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Coping with Inefficiencies in a New Economic Geography Model

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Abstract

This article introduces a social planner version of a model central to the New Economic Geography for explicitly answering whether the symmetric equilibrium outcome of the decentralized market economy is socially desirable. We find that savings incentives are too weak, resulting in an inefficiently low capital stock and therefore an inadequate number of product varieties. The optimal subsidy and taxation scheme to remedy these distortions resulting from the monopolistic competition structure is shown to be a sales subsidy financed by a lump-sum tax that results in marginal cost pricing. Interestingly, implementing this optimal policy might actually destroy the stability of the symmetric equilibrium and result in unintended agglomeration processes.

Keywords

New economic geography, constructed capital model, social planner, regional policy, agglomeration.

JEL classification: F12, R12, R50

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Coping with Inefficiencies in a New Economic Geography Model

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1 Introduction

Models of the New Economic Geography (NEG) analyze conditions under which agglomeration can occur endogenously even with initially symmetric regions. Prototype models involve "two sectors that are different in terms of (i) technology (constant vs. increasing returns) and (ii) market structure (perfect vs. monopolistic competition)" (Ottaviano and Thisse, 2001, p.162) and with respect to the variability of locally available productive factors. The different availability of productive factors occurs through either factor migration or differences in factor accumulation. A central parameter determining the strength of these forces is the so-called trade freeness. It is argued that for low values of trade freeness decentralized market processes lead to a symmetric equilibrium in which economic activity is evenly spread over the space, and that for high values of trade freeness the symmetric equilibrium loses its stability and decentralized market processes lead to a core-periphery equilibrium, in which economic activity is agglomerated in one of the regions. Stability properties of equilibria depending on trade freeness are therefore at the core of a typical NEG argument.

The models were often used to assess the impact of various policy measures including tariffs, free-trade agreements, customs unions, taxes, subsidies, public expenditures on infrastructure, transport systems and research and development on the regional distribution of economic activity and welfare (see Baldwin et al., 2003, for an overview). However, only few papers have explicitly addressed the question whether the properties of the decentralized market equilibria are socially desirable and how an optimal policy should be designed. This is astonishing since the very core of a prototype NEG model structure involves several inefficiencies — in addition to the monopolistic distortion, the change in the locally available amounts of productive factors involves pecuniary externalities that are welfare-relevant in the given context of imperfect competition.

A central stream of papers in this field — Ottaviano and Thisse (2001), Ottaviano et al. (2002), Ottaviano and Thisse (2002), Tabuchi and Thisse (2002) and more recently Pflüger and Südekum (2008) (linking their analysis to Helpman, 1998) — introduce a specific variant of a social planner, in particular one which imposes marginal cost pricing, uses lump-sum transfers to pay for losses involved and chooses the spatial allocation of the mobile factor such that the sum of the indirect utilities is maximized.[1]

Ottaviano and Thisse (2001), Ottaviano et al. (2002) and Pflüger and Südekum (2008) interpret this divergence as opening up room for regional policy interventions without specifying them in detail; Ottaviano et al. (2002) are a bit more explicit and argue for restricting factor mobility when market processes would produce over-agglomeration, i.e., agglomeration in a parameter range within which the symmetric equilibrium exhibits a higher social welfare. Alternatively, they suggest interregional transfers to compensate the periphery in a similar vein to Tabuchi and Thisse (2002), p.173, who also mention interregional income transfers. However, none of these studies explicitly derives policy recommendations on the basis of the model analysis.

Distributive issues are pervasive in NEG models, since the utility level of the immobile workers left behind in the periphery is lower than the utility level in the core region. Ottaviano et al. (2002) as well as Tabuchi and Thisse (2002) explicitly analyze the welfare position of different groups. In such a situation, the utilization of a social welfare function is not unproblematic. Charlot et al. (2006), pointing out that the simple utilitarian social welfare function actually reflects indifference to inequality, suggest using the more general CES specification that is able to represent a wide range of societal attitudes toward inequality. In addition, they apply compensation criteria (cf. Robert-Nicoud, 2006; Kranich, 2009) in order to directly rank the two possible market outcomes, namely the symmetric, dispersed equilibrium and a CP equilibrium. They show that the result heavily depends on attitudes toward inequality. For plausible parameter values they show that the market might lead to over-agglomeration. Again, policy implications are not at their focus. Similar to the papers reviewed above, they cautiously recommend not to intervene in agglomerative processes, but to use interregional transfers to compensate ex post for the lower utility levels in the periphery. The reason given for that position is worth quoting: "we find it hard to recommend a move from a stable equilibrium, such as agglomeration, to a socially preferred unstable equilibrium, such as dispersion." (Charlot et al., 2006, p. 343).

The papers reviewed so far use a special variant of a social planner: it imposes marginal cost pricing and uses lump-sum taxes to compensate for the associated losses. In addition, policy recommendations are fairly general, neither derived from the model analysis nor explicitly implemented into the decentralized model. By contrast, Hadar and Pines (2004)[2] who use a related urban-economics framework introduce a social planner that cannot directly impose marginal cost pricing, derive explicit solutions for the decentralized market

[1] In addition, Ottaviano et al. (2002) and Pflüger and Südekum (2008) also analyze a second-best solution for the social planner, i.e., a solution in which the social planner is assumed not to change market prices, but only to optimally choose the factor location. These authors derive parameter ranges (in particular for trade freeness) for which the symmetric and the core-periphery (CP) equilibrium are welfare maximizing and they show that those ranges do not necessarily coincide with the parameter ranges for which the respective type of equilibrium is the stable outcome of the decentralized market processes. Note, however, that they do not consider stability issues in the social planner solution.

[2] Their model has no agricultural sector, only a diversified manufacturing goods sector; labor is mobile between regions and thus one region can be totally depopulated. In addition, and similar to Helpman (1998), both regions are endowed with immobile land that directly enters the utility function.

and the social planner version and analyze their differences in detail. Interestingly, they also present (simulation) results for a decentralized market model in which the social planner has introduced taxes and subsidies in order to eliminate those differences.[3] The augmented model still exhibits multiple, stable equilibria that involve different utility levels. Therefore, implementing this tax and subsidy scheme alone does not automatically lead the economy to the social welfare maximum. In addition, there exist parameter ranges for which the market economy without taxes and subsidies produces utility levels in between the utility levels of the model with the taxation and subsidy scheme — therefore it is possible that the policy intervention designed to increase efficiency actually leads to a lower utility level than without it.

In our paper we follow an approach similar to Hadar and Pines (2004) and introduce a social planner into an NEG model which directly chooses quantities (allocation of productive factors, allocation of outputs, given preferences and given technology) and does not impose any price mechanism. We derive an optimal policy scheme and show analytically that its implementation changes the stability properties.

For our analysis, we use the constructed capital model of Baldwin (1999) since it involves a fully-fledged intertemporal micro-foundation but nevertheless shows a considerable degree of analytical tractability. We recapitulate the properties of the equilibria in a decentralized market economy and specify the corresponding social planner problem with a simple utilitarian welfare function. We show that the symmetric equilibrium is a fixed point both for the decentralized market setting as well as for the social planner framework. However, the solution differs between the two models opening up room for policy measures. We derive an optimal subsidy/taxation scheme in the sense that its implementation would adjust the solution reached in a decentralized market economy in the symmetric equilibrium, to the solution of the social planner problem. Not surprisingly, it turns out that the optimal policy is a sales subsidy financed by a lump-sum tax that results in marginal cost pricing. In addition, we show that implementing this optimal policy into the decentralized market economy may actually destroy stability of the symmetric equilibrium in the decentralized economy. Thus starting from a symmetric equilibrium (corresponding to equity considerations), the attempt to increase economic efficiency actually triggers a dynamic process that leads away from this equitable situation.[4]

The paper proceeds as follows. In Section 2 we review the constructed capital model and introduce its social planner version to be able to compare symmetric equilibrium outcomes. Section 3 shows how to internalize the observed inefficiencies by a suitable tax and subsidy regime and also deals with dynamic consequences of this policy strategy. Finally, Section 4 concludes.

[3] Not surprisingly, the social planner uses a sales subsidy to counter the monopolistic competition distortion financed by a lump-sum tax.

[4] As discussed in Charlot et al. (2006), the utilitarian welfare approach implies indifference to inequality and hence biases the results toward agglomeration. In our context this is less problematic since our focus is on the symmetric equilibrium and hence inequality is not an issue.

2 The constructed capital model and its social planner version

Suppose that the economy can be described by the constructed capital model of Baldwin (1999). It consists of two symmetric regions, home (H) and foreign (F)[5] with identical production technologies, preferences of individuals and labor endowments. We normalize the population size (labor endowment) to 1. In each region there are three economic sectors: the perfectly competitive agricultural sector produces food (z), being a homogeneous good according to the following production function with a unit input coefficient

$$Y_z(t) = L_z(t), \tag{1}$$

where total labor $L_z(t)$ devoted to the agricultural sector is the only input factor and $Y_z(t)$ denotes agricultural output. The perfectly competitive investment sector produces capital (K) out of households' savings according to the production function

$$I(t) = \frac{L_I(t)}{F} \tag{2}$$

where $I(t)$ denotes investment of new capital goods, $L_I(t)$ denotes the labor devoted to the investment sector and F is the exogenous unit input coefficient. Finally, the monopolistically competitive manufacturing sector (cf. Dixit and Stiglitz, 1977) produces differentiated varieties (m), using labor as variable input with a unit input coefficient and exactly one unit of capital, which can be viewed as a machine, as fixed input. The variable production function thus reads

$$y_m(i,t) = L_m(i,t) \tag{3}$$

where $L_m(i,t)$ denotes labor devoted to one variety of the manufacturing sector and $y_m(i,t)$ refers to output of one product variety. Due to the fixed costs, a continuum of varieties $i \in (0, K]$ is produced at home and a continuum of varieties $j \in (0, K^*]$ is produced abroad. Food can be costlessly traded between the two regions (or countries), whereas trade of manufactures involves iceberg transport costs such that $\tau \geq 1$ units of a certain good have to be shipped in order to sell one unit abroad (see e.g., Baldwin et al., 2003). The failure rate of a machine is $\delta > 0$ and independent of the machine's age. Consequently, the law of large numbers implies that the overall depreciation rate of capital is given by δ as well. The next subsection describes the most important features of the decentralized solution, while we consider the social planner version in the subsequent subsection.

[5] In general an asterisk corresponds to foreign variables. Only in the case of consumption we additionally use H and F to distinguish between home and foreign production and an asterisk to denote foreign consumption (i.e., c_m^{*H} denotes foreign consumption of a manufactured variety produced at home).

2.1 The Baldwin (1999) model

In the constructed capital model, the representative individual maximizes its discounted stream of lifetime utility

$$\int_0^\infty e^{-\rho t} \ln \left[(C_z(t))^{1-\alpha} (C_m^{agg}(t))^\alpha \right] dt, \tag{4}$$

where $\rho > 0$ is the rate of pure time preference, $0 < \alpha < 1$ is the manufacturing share of consumption and

$$C_m^{agg}(t) \equiv \left[\int_0^{K(t)} \left(c_m^H(i,t) \right)^{\frac{\sigma-1}{\sigma}} di + \int_0^{K^*(t)} \left(c_m^F(j,t) \right)^{\frac{\sigma-1}{\sigma}} dj \right]^{\frac{\sigma}{\sigma-1}}$$

represents consumption of the CES composite with $\sigma > 1$ denoting the elasticity of substitution between varieties. The law of motion for capital is given by

$$\dot{K}(t) = \frac{w(t) + \pi(t)K(t) - E(t)}{w(t)F} - \delta K(t), \tag{5}$$

where $w(t)$ denotes the wage of an individual inelastically supplying all its efficiency units of labor on the labor market, $\pi(t)$ is the capital rental rate and $E(t)$ are total expenditures for consumption defined as

$$\begin{aligned} E(t) \quad \equiv \quad & p_z(t)C_z(t) + \int_0^{K(t)} p_m^H(i,t)c_m^H(i,t)di + \\ & \int_0^{K^*(t)} p_{m,\tau}^F(j,t)c_m^F(j,t)dj. \end{aligned}$$

In this expression $p_z(t)$ is the price of the agricultural good, $p_m^H(i,t)$ the price of a manufactured variety produced at home and $p_{m,\tau}^F(j,t)$ refers to the price of a manufactured variety produced abroad with τ indicating the dependence on transport costs.

Solving the utility maximization problem leads to the consumption Euler equation

$$\frac{\dot{E}(t)}{E(t)} = \frac{\pi(t)}{Fw(t)} - \delta - \rho, \tag{6}$$

where $\pi(t)$ denotes the capital rental rate per unit of capital and $\pi(t)/(Fw(t))$ is a suitably defined rate of profit. Equation (6) states that consumption expenditure growth is positive if and only if the profit rate exceeds the rate of depreciation plus the discount rate.

Postulating that each manufacturing firm has to purchase one machine as fixed input and assuming free entry, Baldwin (1999) eventually arrives at the following expressions for the capital rental rates per unit of capital in both economies :

$$\pi = \left(\frac{\alpha}{\sigma}\right)\left(\frac{E}{K+\phi K^*} + \frac{E^*\phi}{\phi K + K^*}\right), \tag{7}$$

$$\pi^* = \left(\frac{\alpha}{\sigma}\right)\left(\frac{E^*}{K^*+\phi K} + \frac{E\phi}{\phi K^* + K}\right), \tag{8}$$

where the right-hand sides are operating profits as functions of home and foreign expenditures and the aggregate capital stocks. Moreover, $\phi \equiv \tau^{1-\sigma}$ is a measure of openness between the two regions with $\phi = 0$ indicating prohibitive trade barriers and $\phi = 1$ referring to free trade. Obviously, operating profits increase if expenditures increase (demand effect) and they decrease if the capital stock — and thus the number of firms — increases (competition effect). As Baldwin (1999) shows, an informal way of checking the stability properties of the symmetric equilibrium analyzes the response of profits to an accidental shift of one capital unit between the two regions. If this shift leads to a decrease in profits in the receiving region, i.e., the competition effect dominates, then the symmetric equilibrium is stable. If, on the other hand, shifting capital leads to an increase in profits in the receiving region, i.e., the demand effect dominates[6], then the symmetric equilibrium is unstable.

Using these equilibrium capital rental rates — together with the equilibrium wages pinned down by profit maximization in the agricultural sector[7] — in the laws of motion for capital and expenditures implies that the equilibrium dynamics can be fully described by the following four-dimensional dynamic system in the variables E, E^*, K and K^*

$$\begin{aligned}
\dot{K} &= \left[\frac{\alpha}{\sigma F}\left(\frac{E}{K+\phi K^*} + \frac{\phi E^*}{\phi K + K^*}\right) - \delta\right] K + \frac{1}{F} - \frac{E}{F}, \\
\dot{K^*} &= \left[\frac{\alpha}{\sigma F}\left(\frac{E^*}{K^*+\phi K} + \frac{\phi E}{\phi K^* + K}\right) - \delta\right] K^* + \frac{1}{F} - \frac{E^*}{F}, \\
\dot{E} &= E\left[\frac{\alpha}{\sigma F}\left(\frac{E}{K+\phi K^*} + \frac{\phi E^*}{\phi K + K^*}\right) - \rho - \delta\right], \\
\dot{E}^* &= E^*\left[\frac{\alpha}{\sigma F}\left(\frac{E^*}{K^*+\phi K} + \frac{\phi E}{\phi K^* + K}\right) - \rho - \delta\right].
\end{aligned} \tag{9}$$

The symmetric outcome with $E = E^*$ and $K = K^*$ is a steady state with the equilibrium capital stock and expenditures given by

$$K^{DM}_{sym} = \frac{\alpha}{F(\delta\sigma + \rho(\sigma - \alpha))}, \tag{10}$$

$$E^{DM}_{sym} = \frac{\sigma\,(\delta + \rho)}{(\delta\sigma + \rho(\sigma - \alpha))}. \tag{11}$$

In Section 2.3, we compare this symmetric equilibrium of the decentralized model (DM) to the corresponding expression in the social planner model (SP) to find out how potential

[6] A higher capital stock implies higher income and thus raises expenditures in the respective region.

[7] Perfect inter-sectoral labor mobility, a labor input coefficient of one in the agricultural sector and the assumption that there is free trade in agricultural goods implies that $w(t) = w(t)^* = 1$.

inefficiencies impact upon the equilibrium capital stock.

2.2 The social planner version

In this section we introduce a social planner model corresponding to Baldwin (1999)'s decentralized set-up. The major difference to the decentralized model is that an omniscient benevolent social planner maximizes the aggregate utility of all individuals in both countries. In so doing consumption has to be optimally chosen subject to the resource constraint but without the need of considering prices.

Using the production functions shown in Equations (1), (2) and (3) and labor market clearing leads to the following non-negativity constraint[8]

$$I(t) = \frac{1 - Y_z(t) - K y_m(i,t)}{F} \geq 0 \tag{12}$$

guaranteeing a non-negative capital stock in both regions. Market clearing in the agricultural and manufacturing sectors implies

$$\begin{aligned} Y_z(t) &= c_z^H(t) + c_z^{*H}(t), \\ y_m(i,t) &= c_m^H(i,t) + \tau c_m^{*H}(i,t), \\ Y_z^*(t) &= c_z^{*F}(t) + c_z^F(t), \\ y_m^*(j,t) &= c_m^{*F}(j,t) + \tau c_m^F(j,t), \end{aligned} \tag{13}$$

where all consumption values are non-negative. Using Equations (12) and (13), we arrive at the following capital dynamics

$$\begin{aligned} \dot{K} &= \frac{1}{F}\Big[1 - \big(c_z^H + c_z^{*H} + K(c_m^H(i) + \tau c_m^{*H}(i))\big)\Big] - \delta K, \\ \dot{K}^* &= \frac{1}{F}\Big[1 - \big(c_z^{*F} + c_z^F + K^*(c_m^{*F}(j) + \tau c_m^F(j))\big)\Big] - \delta K^*, \end{aligned} \tag{14}$$

where $K(0), K^*(0) \geq 0$.

Using a simple utilitarian welfare function, the social planner therefore maximizes

$$\int_0^\infty e^{-\rho t}\Big[\ln((C_z(t))^{1-\alpha}(C_m^{agg}(t))^\alpha) + \ln((C_z^*(t))^{1-\alpha}(C_m^{*,agg}(t))^\alpha)\Big]\ dt, \tag{15}$$

subject to the capital dynamics in (14) and the non-negativity constraint given in (12). The optimal consumption allocations thus also determine the optimal division of labor between sectors.

The derivations of the results can be found in Appendix A. From the first-order conditions it is obvious that agricultural consumption is equal in both countries, while its production can differ. Let $\eta \in [-1, 1]$ be the share of total agricultural output produced

[8] Note that we use the same expressions for K, E and the consumption variables as in the decentralized model. However, the values can differ due to the different setup.

in region F. Using the first-order conditions, Equation (14) can be transformed into (for the foreign region we obtain an analogous expression)

$$\dot{K} = \frac{1}{F} - \frac{(1-\alpha)E}{F}(1-\eta) - \frac{\alpha E}{F}\Big(\frac{K}{K + K^*\phi} + \frac{K\phi}{K\phi + K^*}\Big) - \delta K. \tag{16}$$

Note that η cannot be determined explicitly but depends endogenously on the complementary slackness conditions.

Due to the Cobb-Douglas specification of the aggregate utility function, we can rewrite the dynamics of agricultural consumption in terms of expenditures by using $C_z = (1-\alpha)E$. The law of motion of expenditures thus reads

$$\dot{E} = -(\rho+\delta)E - \frac{\alpha}{\sigma-1}\frac{E}{F}\Big(\frac{E}{K + K^*\phi} + \frac{E^*\phi}{K\phi + K^*}\Big) + \dot{\mu} - \mu\frac{\rho+\delta}{(1-\alpha)^2}, \tag{17}$$

where μ is the Lagrange multiplier of the non-negativity constraints. Equations (16) and (17) combined with the corresponding equations for the foreign regions and the dynamic equations for the Lagrange multipliers therefore constitute the dynamic system in the social planner model (explicitly written down in Appendix A) that corresponds to the decentralized dynamic system defined by (9).

The symmetric equilibrium can finally be evaluated analytically with capital stocks and expenditures at home and abroad given by

$$K^{SP}_{sym} = \frac{\alpha}{F(\alpha\delta + (\rho+\delta)(\sigma-1))}, \tag{18}$$

$$E^{SP}_{sym} = \frac{(\rho+\delta)(\sigma-1)}{\alpha\delta + (\rho+\delta)(\sigma-1)}. \tag{19}$$

When comparing the decentralized and the social planner economies we focus on comparing these steady-state values because the indeterminacy of η for $K \neq K^*$ renders stability analyses of the social planner model impossible.

2.3 Comparison between the decentralized and the social planner symmetric equilibrium

In this section we compare the symmetric equilibrium of the decentralized model to its social planner counterpart. In particular, we are interested in the employment structure between the three sectors, i.e., investment goods, manufactured goods and agricultural goods. Employment in the investment goods sector is equal to $K\delta F$. From Equations (10) and (18) the difference between the equilibrium capital stocks equals

$$K^{SP}_{sym} - K^{DM}_{sym} = \frac{1}{\alpha F}\frac{(\rho+\delta)(1-\alpha)}{(\delta\alpha + (\rho+\delta)(\sigma-1))\,(\delta\sigma + \rho(\sigma-\alpha))}. \tag{20}$$

Since $0 < \alpha < 1 < \sigma$, the capital stock, i.e., the number of varieties a social planner would choose in the symmetric equilibrium, is higher than in a decentralized market economy.

Therefore, in a decentralized economy, employment in the investment goods sector is too small and employment in the two consumption goods sectors is too high.

Turning to the employment structure of the consumption goods sectors, first note that output levels per variety do not differ between the social planner model and the decentralized framework, i.e., we have[9]

$$y_m^{SP}(i) = y_m^{DM}(i) \quad = \quad F(\rho+\delta)(\sigma-1). \tag{21}$$

Employment in the agricultural sector is equal to $(1-\alpha)E$, while in the manufacturing sector it is $Ky_m(i)$ and in the investment sector it is $K\delta F$. Comparing employment in the two consumption goods sectors yields

$$\frac{K_{sym}^{DM} y_m^{DM}(i)}{(1-\alpha)E_{sym}^{DM}} = \frac{\sigma-1}{\sigma}\frac{\alpha}{1-\alpha} < \frac{\alpha}{1-\alpha} = \frac{K_{sym}^{SP} y_m^{SP}(i)}{(1-\alpha)E_{sym}^{SP}}. \tag{22}$$

Thus employment in the manufactured goods sector relative to employment in the agricultural sector is lower in a decentralized market economy than in the social planner solution.

These results tie in neatly with the discussion of efficiency issues in models with monopolistic competition and an endogenously evolving number of varieties (see Dixit and Stiglitz, 1977; Judd, 1985; Mankiw and Whinston, 1986; Bilbiie et al., 2008) which identifies various potential sources of inefficiencies. First, the consumption commodities, i.e., the commodities entering the utility function, are associated with different mark-ups since the industrial sector is monopolistically competitive, while the agricultural sector is perfectly competitive. Therefore, in the decentralized economy, the marginal rate of substitution between the industrial and the agricultural sector will not be equal to the associated marginal rate of transformation and the relative size of these two sectors is expected to be inefficient. In particular, the share of the commodity with the higher mark-up, i.e., the share of the industrial commodity, is too small. This has been discussed already in Dixit and Stiglitz (1977) and is what we observe in Equations (20), (21) and (22). Second, the decision to create new varieties, i.e., the entry decision (or in our case the savings decision), can be inefficient or excessive because the individual decision neither takes into account the positive effect of the increased product variety on the consumer surplus nor the negative pecuniary effect on the profits of producers of already existing varieties. Inspection of Equation (20) suggests that in our case the product variety created in a decentralized economy is insufficient, thus the effect via the consumer surplus seems to dominate.

In the following we explore these issues in greater depth. We implement a fairly general taxation and subsidy scheme into the model of the decentralized market economy. In Section 3.1, we analyze whether and how it is possible to replicate the social planner's equilibrium outcome. In Section 3.2, we focus on the question whether and how our

[9]This can be obtained by inserting the values of the symmetric equilibrium for K and E into the first-order conditions.

taxation/subsidy scheme affects the stability properties of the (symmetric) equilibrium.

3 Internalization of the inefficiencies

3.1 A suitable subsidy and tax regime

Following the above discussion of potential inefficiencies, we allow for a sales subsidy on industrial commodities for influencing the sectoral split between industry and agriculture and a savings subsidy in order to control the entry/savings decision. In particular, we assume that policymakers can implement saving subsidies via the unit input coefficients in the investment sectors F and F^*, denoted by s_k and s_k^* respectively, as well as subsidies on each unit of the manufactured good sold, denoted by s_m and s_m^*. These subsidies are in turn financed by lump-sum taxes T and T^*. This tax and subsidy scheme changes the Baldwin (1999) model in two distinct ways. First, the representative individual now faces a modified utility optimization problem. In particular, it solves

$$\begin{aligned} \max_E U &= \int_0^\infty e^{-\rho t} \ln\left[\frac{E}{P}\right] dt \\ s.t. \quad \dot{K} &= \frac{w + \pi K - E - T}{w s_k F} - \delta K, \end{aligned} \tag{23}$$

where P is the perfect price index translating expenditures into indirect utility.[10] The difference to the decentralized model of Baldwin (1999), i.e., to Equations (4) and (5), is only due to the incorporation of the lump-sum tax and the savings subsidy. Solving this problem yields the consumption Euler equation as[11]

$$\frac{\dot{E}}{E} = \frac{\pi}{w s_k F} - \delta - \rho. \tag{24}$$

The second modification of the Baldwin (1999) model due to the implementation of a tax and subsidy scheme concerns the manufacturing firms' optimization problem which must account for the sales subsidies and thus can be reformulated as[12]

$$\begin{aligned} \max_{p_m^H, p_{m,\tau}^F} \quad & \left(s_m p_m^H(i) - w\right) c_m^H(i) + \left(s_m p_{m,\tau}^F(i) - \tau w\right) c_m^{H*}(i) \\ s.t. \quad & c_m^H(i) = \frac{\alpha E (p_m^H(i))^{-\sigma}}{P_m}, \\ & c_m^{H*}(i) = \frac{\alpha E^* (p_{m,\tau}^F)^{-\sigma}}{P_m^*}, \end{aligned} \tag{25}$$

where $P_m(t) \equiv \int_0^{K(t)} (p_m^H(i,t))^{1-\sigma} di + \int_0^{K^*(t)} (p_{m,\tau}^F(j,t))^{1-\sigma} dj$ and similarly $P_m^*(t) \equiv$

[10] We only present the analysis for the home region here. The foreign region's equations can be easily derived using symmetry.

[11] Detailed derivations are contained in Appendix B.

[12] Note that the home country subsidizes all goods produced at home equally irrespective of where they are consumed.

$\int_0^{K^*(t)} (p_m^H(j,t))^{1-\sigma} dj + \int_0^{K(t)} (p_{m,\tau}^F(i,t))^{1-\sigma} di$ denote the price indices of manufactured goods in the two countries. Manufacturing firms maximize operating profits defined as revenues from selling the variety to the home and foreign region minus variable production costs (taking into account the effect of transport costs) subject to the optimal demands that result from the individuals' utility maximization problem.[13] Substituting optimal demands into operating profits, taking first-order conditions and rearranging yields optimal prices

$$p_m^H(i,t) = \frac{\sigma}{(\sigma-1)s_m} w, \tag{26}$$

$$p_{m,\tau}^F(i,t) = \frac{\sigma}{(\sigma-1)s_m} w\tau. \tag{27}$$

Using these pricing rules and simplifying eventually yields operating profits and thus regional rental rates in the case of a sales subsidy system as

$$\pi = \left(\frac{s_m E}{K+\phi K^*} + \frac{s_m \phi E^*}{\phi K + K^*}\right)\left(\frac{\alpha}{\sigma}\right), \tag{28}$$

$$\pi^* = \left(\frac{s_m^* E^*}{K^*+\phi K} + \frac{s_m^* \phi E}{\phi K^* + K}\right)\left(\frac{\alpha}{\sigma}\right). \tag{29}$$

Note that the sales subsidy aimed at changing the relative employment structure between the two consumption goods sectors also changes the regional rental rates and thus the incentive for the savings/entry decision.

Using the equilibrium rental rates of Equations (28) and (29) in the Euler equations and the capital accumulation equations[14] finally yields the following dynamic system under a tax and subsidy scheme

$$\begin{aligned}
\dot{K} &= \left[\frac{\alpha}{\sigma s_k F}\left(\frac{s_m E}{K+\phi K^*} + \frac{s_m \phi E^*}{\phi K + K^*}\right) - \delta\right] K + \frac{1}{s_k F} - \frac{E}{s_k F} - \frac{T}{s_k F}, \\
\dot{E} &= E\left[\frac{\alpha}{\sigma s_k F}\left(\frac{s_m E}{K+\phi K^*} + \frac{s_m \phi E^*}{\phi K + K^*}\right) - \rho - \delta\right], \\
\dot{K}^* &= \left[\frac{\alpha}{\sigma s_k^* F}\left(\frac{s_m^* E^*}{K^*+\phi K} + \frac{s_m^* \phi E}{\phi K^* + K}\right) - \delta\right] K^* + \frac{1}{s_k^* F} - \frac{E^*}{s_k^* F} - \frac{T^*}{s_k^* F}, \\
\dot{E}^* &= E^*\left[\frac{\alpha}{\sigma s_k^* F}\left(\frac{s_m^* E^*}{K^*+\phi K} + \frac{s_m^* \phi E}{\phi K^* + K}\right) - \rho - \delta\right].
\end{aligned} \tag{30}$$

The symmetric outcome with equal capital stocks and expenditure levels can be shown to

[13] We ignore fixed costs in the derivations because they do not influence the first-order conditions.

[14] Note that the equilibrium wage is still equal to one.

still constitute a steady state characterized by the following equilibrium values

$$K_{sym}^{SUB} = \frac{s_m \alpha (T-1)}{F s_k (s_m \alpha \rho - \sigma(\delta+\rho))}, \tag{31}$$

$$E_{sym}^{SUB} = \frac{\sigma(T-1)(\delta+\rho)}{s_m \alpha\rho - \sigma(\delta+\rho)}. \tag{32}$$

To calculate the optimal subsidy rates, we equalize these steady-state outcomes to the corresponding ones of the social planner problem represented by Equations (18) and (19)). In addition we impose a balanced governmental budget[15]

$$\underbrace{(1-s_k)\, F\delta K}_{capital\ subsidy} + \underbrace{(s_m - 1)\, \alpha E}_{sales\ subsidy} = T. \tag{33}$$

Solving the resulting system of equations yields

$$s_k = 1, \tag{34}$$

$$s_m = \frac{\sigma}{\sigma-1}, \tag{35}$$

$$T = \frac{\alpha(s_k\delta+\rho)}{s_k(\rho(\sigma-1)+\delta(\alpha+\sigma-1))} \tag{36}$$

which fully describes the tax and subsidy scheme that must be implemented to replicate the symmetric equilibrium outcome of the social planner. Note that there are no savings subsidies paid.

These results again neatly tie in with the discussion of market inefficiencies in monopolistically competitive markets with endogenous product variety. As shown above, employment in the manufactured goods sector is too small; the sales subsidy corrects for this inefficiency. At the same time, it increases savings/entry incentives (see Equations (28) and (29)) and no further savings/entry subsidy is required. With Dixit-Stiglitz preferences the "consumer surplus effect" and the "profit destruction effect" perfectly balance each other and the decentralized entry/savings decision is efficient without additional policy interventions once the sectoral structure has been adjusted to the efficient one (see Bilbiie et al., 2008; Grossman and Helpman, 1991, p.15). This result is due to Dixit-Stiglitz preferences, where the parameter σ at the same time relates to market power (that lies at the root of the sectoral distortion) *and* to the valuation of product diversity (that is related to the entry/savings distortion)[16]. For this particular utility function the monopolistic mark-up Ξ is equal to the marginal valuation of product diversity measured in elasticity form, i.e.,

$$\Xi = \frac{\sigma}{\sigma-1} = \frac{\partial C_m^{agg}}{\partial K}\frac{K}{C_m^{agg}}.$$

[15] Note that the steady state is symmetric, so we just have to impose a balanced budget for one region.

[16] See Benassy (1996), and Heijdra (2009) for models of monopolistic competition with a CES utility function that does not involve this special property.

3.2 Dynamic aspects of the internalization strategy

To analyze the effects of the internalization strategy on the stability properties of the symmetric equilibrium, we linearize the dynamic system given in Equation (30) around the symmetric steady state given in Equations (31) and (32) and then evaluate the eigenvalues of the corresponding 4×4 Jacobian matrix

$$\begin{pmatrix} j11 & j12 & j13 & j14 \\ j21 & j22 & j23 & j24 \\ j13 & j14 & j11 & j12 \\ j23 & j24 & j21 & j22 \end{pmatrix}, \tag{37}$$

whose entries are given in Appendix C. Solving the characteristic equation yields the following four eigenvalues

$$eig1 = \frac{1}{2}\left(\rho - \sqrt{\frac{4\sigma(\delta+\rho)^2}{s_m \alpha} - \rho(4\delta+3\rho)}\right), \tag{38}$$

$$eig2 = \frac{1}{2}\left(\rho + \sqrt{\frac{4\sigma(\delta+\rho)^2}{s_m \alpha} - \rho(4\delta+3\rho)}\right), \tag{39}$$

$$eig3 = \frac{1}{2(\phi+1)^2}\left(r - \sqrt{r^2 - \Omega + \frac{4\sigma\left(\phi^2-1\right)^2(\delta+\rho)^2}{s_m \alpha}}\right), \tag{40}$$

$$eig4 = \frac{1}{2(\phi+1)^2}\left(r + \sqrt{r^2 - \Omega + \frac{4\sigma\left(\phi^2-1\right)^2(\delta+\rho)^2}{s_m \alpha}}\right), \tag{41}$$

where

$$\begin{aligned} r &\equiv \rho(1-\phi^2) + 4\phi\rho + 2\phi\delta(1-\phi) > 0, \\ \Omega &\equiv 4\rho(1-\phi^2)(\phi+1)^2(\rho+\delta) > 0. \end{aligned}$$

The signs and nature of these eigenvalues fully characterize the system's local dynamics around the symmetric equilibrium. By varying the subsidy rate s_m[17] we are now able to assess the dynamic stability effects of our tax and subsidy scheme on the symmetric equilibrium. In particular, setting $s_m = 1$ implies that we are back in the decentralized model of Baldwin (1999). In this case the four eigenvalues are real for all possible parameter values; Eigenvalue 1 is always negative, while Eigenvalues 2 and 4 are always positive. Eigenvalue 3 switches sign depending on the specific parameter values. In particular, Baldwin (1999) shows that there exists a break point value for the trade freeness $\phi_{breakDM}$, with DM referring to the decentralized model:

$$\phi_{breakDM} = 1 - \frac{2\rho\alpha}{\sigma(\rho+\delta)+\rho\alpha}.$$

[17] Note that both s_k and T do not show up in the eigenvalues.

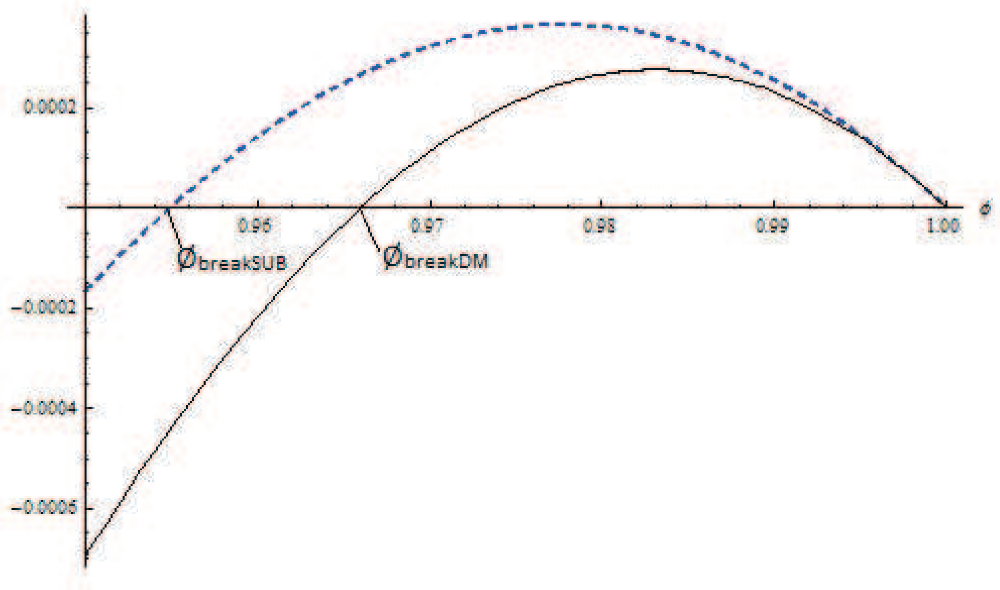
Eigenvalue 3 is negative and thus the symmetric equilibrium is (saddle-path) stable for values of the trade freeness below that threshold and unstable otherwise, giving rise to agglomeration tendencies. Intuitively, lower trade costs (i.e., freer trade) strengthen the demand effect relatively to the competition effect and thus foster agglomeration tendencies.

On the other hand, if we set $s_m = \sigma/(\sigma-1) > 1$, we can analyze the stability properties of the symmetric equilibrium for the case of a tax and subsidy scheme that yields the social planner symmetric equilibrium outcome. Interestingly, the properties of the eigenvalues change quite dramatically. In particular, for certain parameter ranges two or even all four eigenvalues can become complex. However, a careful analysis of the eigenvalues (see Appendix D) reveals that also in this case it is possible to derive a unique break point value $\phi_{breakSUB}$, with SUB referring to the subsidy and tax scheme:

$$\phi_{breakSUB} = 1 - \frac{2\rho\alpha}{(\sigma-1)(\rho+\delta)+\rho\alpha}.$$

Again, the symmetric equilibrium is (saddle-path) stable for values of the trade freeness below that threshold and unstable otherwise.

Figure 1: Eigenvalue 3 with internalization (dashed) and without internalization (solid) depending on trade freeness ϕ



It is important to note that $\phi_{breakSUB} < \phi_{breakDM}$ as illustrated in Figure 1 which plots Eigenvalue 3 for varying levels of trade openness for both $s_m = 1$ and $s_m = \frac{\sigma}{\sigma-1}$ using as a plausible choice of parameters $\rho = 0.015$, $\delta = 0.05$, $\sigma = 4$ and $\alpha = 0.3$ (cf. Auerbach and Kotlikoff, 1987; Krugman, 1991; Krugman and Venables, 1995; Martin and Ottaviano, 1999; Puga, 1999).[18]. Therefore, a range of trade barriers exists for which $\phi_{breakSUB} < \phi < \phi_{breakDM}$, within which the symmetric equilibrium is stable, but becomes unstable as soon as the subsidy and tax scheme is implemented. Intuitively, the sales subsidy strengthens the demand effect relative to the competition effect and thus the forces fostering concentration of economic activity. Thus, implementing the tax and subsidy scheme in such a situation might not be improving welfare because this perturbation will lead the economy away from the social planner symmetric equilibrium outcome and result in unintended agglomeration processes.

4 Conclusion

We set up a social planner version of the constructed capital model due to Baldwin (1999) to show that the symmetric equilibrium of the decentralized model is associated with a less than the efficient amount of capital and manufactured varieties and with a too low (high) employment share in the manufacturing (agricultural) sector. We show that an optimal subsidy/taxation scheme allowing economic policy to establish the social efficient outcome involves lump-sum taxation and a sales subsidy in the manufacturing sector. The latter increases (decreases) output and employment in the manufacturing (agricultural) sector; at the same time, it increases savings/entry incentives and thus leads to a higher product variety.

In an NEG framework, stability properties of the equilibria play a crucial role. We show that introducing the optimal policy scheme profoundly changes the eigenvalues of the Jacobian matrix describing the linearized system around the steady state. In particular, it narrows the parameter range for which the symmetric outcome is stable. We show that a parameter range exists for which the symmetric equilibrium is stable, but loses its stability once the optimal policy scheme is implemented. Thus, the policy interventions give rise to unintended agglomeration processes. More generally, these findings indicate that the standard strategy to internalize inefficiencies by only matching the equilibrium outcomes without taking into account the associated changes in stability properties might be highly misleading.

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[18] Note that in both cases Eigenvalue 1 is always negative, while Eigenvalues 2 and 4 are always positive.

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Appendix

A Derivations of the social planner model

The problem of the social planner is to maximize the following Benthamite intertemporal utility function

$$\int_0^{\infty} e^{-\rho t}\Big[\ln\Big((c_z^H + c_z^F)^{1-\alpha}(K(c_m^H)^{\frac{\sigma-1}{\sigma}} + K^*(c_m^F)^{\frac{\sigma-1}{\sigma}})^{\frac{\alpha\sigma}{\sigma-1}}\Big) + \ln\Big((c_z^{*,H} + c_z^{*,F})^{1-\alpha}(K(c_m^{*,H})^{\frac{\sigma-1}{\sigma}} + K^*(c_m^{*,F})^{\frac{\sigma-1}{\sigma}})^{\frac{\alpha\sigma}{\sigma-1}}\Big)\Big]\ dt \tag{42}$$

subject to the capital dynamics

$$\begin{aligned} \dot{K} &= \frac{1}{F}\Big[1 - \big(c_z^H + c_z^{*,H} + K(c_m^H + \tau c_m^{*,H})\big)\Big] - \delta K, \\ \dot{K}^* &= \frac{1}{F}\Big[1 - \big(c_z^* + m_z + K^*(c_m^{*F} + \tau c_m^F)\big)\Big] - \delta K^*, \end{aligned} \tag{43}$$

where $K(0), K^*(0) \geq 0$, and the control-state constraints

$$\begin{aligned} 1 &\geq c_z^H + c_z^{*,H} + K(c_m^H + \tau c_m^{*,H}), \\ 1 &\geq c_z^{*,F} + c_z^F + K^*(c_m^{*F} + \tau c_m^F). \end{aligned} \tag{44}$$

The controls have to be non-negative, i.e., $c_z^H, c_z^{*,F}, c_z^F, c_z^{*,H}, c_m^H, c_m^{*,F}, c_m^F, c_m^{*,H} \geq 0$. The Hamiltonian of the problem reads

$$\begin{aligned} \mathcal{H} = & \Big[\ln\Big((c_z^H + c_z^F)^{1-\alpha}(K(c_m^H)^{\frac{\sigma-1}{\sigma}} + K^*(c_m^F)^{\frac{\sigma-1}{\sigma}})^{\frac{\alpha\sigma}{\sigma-1}}\Big) + \\ & \ln\Big((c_z^{*,H} + c_z^{*,F})^{1-\alpha}(K(c_m^{*,H})^{\frac{\sigma-1}{\sigma}} + K^*(c_m^{*,F})^{\frac{\sigma-1}{\sigma}})^{\frac{\alpha\sigma}{\sigma-1}}\Big)\Big] \\ & +\lambda\Big[\frac{1}{F}\Big[1 - \big(c_z^H + c_z^{*,H} + K(c_m^H + \tau c_m^{*,H})\big)\Big] - \delta K\Big] + \\ & +\lambda^*\Big[\frac{1}{F}\Big[1 - \big(c_z^{*,F} + c_z^F + K^*(c_m^{*F} + \tau c_m^F)\big)\Big] - \delta K^*\Big] + \\ & +\mu\Big[1 - c_z^H - c_z^{*,H} - K(c_m^H + \tau c_m^{*,H})\Big] \\ & +\mu^*\Big[1 - c_z^* - m_z - K^*(c_m^{*F} + \tau c_m^F)\Big], \end{aligned} \tag{45}$$

where λ and λ^* are the adjoint variables for the corresponding capital dynamics. The Lagrange multipliers of the constraints (44) are denoted as μ and μ^*. By applying the

Maximum Principle we get the first-order conditions

$$
\begin{aligned}
\mathcal{H}_{c_z^H} &= (c_z^H + c_z^F)^{-1}(1-\alpha) - \big(\frac{\lambda}{F} + \mu\big) \leq 0, \\
\mathcal{H}_{c_z^F} &= (c_z^H + c_z^F)^{-1}(1-\alpha) - \big(\frac{\lambda^*}{F} + \mu^*\big) \leq 0, \\
\mathcal{H}_{c_z^{*,F}} &= (c_z^{*,H} + c_z^{*,F})^{-1}(1-\alpha) - \big(\frac{\lambda^*}{F} + \mu^*\big) \leq 0, \\
\mathcal{H}_{c_z^{*,H}} &= (c_z^{*,H} + c_z^{*,F})^{-1}(1-\alpha) - \big(\frac{\lambda}{F} + \mu\big) \leq 0, \\
\mathcal{H}_{c_m^H} &= \alpha K\big(K(c_m^H)^{\frac{\sigma-1}{\sigma}} + K^*(c_m^F)\big)^{-1}(c_m^H)^{-\frac{1}{\sigma}} - \big(\frac{\lambda}{F} + \mu\big)K \leq 0, \\
\mathcal{H}_{c_m^F} &= \alpha K^*\big(K(c_m^H)^{\frac{\sigma-1}{\sigma}} + K^*(c_m^F)\big)^{-1}(c_m^F)^{-\frac{1}{\sigma}} - \big(\frac{\lambda^*}{F} + \mu^*\big)K^*\tau \leq 0, \\
\mathcal{H}_{c_m^{*,F}} &= \alpha K^*\big(K(c_m^{*,H})^{\frac{\sigma-1}{\sigma}} + K^*(c_m^{*,F})\big)^{-1}(c_m^H)^{*-\frac{1}{\sigma}} - \big(\frac{\lambda^*}{F} + \mu^*\big)K^* \leq 0, \\
\mathcal{H}_{c_m^{*,H}} &= \alpha K\big(K(c_m^{*,H})^{\frac{\sigma-1}{\sigma}} + K^*(c_m^{*,F})\big)^{-1}(c_m^F)^{*-\frac{1}{\sigma}} - \big(\frac{\lambda}{F} + \mu\big)K\tau \leq 0
\end{aligned} \tag{46}
$$

which imply that agricultural consumption is equal in both countries and that $\lambda/F + \mu = \lambda^*/F + \mu^*$. By taking the derivative of the Hamiltonian with respect to capital we obtain the adjoint Equations:

$$
\begin{aligned}
\dot{\lambda} &= \Big(\rho + \delta + \frac{1}{F}(c_m^H + \tau c_m^{*,H})\Big)\lambda - \frac{\alpha\sigma}{\sigma-1}\Big(K(c_m^H)^{\frac{\sigma-1}{\sigma}} + K^*(c_m^F)^{\frac{\sigma-1}{\sigma}}\Big)^{-1}(c_m^H)^{\frac{\sigma-1}{\sigma}} \\
&\quad -\frac{\alpha\sigma}{\sigma-1}\Big(K(c_m^F)^{*\frac{\sigma-1}{\sigma}} + K^*(c_m^H)^{*\frac{\sigma-1}{\sigma}}\Big)^{-1}(c_m^F)^{*\frac{\sigma-1}{\sigma}} + \mu(c_m^H + \tau c_m^{*,H}), \\
\dot{\lambda}^* &= \Big(\rho + \delta + \frac{1}{F}(c_m^{*,F} + \tau c_m^F)\Big)\lambda^* - \frac{\alpha\sigma}{\sigma-1}\Big(K(c_m^H)^{\frac{\sigma-1}{\sigma}} + K^*(c_m^F)^{\frac{\sigma-1}{\sigma}}\Big)^{-1}(c_m^F)^{\frac{\sigma-1}{\sigma}} \\
&\quad -\frac{\alpha\sigma}{\sigma-1}\Big(K(c_m^F)^{*\frac{\sigma-1}{\sigma}} + K^*(c_m^H)^{*\frac{\sigma-1}{\sigma}}\Big)^{-1}(c_m^H)^{*\frac{\sigma-1}{\sigma}} + \mu^*(c_m^{*,F} + \tau c_m^F),
\end{aligned} \tag{47}
$$

together with the complementary slackness conditions

$$
\begin{aligned}
0 &= \mu\Big[1 - c_z^H - c_z^{*,H} - K(c_m^H + \tau c_m^{*,H})\Big], \\
0 &= \mu^*\Big[1 - c_z^{*,F} - c_z^F - K^*(c_m^{*F} + \tau c_m^F)\Big],
\end{aligned} \tag{48}
$$

with $\mu \geq 0$ and $\mu^* \geq 0$.

Using Equations (43), (48) and the first-order conditions we arrive at the following system

$$
\begin{aligned}
\dot{K} &= \frac{1}{F} - \frac{1-\alpha}{F}\Big(\frac{\lambda}{F}+\mu\Big)^{-1}(1-\eta) \\
&\quad -\frac{\alpha}{F}\Big(\frac{\lambda}{F}+\mu\Big)^{-1}\Big(\frac{K}{K+K^*\phi}+\frac{K\phi}{K\phi+K^*}\Big)-\delta K, \\
\dot{K}^* &= \frac{1}{F} - \frac{1-\alpha}{F}\Big(\frac{\lambda^*}{F}+\mu^*\Big)^{-1}(1+\eta) \\
&\quad -\frac{\alpha}{F}\Big(\frac{\lambda^*}{F}+\mu^*\Big)^{-1}\Big(\frac{K^*}{K\phi+K^*}+\frac{K^*\phi}{K+K^*\phi}\Big)-\delta K^*, \\
\dot{\lambda} &= (\rho+\delta)\lambda - \frac{\alpha}{\sigma-1}\Big(\frac{1}{K+K^*\phi}+\frac{\phi}{K\phi+K^*}\Big), \\
\dot{\lambda}^* &= (\rho+\delta)\lambda^* - \frac{\alpha}{\sigma-1}\Big(\frac{1}{K\phi+K^*}+\frac{\phi}{K+K^*\phi}\Big).
\end{aligned}
\tag{49}
$$

Together with the laws of motion for μ and μ^*, this system can be rewritten as one in the variables K, E and μ.

B Derivations under a tax and subsidy scheme

The Hamiltonian of the modified utility optimization problem is

$$
H(E,K,\lambda,t) = \ln\left[\frac{E}{P}\right] + \lambda\left(\frac{w+\pi K-E-T}{ws_kF}-\delta K\right). \tag{50}
$$

The first-order conditions of the problem associated with Equation (50) are given by

$$
\frac{\partial H}{\partial E} \overset{!}{=} 0 \quad\Rightarrow\quad \frac{1}{E} = \frac{\lambda}{ws_kF}, \tag{51}
$$

$$
\frac{\partial H}{\partial K} \overset{!}{=} \rho\lambda - \dot{\lambda} \quad\Rightarrow\quad \frac{\dot{\lambda}}{\lambda} = -\frac{\pi}{ws_kF}+\rho+\delta, \tag{52}
$$

$$
\frac{\partial H}{\partial \lambda} \overset{!}{=} \dot{K} \quad\Rightarrow\quad \frac{w+\pi K-E-T}{ws_kF}-\delta K = \dot{K} \tag{53}
$$

and the standard transversality condition. Taking the time derivative of Equation (51) under the assumption that w and s_k are time-independent and combining it with Equation (52) yields the consumption Euler equation

$$
\frac{\dot{E}}{E} = \frac{\pi}{ws_kF} - \delta - \rho.
$$

C Intermediate results for the stability analysis

The Jacobian matrix given in Equation (37) has the following entries

$$
\begin{aligned}
J_{11} &= -\frac{\delta\phi^2+\delta-2\rho\phi}{(\phi+1)^2}, \\
J_{12} &= -\frac{-s_m\alpha+\sigma\phi+\sigma}{Fs_k\sigma\phi+Fs_k\sigma}, \\
J_{13} &= -\frac{2\phi(\delta+\rho)}{(\phi+1)^2}, \\
J_{14} &= \frac{s_m\alpha\phi}{Fs_k\sigma\phi+Fs_k\sigma}, \\
J_{21} &= -\frac{Fs_k\sigma\left(\phi^2+1\right)(\delta+\rho)^2}{s_m\alpha(\phi+1)^2}, \\
J_{22} &= \frac{\delta+\rho}{\phi+1}, \\
J_{23} &= -\frac{2Fs_k\sigma\phi(\delta+\rho)^2}{s_m\alpha(\phi+1)^2}, \\
J_{24} &= \frac{\phi(\delta+\rho)}{\phi+1}.
\end{aligned}
$$

D Properties of the eigenvalues and implications for stability

It can be shown that Eigenvalues 1 and 2 are complex if $\sigma < \sigma_{c12}$ with

$$\sigma_{c12} = 1+\frac{\alpha\rho(3\rho+4\delta)}{4(\rho+\delta)^2},$$

while Eigenvalues 3 and 4 are complex if $\sigma < \sigma_{c34}$ with

$$\sigma_{c34} = 1+\alpha\frac{\rho(3\rho+4\delta)}{4(\rho+\delta)^2}-\alpha\frac{\phi(2\rho+\delta)}{(\rho+\delta)\left(1-\phi^2\right)^2}(\phi-\varphi_1)(\phi-\varphi_2)(\phi-\varphi_3)$$

and

$$
\begin{aligned}
\varphi_1 &= \frac{\delta}{2\rho+\delta} < 1, \\
\varphi_2 &= \frac{\delta}{2(\rho+\delta)}+\sqrt{\frac{\delta^2}{[2(\rho+\delta)^2]^2}-\frac{\rho}{\rho+\delta}}, \\
\varphi_3 &= \frac{\delta}{2(\rho+\delta)}-\sqrt{\frac{\delta^2}{[2(\rho+\delta)^2]^2}-\frac{\rho}{\rho+\delta}}.
\end{aligned}
$$

Note that in case of complex eigenvalues, the real parts in Equations (38)-(41) are positive.

In case of real eigenvalues, we see from equations (38)-(41) that an introduction of the subsidy decreases Eigenvalues 2 and 4. Nevertheless, since $\rho > 0$ and $r > 0$, Eigenvalues 2 and 4 remain positive. By contrast, the introduction of the subsidy increases Eigenvalues 1 and 3. Eigenvalue 1 is real and positive if $\sigma_{c12} < \sigma < \sigma_{p1}$ with

$$\sigma_{p1} = 1 + \frac{\rho\alpha}{\rho + \delta}.$$

Eigenvalue 3 is real and positive if $\sigma_{c34} < \sigma < \sigma_{p3}$ with

$$\sigma_{p3} = 1 + \frac{1+\phi}{1-\phi}\frac{\rho\alpha}{\rho + \delta}.$$

The following properties can be shown to hold

$$\sigma_{c12} < \sigma_{p1} < \sigma_{p3},$$
$$\sigma_{c34} < \sigma_{p1} < \sigma_{p3}.$$

Depending on the parameters, $\sigma_{c12} > \sigma_{c34}$ or $\sigma_{c12} < \sigma_{c34}$ may hold. Therefore, depending on σ we can summarize the properties of the eigenvalues in Table 1 for $\sigma_{c12} < \sigma_{c34}$ and in Table 2 for $\sigma_{c12} > \sigma_{c34}$. This shows that the symmetric equilibrium is saddle-path stable for $\sigma_{p3} < \sigma$ and unstable in all other cases. Solving for ϕ, this condition results in

$$\phi < \phi_{breakSUB} := 1 - \frac{2\rho\alpha}{(\sigma - 1)(\rho + \delta) + \rho\alpha}.$$

$\sigma_{c12} < \sigma_{c34} < \sigma_{p1} < \sigma_{p3} < \sigma$	eig1 and eig3: real, negative eig2 and eig 4: real and positive
$\sigma_{c12} < \sigma_{c34} < \sigma_{p1} < \sigma < \sigma_{p3}$	eig1: real, negative eig2, eig3 and eig 4: real and positive
$\sigma_{c12} < \sigma_{c34} < \sigma < \sigma_{p1} < \sigma_{p3}$	eig1, eig2, eig3 and eig 4: real and positive
$\sigma_{c12} < \sigma < \sigma_{c34} < \sigma_{p1} < \sigma_{p3}$	eig1 and eig2: real and positive eig3 and eig 4: complex with positive real parts
$\sigma < \sigma_{c12} < \sigma_{c34} < \sigma_{p1} < \sigma_{p3}$	eig1, eig2, eig3 and eig 4: complex with positive real parts

Table 1: Summary of the properties for the eigenvalues for $\sigma_{c12} < \sigma_{c34}$

$\sigma_{c34} < \sigma_{c12} < \sigma_{p1} < \sigma_{p3} < \sigma$	eig1 and eig3: real, negative eig2 and eig 4: real and positive
$\sigma_{c34} < \sigma_{c12} < \sigma_{p1} < \sigma < \sigma_{p3}$	eig1: real, negative eig2, eig3 and eig 4: real and positive
$\sigma_{c34} < \sigma_{c12} < \sigma < \sigma_{p1} < \sigma_{p3}$	eig1, eig2, eig3 and eig 4: real and positive
$\sigma_{c34} < \sigma < \sigma_{c12} < \sigma_{p1} < \sigma_{p3}$	eig1 and eig 2: complex with positive real parts eig3 and eig4: real and positive
$\sigma < \sigma_{c34} < \sigma_{c12} < \sigma_{p1} < \sigma_{p3}$	eig1, eig2, eig3 and eig 4: complex with positive real parts

Table 2: Summary of the properties for the eigenvalues for $\sigma_{c12} > \sigma_{c34}$

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