Dynamic Induction Machine Model Accounting for Stator and Rotor Slotting

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Abstract — **A method for dynamic modelling of induction machine with a doubly slotted air gap is proposed and implemented for the case of a cage induction motor. The described method is easily extensible to wound rotor machines. A numerical description of the air gap permeance is provided that takes into account a slotted stator and rotor structure as well as their mutual, time and space dependant positions as a function of rotor rotation. The multiple coupled circuit model approach is used with the modified winding function in order to calculate the inductance of all motor windings. The developed model is general in nature and could be used for the analysis of different dynamic regimes of induction machine, particularly different combinations of stator and rotor slot numbers. Model validation is provided by stator current spectrum analysis of a standard four pole induction motor with *S*=36 and *R*=32 slots. The experimental results presented clearly support these findings.**

Index Terms — **Induction machine, Slot permeance, Rotor slot harmonics, Principal slot harmonics, Modified Winding Function Approach.**

I. INTRODUCTION

Rotor slot harmonics (RSH) and their treatment can be considered as an example of how the passage of time and development of technology changes the perspective from which technical phenomena are viewed. A primary consideration of the induction motor designer is the design of a motor with good starting capabilities i.e. good starting torque with as small as possible starting current, whilst endeavouring to deliver smooth and noiseless operation at rated conditions. This target is as valid today as it was a century ago. In the first instance, this means that the designer must make the choice of optimal stator and rotor slots number configurations, or more precisely, make a choice of optimum rotor slot (bar) number for an (ordinarily) predefined and fixed number of stator slots.

During the previous century many rules for determining the optimum number of rotor bars for a given number of stator slots and pole pair numbers have been derived, [1], [2], [3]. More precisely, there are "forbidden" combinations of the number of stator and rotor slots that coincide with the existence of both RSH in the stator current spectrum. Therefore, intuitively or not, induction motor designers always tried to eliminate the chance of strong RSH occurrence. Today, however, the existence of RSH is looked on more favourably than before, particularly from the perspective of sensorless speed estimation, [4], [5], but also from the condition monitoring and diagnosis point of view, [6]. Indeed it is even the case that the selection criteria for the commercially available motors can be predicated on the basis of pronounced RSH that are adequate for speed sensorless applications, [7].

Stator and rotor slot permeance is well defined and analyzed in a number of textbooks and papers, from different points of view, [1]-[3], [8]. However, it is only recently the case that this effect has been incorporated into the dynamic model of the induction machine, [9]. In [9] the air gap permeance was described using analytical expressions, i.e. the first slot permeance harmonic.

This paper proposes a method for dynamic modelling of the induction motor whilst taking into account a real profile of the double slotted air gap structure. The inverse air gap function is completely numerically described and the modified winding function approach (MWFA) is used for the calculation of all of the motor winding inductances. The proposed model easily takes into account different stator and rotor slot number combinations, different slot mouth widths, and skewing of rotor bars, [10]. These possibilities give a wide area of application for the model.

II. AIR-GAP PERMEANCE MODELLING

As is well known, the MWFA allows the calculation of the self and mutual inductances of all the windings in the machine in the case of non uniform air gap along the motor circumference, [11]. If the winding function of the winding *A*, the turn function of the winding *B* and the air gap permeance function $P(\theta)$ are known, then the mutual inductance between these two windings is given by,

$$L_{AB} = \mu_0 r l \int_0^{2\pi} P(\theta) N_A(\theta_A) n_B(\theta_B) d\theta \qquad (1)$$

where r is the average radius of the air gap, l is the length of the stack and θ is the angular position of the rotor with respect to a stator reference (the mechanical angle). The mechanical angles θ_A and θ_B represent the angular position of winding A and B on stator or rotor from a referent position.

As a consequence of the stator and rotor slotting, the air gap length i.e. the air gap permeance function $P(\theta)$, becomes a function of the rotor position.

This function can be included in (1) on the basis that the total air gap length can be observed as a sum of two air gaps that are separated by the imaginary circle placed in the middle of the air gap between the stator and rotor as shown in Fig 1.
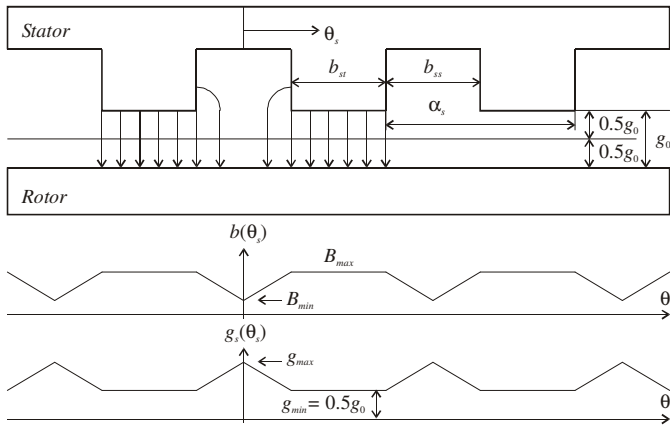

Fig. 1. Stator slot geometry, magnetic flux density and air gap length profile. Rotor is assumed smooth.

The first air gap is the air gap seen from the stator side, which is independent of rotor position. The second air gap is the air gap seen from rotor side which rotates with the rotor. The total air gap length is the sum of these two air gaps at any instant in time. By rotating the rotor in a step-by-step fashion and integrating (1) the inductances of all the machine windings can be calculated. A linear drop of the air gap permeance i.e. a linear rise of air gap length is assumed under the slots, in accordance with the results presented in [12] for the saturated slot bridge in closed slot machines.

If it is assumed that the rotor shown in Fig 1 is smooth then the following function for the air-gap length under a single stator slot pitch could be defined as,

$$g_{sp}(\theta_s) = \begin{cases} g_{s\max} + \dfrac{0.5g_0 - g_{s\max}}{0.5(b_{ss}/r)}\theta_s, & 0 < \theta_s \le 0.5(b_{ss}/r) \\[2ex] 0.5g_0, & 0.5(b_{ss}/r) < \theta_s \le \alpha_s - 0.5(b_{ss}/r) \\[2ex] 0.5g_0 + \dfrac{g_{s\max} - 0.5g_0}{0.5(b_{ss}/r)}(\theta_s - \alpha_s + 0.5(b_{ss}/r)), \\[1ex] \qquad\qquad \alpha_s - 0.5(b_{ss}/r) < \theta_s \le \alpha_s \end{cases}$$

$$(2)$$

where b_{ss} is the stator slot width and $\alpha_s = 2\pi/S$ is the stator slot

pitch. The maximum value of the "stator" air-gap length is given approximately by [13],

$$g_{s\max} = \frac{2u_s}{1 + u_s^2} g_0 \qquad (3)$$

and

$$u_s = \frac{b_{ss}}{2g_0} + \sqrt{1 + \left(\frac{b_{ss}}{2g_0}\right)^2} \qquad (4)$$

and the resultant air-gap length function along the stator circumference can be formed by S functions which define air gap length under one stator slot pitch (2) given by:

$$g_s = \left[\underbrace{\begin{array}{cccccccc} g_{sp} & g_{sp} & g_{sp} & g_{sp} & \cdots & g_{sp} & g_{sp} \end{array}}_{S\ times}\right] \qquad (5)$$

Similarly, assuming a smooth stator and taking into account the rotor slots, the resultant air-gap length function along the rotor circumference can be formed from R functions under one rotor slot pitch given by:

$$g_r = \left[\underbrace{\begin{array}{cccccccc} g_{rp} & g_{rp} & g_{rp} & g_{rp} & \cdots & g_{rp} & g_{rp} \end{array}}_{R\ times}\right] \qquad (6)$$

For known air gap functions it is straightforward to define the inverse air-gap function i.e. air gap permeance function. Of course, it must be considered that the "rotor part" of the air gap rotates with the rotor. For a given rotor position, the air-gap permeance function is,

$$P(\theta) = \frac{1}{g_s + g_r} \qquad (7)$$

During the formulation of the above functions one should bear in mind that the two air gap functions should be of the same length. For example, if $S=36$ and $R=32$, the g_s and g_r vectors should have $36 \cdot 32 = 1152$ or 2304 or 3456 entries etc.

Fig 2 shows the "stator" and "rotor" air-gap length, resultant air-gap length and the air-gap permeance function for the rotor position defined by $\theta=0$ rad (the position where the 1st stator slot and 1st rotor slot are in opposition).

III. INDUCTANCE CALCULATION

The inductances of all windings in the machine can be calculated using the modified winding function approach as described in [11].

A) Stator winding self and mutual inductances

Stator winding inductances are calculated numerically,

using expression (1). One rotor revolution can be discretized in *H* steps hence $d\theta=2\pi/H$. In rotating the rotor in a step-by-step fashion, the air-gap permeance function is defined at each step by (7), the stator winding function is defined,

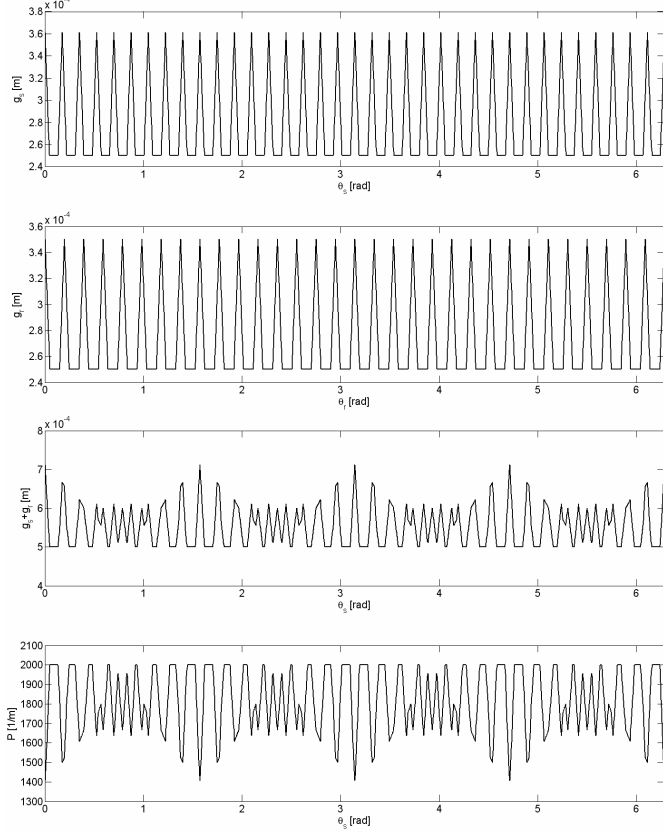$$N(\theta)=n(\theta)-\frac{\langle P(\theta)\cdot n(\theta)\rangle}{\langle P(\theta)\rangle} \qquad (8)$$

Fig. 2. Upside down: "stator" air-gap length, "rotor" air-gap length, resultant air-gap length and air-gap permeance function: $\theta=0$, *S*=36, *R*=32, $b_{ss}=0.5\alpha_s$, $b_{rs}=0.5\alpha_r$, g_0=0.5mm.

and the inductance is calculated by (1). In this way, the self and mutual inductances of the stator phases can be organized in look-up tables. For every entry in the matrix of stator winding self and mutual inductances, [L_{ss}] is a vector of length *H*, where $H=n\times S\times R$ and *n* is an integer that represents the number of angle samples at which inductances are calculated. As a result of this process, the self inductance profile for the stator phase winding *A* is obtained, as shown in Fig 3. The derivative of this function, needed for electromagnetic torque calculation, is also shown on the same figure.

B) Stator – rotor mutual inductance

In order that all elements of the matrix [L_{sr}] can be known, it is enough to know only three look up tables: the mutual inductance between phase *A* and one rotor loop; the mutual inductance between phase *B* and the same rotor loop and the mutual inductance between phase *C* and the same rotor loop. All other mutual inductances can be obtained form these three

look-up tables by taking into account an appropriate space shift between rotor loops. This task can be solved in the same manner as for stator inductances. However, the rotation of the rotor in a step-by-step fashion means shifting not only the rotor part of air-gap function but also the rotor turn or winding function. Fig 5 shows the mutual inductance profile between phase *A* and first rotor loop as well as its derivative. The same profiles are for other two stator phases, appropriately shifted in space.
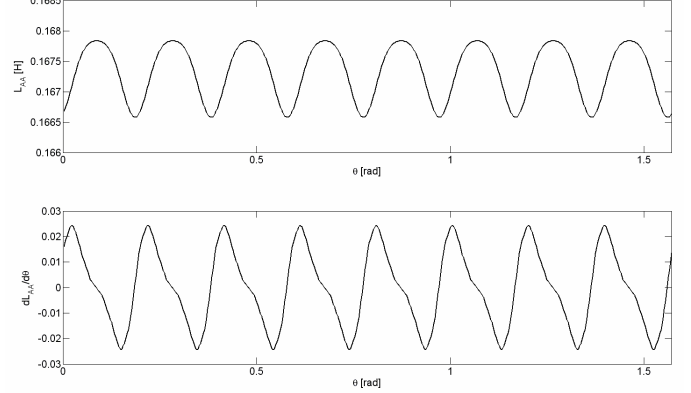

Fig. 3. Phase winding self inductance and its derivative, under one pole: *S*=36, *R*=32, $b_{ss}=0.5\alpha_s$, $b_{rs}=0.5\alpha_r$, g_0=0.5mm.

Fig 4 shows the mutual inductance between phases *A* and *B* as well as derivative of this profile as a function of rotor position. The mutual inductance profile for phases *A* and *C* or *B* and *C* has the same profile but are shifted in space.
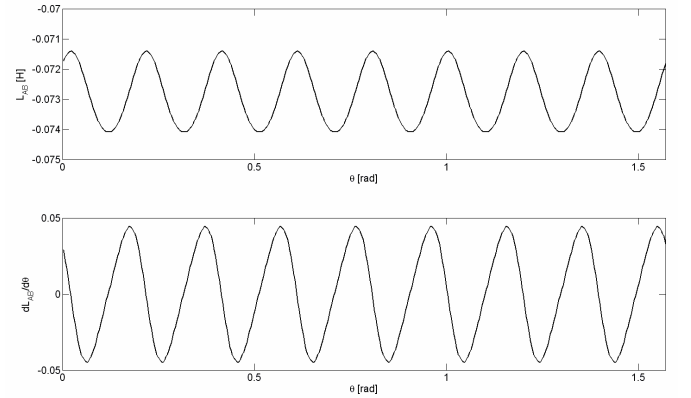

Fig. 4. Phase windings mutual inductance and its derivative, under one pole: *S*=36, *R*=32, $b_{ss}=0.5\alpha_s$, $b_{rs}=0.5\alpha_r$, g_0=0.5mm.

C) Rotor winding self and mutual inductances

The most difficult task in the modelling of an induction motor taking into account stator and rotor slot permeances is the calculation of rotor loop self and mutual inductances. During the rotation of the rotor, the rotor loop experiences a different picture of air gap profile at each discrete instant in time. Moreover, in the general case, the mutual-inductance between, for example, rotor loop 1 and rotor loop 2 have a different profile from the mutual inductance profile between any other two loops.
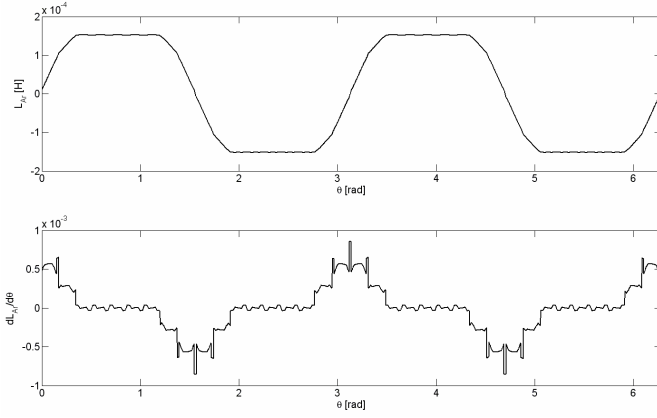
Fig. 5. Phase winding – rotor loop mutual inductance and its derivative: S=36, R=32, b_{ss}=0.5α_s, b_{rs}=0.5α_r, g_0=0.5mm.

However, by using the modified winding function approach this task can be solved and these inductances can be numerically calculated.

The first step is to form the turn functions for all of the rotor loops. Then, for each step, the air-gap permeance function, new winding function and inductance is defined as has been previously described. This is repeated for each step in the step-by-step rotor rotation. As the rotor has a significant number of rotor loops this process of calculation could take some time, however it is negligible in comparison with FEM techniques. For example, for R=32, with H=36×32, the time for calculation of this matrix is around 5 minutes on a 2GHz Pentium IV PC running on Windows XP Professional with 256MB RAM.

Using the manner described above, the rotor self and mutual inductance matrix [L_{rr}] becomes a three dimensional matrix with dimensions $R×R×H$, i.e. every entry of this matrix becomes a vector (look-up table) of length H.

Fig 6 shows the self inductance profile for first rotor loop and their derivative. Self inductances of the other rotor loops are of the same shape but with the appropriate space shift. For other combinations of stator and rotor slots number the rotor self-inductance will have different profile.

Figs 7 and 8 show the mutual inductance profiles between the first rotor loop and two other different rotor loops. As can be observed, in the general case the mutual inductance profiles are different for each different rotor loop.
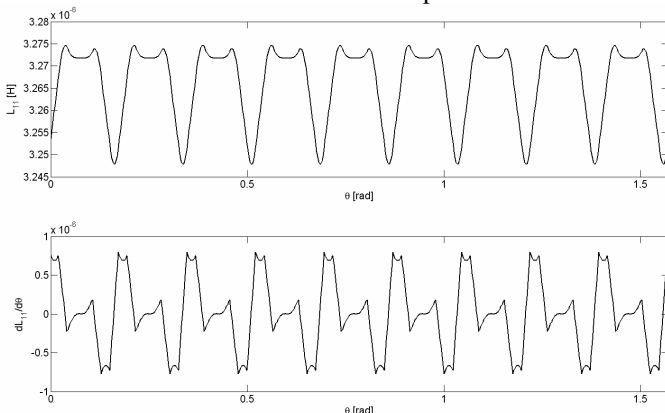
Fig. 6. Self inductance of the 1st rotor loop as a function of the rotor position under one pole and its derivative: S=36, R=32, b_{ss}=0.5α_s, b_{rs}=0.5α_r, g_0=0.5mm.

Fig. 7. Mutual inductance between 1st and 5th rotor loop as a function of the rotor position under one pole and its derivative: S=36, R=32, b_{ss}=0.5α_s, b_{rs}=0.5α_r, g_0=0.5mm.
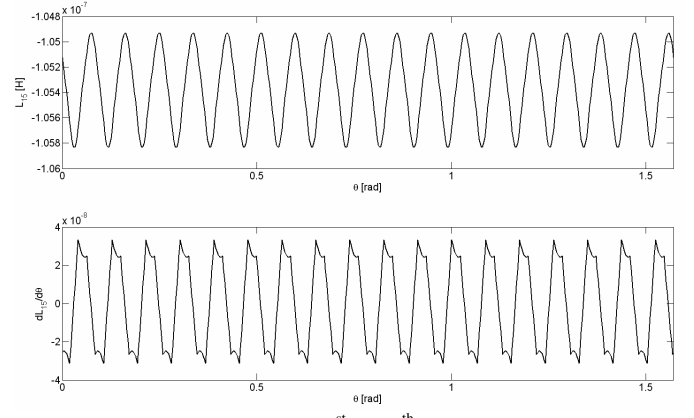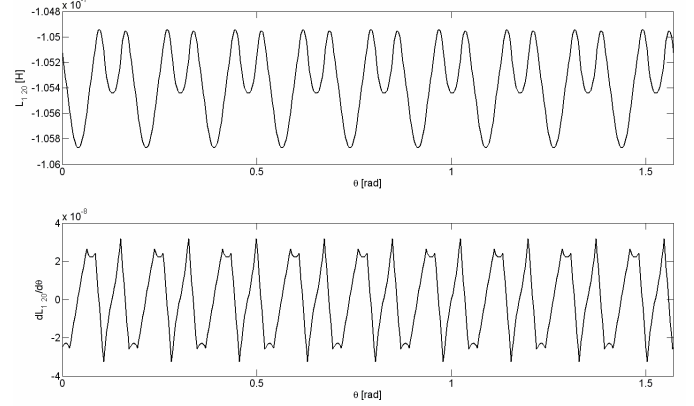
Fig. 8. Mutual inductance between 1st and 20th rotor loop as a function of the rotor position under one pole and its derivative: S=36, R=32, b_{ss}=0.5α_s, b_{rs}=0.5α_r, g_0=0.5mm.

At first sight it looks like $R×R$ different look-up tables are needed; however, given the linear magnetic circuit mutual inductance between circuits 1 and 2 is the same as for that between 2 and 1, and given that the self inductance profiles for different loops are the same, only $R×R−(R−1)(1+0.5·R)$ look up tables are needed (i.e. 497 for R=32 which is significantly lower than 32×32=1024).

IV. RESULTS AND DISCUSSION

Using the proposed model a four pole cage induction motor with S=36 slots and R=32 bars was analyzed (the motor's details are given in the Appendix). In the stator current spectrum of this motor only one of the rotor slot harmonics could exist: the upper rotor slot harmonic, [14]. Additionally, this motor has a number of stator and rotor slots such that the slot permeance will also have a very significant influence on the same RSH amplitude because the fundamental stator magnetomotive force (MMF) wave through the slot permeance produces upper RSH. It might reasonably be expected that this upper principal slot harmonic is very prominent in the stator current spectrum.

These observations are validated by the results from the numerical model. Fig 9 shows the stator current spectrum for an unloaded motor in three different cases: for a smooth air

gap; a slotted air gap with slot opening of 10% of the slot pitch, and for the case of open slots on stator as well as on the rotor. The influence of the slot permeance is obvious and rather high. Interestingly, the influence of the slot permeance on the RSH amplitude is almost independent of the slot opening in the no-load regime. It should be noted that the amplitude of the stator current (in this regime it is the magnetizing current) rises with width of slot opening as could be expected.

In the case of loaded motor as shown in Fig 10, the opening of the slots has an influence on the slot harmonic amplitude. In the case of open slots the slot harmonic amplitude is 100% higher when compared with the smooth air gap case.
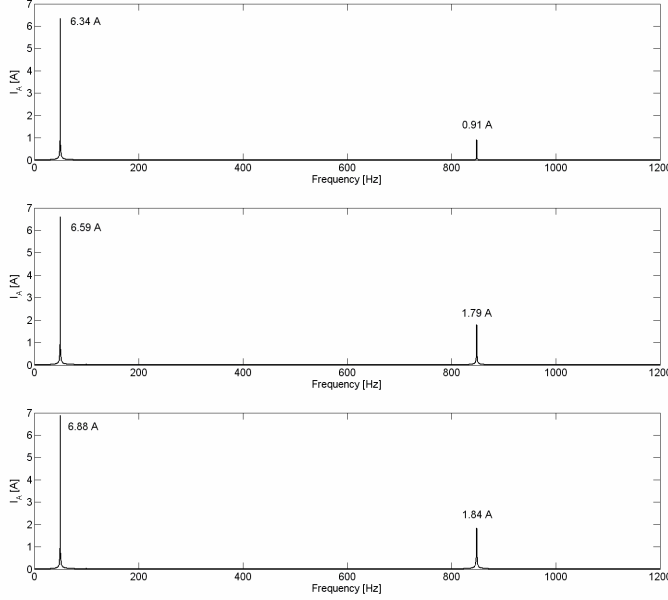

Fig. 9. Stator line current spectrum of the unloaded motor. Top: smooth air gap; Middle: slotted air gap ($b_{ss}=0.1\alpha_s$, $b_{rs}=0.1\alpha_r$); Bottom: slotted air gap ($b_{ss}=0.5\alpha_s$, $b_{rs}=0.5\alpha_r$).
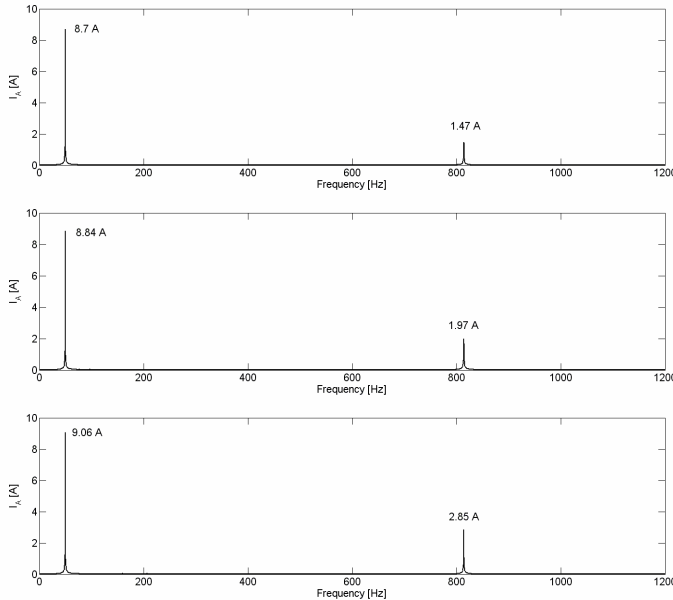

Fig. 10. Stator line current spectrum of loaded motor, $s=4.32\%$. Top: smooth air gap; Middle: slotted air gap ($b_{ss}=0.1\alpha_s$, $b_{rs}=0.1\alpha_r$); Bottom: slotted air gap ($b_{ss}=0.5\alpha_s$, $b_{rs}=0.5\alpha_r$).

Experimental result clearly supports the findings from the numerical model. Fig 11 shows the stator current spectrum of a cage induction motor with $S=36$ slots, $R=32$ slots and $p=2$. The upper PSH in this motor is the most significant higher harmonic in the stator current spectrum, predominantly due to the slot permeance effect. Therefore, for this motor, it can be concluded that slot permeance is the predominant cause of the existence of rotor slot harmonic in the stator current spectrum.
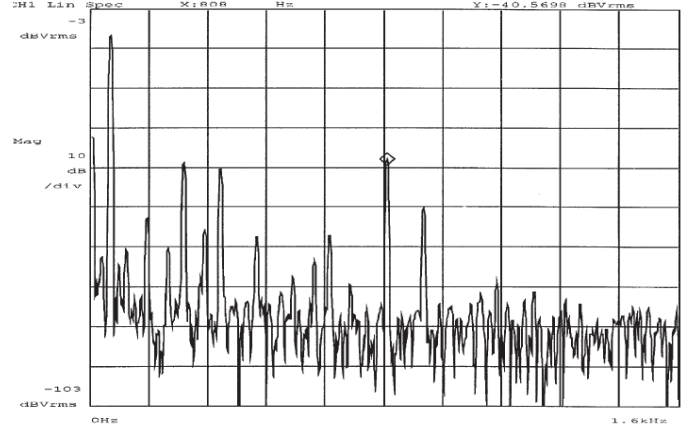

Fig. 11. Experimentally obtained stator line current spectrum: $S=36$, $R=32$, $p=2$ @ $s=5.25\%$. Only upper PSH exists at 808Hz as the most prominent harmonic component in the spectra!

V. CONCLUSION

A method for the dynamic modelling of the induction motor taking into account a real profile of a double slotted air gap is presented in this paper. The proposed method is implemented in the case of a cage induction motor but can easily be implemented for the case of wound rotor motors as well. A numerically described air gap permeance takes into account slotted stator and rotor structure as well as their mutual, time and space dependant position as a function of rotor rotation. A multiple coupled circuit model and a modified winding function approach is used in order to calculate the inductance of all motor windings. The model is general in nature and could be used for the analysis of different dynamic regimes of induction motor, particularly different combinations of stator and rotor slot numbers. Model validation is provided through the stator current spectrum analysis of a standard four pole induction motor with $S=36$ and $R=32$ slots. The presented experimental stator current spectrum clearly support findings from the numerical model.

ACKNOWLEDGEMENT

APPENDIX

Machine parameters: $P_r=3$kW, Y, $U=400$V, $I_r=6.5$A, $n_r=1420$rev/min, 50Hz, $p=2$, $l=0.15$m, $r=0.05$m, $S=36$ stator slots, $R=32$ rotor bars, $w=15$ turns per

coil, g_0=0.5mm, J=0.0113 kgm^2, $R_{s(phase)}$=1.3Ω, R_b=200μΩ, R_e=10μΩ, L_b=10nH, L_e=2nH, γ=2π/36 (angle of skewing of rotor bars).

Winding scheme of phase A under one pair of poles:
A_1-1-9'-2-10'-3-11'-20-12'-19-11'-18-10'-X_1

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