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Тезисы докладов

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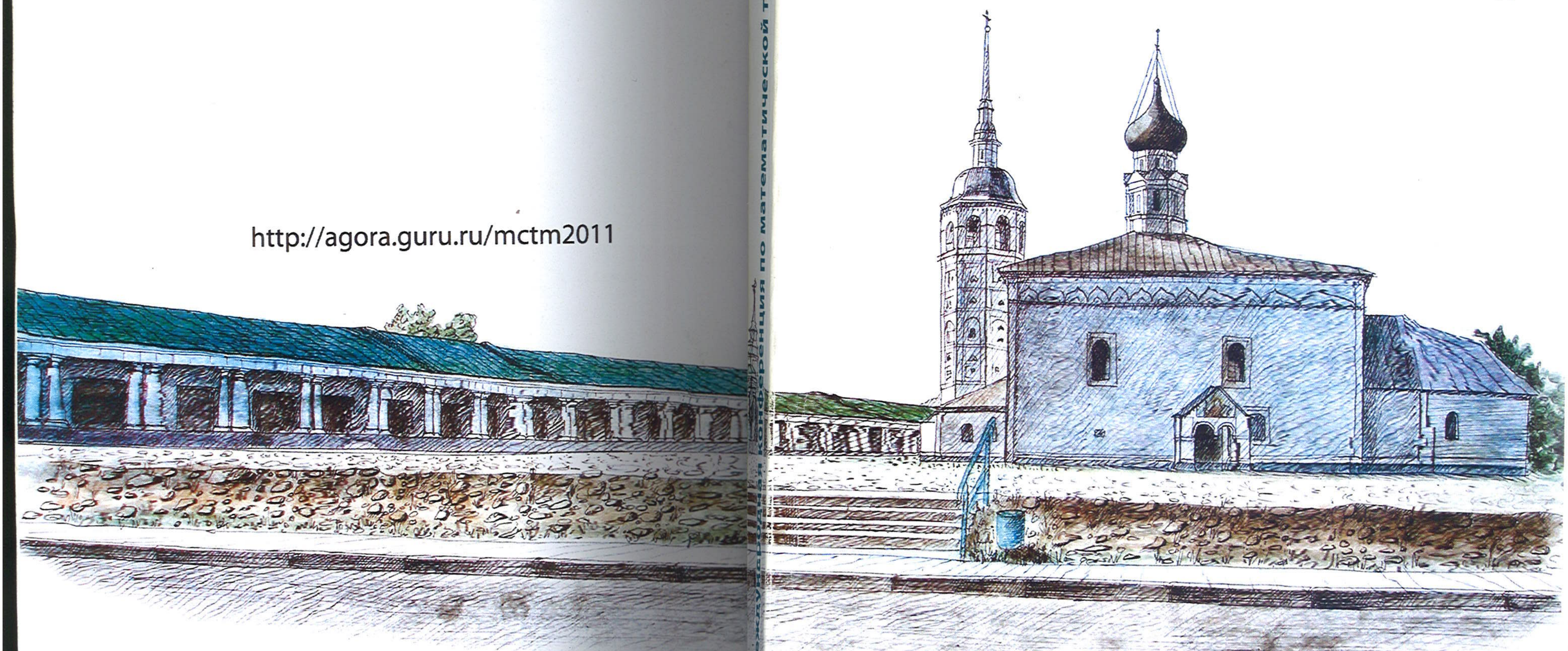

Steklov Mathematical Institute of the RAS

Vladimir State University
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INTERNATIONAL CONFERENCE ON MATHEMATICAL CONTROL THEORY AND MECHANICS

Abstracts

Suzdal 2011

Математический институт им. В. А. Стеклова РАН

Владимирский государственный университет
им. А.Г. и Н.Г. Столетовых

МЕЖДУНАРОДНАЯ КОНФЕРЕНЦИЯ ПО МАТЕМАТИЧЕСКОЙ ТЕОРИИ УПРАВЛЕНИЯ И МЕХАНИКЕ

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Regular Dynamics of a Hula-hoop

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Hula-hoop is a popular toy – a thin hoop that is twirled around the waist, limbs or neck. To twirl a hula-hoop the waist of a gymnast carries out a periodic motion in the horizontal plane. For the sake of simplicity we assume that the waist of the gymnast is round and its center moves in the horizontal plane along an elliptic trajectory $x = a\sin\omega t$, $y = b\cos\omega t$ close to a circle ($a \approx b$) with frequency ω, semi-major axis a, and semi-minor axis b. Then in new time $\tau = \omega t$ we have the equation for angle φ that determines the position of the hula-hoop center with respect to the waist

$$\ddot{\varphi} + \gamma\dot{\varphi} + \mu\cos(\varphi - \tau) = \varepsilon\cos(\varphi + \tau) \tag{1}$$

along with the condition for the hula-hoop not to separate from the waist during its motion

$$\dot{\varphi}^2 - 2\mu\sin(\varphi - \tau) + 2\varepsilon\cos(\varphi + \tau) > 0, \tag{2}$$

with dimensionless parameters $\gamma = \frac{k}{2mR^2\omega}$, $\mu = \frac{a+b}{4(R-r)} > 0$, $\varepsilon = \frac{a-b}{4(R-r)} \geq 0$, where k is the coefficient of viscous friction, r is the radius of the waist, R and m are the radius and mass of the hula-hoop.

In [1] the periodic motion of the gymnast's waist along only one axis was considered ($b = 0$). In the present study, we consider the hula-hoop excitation along two axes corresponding to an elliptic trajectory as in [2]. But in contrast to previous works [1],[2] we do not require that all parameters of excitation μ and dissipation γ are small. We assume only ε to be small like in our work [3] which we extend here.

When the waist moves along a circle ($a = b$, i.e. $\varepsilon = 0$) we have exact solutions of (1)

$$\varphi = \tau + \varphi_0, \quad \varphi_0 = \pm\arccos(-\gamma/\mu) \mod 2\pi \tag{3}$$

corresponding to the hula-hoop rotation with a constant angular velocity equal to the excitation frequency, provided that $|\gamma| \leq \mu$. According to the *Lyapunov's theorem on the stability based on a linear approximation solution* with the use of Routh-Hurwitz criterion we obtain asymptotic stability conditions $\gamma > 0$ and $\sin\varphi_0 < 0$ from which the inequalities $0 < \gamma < \mu$ follow.

Inseparability condition (2) takes the form $1 - 2\mu\sin\varphi_0 > 0$. Hence, rotation (3) with $\varphi_0 = -\arccos(-\gamma/\mu)$ is asymptotically stable and inseparable, while that of with $\varphi_0 = \arccos(-\gamma/\mu)$ is unstable and inseparable only if $\mu < \sqrt{1/4 + \gamma^2}$.

When the waist center trajectory has small ellipticity ($a > b$, i.e. $\varepsilon > 0$) we use simple perturbation method assuming that solution of (1) can be expressed in series $\varphi = \tau + \varphi_0 + \varepsilon\varphi_1(\tau) + \ldots$ of small parameter ε. After substitution of this series in (1) and grouping the terms by powers of ε we obtain φ_0 like in (3). Taking $\varphi_0 = -\arccos(-\gamma/\mu)$ corresponding to the stable solution (3) of the unperturbed system we derive

$$\varphi_1(\tau) = \frac{2\gamma\sin(\varphi_0 + 2\tau) - \left(4 - \sqrt{\mu^2 - \gamma^2}\right)\cos(\varphi_0 + 2\tau)}{3\gamma^2 + \mu^2 - 8\sqrt{\mu^2 - \gamma^2} + 16}.$$

Inseparability condition (2) leads in the first approximation to the following inequality

$$\varepsilon < \frac{1 + 2\sqrt{\mu^2 - \gamma^2}}{2}\sqrt{\frac{\mu^2 + 3\gamma^2 - 8\sqrt{\mu^2 - \gamma^2} + 16}{\mu^2 + 8\gamma^2 - 12\sqrt{\mu^2 - \gamma^2} + 36}}.$$

The hula-hoop can rotate in both direction only if all parameters μ, γ, and ε are small since the coexistence condition for both direct and inverse rotations has the form $0 < \gamma < \min\{\varepsilon, \mu\}$. In physical variables the coexistence condition takes the form

$$0 < 2k\frac{R - r}{R^2\omega m} < a - |b|$$

meaning that the trajectory of the waist should be sufficiently prolate.

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OPTIMAL CONTROL OF CONTINUOUS-DISCRETE SYSTEMS WITH MULTIPLE INSTANTANEOUS SWITCHINGS OF THE DESCRETE PART[36]

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We consider the continuous-discrete system with the continuous part dynamics determined by differential equations, and with the behavior of the discrete part determined by the recurrent equations. Multiple instantaneous switches at given pace time instants are allowed. These systems, which are called hybrid, takes an intermediate place between discrete-continuous systems with single switches [1] and logic-dynamical systems [2].

We describe various settings of the problem, when the number of switches of the discrete part is either given, either is totally bounded over all pace switching instants, or is to be found within the solution of the optimization problem.

The equations for the variations of the functionals, defined on the trajectories of these systems and prove the necessary optimality conditions. The relevant practical examples are considered.

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THE LAPLACE-BELTRAMI OPERATOR IN ALMOST-RIEMANNIAN GEOMETRY

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Two-dimensional almost-Riemannian structures are generalized Riemannian structures on surfaces for which a local orthonormal frame is given by a Lie bracket generating pair of vector fields that can become collinear. They appears naturally in optimal control problems linear in the control and with quadratic cost. Generically, the singular set is an embedded one dimensional manifold and there are three type of points: Riemannian points where the two vector fields are linearly independent, Grushin points where the two vector fields are collinear but their Lie bracket is not and tangency points where the two vector fields and their Lie bracket are collinear and the missing direction is obtained with one more bracket, the last one being isolated. In this talk we study the Laplace-Beltrami operator on such a structure. In the case of a compact orientable surfaces without tangency points, we prove that the Laplace-Beltrami operator is essentially self-adjoint. As a consequence a quantum particle in such a structure cannot cross the singular set and the heat cannot flow through the singularity. This is an interesting phenomenon since when approaching the singular set (i.e. where the vector fields become