

Performance Analysis of LTE Downlink under Symbol Timing Offset

Qi Wang, Michal Šimko and Markus Rupp

Institute of Telecommunications, Vienna University of Technology
 Gusshausstrasse 25/389, A-1040 Vienna, Austria
 Email: {qwang, msimko, mrupp}@nt.tuwien.ac.at
 Web: <http://www.nt.tuwien.ac.at/research/mobile-communications>

Abstract—In this paper, we evaluate the performance of a standardized OFDM system, namely Long Term Evolution (LTE) downlink, with imperfect symbol timing. A closed form expression of the post-equalization Signal to Interference and Noise Ratio (SINR) is derived and compared with results obtained from a standard compliant simulator. Also, we analyzed the channel estimation performance when Symbol Timing Offset (STO) occurs. This work reveals the impact of imperfect synchronization on the link performance. It also allows an accurate and realistic modeling of the physical layer behavior, which can be applied to reproduce results from time-consuming link level simulations.

I. INTRODUCTION

Orthogonal Frequency Division Multiplexing (OFDM) has become a dominant physical layer technique of modern wireless communication systems thanks to its high spectrum efficiency and robustness to frequency selective fading channels [1]. In order to eliminate Inter-Symbol Interference (ISI), a guard interval Cyclic Prefix (CP) is appended at the beginning of each OFDM symbol. This, to some extent, provides the system a certain tolerance to symbol timing errors. However, the valid symbol timing region shrinks as the maximum channel delay increases.

Plenty of techniques can be found in literature on symbol timing error estimation [2–5]. Their estimation performance are usually evaluated in terms of lock-in probability, namely the probability that the estimated symbol timing falls within the valid symbol timing region. In [6], a mathematical analysis of the impact of timing synchronization error was presented. However, the evaluation is in terms of the SINR after the Fast Fourier Transform (FFT) at the receiver. Although this metric reflects the performance loss, in order to evaluate the link performance of a realistic communications system, other aspects in the receiver need to be considered, e.g., channel estimation and equalization.

In this work, we consider the downlink of LTE with a STO at sampling period level and analytically derive a closed form expression of the post-equalization SINR as a function of the symbol timing error. Additionally, the impact of the STO on the channel estimation performance is investigated. Afterwards, these results are validated using standard compliant simulations [7].

The paper is organized as follows. In Section II, we describe a mathematical model as a basis of the analysis. A post-

equalization SINR model is presented in Section III. In Section IV, we discuss the estimation performance of a Least Squares (LS) channel estimator under STO. Numerical results are provided in Section V. Section VI concludes the work.

II. SYSTEM MODEL

In this section, a system model is described. We consider an STO of θ sampling periods, i.e., $\theta \in \mathbb{Z}$. The sampling time index is referred to as n , the OFDM symbol index as l and the subcarrier indices as k and p . Among the N subcarriers, N_{tot} are occupied by data symbols. Given a multi-path channel

$$h_n = \sum_{\tau=0}^L c_\tau \cdot \delta_K(n - \tau), \quad (1)$$

the data transmission in the OFDM symbol l in the frequency domain for LTE Single-Input Single-Output (SISO) downlink can be described by

$$\begin{aligned} \underline{R}_l &= P^H FWH^{(t)} \cdot MF^H P \underline{X}_l + \\ &\quad + \underbrace{P^H FWH^{\text{ISI}} H^{(t)} \cdot MF^H P \underline{X}_{\text{int}}}_{\text{ISI}} + \underline{V}_l, \\ &= A \underline{X}_l + B \underline{X}_{\text{int}} + \underline{V}_l, \end{aligned} \quad (2)$$

with

$$A = P^H FWH^{(t)} \cdot MF^H P, \in \mathbb{C}^{N_{\text{tot}} \times N_{\text{tot}}}, \quad (4)$$

$$B = P^H FWH^{\text{ISI}} H^{(t)} \cdot MF^H P, \in \mathbb{C}^{N_{\text{tot}} \times N_{\text{tot}}}. \quad (5)$$

where \underline{X}_l denotes the desired data symbol vector, $\underline{X}_{\text{int}}$ the interfering signal vector, \underline{V}_l the Additive White Gaussian Noise (AWGN) vector and \underline{R}_l the received data symbol vector. Matrices A and B can be interpreted as the transfer functions for the desired signal \underline{X}_l and the interfering signal $\underline{X}_{\text{int}}$. Specifically, zero subcarriers are padded by the matrix P of size $N \times N_{\text{tot}}$ to eliminate inter-band interference. Matrices F (F^H) is the corresponding $N \times N$ FFT (IFFT) matrix. The insertion of the CP is fulfilled by the matrix M of size $(N + N_g) \times N$ where N_g denotes the CP length.

The convolution in the time domain is described as a Toeplitz matrix composed by the channel coefficients in Equation (1)

$$H^{(t)} = \begin{bmatrix} c_0 & 0 & \cdots & \cdots & 0 \\ c_1 & c_0 & 0 & \cdots & 0 \\ \vdots & \ddots & \ddots & \ddots & \vdots \\ 0 & \ddots & 0 & c_L & c_{L-1} \\ 0 & \cdots & \cdots & 0 & c_L \end{bmatrix} \quad (6)$$

which has a dimension of $(N + N_g + L) \times (N + N_g)$.

On the receiver side, the signal vector that is fed into the IFFT unit is extracted by a windowing matrix W and W^{ISI} of size $N \times (N + N_g + L)$. In the ideal timing case $\theta = 0$,

$$W = [\mathbf{0}_{N \times N_g} \quad \mathbf{I}_{N \times N} \quad \mathbf{0}_{N \times L}], \quad W^{\text{ISI}} = \mathbf{0}_{N \times (N + N_g + L)}. \quad (7)$$

We denote the entries in matrix A as a_{ij} and in B as b_{ij} . Given Equation (3), the received signal on subcarrier k in OFDM symbol l can be written as

$$R_{l,k} = a_{kk}X_{l,k} + \underbrace{\sum_{p \neq k} a_{kp}X_{l,p}}_{\text{ICI}} + \underbrace{\sum_p b_{kp}X_{\text{int},p}}_{\text{ISI}} + V_{l,k}. \quad (8)$$

In case of ideal symbol timing, transfer matrix A becomes diagonal and matrix B all-zero. Therefore, neither Inter-Carrier Interference (ICI) nor ISI occurs.

In the following, we extend Equation (8) to a $N_R \times N_T$ Multiple-Input Multiple-Output (MIMO) case. The transfer functions $A^{(m,q)}$ and $B^{(m,q)}$ from Transmitter (TX) q to Receiver (RX) m are constructed using the corresponding channel coefficients $h_n^{(m,q)}$. Thus, we obtain a vector-matrix form

$$\mathbf{r}_{l,k} = \mathbf{a}_{kk}\mathbf{x}_{l,k} + \underbrace{\sum_{p \neq k} \mathbf{a}_{kp}\mathbf{x}_{l,p}}_{\text{ICI}} + \underbrace{\sum_p \mathbf{b}_{kp}\mathbf{x}_{\text{int},p}}_{\text{ISI}} + \mathbf{v}_{l,k}, \quad (9)$$

where $\mathbf{r}_{l,k}$ ($\mathbf{v}_{l,k}$) denotes the $N_R \times 1$ received signal (noise) vector, $\mathbf{x}_{l,k}$ the $N_T \times 1$ transmitted signal vector. The transfer functions for subcarrier k are of dimension $N_R \times N_T$, given as

$$\mathbf{a}_{kk} = \begin{bmatrix} a_{kk}^{(1,1)} & \cdots & a_{kk}^{(1,N_T)} \\ \vdots & \ddots & \vdots \\ a_{kk}^{(N_R,1)} & \cdots & a_{kk}^{(N_R,N_T)} \end{bmatrix}, \quad (10)$$

$$\mathbf{b}_{kp} = \begin{bmatrix} b_{kp}^{(1,1)} & \cdots & b_{kp}^{(1,N_T)} \\ \vdots & \ddots & \vdots \\ b_{kp}^{(N_R,1)} & \cdots & b_{kp}^{(N_R,N_T)} \end{bmatrix}. \quad (11)$$

We distinguish two cases, namely STO to the right (late timing) and STO to the left (early timing). Correspondingly, $\underline{X}_{\text{int}}$ is either \underline{X}_{l+1} or \underline{X}_{l-1} .

A. STO to the right ($\theta < 0$)

In the late timing case, the FFT window shifts to the right and takes in the interference from the successive OFDM symbol. Such a process can be modeled by applying

$$W = [\mathbf{0}_{N \times (N_g - \theta)} \quad \mathbf{I}_{N \times N} \quad \mathbf{0}_{N \times (L + \theta)}] \quad (12)$$

$$W^{\text{ISI}} = \begin{bmatrix} \mathbf{0} & \mathbf{0} \\ \mathbf{I}_{(-\theta) \times (-\theta)} & \mathbf{0} \end{bmatrix} \quad (13)$$

to Equations (2), (4), and (5).

B. STO to the left ($\theta > N_g - L$)

When STO occurs to the left, no interference is induced as long as the FFT window does not embrace the tail of the previous OFDM symbol. Otherwise, there are $d = \theta + L - N_g$ samples in the desired OFDM symbol corrupted by the previous one. Similarly, this can be modeled by applying W in Equation (12) and

$$W^{\text{ISI}} = \begin{bmatrix} \mathbf{0} & \mathbf{I}_{d \times d} \\ \mathbf{0} & \mathbf{0} \end{bmatrix}. \quad (14)$$

III. POST-EQUALIZATION SINR

The post-equalization SINR is a metric of importance for a transmission system which employs linear spatial equalizers at the receiver, because it directly determines the theoretically possible throughput I via Shannon's formula:

$$I = \log_2 \det(1 + \text{SINR}). \quad (15)$$

In this section, a closed form expression of the post-equalization SINR of a LTE downlink transmission under symbol timing offset is derived.

In an LTE system with coherent detection, channel estimation is performed with the help of the standardized reference signal defined in [1]. Nevertheless, given the received signal in Equations (8) and (9), a simple linear channel estimator is merely able to estimate the diagonal elements of the effective channel matrix A , namely \mathbf{a}_{kk} . Starting with a simple assumption of perfect channel knowledge, the Zero Forcing (ZF) equalizer can be expressed as

$$\mathbf{g}_{l,k} = \mathbf{a}_{kk}^\dagger = (\mathbf{a}_{kk}^H \mathbf{a}_{kk})^{-1} \cdot \mathbf{a}_{kk}^H. \quad (16)$$

The output signal of such an equalizer can be written as

$$\begin{aligned} \hat{\mathbf{x}}_{l,k} &= \mathbf{g}_{l,k} \cdot \mathbf{r}_{l,k} \\ &= \mathbf{x}_{l,k} + \underbrace{\mathbf{g}_{l,k} \sum_{p \neq k} \mathbf{a}_{kp}\mathbf{x}_{l,p}}_{\text{ICI}} + \underbrace{\mathbf{g}_{l,k} \sum_p \mathbf{b}_{kp}\mathbf{x}_{\text{int},p}}_{\text{ISI}} + \underbrace{\mathbf{g}_{l,k}\mathbf{v}_{l,k}}_{\text{noise}}. \end{aligned} \quad (17)$$

Define

$$\sigma_{\text{ICI}}^2 = \mathbb{E} \left\{ \sum_{p \neq k} \|\mathbf{a}_{kp}\mathbf{x}_{l,p}\|_F^2 \right\} = \sum_{p \neq k} \sigma_s^2 \|\mathbf{a}_{kp}\|_F^2, \quad (18)$$

$$\sigma_{\text{ISI}}^2 = \mathbb{E} \left\{ \sum_p \|\mathbf{b}_{kp}\mathbf{x}_{\text{int},p}\|_F^2 \right\} = \sum_p \sigma_s^2 \|\mathbf{b}_{kp}\|_F^2, \quad (19)$$

we obtain a closed form expression of the SINR of the output signal from the equalizer, expressed as

$$\begin{aligned} \text{SINR}_{l,k} &= \frac{\mathbb{E}\{\|\mathbf{x}_{l,k}\|_2^2\}}{\mathbb{E}\{\|\hat{\mathbf{x}}_{l,k} - \mathbf{x}_{l,k}\|_2^2\}} \\ &= \frac{\sigma_s^2 \cdot N_T}{(\sigma_{\text{ICI}}^2 + \sigma_{\text{ISI}}^2 + \sigma_v^2) \cdot \text{tr}\{\mathbf{g}_{l,k} \mathbf{g}_{l,k}^H\}} \\ &= \frac{\sigma_s^2 \cdot N_T}{(\sigma_{\text{ICI}}^2 + \sigma_{\text{ISI}}^2 + \sigma_v^2) \cdot \text{tr}\{(\mathbf{a}_{kk}^H \mathbf{a}_{kk})^{-1}\}}, \end{aligned} \quad (20)$$

where σ_s^2 and σ_v^2 denote the signal and the noise power. Although the STO is not a time-variant distortion, it also introduces ICI and ISI which degrades the link performance significantly.

IV. IMPACT ON CHANNEL ESTIMATOR

In the previous analysis, a perfect channel knowledge was assumed for simplicity. Only ICI and ISI that is imposed to the received signal itself was taken into account. In fact, another significant effect of these interference is that they also degrade the performance of the channel estimator. Afterwards, in the overall SINR expression, the impact of this estimation error is further enhanced. In this section, the estimation performance of a typical LS channel estimator is investigate in the context of LTE downlink.

In Rel-8 [9], cell-specific Reference Signals (RSs) are utilized in channel estimation for both demodulation and feedback calculation. Let \mathcal{K} denote the set of RS positions, a simple LS channel estimator on these position $(l', k') \in \mathcal{K}$ can be expressed as

$$\hat{\mathbf{h}}_{l',k'}^{\text{LS}} = \arg \min_{\hat{\mathbf{h}}_{l',k'}} \left\| \mathbf{r}_{l',k'} - \hat{\mathbf{h}}_{l',k'} \mathbf{x}_{l',k'} \right\|_2^2 = \mathbf{r}_{l',k'} \cdot \mathbf{x}_{l',k'}^\dagger. \quad (21)$$

Given the received signal shown in Equation (9), on the RS positions, the Mean Squared Error (MSE) of the LS estimation is given as

$$\begin{aligned} \sigma_{e,\text{RS}}^2 &= \mathbb{E}\left\{ \|\mathbf{h}_{l',k'} - \hat{\mathbf{h}}_{l',k'}\|_2^2 \right\} \\ &= \underbrace{\sum_{p \neq k} \|a_{kp}\|^2}_{\text{ICI}} + \underbrace{\sum_p \|b_{kp}\|^2}_{\text{ISI}} + \frac{\sigma_v^2}{\sigma_s^2 N_T}. \end{aligned} \quad (22)$$

On the data symbol positions, channel estimates are obtained by linear interpolation. Their theoretical MSE performance has been provided in [8] as

$$\sigma_{e,\text{data}}^2 = c_e \cdot \sigma_{e,\text{RS}}^2 + d, \quad (23)$$

where c_e is a scalar determined by the RS structure. The factor d depends on the channel autocorrelation matrix as well as the RS structure. Knowing that the ICI and ISI in Equation (22) are dependent on the channel Power Delay Profile (PDP) and the STO, a saturation in the MSE performance at the higher Signal-to-Noise Ratio (SNR) regime is expected.

TABLE I
SIMULATION PARAMETERS

Parameter	Value
Bandwidth	1.4 MHz
FFT size (N)	128
Number of data subcarriers (N_{tot})	72
CP length (N_g)	normal [9]
Sampling frequency	1.92 MHz
Subcarrier spacing	15 kHz
Transmission setting	$1 \times 1, 2 \times 2$ OLSM
Modulation & coding	adaptive
Channel model	ITU PedB [10]
Channel state information	perfect/LS
Equalizer	ZF
Channel Quality Indicator (CQI) feedback	optimal

TABLE II
PDP OF ITU PEDESTRIAN B CHANNEL MODEL [10]

Excess tap delay (ns)	0	200	800	1200	2300	3700
Relative power (dB)	0	-0.9	-4.9	-8.0	-7.8	-23.9

In addition, when there is an STO, the Linear Minimum Mean Squared Error (LMMSE) estimator which normally outperforms the LS estimator needs to be treated with care. Given the effective channel transfer function in Equation (4), the second order statistics of the effective channel becomes STO-dependent. Certain adjustments in the LMMSE estimator are necessary.

V. NUMERICAL RESULTS

In this section, we validate our analytical solution by comparing results with those from the standard compliant simulations using the Vienna LTE Link Level simulator [7]. Simulation parameters are listed in Table I and Table II.

A. Post-equalization SINR

In order to validate Equation (20), a Monte Carlo simulation was carried out using 500 LTE subframes. With these 500 channel realizations, the post-equalization SINRs were calculated using the closed form expression. Since the system performance is more sensitive to the interference at high SNR region, SNR was chosen to be fixed at 30 dB. A series of STO $\theta \in [-5, 15]$ was introduced. Corresponding results are shown in Figure 1 in terms of the so-called wide-band SINR which is averaged over 72 data subcarriers.

In Figure 1, the curve obtained from the closed form expression agrees well with the one from the Monte Carlo simulation. In the regard of $\theta < 0$, namely a late timing occurs, a sharp drop can be observed in post-equalization SINR; while on the other hand, where an early timing occurs, the CP alleviate the situation. Although the CP has a length of $4.7 \mu\text{s}$, corresponding to 9 sampling periods in this case, given the channel PDP in Table II, ISI arises at the head of the CPs. Therefore, an SINR degradation appears after merely five samples. The 95% confidence intervals are relatively large which can be explained by the strong frequency selectivity induced by the multi-path channel.

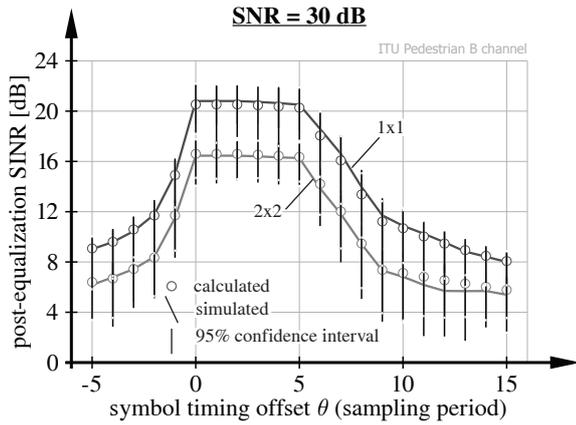


Fig. 1. Comparison of the calculated and simulated post-equalization SINR under STOs.

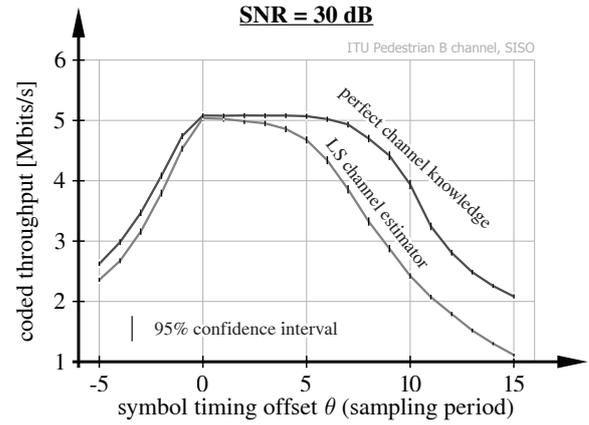


Fig. 3. Coded throughput of a SISO transmission under STOs.

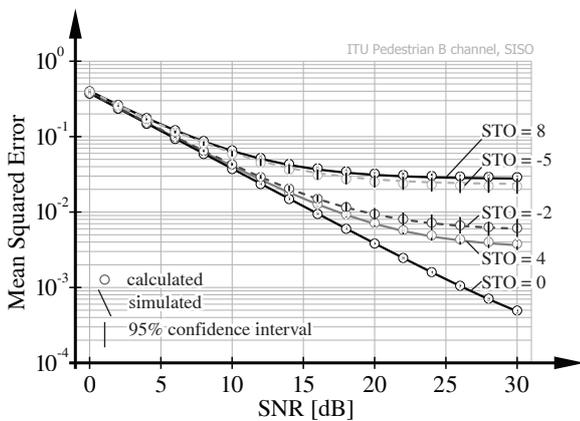


Fig. 2. MSE performance of an LS channel estimator under various STOs: $\theta = \{-5, -2, 0, 4, 8\}$.

B. Channel Estimation Performance

Figure 2 presents the MSE performance of an LS channel estimator with linear interpolation under different levels of STOs. Both late and early timing were considered. As indicated in Equation (22), a saturation can be observed due to the ICI and ISI. Interestingly, it is noticed that for $\theta = 4$, namely a timing offset of four sampling periods within the ISI-free region, a saturation also occurs due to the ICI.

When ZF equalization is performed using imperfect channel estimates, the energy from the estimation error could be further enhanced. This effect was investigated by a throughput simulation at 30 dB SNR. Results are shown Figure 3. Compared to the perfect channel knowledge case, a further loss can be found. In order to model this effect analytically, an appropriate mathematical model for the channel estimation error must be chosen.

VI. CONCLUSION

In this work, we analytically derived an expression of the post-equalization SINR for the LTE downlink with imper-

fect symbol timing synchronization. A comparison is made between statistical simulation results and calculation results using a closed form expression in which the data and noise realizations are averaged. This work allows an accurate and realistic modeling of the physical layer behavior, which can be applied to system level simulations [11].

Additionally, the investigation on the channel estimation performance shows that the STO is a significant source of ICI as well, although it is not a time-variant distortion. In order to model the SINR loss due to the channel estimation error, a valid characterization of the imperfect channel knowledge needs to be chosen carefully, which is considered to be a future work.

ACKNOWLEDGMENTS

Funding for this research was provided by the fFORTE WIT - Women in Technology Program of the Vienna University of Technology. This program is co-financed by the Vienna University of Technology, the Ministry for Science and Research and the fFORTE Initiative of the Austrian Government. Also, this work has been co-funded by the Christian Doppler Laboratory for Wireless Technologies for Sustainable Mobility.

REFERENCES

- [1] 3GPP, "Technical specification group radio access network; (E-UTRA) and (E-UTRAN); overall description; stage 2," Tech. Rep., Sep. 2008. [Online]. Available: <http://www.3gpp.org/ftp/Specs/html-info/36300.htm>
- [2] T. Schmidl and D. Cox, "Robust frequency and timing synchronization for OFDM," *IEEE Transactions on Communications*, vol. 45, no. 12, pp. 1613–1621, Dec. 1997.
- [3] H. Minn, V. K. Bhargava, and K. B. Letaief, "A robust timing and frequency synchronization for OFDM systems," *IEEE Trans. on Wireless Com.*, vol. 2, no. 4, pp. 822–839, Jul. 2003.
- [4] H. Bölcskei, "Blind estimation of symbol timing and carrier frequency offset in wireless OFDM systems," *IEEE Trans. on Communications*, vol. 49, no. 6, pp. 988–999, Jun. 2001. [Online]. Available: <http://www.nari.ee.ethz.ch/commth/pubs/p/synch>
- [5] Q. Wang, M. Šimko, and M. Rupp, "Modified symbol timing offset estimation for OFDM over frequency selective channels," in *Proceeding of IEEE 74th Vehicular Technology Conference (VTC2011-Fall)*, San Francisco, USA, Sep. 2011.
- [6] Y. Mostofi and D. C. Cox, "Mathematical analysis of the impact of timing synchronization errors on the performance of an OFDM system,"

IEEE Transactions on Communications, vol. 54, no. 2, pp. 226–230, Feb. 2006.

- [7] C. Mehlführer, J. C. Ikuno, M. Šimko, S. Schwarz, M. Wrulich, and M. Rupp, “The Vienna LTE Simulators - Enabling Reproducibility in Wireless Communications Research,” *EURASIP Journal on Advances in Signal Processing*, 2011.
- [8] M. Šimko, S. Pendl, S. Schwarz, Q. Wang, J. C. Ikuno, and M. Rupp, “Optimal pilot symbol power allocation in LTE,” in *Proceeding of IEEE 74th Vehicular Technology Conference (VTC2011-Fall)*, San Francisco, USA, Sep. 2011.
- [9] 3GPP, “Technical specification group radio access network; (E-UTRA) and (E-UTRAN); physical channels and modulation; (release 8),” Tech. Rep., Sep. 2008. [Online]. Available: <http://www.3gpp.org/ftp/Specs/html-info/36211.htm>
- [10] “Recommendation ITU-R M.1225: Guidelines for evaluation of radio transmission technologies for IMT-2000,” Tech. Rep., 1997.
- [11] J. C. Ikuno, M. Wrulich, and M. Rupp, “System level simulation of LTE networks,” in *Vehicular Technology Conference (VTC-Spring)*, Taipei, May 2010.

```
@InProceedings{WSA2012_Qi,  
author = {Qi Wang and Michal Šimko and Markus Rupp},  
title = {Performance Analysis of {LTE} Downlink under Symbol Timing Offset },  
booktitle = {Proceeding of 16th ITG Workshop on Smart Antennas (WSA2012)},  
month = mar,  
year = 2012,  
address = {Dresden, Germany},  
}
```