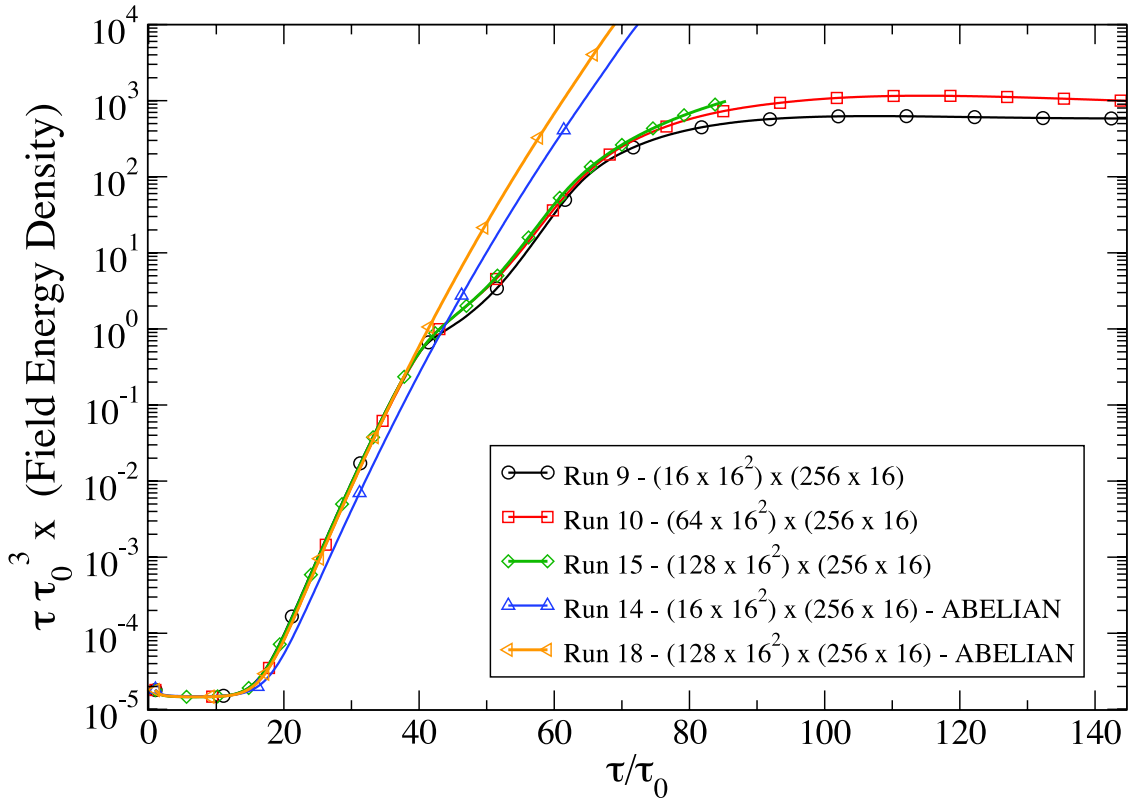
Expanding 1D+3V non-Abelian plasma



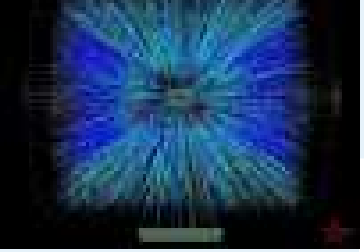
Fourier spectrum of the color-traced conjugate field momentum obtained from non-Abelian run with FGM initial conditions.



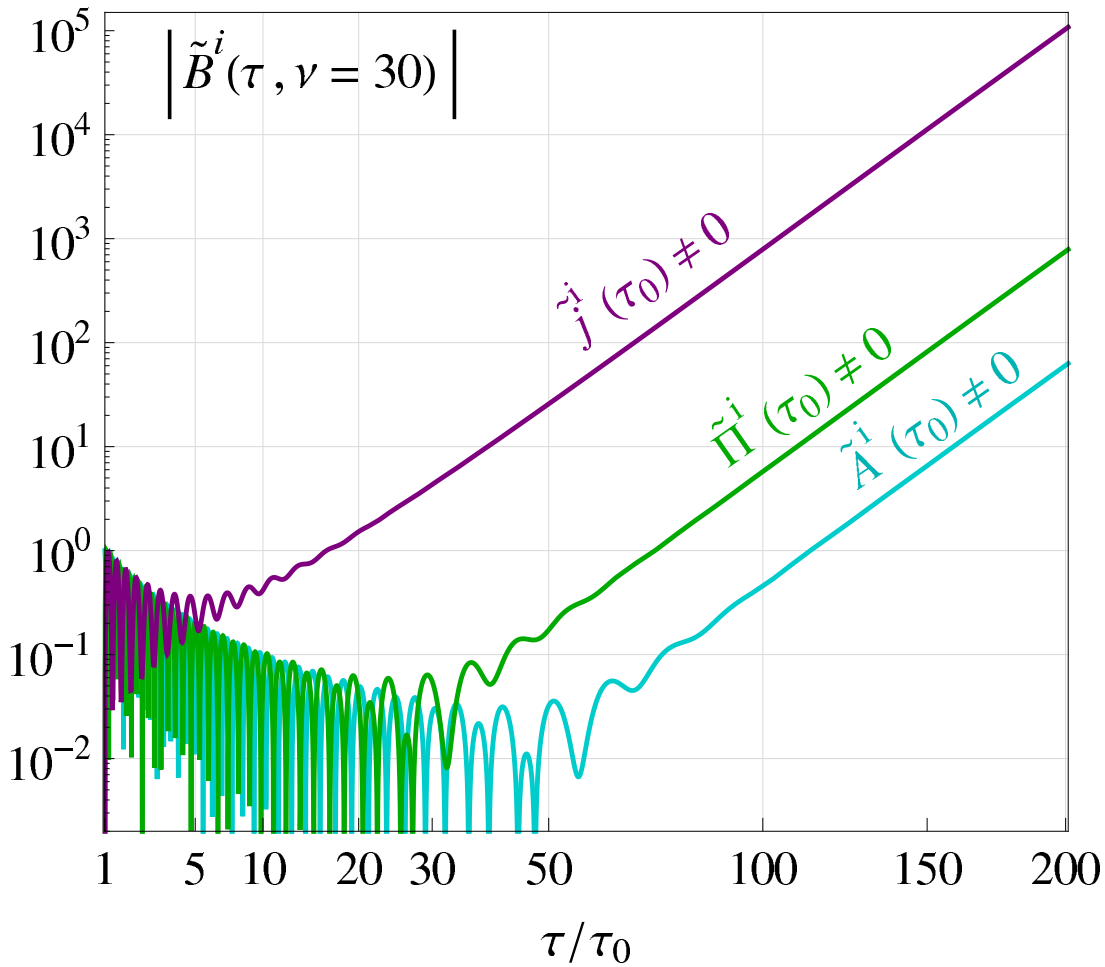
Expanding 3D+5V plasma



Preliminary runs from the HEL 3d codes in Abelian and non Abelian setup with different lattice sizing's in the longitudinal η direction, but identical transverse size and $\mathcal{W}$ auxiliary field numbers.

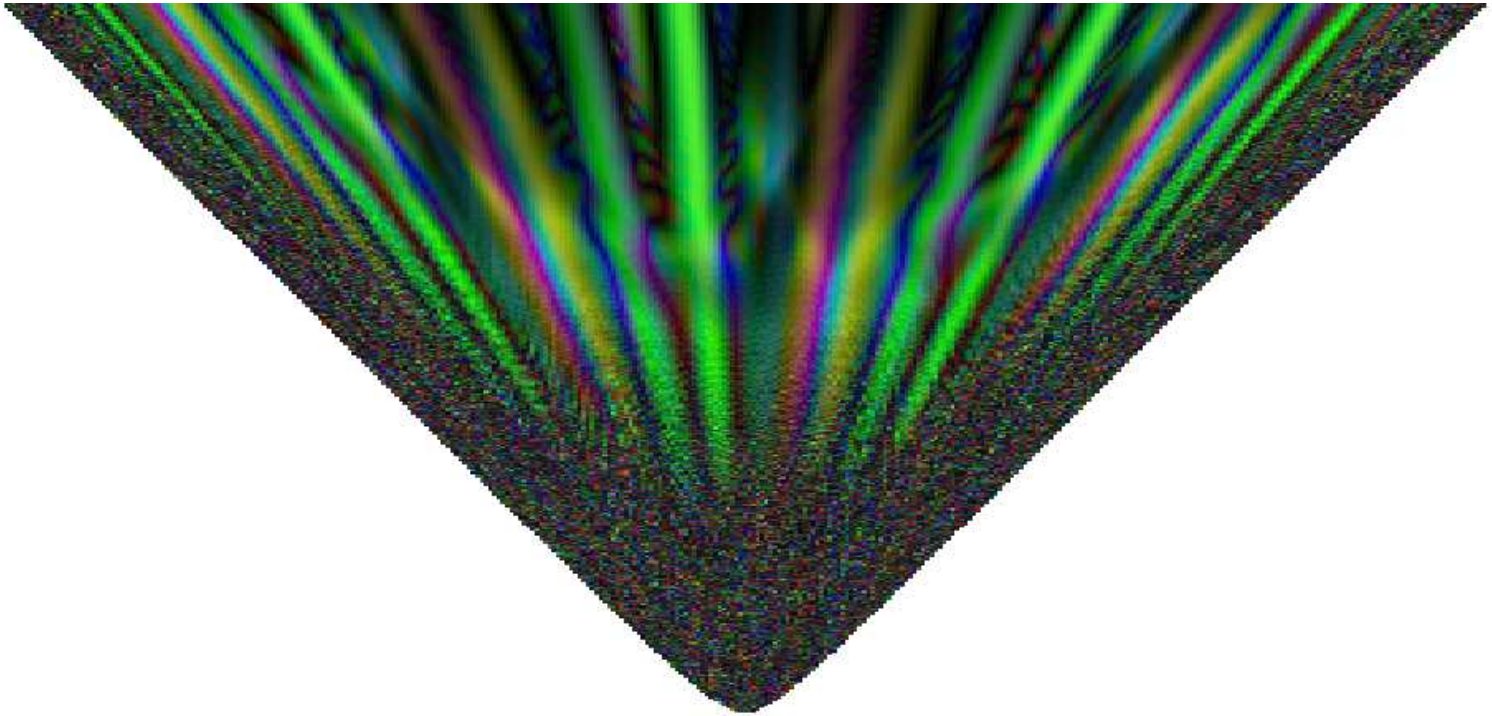


Unstable transverse modes

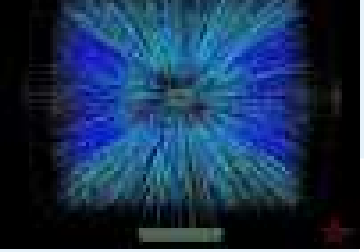


Influence of different initial conditions for a specific mode with $\nu = 30$

Expanding 1D+3V non-Abelian plasma



Visualization of the space-time development of color correlations in a non-Abelian plasma instabilities in Bjorken expansion.



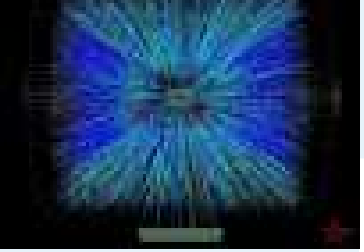
Conclusions

Non-abelian plasma instabilities accelerate isotropization and thermalization of the Quark Gluon Plasma.

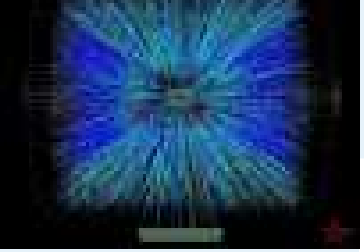
Large amplitude turbulent field configurations can have an important effect on Quark Gluon Plasma transport such as momentum broadening, energy loss, plasma viscosity, ...

In the 1D+3V Hard Expanding Loop (HEL) 1D we found that the exponential (in $\sqrt{\tau}$) growth in the Abelian (weak-field) phase is only mildly weakened when nonlinearities through non-Abelian self-interactions of the collective fields set in.

The previous 1D HEL code has been extended to full 3D+5V. Final results including different initial conditions are being computed.



Thank you.



Backup - Equation of motions

Conjugate Momenta

$$\partial_\tau E_i = +\tau j^i + \frac{1}{\tau} D_\eta^2 A^i + \tau g^2 i[A^{j\neq i}, i[A^{j\neq i}, A^i]] \tag{17}$$

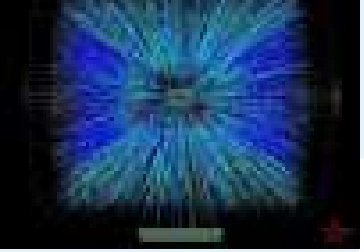
$$\partial_\tau E^\eta = -\tau j^\eta + \frac{ig}{\tau}[A^i, D_\eta A^i] \tag{18}$$

Gauss law

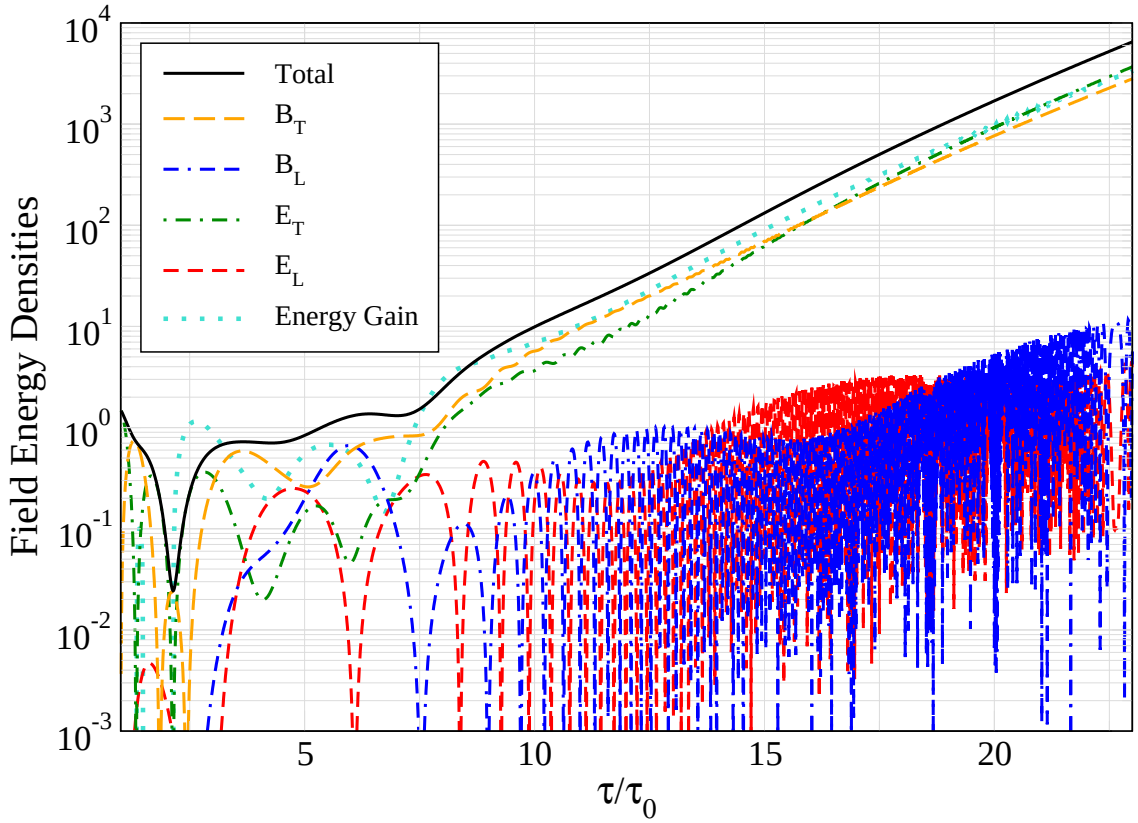
$$j^\tau = +\frac{1}{\tau} D_\eta E^\eta - \frac{ig}{\tau}[A_i, E^i] \tag{19}$$

with

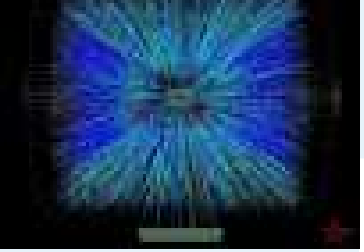
$$E^i \equiv \tau \partial_\tau A_i, \quad E^\eta \equiv \frac{1}{\tau}\partial_\tau A_\eta \tag{20}$$



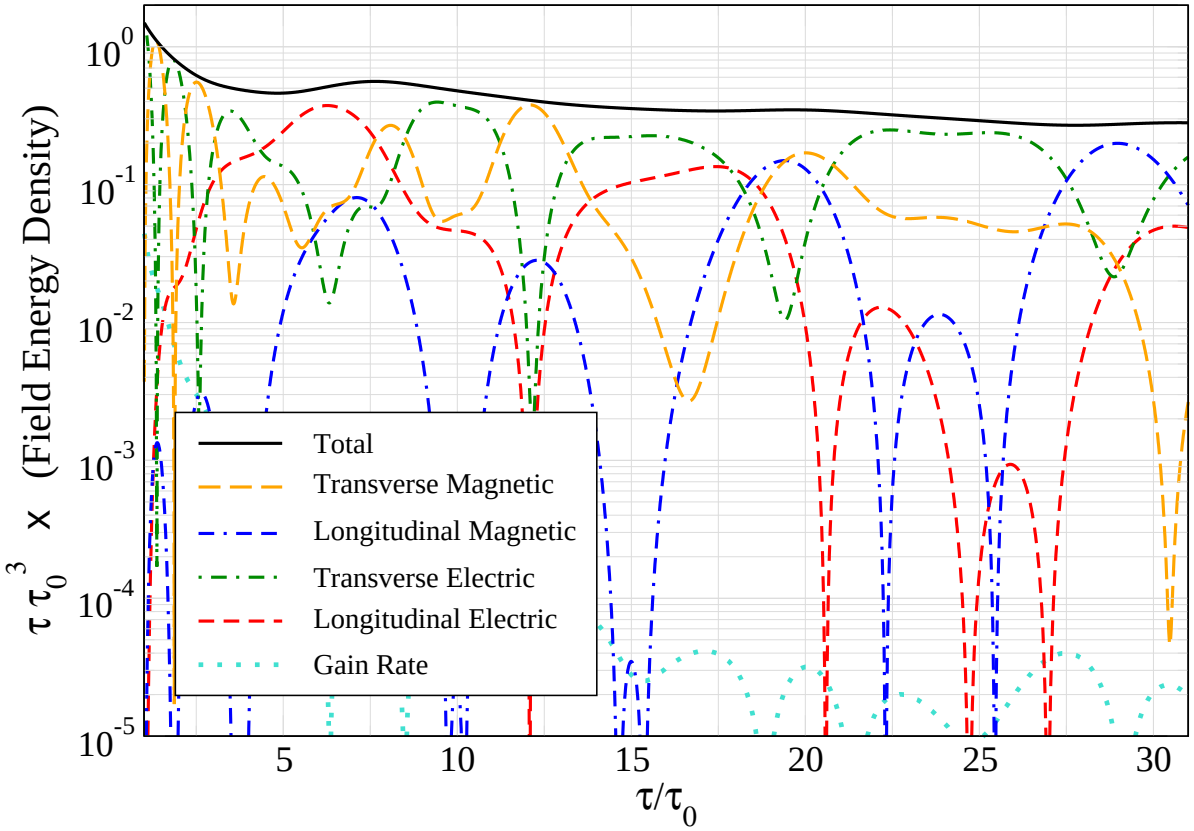
Backup - Expanding 1D+3V non-Abelian plasma



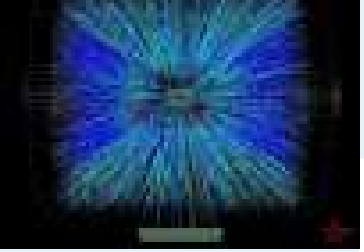
The proper-time dependence of the chromo-field energy densities from a run with a single non-Abelian mode seeded with random noise.



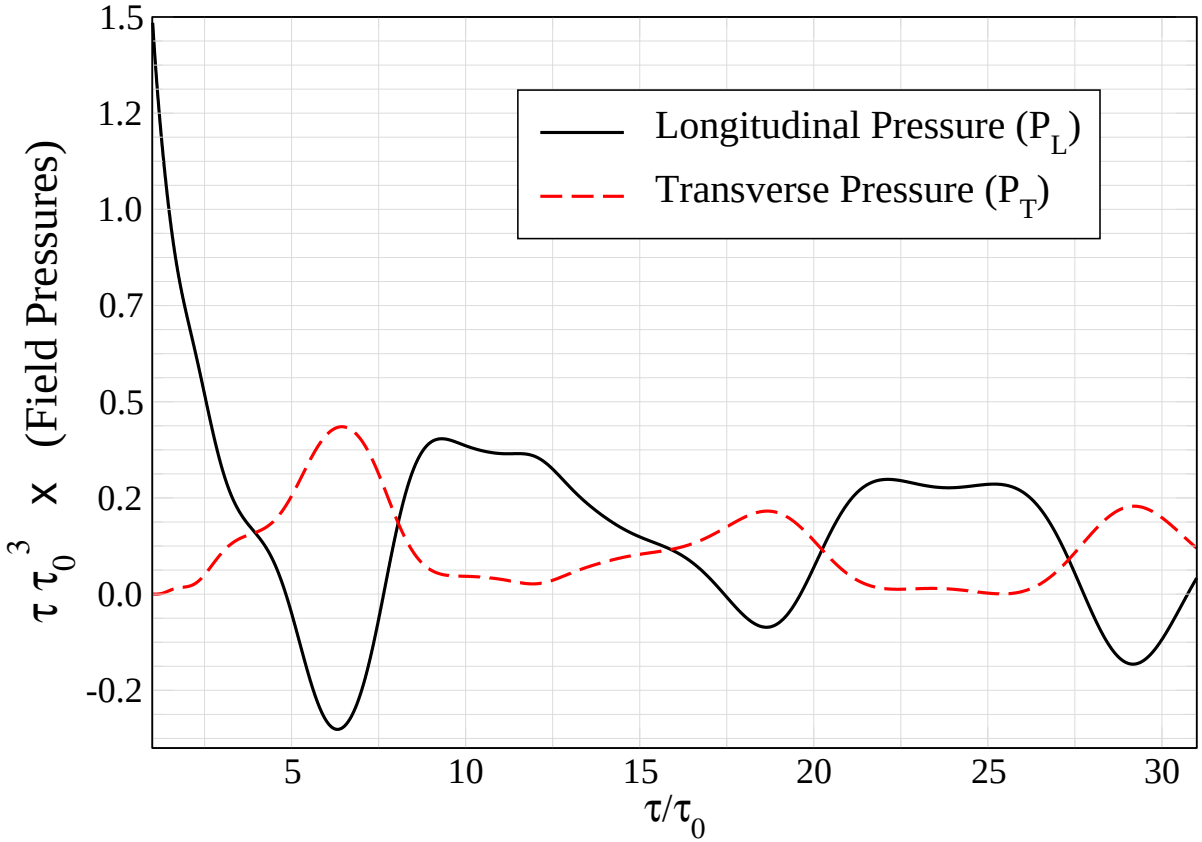
Backup - Expanding 1D+3V non-Abelian plasma



The proper-time dependence of the chromo-field energy densities with a single non-Abelian mode with decoupled hard particle currents ($j = 0$).



Backup - Expanding 1D+3V non-Abelian plasma



The proper-time dependence of the longitudinal and transverse pressure with a single non-Abelian mode with decoupled hard particle currents ($j = 0$).