TURBULENT SEPARATION FROM A BLUFF BODY: THE TRAILING-EDGE CASE

Bernhard Scheichl*† & Alfred Kluwick*
*Institute of Fluid Mechanics and Heat Transfer, Vienna University of Technology, A-1040 Vienna, Austria
†AC²T research Gmbh (Austrian Center of Competence for Tribology), A-2700 Wiener Neustadt, Austria

Summary A recent asymptotic theory of massive separation of an incompressible two-dimensional time-mean turbulent boundary layer from the smooth impervious surface of a rigid bluff body for large Reynolds numbers is extended insofar as the level of turbulence intensity is increased and separation correspondingly delayed towards a stagnation point. In contrast to previous studies, the associated strictly attached potential flow is perturbed such that the free streamlines detach smoothly, yielding a self-consistent flow description.

MOTIVATION

It is demonstrated in [1] that a rigorous flow description of bluff-body separation, under the prerequisites given in the summary above, requires the boundary layer to exhibit a level of turbulence intensity of a smaller order of magnitude compared with that referring to fully developed turbulence. However, the definite position of separation still poses an intriguing long-standing question, and it is unclear how the turbulence intensity increases when separation is assumed to take place as far downstream as possible. This situation was scrutinised in the pioneering work [2], where (i) fully developed turbulence and (ii) fully attached imposed potential flow were postulated. Unfortunately, conditions (i) and (ii) have not led to a complete self-consistent theory so far; see [3]. Here light is shed on this specific challenging problem.

In the following all time-averaged flow quantities are non-dimensional with a semi-diameter of the obstacle, providing a reference length $\tilde{L}$ for the flow on the body scale, the speed $\tilde{U}$ of the unperturbed oncoming flow, and the constant fluid density. Let x, y, u, v denote natural coordinates along and perpendicular to the body surface (with origin in the front stagnation point) and the velocity components in x- and y-direction, respectively. The Reynolds number *Re* formed with the constant kinematic viscosity $\tilde{\nu}$ of the fluid is assumed to take on arbitrarily large values: $Re = \tilde{U}\tilde{L}/\tilde{\nu} \gg 1$.

POTENTIAL FLOW ENCOMPASSING A SMALL CUSPIDAL DEAD-WATER REGION

In [1] the inviscid-flow limit is sought in the one-parameter family of Helmholtz–Kirchhoff flows, confining a dead-water zone that extends to infinity and distinguished by a (non-negative) parameter k: $[u,v] \sim [u_0,v_0](x,y;k)$ as $Re \to \infty$ and k conveniently chosen such that the position $x = x_D(k)$ of flow detachment is shifted further downstream for increasing values of k. Here the flow speed u_D on the free parts of the two detaching streamlines that trace out the cavity equals unity. The numerical treatment of this free-streamline potential flow problem advantageously exploits the conformal mapping of the flow region except for the cavity onto the interior of the unit semi-circle.

The cavity becomes more elongated when k is increased. Most important, it is found that the free streamlines are tangent at infinity when x_D reaches a critical value, where the solutions are continuously continued into the class of potential flows that exhibit a closed cavity. The following situation is depicted in figure 1 (*a*) (under the assumption of symmetric flow): once k or, equivalently, x_D is increased further, both u_D and the extent of the now cusp-shaped cavity decrease monotonically. Finally, $u_D \to 0$ as the cavity shrinks up to a rear stagnation point, forming a trailing edge of the body at $x = x_T$, say, associated with a singular limit $x_D \to x_T$ (on either side of x_T). In turn, $[u_0,v_0](x,y;\infty) \sim \lambda[\sigma,y]$ with $\sigma = x_T - x$ and some $\lambda > 0$ as $(\sigma,y) \to (0,0)$, which reveals fully attached potential flow – the case envisaged in [2]. For instance, for the canonical circulation-free circular-cylinder flow the classical result $u_s(x;\infty) = 2\sin x$ is recovered. On the other hand, it is known long since that the surface speed $u_s = u_0(x,0;k)$ admits a universal three-term representation immediately upstream of $x = x_D$ as for

$$s = x_D(k) - x \to 0_+: \quad u_s(x;k)/u_D(k) \sim g(k^2 s) + O(s^{3/2}) \quad \text{with} \quad g(t) = 1 + 2\sqrt{t} + 10\,t/3\,. \tag{1}$$

That is, k is introduced as a measure for the strength of the potential flow singularity at separation. One then finds that the aforementioned singular limit is obtained for $k \to \infty$ but incommensurate with (1) unless it is resolved in a uniformly valid manner by the introduction of a inner region, encompassing the trailing edge. To this end, we adopt the distance $d(k) = x_T - x_D(k)$ along the body surface (on either side of x_T) as a perturbation parameter. The inner limit assumes universal form by introduction of suitably contracted variables $(X,Y) = (-\sigma,y)/d$: $[u_0,v_0]/(\lambda d) \sim [U,V](X,Y)$ as $k \to \infty$ or, equivalently, $d \to 0$. The resulting flow picture in the half-plane $|X| \le \infty$, $0 < Y < \infty$ exhibits a fourfold symmetry with respect to $X = 0$ and $Y = |X|$. Its analysis again benefits from the conformal mapping mentioned above. As essential results, the half of a tetracuspid (astroid) defined by $X^{2/3} + Y^{2/3} = 1$ confines the dead-water region, the value of the rescaled flow speed on the free streamlines $U_f = U(-1,0) = V(0,1)$ is given by $3/4$, and the rescaled surface speed $U_s(X) = U(X,0)$, $X \le -1$, can interestingly be expressed in closed form since $(U_s/U_f - U_f/U_s)(X)$ represents the root of a polynomial of fourth degree. It is displayed in figure 1 (*b*) together with its asymptotes as for

$$S = -1 - X \to 0_+: \quad U_s \sim (3/4)\,g(S/6) + O(S^{3/2})\,, \qquad X \to -\infty: \quad U_s \sim -X + O(1/X^3)\,. \tag{2a, b}$$

Here the first limit agrees with (1) as $k \sim 1/\sqrt{6d}$, and the second expresses the match with the external stagnant flow.

a) Corresponding author. E-mail: bernhard.scheichl@tuwien.ac.at.

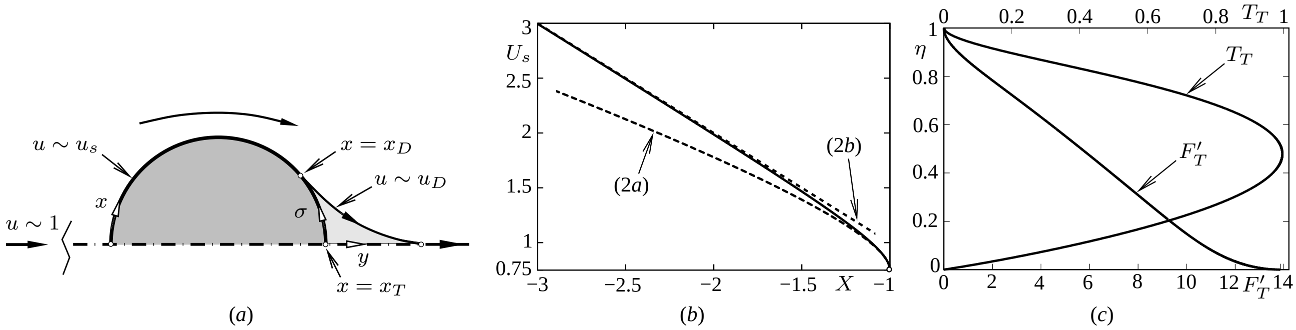

Figure 1: (*a*) flow configuration, (*b*) $U_s(X)$ with asymptotes (2*a*, *b*), (*c*) shape functions of boundary layer subject to stagnant flow.

BOUNDARY LAYER FLOW SUSCEPTIBLE TO TRAILING-EDGE SEPARATION

According to [1], the main part of the incident turbulent boundary layer is to leading order characterised by the small relative velocity deficit $1 - u/u_s \sim \epsilon F_\eta(x, \eta)$, the representation $\alpha\epsilon^2 T(x, \eta)$ of the Reynolds shear stress, and the relatively abrupt outer boundary layer edge at $y = \delta \sim \alpha\epsilon\Delta(x)$. Here we have $\epsilon = \kappa/\log Re$ with the von Kármán constant $\kappa \approx 0.384$, the small gauge function $\alpha(Re)$ measuring the turbulence intensity (reaching unity in the hypothetical limit of fully developed turbulence), $\eta = y/\delta$, and F, T, Δ denoting the stream and the stress function (the latter to be modelled), and the streamwise thickness variation, respectively. According to [2, 3], when $k \to \infty$, both the velocity defect and δ grow rapidly as $[F, T] \sim [3\,F_T(\eta)/(4\,\sigma^2\sqrt{-\log\sigma}),\, 9\,T_T(\eta)/(-16\,\sigma^4\log\sigma)]$, $\Delta \sim 3\sqrt{-\log\sigma}/[\sigma F_T(1)]$ for $\sigma \to 0_+$, where no closure-specific information on T is used. Remarkably, these limiting representations attained near the trailing edge are independent of the upstream history and exactly satisfy the boundary layer equation derived in [4] that applies for a moderately enlarged defect and a correspondingly reduced wall shear stress. In particular, it assumes the self-similar form $2\eta F'_T = F_T(1)\,T_T$ subject to $F'_T(1) = T_T(1) = T_T(0) = 0$, which otherwise applies in the entirely different context of equilibrium flows. Solutions based on the specific algebraic mixing length closure employed in [1, 3, 4] are plotted in figure 1 (*c*). A finite value of $F'_T(0)$, here $F'_T(0) \doteq 13.868$, indicates the onset of a pronounced surface slip correction. Most important, the defect is seen to become a quantity of $O(1)$ when $\sigma = O(\tau)$ with τ defined by $(-2/\log\epsilon)^{1/4}\epsilon^{1/2}$. Simultaneously, the effect of the Reynolds stresses becomes comparatively negligible. A boundary layer approximation, necessary for the establishment of a rational description of the separation process, is only maintained if there $\delta/\tau \ll 1$ or, equivalently, $\alpha \ll -1/\log\epsilon$. This represents a much less restriction than that posed in [1] upon the turbulence intensity of a boundary layer separating already further upstream, i.e. for a moderate value of k. Consequently, the distinguished limit $D = d/\tau = O(1)$ with D measuring the distance between x_D and x_T governs the generic situation: the potential flow discussed above, which resolves the asymptotically small cavity near the trailing edge, drives a predominantly inviscid large-defect boundary layer, where the preserved vorticity is reminiscent of the incident small-defect flow.

One obtains the actual surface slip exerted by the boundary layer in the form $u \sim \tau\lambda\bar{U}_s(X; D)$ for $DX = O(1)$ and $Y/\delta \ll 1$. Bernoulli's theorem evaluated close to the surface gives $D^2U_s^2(X) - \bar{U}_s^2(X; D) = 3F'_T(0)/2$. A self-consistent theory of the local mechanism of inviscid/irrotational interaction (under investigation) demands that $\bar{U}_s = 0$ at the position $X = -1$ of the potential-flow singularity. This sorts out $D = \sqrt{8F'_T(0)/3}$ as a separation criterion and finally yields

$$d \sim 2^{7/4}(F'_T(0)/3)^{1/2}\,\epsilon^{1/2}/(-\log\epsilon)^{1/4}\,. \tag{3}$$

For the above values of κ and $F'_T(0)$ and the circular-cylinder problem, (3) gives $x_D \doteq 130^\circ \ldots 138^\circ$ for $Re = 10^6 \ldots 10^8$, in satisfactory agreement with available measurements, cf. [5], seeing the weak logarithmic dependences on *Re* involved.

FURTHER ASPECTS

Apart from the interaction process, the effects of the near-wall portions of the boundary layer refine the prediction (3) of the separation point. The current research also comprises the merge of the separated shear layers along the branches of the astroid towards a wake flow near $(X, Y) = (0, 1)$ and the recirculating-flow region, shown light-shaded in figure 1 (*a*).

References

[1] Scheichl B., Kluwick A., Smith F.T.: Break-away separation for high turbulence intensity and large Reynolds number. *J. Fluid Mech.* **670**:260–300, 2011.

[2] Neish A., Smith F.T.: On turbulent separation in the flow past a bluff body. *J. Fluid Mech.* **241**:443–467, 1992.

[3] Scheichl B., Kluwick A.: Asymptotic theory of bluff-body separation: a novel shear-layer scaling deduced from an investigation of the unsteady motion. *J. Fluids Struct.* **24**(8):1326–1338, 2008.

[4] Scheichl B., Kluwick A.: Non-unique turbulent boundary layer flows having a moderately large velocity defect: A rational extension of the classical asymptotic theory. *Theor. Comput. Fluid Dyn.*, 2012 (submitted).

[5] Zdravkovich, M. M.: Flow Around Circular Cylinders. A Comprehensive Guide Through Flow Phenomena, Experiments, Applications, Mathematical Models, and Computer Simulations. Vol. 1: Fundamentals. Oxford University Press, Oxford, NY, Tokio 1997.