# Design study of magnet shapes for axial Halbach arrays using 3D Finite Element Analyses

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*Abstract*—Magnet material prices has become an uncertain factor for electric machine development. Most of all, the output of ironless axial flux motors equipped with Halbach magnet arrays depend on the elaborated magnetic flux. Therefore, possibilities to reduce the manufacturing cost without negatively affecting the performance are studied in this paper. Both magnetostatic and transient 3D finite element analyses are applied to compare flux density distribution, elaborated output torque and induced back EMF. It is shown, that the proposed magnet shapes and magnetization pattern meet the requirements. Together with the assembly and measurements of functional linear Halbach magnet arrays, the prerequisite for the manufacturing of axial magnet arrays for an ironless in-wheel hub motor are given. <sup>1</sup>

#### NOMENCLATURE

AFPM	Axial flux permanent magnet machine
CSIRO	Commonwealth Scientific and Industrial Re-
	search Organisation
CFRP	Carbon fibre reinforced plastic
EDM	Electric discharge machining
EM	Electromagnetic calculation
FEA	Finite element analysis
GRP	Glass fibre reinforced plastic
PM	Permanent magnet

# I. INTRODUCTION

Axial flux in-wheel hub motors were often regarded as promising solution for electric car propulsion [1]. However direct-driven in-wheel vehicles are nowadays still mostly restricted to scooters and motor show presentation concept cars. Even though axial flux machines provide high torque to volume ratio [2] the number of realized direct drive prototypes is marginal. The argument of reduced comfort or safety due to added unsprung mass was also rebutted by [3] and [4]. Mechanical requirements to limit the overall weight were addressed in [2] and the application of novel materials were recommend. A water cooled two-stage 25 kW axial-flux motor was presented in 1996 [5]. However, considering both highest

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possible efficiency and minimum weight, the ironless axialflux topology may be the best choice [6]. Today, the most widely-used commercially available axial-flux ironless motor was presented by Lovatt [7] and is in use for solar race cars [8]. Even though the recommendation given by [9] to use Halbach magnet arrays was considered during the design phase, the available motor from CSIRO is equipped with standard ironbacked magnet rings. A comprehensive overview on Halbach permanent magnet applications is given by [10]. The combination of latest light-weight manufacturing techniques, air cored winding arrangement and Halbach array magnet rings may lead to a new benchmark when it comes to highest efficiency and nominal torque to active weight ratio [11]. This paper is focused on the magnet shape selection and forces between magnets in Halbach arrangements for axial flux ironless inwheel motors.

## II. HALBACH ARRAYS - HISTORY AND ANALYTICAL CALCULATION

If a certain changing magnetization pattern is applied to a planar structure such as recording tapes, flux concentration occurs on one side of the specimen. First regarded as "magnetic curiosity" [12] this effect was further elaborated by K. Halbach, a physicist and eponym for the so called Halbach effect [13]. The first applications were multipole arrangements and undulators for synchrotron storage rings [14]. Planar Halbach magnet arrays are today applied in planar actuators for nanolithography used in semiconductor industry [15].

The main advantages of the Halbach magnet arrangement are a

- stronger fundamental field compared to conventional PM arrays,
- · heavy-weight backing steel elements can be omitted and
- therefore no iron losses occur,
- the magnetic flux density is more sinusoidal and,
- there are very low back-side fields [16].

The general description of the Halbach magnetization pattern was given by [12], a simple superposition of two trigonometric functions

$$M_x = \hat{M} \sin\left(\frac{2\pi x}{\lambda}\right) \tag{1a}$$

$$M_y = \hat{M} \cos\left(\frac{2\pi x}{\lambda}\right) \tag{1b}$$



Fig. 1. Halbach array with 4 single magnets per wavelength,  $n_m = 4$ , magnetization pattern and magnetic flux density characteristics

where  $\lambda$  denotes the wavelength and  $\hat{M}$  the magnetization amplitude. However, this approach cannot be applied to rareearth anisotropic magnet material with their so called easy axis characteristics. The predominated direction elaborated during the manufacturing process requires a description on magnet segment basis, as depicted in Fig. 1. The magnetic flux density peak value at the active surface is

$$\hat{B} = B_r \left( 1 - e^{\frac{-2\pi h_m}{\lambda}} \right) \operatorname{si}\left(\frac{\pi}{n_m}\right).$$
(2)

The example given in Fig. 1 is defined with the magnet height  $h_m$  and the number of magnets per wavelength  $n_m = 4$ . The rectangular shape yields from the wavelength  $\lambda = 8 \cdot h_m$ ,  $B_r$  represents the remanent magnetic flux density. For the distance  $\delta$  between two facing Halbach arrays, as depicted in Fig. 2 (i), the flux density in normal direction is [16]:

$$B_z(x,z) = \hat{B}\sin\left(\frac{2\pi x}{\lambda}\right) \frac{\cosh\left(\frac{2\pi z}{\lambda}\right)}{\cosh\left(\frac{\pi\delta}{\lambda}\right)}$$
(3)

The magnets with vertical magnetization are considered as pole magnets and the horizontal magnets as gap magnets, respectively. For the equations, the coordinate system is placed in the center of the air-gap. Augmenting the number of magnet segments from the minimum number  $n_m = 4$  to 6 segments leads to a slight increase of the peak flux density by 6 percent. However, two practical problems arise in this case. First, the production of magnets with an easy axis of  $60^{\circ}$ is very cost-intensive and second, the handling effort during assembly is increased as well. Therefore four magnet pieces per wavelength are considered for the proposed design.

The effect of different magnet width to height ratios on the peak value according equation (2) and correlation to the airgap distance was studied both analytically and by 2D FEA. An example is given in Fig. 2 (ii).

# III. HALBACH ARRAY DESIGN FOR AN AFPM

The main electro mechanic parts within an ironless AFPM, as shown in Fig. 3, are the air cored winding and two Halbach



Fig. 2. Double sided Halbach magnet array arrangement without back iron, (i) magnetization pattern, (ii) resulting field pattern



Fig. 3. Motor main parts cross section: (i) light metal hub, (ii) bearings, (iii) two equal CFRP half rims, (iv) air cored winding with GRP connection to the hub, (v) two Halbach magnet array rings, (vi) brake disk, [11]

TABLE I MOTOR DESIGN SPECIFICATION

Description	Symbol	Nominal value
Continuous output power	Pnom	3500 W
Peak power (10 s)	$P_{peak}$	$5 \cdot P_{nom}$
Speed at 80 km/h	$n_n$	660 rpm
Continuous torque	T <sub>nom</sub>	55 Nm
1 Peak torque (10 s)	$T_{peak}$	$5 \cdot T_{nom}$
Nominal torque/active weight		4.8 Nm/kg
Peak torque/active weight		24 Nm/kg

TABLE II MAIN DIMENSIONS

Description	Symbol	Nominal value
Outer diameter	$D_o$	416 mm
Inner diameter	$D_i$	306 mm
Mean diameter	$D_m$	306 mm
Magnet length	$l_m$	55 mm
Magnet height	$h_m$	8.6 mm
Magnet width	$w_m$	equ. (4a)-(4e)



Fig. 4. Magnet shape trapezoid-trapezoid array - TT

magnet array rings. Considering the geometric limitations, the winding concept and the magnet array were analytically designed to meet the motor specification given in Table I. The winding axial width was set to 5 mm with 1.5 mm air-gap on both sides. For the given diameter and magnet height, an ideal quadratic magnet would lead to around 65 Poles, therefore an approximately 20 percent decrease on peak flux density was accepted in order to reduce the number of poles. The magnet width to height ratio  $\gamma_m = w_m/h_m$  was set to 1.57. The magnet ring dimensions are summarized in Table II.

## IV. HALBACH MAGNET ARRAY - SHAPE VARIATION

The manufacturing of classic ring segment magnets on prototype level, would require EDM for an precise but quite expensive result. Additionally for high pole machines, the difference between narrow ring segment and trapezoid magnets is considered to be marginal. But even if the magnet array consists of trapezoid magnets, due to the small production volume in the prototype phase, the manufacturing costs per magnet are an important factor. To meet this challenge, three different magnet shapes and arrangements, respectively, were developed and evaluated. The already mentioned trapezoid magnet ring, where both pole and gap magnet are of the same dimension, as shown in Fig. 4, subsequently regarded as TT (trapezoid - trapezoid). Actually, the trapezoid shape has also to be manufactured based on the requirements from mechanical and electromagnetic view, therefore the measure to reduce cost is to replace every other magnet by a standard



Fig. 5. Magnet shape trapezoid-rectangular - TR

rectangular shaped magnet. This arrangement as depicted in Fig. 5 facilitates two options to apply either the rectangular or the trapezoid magnet as pole. These three options, TT, TR Rpole (rectangular pole magnet) and TR Tpole (trapezoid pole magnet) were evaluated.

Based on the parameters,

$$w_m = \frac{\tan\left(\frac{\alpha_p}{2}\right)}{1 + \cos\left(\frac{\alpha_p}{2}\right)} D_m \tag{4a}$$

$$w_{tt_i} = \tan\left(\frac{\alpha_p}{2}\right) D_i \tag{4b}$$

$$w_{tt_o} = \tan\left(\frac{\alpha_p}{2}\right) D_o \tag{4c}$$

$$w_{tr_i} = \tan\left(\frac{\alpha_p}{2}\right) D_i - \frac{\omega_m}{\cos\left(\frac{\alpha_p}{2}\right)} \tag{4d}$$

$$w_{tr_o} = \tan\left(\frac{\alpha_p}{2}\right) D_o - \frac{w_m}{\cos\left(\frac{\alpha_p}{2}\right)} \tag{4e}$$

the model for each arrangement was designed and applied to a 3D EM FEA software. Taking advantage of the periodicity, the arrangement was reduced to one pole pair. The section model including the winding is shown in Fig. 6.

# V. 3D ELECTROMAGNETIC FINITE ELEMENT ANALYSES

## A. Magnetostatic solver

In the magnetostatic domain, Maxwell's equations

$$\nabla \times \mathbf{H} = \mathbf{J} \tag{5}$$

$$\nabla \cdot \mathbf{B} = 0 \tag{6}$$

are solved together with the constitutive equation  $\mathbf{B} = \mu \cdot \mathbf{H} = \mu_0 \cdot \mu_r \cdot \mathbf{H}$ . **H** is the magnetic field strength and **B** the magnetic flux density. With the permeability of vacuum  $\mu_0 = 4 \cdot \pi \cdot 10^{-7}$  H/m, the relative permeability  $\mu_r$  for each material, appropriate excitations and boundary conditions, the finite element analysis furnishes all field quantities as



Fig. 6. 3D motor section for electromagnetic FEA, magnet array and winding encapsulation transparent

functions of the Cartesian coordinates (x, y, z) throughout the specimen volume. The following investigations are made:

1) No load: The first assessment was focused on the magnetic flux distribution in the air-gap at no load condition. The winding was masked out and multiple evaluation surfaces and lines were established to compare the results. The model consisted of approximately 160000 second order tetrahedral elements. The most crucial factor for the elaborated output torque is the axial magnetic field in the pole area, therefore the magnitudes of the magnetic flux density were calculated in the middle of the air-gap at the pole center, as shown in Fig. 7. As indicated in the picture, the values at the mean diameter are almost identical because the magnet width at this level is equal for all arrangements. However, close to the inner and outer diameter, the differences between gap and pole magnet width of both TB Tpole and TB Rpole are visible. To review the field solution not only at one point along the circumference, circular arcs were placed in the air-gap center. At the mean diameter, there are no apparent differences, however Fig. 8 shows the characteristics at the diameter  $D_{Teval} = D_i + (D_o - D_i)/6$ (1/6 of the magnet length starting at the inner diameter  $D_i$ ). Only minor deviations are visible. Therefore, from the field waveform point of view, there is no elaborated preference among the considered magnet shapes.

2) No load - forces: The same no load field solution is used to calculate the forces on the individual magnets by using the virtual work method. The surface tetrahedral elements are virtually distorted and from the change in the virtual work and the small deviation, the forces are derived by the software. The three values are in good correlation and no further comparison for the magnet arrangement quality can be drawn from this result. The motor overall attracting force between the magnet rings were calculated to take measure for the future assembly apparatus. Also this result coincides quite well among the three different arrangements. The evaluation



Fig. 7. No load FEA result, axial magnetic flux density distribution at the pole



Fig. 8. No load FEA result, tangential magnetic flux density distribution at  $D_{Teval}$ 

on single magnet basis showed an interesting detail. Obviously, the pole magnets which share the same magnetization direction attract each other, the gap magnet on the other hand does not serve as simple flux bridge without reaction, but the specimen is stressed with a force in the opposite direction towards the outer side of the ring. Especially for the assembly of the magnet ring itself, this detail is a valuable finding and is also discussed in chapter VI regarding manufacturing of Halbach magnet arrays.

*3) Full Load:* Due to almost the same field characteristics, the results for various excitations up to nominal current show only negligible difference in the elaborate output torque among the three arangements for each excitation.



Fig. 9. Transient FEA result, FFT analyses of the back EMF

#### B. Transient solver

The induced voltage at nominal speed  $n_n = 660$  rpm was calculated by the transient solver with a time step of  $40 \,\mu s$ . The FFT spectra of the three arrangements are compared in Fig. 9. The TR Rpole shape results in a slightly reduced first and both higher third and fifth order harmonic components. The single noteworthy deviation between TT and TR Tpole shape is given at the third order harmonic, therefore the TR Tpole shape is considered to be the better choice and will be applied to the prototype machine.

## VI. REALIZATION AND MEASUREMENTS

As mentioned in the previous chapter, during the assembly and in the final configuration within the Halbach array, distracting magnet forces occur within the specimen. Without further measures, the magnets form the staircase arrangement shown in Fig. 10. The shear stress analysis yields to the conclusion that without the presence of a backing material, the shear stress maximum reaches up to 6 MPA, which is quite challenging for some adhesives especially at elevated temperatures. According to this result, a adequate adhesive was chosen to manufacture small scale prototypes to validate the FEA results. During measurements under various temperature conditions, it was observed and later confirmed by 2D and 3D FEA calculations, that rare-earth magnets exhibit an augmented temperature degrading effect compared to standard applications [17]. Samples from different magnet materials were used to manufacture double-sided linear Halbach arrays, as depicted in Fig. 11. The magnetic flux density characteristics were measured along the air-gap distance, an example at 20 °C for three different arrangements, single sided, double sided with an air-gap of 5 mm and 10 mm, is shown in Fig. 12 and compared to 2D FEA.

According to the results from the previous section, the TR Tpole magnet ring configuration was manufactured (cf. Fig. 12) by using trapezoid axial magnetized pole magnets



Fig. 10. Functional model, 3 piece natural balanced Halbach array, two pole magnets and one gap magnet



Fig. 11. Functional models, double sided N40 Halbach magnet arrays ( $2 \times 9$  pieces  $10 \times 10 \times 45$  mm each), (i) 5 mm air-gap, (ii) 10 mm air-gap

and tangential magnetized. The ironless in-wheel hub motor prototype will be finished in August 2012.

### VII. CONCLUSION

Halbach magnet arrays are a beneficial choice to reduce both weight and increase the magnetic flux density and therefore the torque output of ironless axial flux motors. Especially on prototype level, the manufacturing of custom tailored magnets is an important factor. The decrease the overall cost, every second magnet is replaced by an rectangular block magnet. The impact is studied in comparison to a magnet ring consisting of trapezoid magnets. The comparison of magnetic



Fig. 12. Measurement and FEA results, magnetic flux density distribution at 2 mm distance from the surface, Temperature 20  $^\circ C$ 



Fig. 13. Manufactured Halbach array with trapezoid poles and rectangular gap magnets

flux density and elaborated torque for a given motor design showed only minor differences. Based on FFT analyses of the induced voltage at nominal speed, the most suitable choice is the arrangement with trapezoid magnets serving as pole magnet and rectangular magnets in between. Finally, results from both functional linear and ring Halbach magnet arrays are presented, whereby forces and the assembly itself were studied to be able to manufacture the prototype of an ironless in-wheel hub motor.

# BIOGRAPHIES

**Oliver Winter** (S'11) received his Dipl.-Ing. degree in Mechatronics from the University of Linz, Austria, in 2006. After two years in industry he joined the AIT Austrian Institute of Technology in 2009. As Junior Scientist within the business unit Electric Drive Technologies, he is working towards his PhD in electrical engineering. His research interests include modeling of dynamic electromechanic and thermal behavior of mechatronic systems by means of coupled FEA.

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