# Analysis of OBS Burst Assembly Queue with Renewal Input

Tomasz Hołyński and Muhammad Faisal Hayat

***Abstract*—Ongoing research in Optical Burst Switching (OBS) requires more in-depth studies both in theory and in practice before the technology is realized. In OBS paradigm, traffic from access networks is groomed at edge OBS nodes in the forms of large chunks called bursts. This grooming called assembly is crucial in analyzing the overall performance of OBS networks as it affects the design of almost all major functions of OBS nodes. The characteristics of assembled traffic and its effects on OBS performance have been already extensively studied in literature. In this work, the assembled traffic is studied using a transform-based approach, since it is a natural way of analyzing such processes where random variables are summed. The main contribution of this paper is formulation of distributions of burst length and burst inter-departure time in form of Laplace transforms, which are valid for general independent lengths and inter-arrival times of assembled packets. The results can be subsequently inverted analytically or numerically to give full densities or serve as moment generating functions for partial characteristics. A simple method for the distribution of the number of packets in a burst based on discrete Markov chain is provided. Detailed analytical derivations with numerical results are presented for Erlangian traffic and verified by simulations to show good exactness of this approach.**

***Index Terms*—Optical burst switching, burst assembly, hybrid assembly, performance modelling, queueing theory, Laplace transform**

## I. Introduction

THERE is an ever increasing demand for transmission capacity due to increased popularity of new applications requiring large amounts of data exchange. Dense wavelength division multiplexing (DWDM) has promised to cater the needs of future Internet backbones providing huge bandwidth capacities. From the first generation of optical networks with point to point connections, through the second generation with DWDM ring networks, now we are heading towards the third generation with flexible mesh topologies. Therefore, optical networks demand a real change in transfer mode of data as the established packet switching is not realizable in optical domain in the near future with the current state of technology. Optical circuit switching in the form of wavelength-routed networks also cannot provide scalability required to achieve real flexible mesh networks.

Optical burst switching has been proposed as a new paradigm a few years back [1] in attempts to pave the way for an all-optical backbone switching infrastructure. It incorporates prospects of both coarse-grained optical circuit switching and fine-grained optical packet switching and is considered as implementable solution for future all-optical networks.

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Principles of OBS can be briefly summarized as follows. The network is divided into two functional domains, the edge and the core. At the edge, packetized traffic is buffered and assembled into bursts consisting of many packets. As soon as a burst is assembled, it is placed into a transmission queue and a burst control packet is sent out of band over the network along the path determined by a routing protocol. The burst control packet configures switching connections in core nodes just for the time of transmission of the incoming burst. Subsequently, the burst is transmitted over the core without any nodal delays and electronic conversion until it reaches an egress node where disassembly finally takes place. Due to possibility of time contention among different flows, bursts can be lost at the core nodes.

The process of assembly results in a modified type of traffic, a good understanding of which is crucial in practical engineering of OBS networks as it affects many design parameters and functions of OBS nodes at both edge and core [2], [3], [4]. From the viewpoint of performance evaluation, characteristics of this traffic are the main input parameters for the theoretical analysis of core switches (burst losses, optimization of fiber delay lines). Therefore, it is important to dispose with at least approximations of probability distributions of time intervals between bursts and burst length. In a predominant number of studies on OBS core (e.g. [5], [6], [7], [8], [9]) these distributions are assumed to be negative exponential. While this is true for the inter-burst times, due to superposition of many independent flows, it is rather unrealistic when burst length is considered.

Since OBS is still in a pre-deployment phase, one does not know how large on average bursts should be and what degree of variability in their length can be tolerated in practice. Long bursts require more time to be assembled which results in greater delays for single packets. Moreover, they will certainly suffer higher losses and degrade performance of the upper layers due to the need of reordering of the packets delivered out of sequence [10]. On the other hand, shorter bursts, generated at higher rate, will cause more control traffic and unnecessarily load the network.

The analytical tools that can be used to analyze assembly process are queueing theory, renewal theory or some complex models which can be evaluated numerically. Analyzing the assembly process with queueing theoretical approaches is not straightforward as it does not fall under classical queueing discipline. It is because an OBS assembly queue is not strictly a queueing system with a server but it acts rather like a delay element without a server and passes accumulated customers in a batched manner when some criterion is met. Such behavior may have an analogy but it is not completely mappable to the batch service nor to gated vacation models. Therefore, we devise a simple probabilistic technique based on observation of the development of assembly of a single burst and determine probabilities that the $n$-th packet will complete the burst. Subsequently, referring to the trends from classical queueing literature, for the distributions of interest we formulate the solutions in the form of transforms.

Previous studies in this regard have analyzed the assembly process in detail, with focus on the characteristics of assembled burst lengths [11], [12], [13], the burst inter-departure times [14], [13], its impact on different aspects of global network performance, such as link-utilization and blocking probability at intermediate nodes [3], [4], or a combination of some of these aspects. However, to the best of our knowledge, there is no study which have tried to analyzed this burstification process with transform-based approach and general input traffic conditions and only a few studies have paid attention to the distribution of number of packets in burst and actual delay distribution experienced by the packets especially for the most favorite hybrid burst assembly. For example, Zapata et al. [15] analyzed packet delays but only for non-hybrid mechanisms and only average and maximum delays have been evaluated but no other metrics, such as the variance, nor the actual delay distribution. Rodrigo de Vega et al. [16] also analyzed the packet delays and compute the delay of the first packet in the burst, which is upper bound for a packet in burst but do not mention the delay suffered by other packets in that burst and they have also not extended their analysis for hybrid assembly. Hernandez [17] used fixed packet lengths and Poisson arrivals to find out delay distribution of each packet in a burst. To our best knowledge, there is no study on the mentioned probability distributions for general input and general packet length. This work assumes this generality and aims at study of two performance metrics that are most relevant for further analysis of transmission queue and OBS core nodes, namely burst length and inter-departure time in case of hybrid scheme. At this stage of our development, we show exemplar solutions where Erlang distributions of packet length and inter-arrival time are assumed, mainly due to easiness of their transform inversion. However, the method can be used also for more complicated distributions, especially when powerful numerical inversion techniques are applied [18], [19]. The study of the delay of n-th packet can be also treated with this method.

The rest of paper is organized as follows. In Section II, the OBS edge node architecture and burst assembly schemes are briefly described. The analytical model for burst assembly is presented in Section III. In Section IV numerical results with simulations are discussed and Section V concludes the paper.

## II. OBS Burst Assembly

Typically, the edge node consists of a classifier, burst assemblers, burst transmission queues, a routing and wavelength assignment modules and schedulers as shown in Fig.1. Each burst assembler module maintains one separate queue for each different destination/egress node. The classifier distributes the incoming packets, with respect to each packet's destination address, into the respective queues of the burst assembly modules. Based on the burst assembly technique, burst assembler module then assembles bursts consisting of packets headed for a specific egress node. After a burst has been aggregated, the corresponding control packet is generated and sent on the control channel. The assembled bursts wait for transmission in the electronic transmission buffers called burst transmission queues. The decision about scheduling a wavelength channel and time on which a burst is going to be sent is taken by the scheduling unit at the edge node. There are three main schemes that have been categorized in literature for burst assembly: time-based assembly, length-based assembly and hybrid assembly.

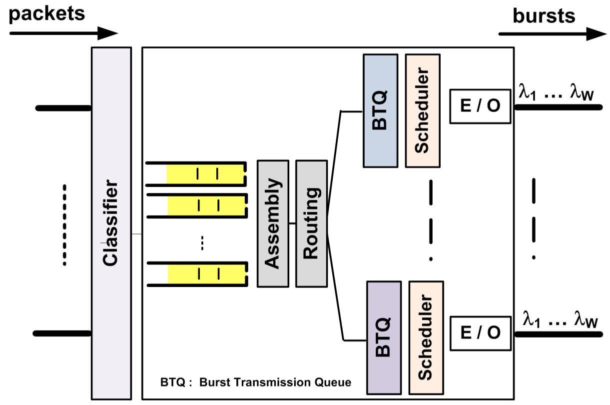


Fig. 1. General architecture of an OBS edge node.

In time-based assembly [11], after receiving the first packet in an assembler queue, a timer is started. Packets are collected in the queue until a defined time-out expires. The collected packets are then assembled into a burst and sent to the transmission buffer. The timer is restarted when a new packet is received in the queue. Therefore in time assembly, bursts are produced in periodic intervals from a single assembler queue, however, their sizes may vary depending on the arrival rate. In length-based assembly [14], [20], packets are collected until the total length of packets exceeds a defined threshold. The last packet that makes the total length equal or greater than threshold triggers the assembly of packets into a burst. Therefore, this kind of assembly generates bursts of approximately equal lengths but variable inter-departures.

The two mentioned have are simple to implement but have the following drawbacks. Monitoring only the time results in undesirably long bursts in high-load scenario, whereas

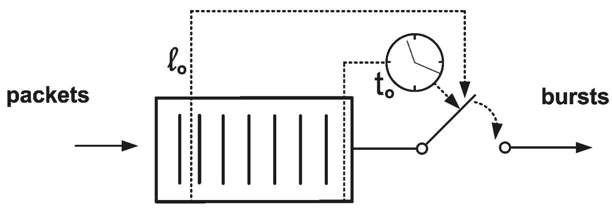


Fig. 2. Model of the hybrid assembly queue.

huge packet delays arise in length-based assembly under low load condition. These problems are overcome with the hybrid mechanism that takes into account both criteria. On arrival of the first packet, the timer is started. The burst is assembled on the basis of the time-out or length exceedance depending upon which event happens first, as schematically presented in Fig. 2. In this work, we have considered the hybrid assembly as it encompasses the two other strategies as special cases.

## III. Analytical Modelling

In this section, we develop an analytical model for hybrid assembly in which we consider a single assembly queue. In the model, packets destined to this queue arrive from a renewal process with general gap distribution and the lengths of packets are general independent random variables. The analysis is based on the observation of arrivals of subsequent packets and summation of their lengths to find probability that the aggregate of $n$ packets exceeds one of the thresholds. Because of the thresholds, the involved distributions and their Laplace transforms (LT) are subjected to truncations from the right, which are here indicated by an auxiliary operator $[...]^*$.

First, we find the probability mass function (pmf) of the number of the packets in a burst and derive general Laplace transforms of burst length and inter-departure time. Then we proceeds with evaluations in case lengths and arrivals are Erlang distributed. The quantities and notation used are listed below.

TABLE I
Notation used in the analysis.

| Symbol | Description |
|---|---|
| $L$ | packet length (random variable) |
| $T_A$ | packet inter-arrival time (random variable) |
| $\ell_o$ | length threshold |
| $t_o$ | time threshold |
| $f_L(\ell)$ | probability distribution function of packet length |
| $f_A(t)$ | probability distribution function of inter-arrival time |
| $\psi(s)$ | Laplace transform of $f_L(\ell)$ |
| $\phi(s)$ | Laplace transform of $f_A(t)$ |
| $f_{L,n}(\ell)$ | pdf of the length of $n$ aggregated packets |
| $f_{A,n}(t)$ | pdf of the time up to ($n$+1)th packet arrival |
| $\psi_n(s)$ | Laplace transform of $f_{L,n}(\ell)$ |
| $\phi_n(s)$ | Laplace transform of $f_{A,n}(t)$ |
| $[\psi(s)]^*$ | Laplace transform of a truncated pdf |
| $q_n$ | prob. that $n$ aggregated packets are shorter than $\ell_o$ |
| $r_n$ | prob. that time up to ($n$+1)th packet arrival is less than $t_o$ |
| $a_n, b_n$ | normalizing probabilities used for truncation of pdfs |
| $p_t$ | prob. that a burst is assembled due to time criterion |
| $p_\ell$ | prob. that a burst is assembled due to length criterion |
| $\pi_n$ | pmf of the number of packets in a assembled burst |
| $f_{BL}(\ell)$ | pdf of length of a assembled burst |
| $f_D(t)$ | pdf of burst inter-departure, LT: $\phi_D(s)$ |
| $f_{ex}(\ell)$ | pdf of the part of the burst part which exceeds $\ell_o$ |
| $\psi_{BL}(s)$ | Laplace transform of $f_{BL}(\ell)$ |
| $\phi_D(s)$ | Laplace transform of $f_D(t)$ |
| $\psi_{ex}(s)$ | Laplace transform of $f_{ex}(t)$ |
| $\phi_A(s)$ | Laplace transform of burst assembly time |

### A. *Model for general independent traffic*

First observation to be made is that under assumption of stationary renewal input and independence of packet lengths, the burst assembly is a regenerative process. This happens due to the fact the timer is reset upon arrival of a first packet making the past realizations irrelevant for the way the current burst is completed. That means that analysis of assembly of a single burst is sufficient for characterization of the whole process.

The development of hybrid assembly can be followed with the help of a discrete Markov chain shown in Fig. 3. The state number represents the number of packets currently aggregated and the absorbing state stands for the assembly completion.

#### *Pmf of the number of packets in a burst* $\pi_n$

Starting with the state no 1 (first arrival and timer reset), the transition probabilities can be explained by the following narration (Fig. 4). The burst will consist of only one packet if either its length exceeds $l_o$ (with probability $P\{L > l_o\}$) or the time up to the next packet arrival is greater than $t_o$ (with probability $P\{T_A > t_o\}$), as shown in Fig. 4a. Since both events are not disjoint, the probability of their union is

$$\begin{aligned} p_1 &= P\{L > l_o\} + P\{T_A > t_o\} \\ &- P\{L > l_o\}P\{T_A > t_o\}. \end{aligned}$$

With the probability $(1 - p_1)$, the burst will consist of at least two packets. The same reasoning is repeated up to the $n$th arrival, upon which one of the thresholds will be exceeded, as depicted in Fig. 4b. Then, the probability $\pi_n$ that burst comprises exactly $n$ packets can be read out from the Markov chain. Then

$$\pi_n = p_n \prod_{i=1}^{n-1}(1 - p_i), \tag{1}$$

with $p_n$ expressed in general by

$$p_n = (1 - q_n) + (1 - r_n) - (1 - q_n)(1 - r_n), \tag{2}$$

whereby the auxiliary probabilities $q_n$ and $r_n$ are calculated by the following integrals

$$q_n = \int_0^{\ell_o} f_{L,n}(\ell)\, d\ell, \quad r_n = \int_0^{t_o} f_{A,n}(t)\, dt. \tag{3}$$

Derivation of the densities $f_{A,n}(t)$ and $f_{L,n}(\ell)$ requires summations of independent random variables that are equivalent to multiplications of their Laplace transforms. The summations related with occurrence of the $n$th packet is done in such a way that the length of the $n$th packet (or the inter-arrival between the $n$th and ($n$+1)-th packet) is added to the already aggregated

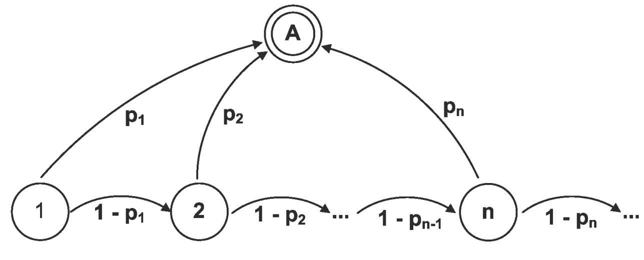


Fig. 3. Markov chain describing the assembly process

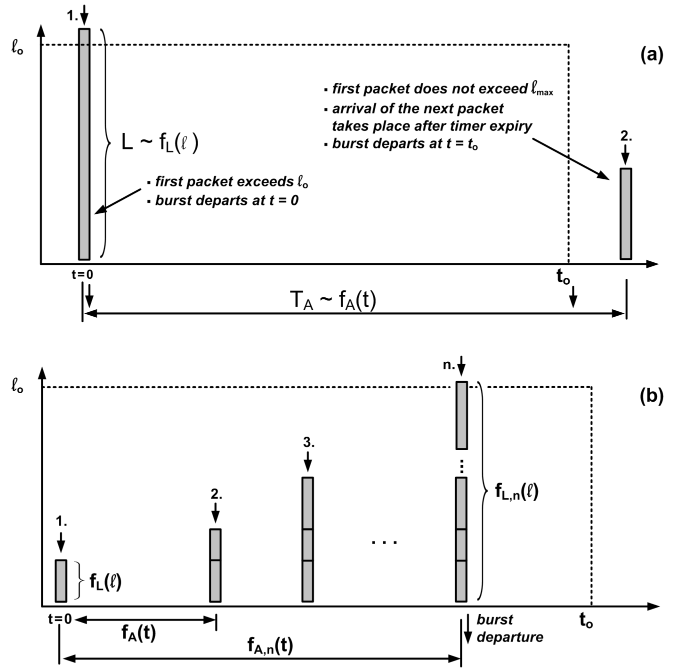


Fig. 4. Examples of hybrid assembly with lengths and time thresholds: (a) two possible realizations of assembly of a burst comprising one packet, (b) assembly of a burst comprising $n$ packets triggered by length exceedance.

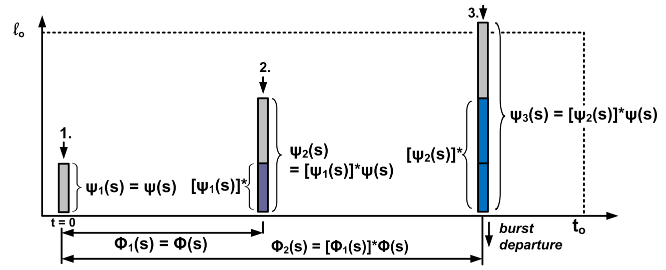


Fig. 5. Description of hybrid burst assembly with Laplace transforms.

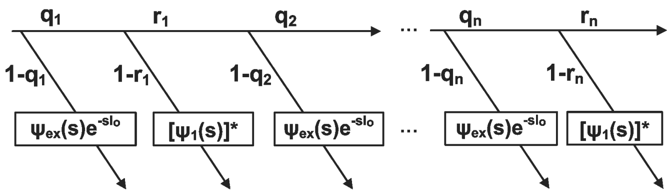


Fig. 6. Phase diagram for formulation of the Laplace transform of burst length $\psi_{BL}(s)$.

portion (or elapsed time) which is truncated at $l_o$ (or $t_o$). Using the Laplace transforms $\psi(s)=\mathcal{L}\{f_L(\ell)\}$, $\phi(s)=\mathcal{L}\{f_A(t)\}$, $\phi_n(s)=\mathcal{L}\{f_{A,n}(t)\}$ and $\psi_n(s)=\mathcal{L}\{f_{L,n}(\ell)\}$ the summation can be expressed by the following recursive relations

$$\begin{aligned}\psi_1(s) &= \psi(s)\\ \psi_n(s) &= [\psi_{n-1}(s)]^*\psi(s) \quad \text{for} \quad n=2,3,...,\infty \end{aligned} \tag{4}$$

$$\begin{aligned}\phi_1(s) &= \phi(s)\\ \phi_n(s) &= [\phi_{n-1}(s)]^*\phi(s) \quad \text{for} \quad n=2,3,...,\infty, \end{aligned} \tag{5}$$

where the operator $[...]^*$ denotes the fact that the Laplace transform is calculated from the density truncated at $\ell_o$ or $t_o$. Understanding of Eq. 4 and 5 is supported by Fig. 5. Usage of these relations can be greatly simplified by the observation that $[\psi_{n-1}(s)]^* = [\psi^{n-1}(s)]^*$ and $[\phi_{n-1}(s)]^* = [\phi^{n-1}(s)]^*$ that leads to the non-recursive expressions

$$\phi_n(s) = [\phi^{n-1}(s)]^*\phi(s) \quad \text{for} \quad n=1,2,...,\infty \tag{6}$$

$$\psi_n(s) = [\psi^{n-1}(s)]^*\psi(s) \quad \text{for} \quad n=1,2,...,\infty. \tag{7}$$

Finally, calculation of the probabilities $q_n$ and $r_n$ requires either analytical or numerical inversion of the transforms $\psi_n(s)$ and $\phi_n(s)$. Note that this operation needs to be performed only on the intervals $[0,\ell_o]$ or $[0,t_o]$, respectively. Knowledge of the probabilities $q_n$ and $r_n$ enables formulation of the transforms of burst length and inter-departure time.

### *Laplace transform of burst length* $\psi_{\mathbf{BL}}(\mathrm{s})$

Now, we are not interested in the probability for a concrete number of packets but rather in finding the probabilities that the assembly is completed by exceeding either of the length- or time threshold. We observe that in the former case the length is composed by a constant portion $l_o$ with transform $e^{-s\ell_o}$ and a random exceeding part with transform $\psi_{ex}(s)$. In the latter case, the burst is formed by sum of $n$ packets with distribution truncated at $l_o$ with the transform $\psi_n(s)$. Considering arrivals of subsequent packets, we employ the probabilities $q_n$ and $r_n$ to construct a phase diagram for the Laplace transform of burst length shown in Fig. 6. The final result is a weighted sum of all possible ways the diagram can be traversed:

$$\begin{aligned}\psi_{BL}(s) &= \sum_{n=1}^{\infty}\left[\prod_{i=1}^{n} q_{i-1}r_{i-1}\right](1-q_n)\psi_{ex}(s)e^{-s\ell_o}\\ &+ \sum_{n=1}^{\infty}\left[\prod_{i=1}^{n} q_i r_{i-1}\right](1-r_n)[\psi_n(s)]^*, \end{aligned} \tag{8}$$

whereby we define $q_0=1$ and $r_0=1$. The transform $\psi_{ex}(s)$ is not easy to derive from the original packet distribution, but if a burst consists of many packets, it can be successfully approximated by the well-known transform of residual lifetime interpreted in the length domain

$$\psi_{ex}(s) \approx \frac{1-\psi(s)}{sE[L]}. \tag{9}$$

If the probability that the burst consists of few packets is relatively small, the effect of this approximation negligible. Moments of the burst length can be computed in the standard way

$$E[BL^k] = (-1)^k \frac{d^k\psi_{BL}(s)}{ds^k}\Big|_{s=0}. \tag{10}$$

### *Laplace transform of burst inter-departure time* $\phi_{\mathbf{D}}(\mathrm{s})$

The inter-departure time is equal to the sum of two periods: the burst assembly time and the period separating the start of the current timer and the departure of the previous burst.

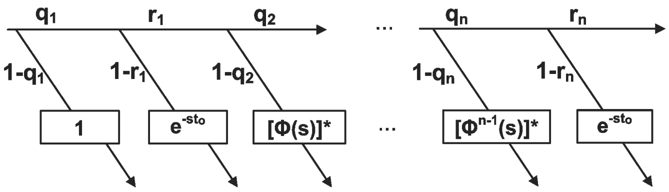


Fig. 7. Phase diagram for formulation of the Laplace transform of burst assembly time $\phi_A(s)$.

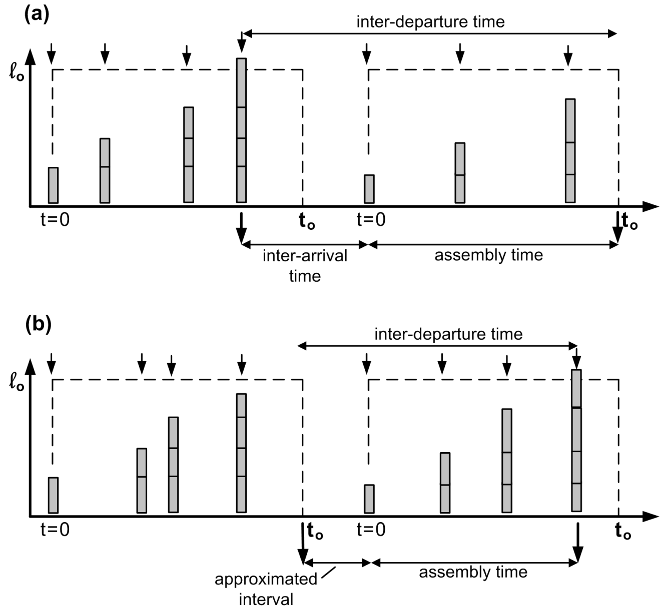


Fig. 8. Two possible realisations of burst inter-departure time.

Formulation of the transform of the assembly time, $\phi_A(s)$, is very similar to that of burst length and is shown by Fig. 7.

If a $n$-packet-burst is completed due to length criterion, with probability $1-q_n$, its assembly time is a sum of $n-1$ inter-arrival times truncated at $t_o( [\phi^{n-1}(s)]^* )$. A burst completed due to timer expiry with probability $1-r_n$, has obviously the assembly time equal to the time threshold ( $e^{-st_o}$ ). Duration of the second period depends upon the fact whether the previous burst departed due to length or time exceedance. In the former case, the separating period is a full inter-arrival time, in the latter it can be again approximated by residual life time of $T_A( [\phi_{res}(s)] )$. Probabilities associated with both events explained in Fig. 8 equal $p_\ell$ and $p_t$, respectively.

$$\begin{aligned} p_\ell &= \sum_{n=1}^{\infty}\left[\prod_{i=1}^{n} q_{i-1}r_{i-1}\right](1-q_n) \\ p_t &= \sum_{n=1}^{\infty}\left[\prod_{i=1}^{n} q_{i}r_{i-1}\right](1-r_n). \end{aligned} \tag{11}$$

Then the transform of burst inter-departure time is

$$\phi_D(s) = \phi_A(s)\Big[p_\ell\phi(s) + p_t\phi_{res}(s)\Big], \tag{12}$$

where

$$\begin{aligned} \phi_A(s) &= \sum_{n=1}^{\infty}\left[\prod_{i=1}^{n} q_{i-1}r_{i-1}\right](1-q_n)[\phi^{n-1}(s)]^* \\ &+ \sum_{n=1}^{\infty}\left[\prod_{i=1}^{n} q_{i}r_{i-1}\right](1-r_n)e^{-st_o}, \end{aligned} \tag{13}$$

where again $q_0 = 1$ and $r_0 = 1$.

$$E[T_D^k] = (-1)^k \frac{d^k\phi_D(s)}{ds^k}\Big|_{s=0}. \tag{14}$$

All the above considerations are valid for general conditions.

### B. *Solutions for Erlangian traffic*

In the sequel, packet length and inter-arrival time have Erlang$_k$ and Erlang$_m$ densities, respectively.

$$\begin{aligned} f_L(\ell) &= \frac{(k\varepsilon)^k}{(k-1)!}\ell^{k-1}e^{-k\varepsilon\ell} \qquad \psi(s) = \left(\frac{k\varepsilon}{k\varepsilon+s}\right)^k \\ f_A(t) &= \frac{(m\lambda)^m}{(m-1)!}t^{m-1}e^{-m\lambda t} \qquad \phi(s) = \left(\frac{m\lambda}{m\lambda+s}\right)^m, \end{aligned}$$

where $\lambda$ is the mean arrival rate and $\varepsilon$ is reciprocal of the mean packet length

$$\lambda = \frac{1}{E[T_A]} \qquad \varepsilon = \frac{1}{E[L]}. \tag{15}$$

***Calculation of the probabilities*** $q_n$, $r_n$, $\pi_n$

Subsequent derivations are shown for the length, that is their concern the probabilities $q_n$. The procedure is identical for time and $r_n$. To start with, we express the density resulting from addition of $n-1$ Erlang$_k$ variables as

$$f_{L,n-1}(\ell) = \frac{(k\varepsilon)^{k(n-1)}}{(k(n-1)-1)!}\ell^{k(n-1)-1}e^{-k\varepsilon\ell}. \tag{16}$$

The Laplace transform of the truncated density $f_{L,n-1}(\ell)$ can be calculated as follows [1]

$$\begin{aligned} [\psi^{n-1}(s)]^* &= \int_0^{\ell_o}\frac{1}{a_{n-1}}\frac{(k\varepsilon)^{k(n-1)}}{(k(n-1)-1)!}\ell^{k(n-1)-1}e^{-k\varepsilon\ell}e^{-s\ell}d\ell \\ &= \frac{1}{a_{n-1}}\frac{(k\varepsilon)^{k(n-1)}}{(k\varepsilon+s)^{k(n-1)}} \\ &- \left[\frac{(k\varepsilon)^{k(n-1)}}{a_{n-1}}\sum_{i=0}^{k(n-1)-1}\frac{(\ell_o)^i e^{-k\varepsilon\ell_o}}{i!(k\varepsilon+s)^{k(n-1)-i}}\right]e^{-s\ell_o}, \end{aligned} \tag{17}$$

[1] In this section, calculation of definite integrals of the type $\int_0^a x^n e^{-bx}\,dx$ is required. Applying multiple integrations by parts, one obtains

$$\int_0^a x^n e^{-bx}\,dx = \frac{n!}{b^{n+1}}\left[1-\sum_{i=0}^{n}\frac{(ab)^i}{i!}e^{-ab}\right]$$

See for example [21] on page 670.

where $a_{n-1}$ is the normalizing factor needed due to truncation at $\ell_o$

$$a_{n-1} = \int_0^{\ell_o} \frac{(k\varepsilon)^{k(n-1)}}{(k(n-1)-1)!} \ell^{k(n-1)-1} e^{-k\varepsilon\ell}\, d\ell = 1 - \sum_{i=0}^{k(n-1)-1} \frac{(k\varepsilon\ell_o)^i}{i!} e^{-k\varepsilon\ell_o}. \quad (18)$$

Then

$$\psi_n(s) = [\psi^{n-1}(s)]^* \psi(s) \quad (19)$$
$$= \frac{1}{a_{n-1}} \frac{(k\varepsilon)^{kn}}{(k\varepsilon+s)^{kn}} - \left[\frac{(k\varepsilon)^{kn}}{a_{n-1}} \sum_{i=0}^{k(n-1)-1} \frac{(\ell_o)^i e^{-k\varepsilon\ell_o}}{i!(k\varepsilon+s)^{kn-i}}\right] e^{-s\ell_o}$$

To find the probability $q_n$ according to Eq. 3, the inversion of $\psi_n(s)$ is needed on the interval $[0, \ell_o]$. By observing that the second complicated term in Eq. 19 has no contribution to the inversion below $\ell_o$ (due to the transform shift theorem), we invert only the first one:

$$f_{L,n}(\ell)_{[0,\ell_o]} = \mathcal{L}^{-1}\left\{\frac{1}{a_{n-1}} \frac{(k\varepsilon)^{kn}}{(k\varepsilon+s)^{kn}}\right\} = \frac{1}{a_{n-1}} \frac{(k\varepsilon)^{kn}}{(kn-1)!} \ell^{kn-1} e^{-k\varepsilon\ell} \quad (20)$$

and finally

$$q_n = \frac{1}{a_{n-1}} \left[1 - e^{-k\varepsilon\ell_o} \sum_{i=0}^{kn-1} \frac{(k\varepsilon\ell_o)^i}{i!}\right]. \quad (21)$$

By identical procedure we find $r_n$ together with the associated normalizing factor $b_{n-1}$.

$$r_n = \frac{1}{b_{n-1}} \left[1 - e^{-m\lambda t_o} \sum_{i=0}^{mn-1} \frac{(m\lambda t_o)^i}{i!}\right] \quad (22)$$

The pmf of the number of packets in a burst is now found by Eq. 1 and 2.

***Pdf of burst length*** $\mathbf{f_{BL}}(\ell)$

According to Eq. 8, this derivation involves the transforms $\psi_{ex}(s)$ and $[\psi_n(s)]^*$. The first one we approximate by the transform of residual life which is:

$$\psi_{ex}(s) \approx \frac{1}{k} \sum_{j=0}^{k} \left[\frac{k\varepsilon}{(k\varepsilon+s)}\right]^{j+1} \quad (23)$$

and the second we readily obtain from Eq. 17 substituting $n-1$ by $n$. After insertion of the transforms into Eq. 8, we can obtain the pdf of burst length by means of simple analytical inversion involving the shift property:

$$f_{BL}(\ell) = \sum_{n=1}^{\infty} \left[\prod_{i=1}^{n} q_{i-1} r_{i-1}\right] (1-q_n) \times \left[\sum_{j=0}^{k-1} \frac{\varepsilon\left[k\varepsilon(\ell-\ell_o)\right]^j}{j!} e^{-k\varepsilon(\ell-\ell_o)} u(\ell-\ell_o)\right] + \sum_{n=1}^{\infty} \left[\left[\prod_{i=1}^{n} q_i r_{i-1}\right] \frac{(1-r_n)}{a_n} \frac{(k\varepsilon)^{kn}}{(kn-1)!} \ell^{kn-1} e^{-k\varepsilon\ell}\right], \quad (24)$$

where $u(\ell)$ is the unit step function.

***Pdf of burst inter-departure time*** $\mathbf{f_D(t)}$

This derivation is done by analogy to that of the burst length pdf, but because of the multiplication of transforms in Eq.12 the final inversion gives rather complicated expression, Eq. 25. However, there is no practical need for detailed knowledge of this function. Usually, in an edge node, output streams of many assembly queues are merged before they are directed to the transmission buffer(s). Since the single departure stream is nearly renewal, we infer that the total departure process tends to a Poisson process as the number of merged streams increases.

This effect was proved in simulation where a number of independent assembly queues fed by uncorrelated renewal inputs was implemented. Fig. 10 shows the results for 10 queues with nearly negative exponential density irrespectively of the type of packet inter-arrival density assumed.

## IV. NUMERICAL EXAMPLES

Fig. 11 shows how the number of packets in a burst varies when configuration of thresholds is changed in case of purely Poisson traffic. The probability mass is symmetrically concentrated around the mean. Fig. 12 depicts the situation when the thresholds allow much more packets to be assembled. With

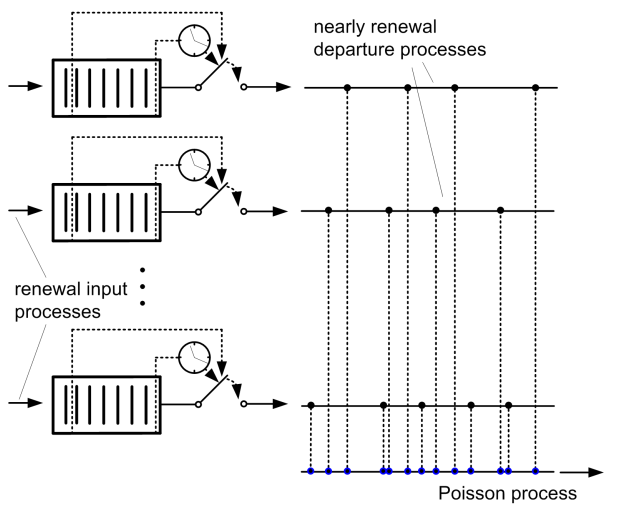


Fig. 9. Superposition of the departure streams from multiple assembly queues in the edge node.

$$
\begin{aligned}
f_D(t) \;=\;& p_\ell \sum_{n=1}^{\infty} \left[ \left[ \prod_{i=1}^{n} q_{i-1} r_{i-1} \right] (1-q_n) \left[ \frac{1}{b_{n-1}} \frac{(m\lambda)^{mn}}{(mk-1)!} \, t^{mn-1} e^{-m\lambda t} - \frac{(m\lambda)^{mn}}{b_{n-1}} \sum_{i=0}^{m(n-1)-1} \frac{t_o^i}{i!(mn-i-1)!} (t-t_o)^{mn-i-1} e^{-m\lambda(t-t_o)} u(t-t_o) \right] \right] \\
&+ p_\ell p_t \left[ \frac{(\lambda)^m}{(m-1)!} e^{-m\lambda(t-t_o)} u(t-t_o) \right] + p_t \sum_{n=1}^{\infty} \left[ \left[ \prod_{i=1}^{n} q_{i-1} r_{i-1} \right] (1-q_n) \sum_{j=1}^{m} \left[ \frac{1}{b_{n-1}} \frac{(m\lambda)^{m(n-1)+j}}{(m(n-1)+j-1)!} t^{m(n-1)+j-1} e^{-m\lambda t} \right. \right. \qquad (25) \\
&\left. \left. - \frac{(m\lambda)^{m(n-1)+j}}{m b_{n-1}} \sum_{i=0}^{m(n-1)-1} \frac{t_o^i e^{-m\lambda t_o}}{i!(m(n-1)-i+j-1)!} (t-t_o)^{m(n-1)-i+j-1} e^{-m\lambda(t-t_o)} u(t-t_o) \right] \right] + p_t^2 \lambda \sum_{i=0}^{m-1} \left[ \frac{m\lambda(t-t_o)^i}{i!} e^{-m\lambda(t-t_o)} u(t-t_o) \right]
\end{aligned}
$$

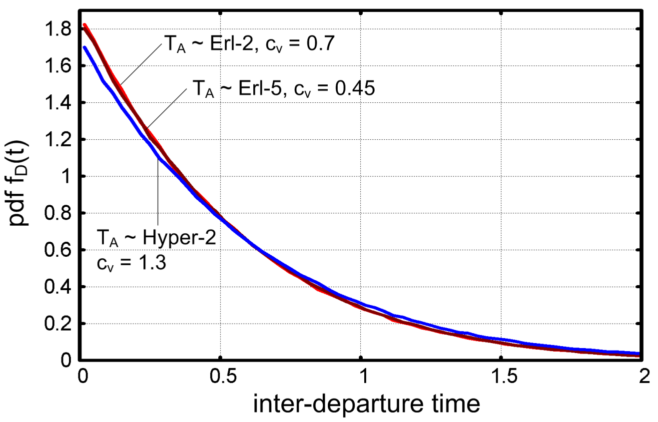


Fig. 10. Simulation results of the superposition of inter-departures processes from 10 independent assembly queues with thresholds $l_o$=5 and $t_o$=5 for different distributions of packet inter-arrival times $T_A$, whereby $\varepsilon$=1 and $\lambda$=1

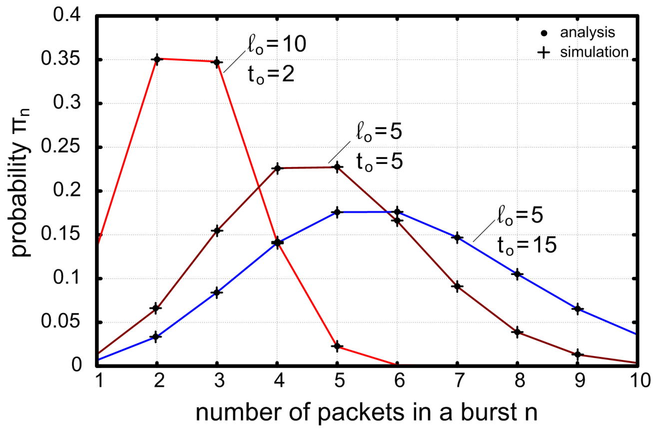


Fig. 11. Pmf of the number of packets in a burst for various values of thresholds. Packet lengths and inter-arrival times exponentially distributed with $\varepsilon$=1 and $\lambda$=1.

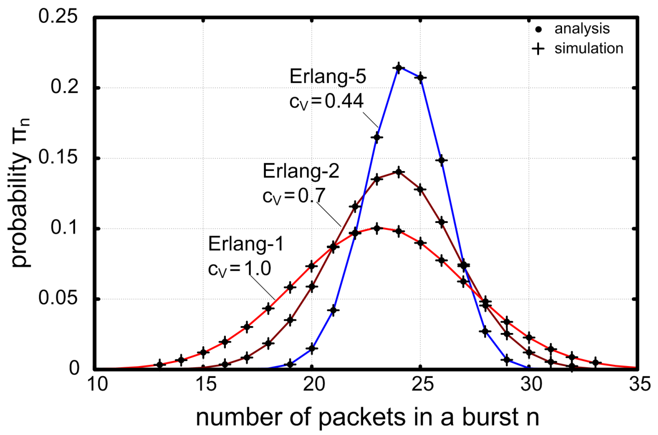


Fig. 12. Pmf of the number of packets in a burst for Erlangian traffic (length and time) with different coefficients of variation, $\varepsilon$ =1, $\lambda$=1, $t_o$=25, $l_o$=25.

decreasing variance of the input distributions, the analyzed pmf tends to be more and more deterministic.

In Fig.13 density of burst length is plotted for various settings. All three pdfs exhibit discontinuities at the point equal to the length threshold. The parts of the curves below $l_o$ represent realizations of assembly due to time-out expiry. The smoothly decaying peaks, which are approximated by mixtures residual lifetimes of Erlang distributions, are slightly inexact compared to simulations in its initial region but this error vanishes rather fast along the tail.

Fig. 14 presents densities for relatively high number of packets collected. If $l_o = 50$ and $t_o = 15$, we have practically only time-based assembly and expected manifestation of the central limit theorem is observed regarding the pdf. In the converse case, nearly no burst is smaller than $l_o$ resulting with a sharp peak at this point.

## V. Conclusions

We have provided a nearly exact analysis of hybrid burst assembly. Although the output traffic preserves the renewal properties of the input, the distributions of interest turned out to be complex functions of the involved parameters. Nevertheless, they give an important insight into the characteristics of the assembled traffic and could be approximated by simpler tractable distributions. The presented results show mainly that the assembled traffic is highly sensitive to the defined thresholds and if their values are not properly adjusted, the resulting large variances of lengths can severely degrade the performance of OBS core nodes. Finally, the distribution of burst length is very far from negative exponential as it is commonly assumed in the literature of the subject.

## References

[1] C. Qiao and M. Yoo, "Optical Burst Switching (OBS) - a New Paradigm for an Optical Internet," *Journal of High Speed Networks*, vol. 8, no. 1, pp. 69–84, 1999.

[2] X. Yu, J. Li, X. Cao, Y. Chen, and C. Qiao, "Traffic Statistics and Performance Evaluation in Optical Burst Switching Networks," *IEEE/OSA Journal of Lightwave Technology*, vol. 22, no. 12, pp. 2722–2738, 2004.

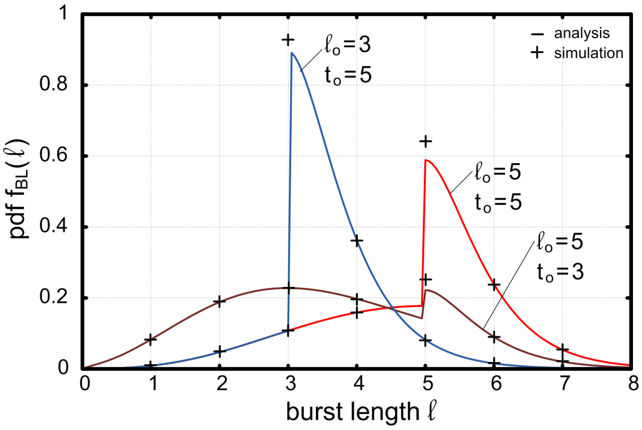


Fig. 13. Pdf of burst length for $Erlang_2$ traffic (length and time) for various values of thresholds, $\varepsilon$=1 and $\lambda$=1.

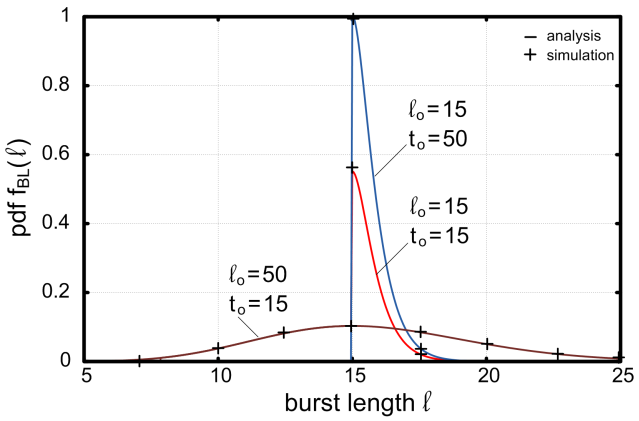


Fig. 14. Pdf of burst length for $Erlang_2$ traffic (length and time) for various values of thresholds, $\varepsilon$=1 and $\lambda$=1.


[3] J. Choi, H. Vu, G. Cameron, M. Zukerman, and M. Kang, "The Effect of Burst Assembly on Performance of Optical Burst Switched Networks," in *ICOIN 2004, Busan, Korea*, 2004.

[4] J. Liu and N. Ansari, "The Impact of the Burst Assembly Interval on the OBS Ingress Traffic Characteristics and System Performance," in *Proc. of IEEE ICC, Paris, France*, 2004.

[5] N. Barakat and E. H. Sargent, "On Teletraffic Applications to OBS," *IEEE Commun. Lett.*, vol. 8, no. 1, pp. 119–121, 2004.

[6] M. Zukerman, E. W. Wong, Z. Rosberg, G. M. Lee, and H. L. Vu, "An Accurate Model for Evaluating Blocking Probabilities in Multi-Class OBS Systems," *IEEE Commun. Lett.*, vol. 8, no. 2, pp. 116–118, 2004.

[7] J. Teng and G. N. Rouskas, "Wavelength Selection in OBS Networks Using Traffic Engineering and Priority-Based Concepts," *IEEE J. Sel. Areas Commun.*, vol. 23, no. 8, pp. 1658–1669, 2005.

[8] N. Akar and E. Karasan, "Exact Calculation of Blocking Probabilities for Bufferless Optical Burst Switched Links with Partial Wavelength Conversion," in *Proc. of First Int. Conf. on Broadband Networks (BROADNETS04)*, 2004.

[9] S. Sarwar, S. Aleksic, and K. Aziz, "Optical Burst Switched (OBS) Systems with Segmentation-Based Dropping," *Eletrotechnik und Informationstechnik*, vol. 125, no. 7-8, pp. 296–300, 2008.

[10] S. Gunreben, "An Optical Burst Reordering Model for Time-Based and Random Selection Assembly Strategies," *Performance Evaluation*, vol. 68, pp. 237–255, 2010.

[11] J. Choi, J. Choi, and M. Kang, "Dimensioning Burst Assembly Process in Optical Burst Switching Networks," *IEICE Trans. Commun.*, vol. E88-B(10), pp. 3855–3863, 2005.

[12] K. Dolzer and C. Gauger, "On Burst Assembly in Optical Burst Switching Networks - a Performance Evaluation of Just-Enough-Time," *Proc. of 17th Int. Teletraffic Congress*, 2001.

[13] A. Rostami and A. Wolisz, "Modeling and Synthesis of Traffic in Optical Burst-Switched Networks," *Journal of Lightwave Technology*, vol. 25, 2007.

[14] K. Leavens, "Traffic Characteristics Inside Optical Burst Switching Networks," in *Proc. of SPIE/IEEE OPTICOMM*, 2002.

[15] A. Zapata and P. Bayvel, "Impact of Burst Aggregation Schemes on Delay in Optical Burst Switched Networks," in *Proc. IEEE/LEOS Annual Meeting, Tucson, Arizona*, 2005.

[16] M. de Vega Rodrigo and J. Gotz, "An Analytical Study of Optical Burst Switching Aggregation Strategies," in *Proc. of Broadnets (Workshop on OBS), San Jose, California*, 2004.

[17] J. Hernandez, J. Aracil, V. Lopez, and J. L. de Vergara, "On the Analysis of Burst-Assembly Delay in OBS Networks and Applications in Delay-Based Service Differentiation," *Photonic Network Commun.*, vol. 14, no. 1, pp. 49–62, 2007.

[18] A. Cohen, *Numerical Methods for Laplace Transform Inversion*. Springer, 2007.

[19] J. Abate and W. Whitt, "The Fourier-Series Method for Inverting Transforms of Probability Distributions," *Queueing Systems*, vol. 10, pp. 5–87, 1992.

[20] X. Yu, Y. Chen, and C. Qiao, "Performance Evaluation of Optical Burst Switching with Assembled Burst Traffic Input," in *Proc. of IEEE Global Telecommunications Conference (Globecom), Taipei*, 2002.

[21] P. Pfeiffer, *Probability for Applications*. Springer, 1990.


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