Longitudinal thermalization via the chromo-Weibel instability

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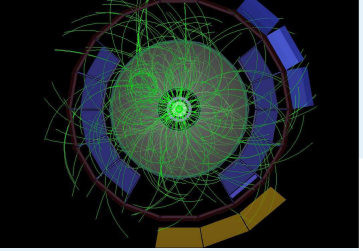
arXiv:1207.5795

Collaborators: Anton Rebhan, Michael Strickland

October 8, 2012



Motivation

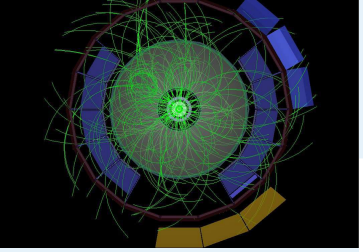
Weakly coupled inspired by Hard Thermal Loops (HTL)

Real-time physical quantities of non-equilibrium processes

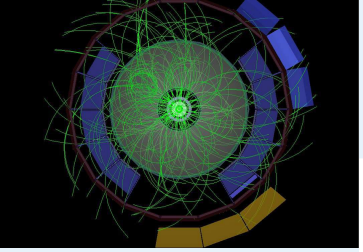
Plasma turbulence affects parton transport (isotropization, jet energy loss, viscosity,..)

Early time dynamics of the quark gluon plasma

Derivation of time scales for the isotropization, thermalization



Hard Expanding Loops (HEL)



Hard-Expanding Loops Assumptions

Free streaming background

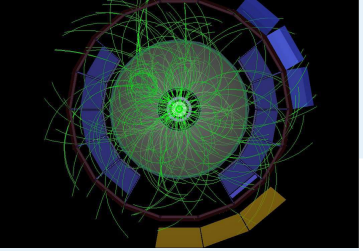
Anisotropy in momentum space

SU(2) particle content

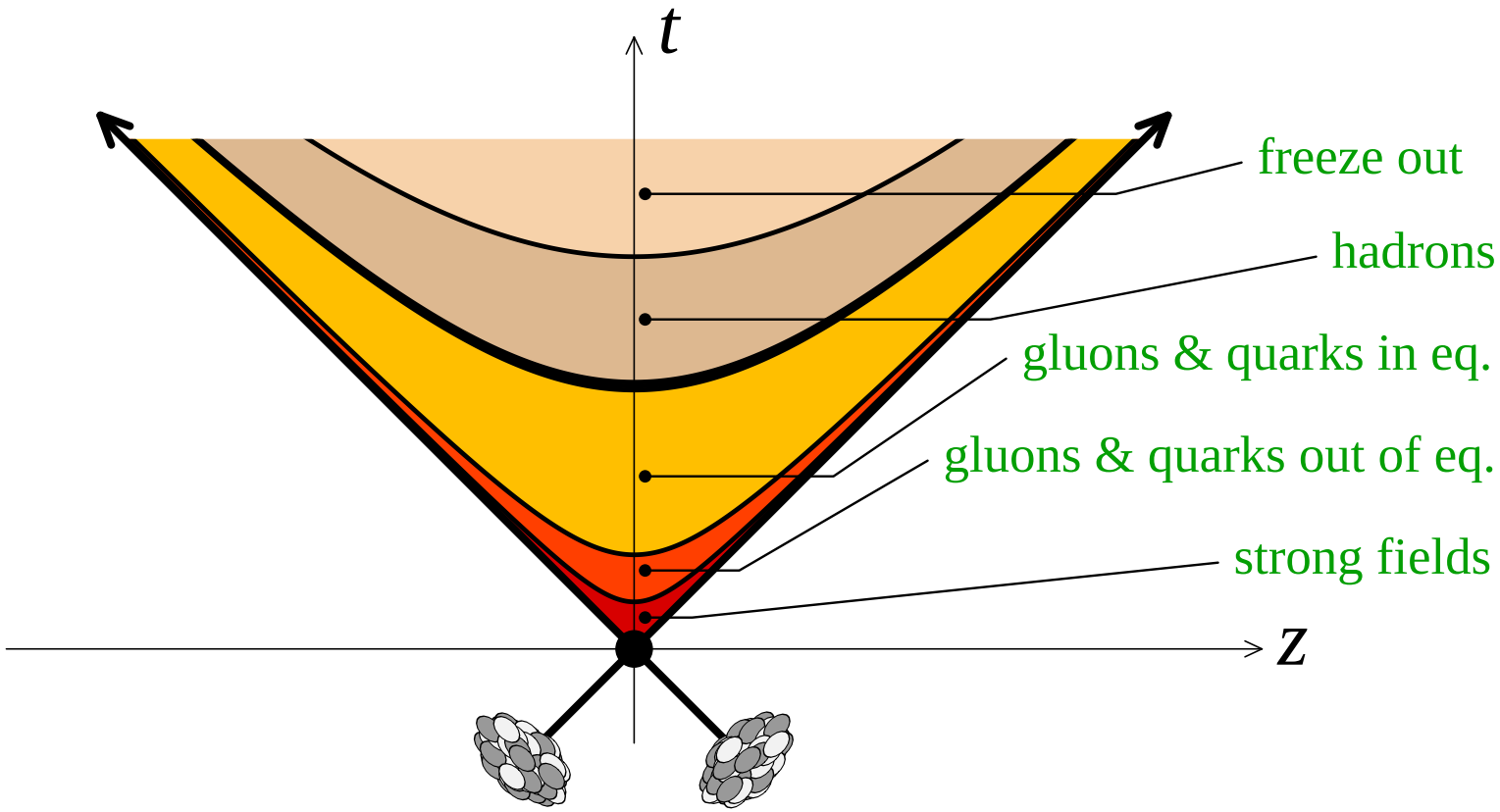
Fixed transverse size

Extrapolate to $\alpha_s \sim 0.3$

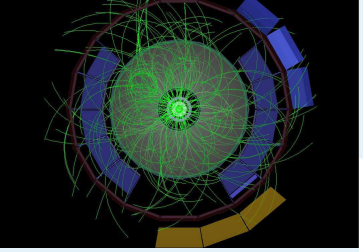
Match CGC $n(\tau_0) \propto Q_s^3 \alpha_s^{-1}$



Stages of the Little Big Bang

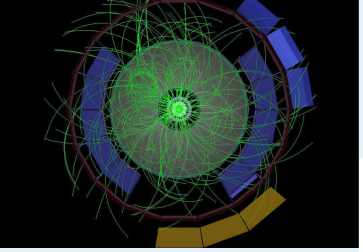


[Gelis 2010] Illustration of the stages of a heavy ion collision. This work focuses on the early phase with strong fields in an out of equilibrium situation.



Scales of weakly coupled QGP

- T: energy of hard particles
- gT: thermal masses, Debye screening mass, Landau damping
- g^2T: magnetic confinement, color relaxation, rate for small angle scattering
- g^4T: rate for large angle scattering, $\eta^{-1}T^4$



Scales of weakly coupled QGP

- T: energy of hard particles
- gT: thermal masses, Debye screening mass, Landau damping, plasma instabilities [Mrowczynski 1988, 1993, ..]
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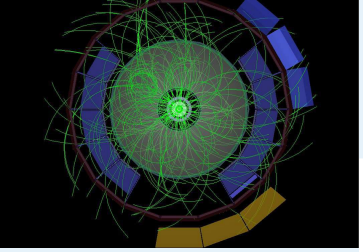
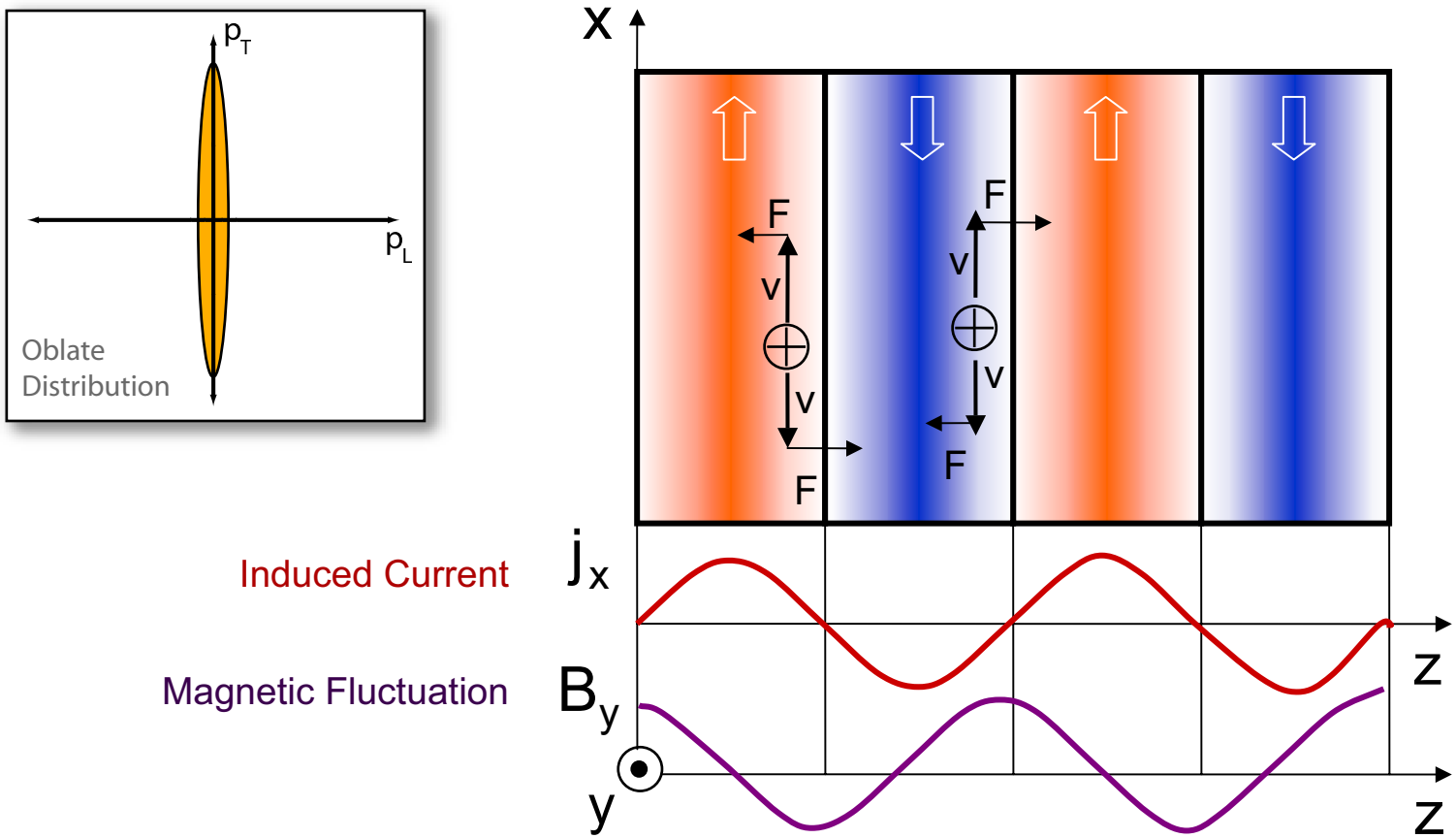
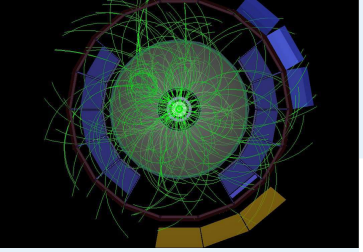


Weibel instabilities



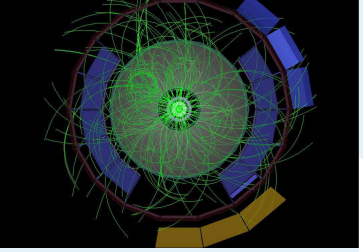
[Strickland 2006]: Illustration of the mechanism of filamentation instabilities.



Hard (Thermal) Loops - Boltzmann - Vlasov

Assuming free streaming, one solves the gauge covariant Boltzmann-Vlasov equation

$$v \cdot D\partial f_a(\mathbf{p}, \mathbf{x}, t) = g v_\mu F_a^{\mu\nu} \partial_\nu^{(p)} f_0(\mathbf{p}, \mathbf{x}, t) \tag{1}$$



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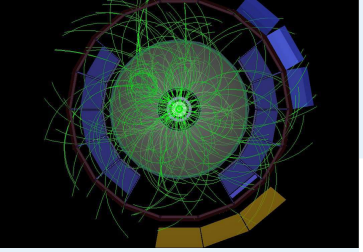
coupled to Yang-Mills equation

$$D_\mu F_a^{\mu\nu} = j_a^\nu = g \int \frac{d^3p}{(2\pi)^3} \frac{p^\mu}{2p^0} \delta f_a(\mathbf{p}, \mathbf{x}, t) \tag{2}$$



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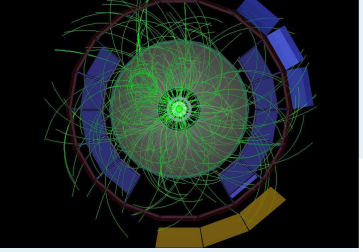
in the HTL approximation

$$g A_\mu \ll |\mathbf{p}_{hard}| \,, \tag{3}$$



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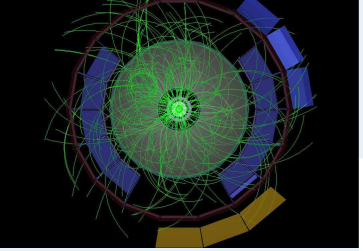
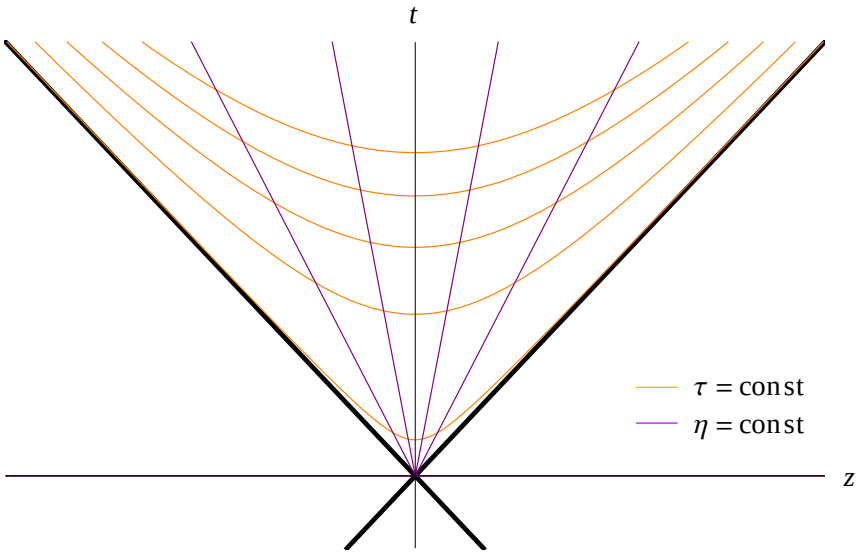
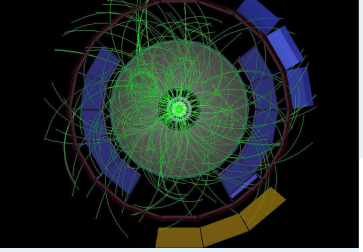
the Romatschke, Strickland background distribution function

$$f_0(p_\perp, \tilde{p}_\eta) = f_{\text{iso}}\left([\mathbf{p^2} + \xi(\tau)(\mathbf{p} \cdot \hat{\mathbf{n}})^2]/p_{\text{hard}}^2(\tau)\right)^{0.5} . \tag{4}$$



Bjorken expansion





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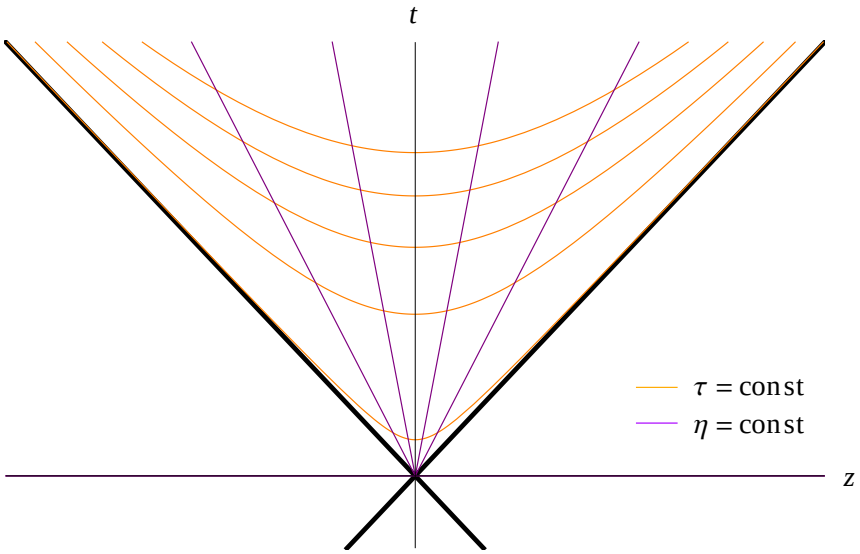
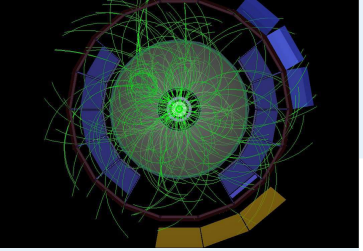


It is convenient to switch to comoving coordinates

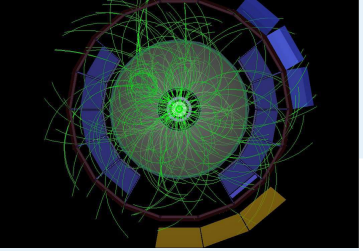
$$t = \tau \cosh \eta \,, \qquad \tau = \sqrt{t^2 - z^2} \,,$$
$$z = \tau \sinh \eta \,, \qquad \eta = \operatorname{arctanh} \frac{z}{t} \,, \tag{5}$$

with the corresponding metric

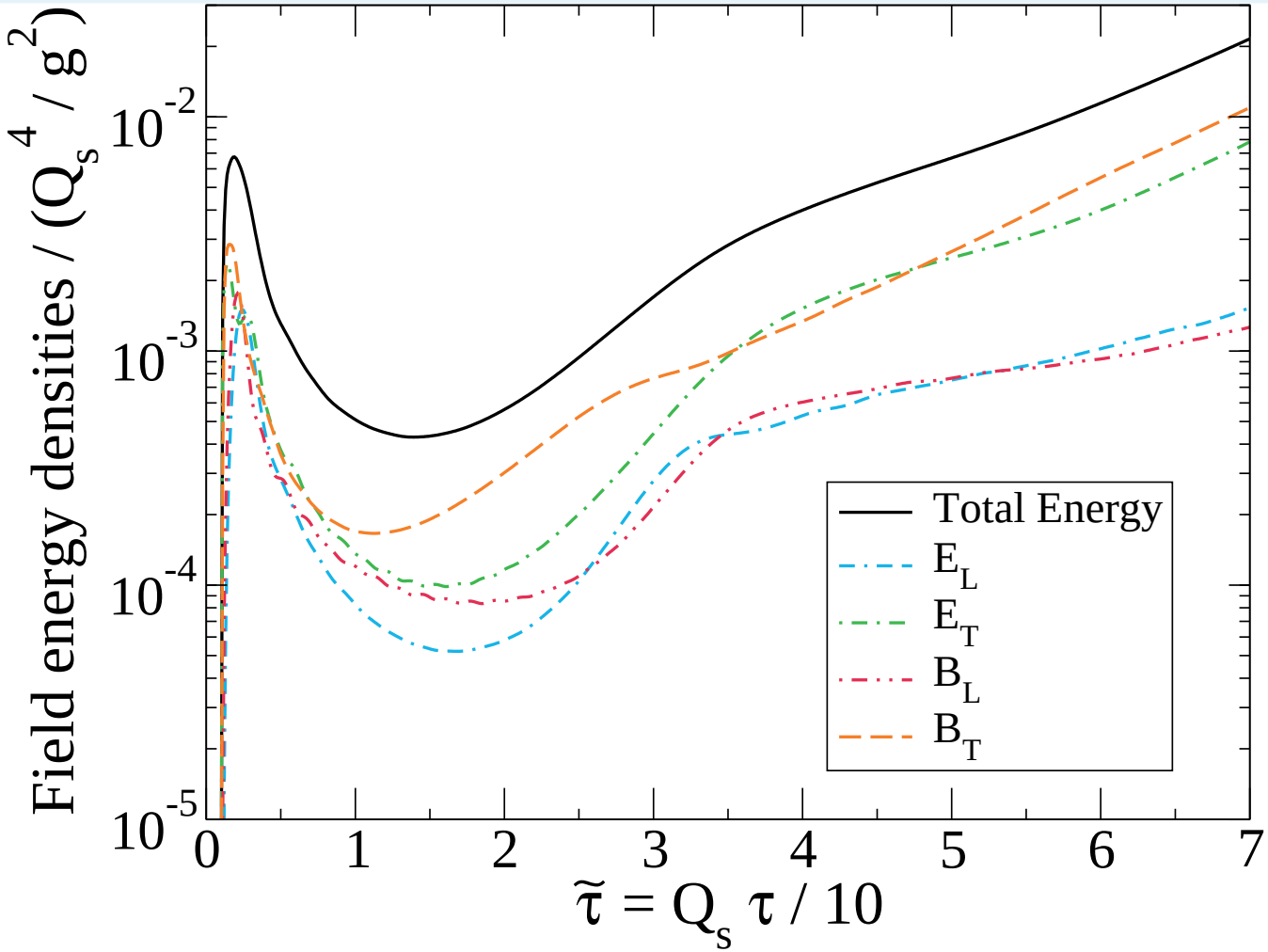
$$ds^2 = d\tau^2 - d\mathbf{x}_\perp^2 - \tau^2 d\eta^2 \,. \tag{6}$$



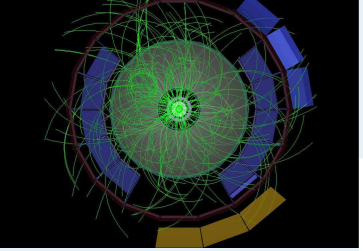
Expanding 3D+3V non-Abelian Plasma Instabilities



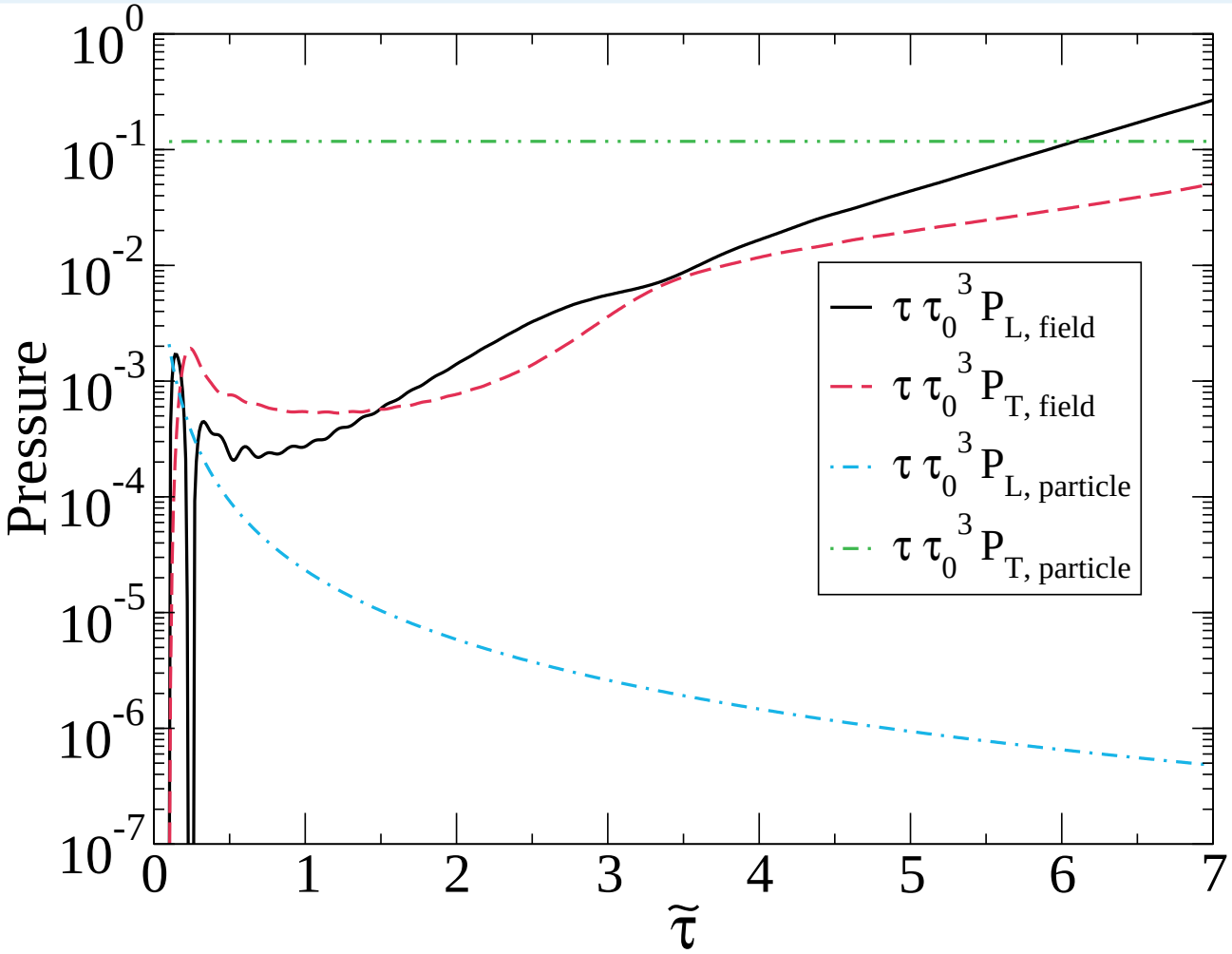
Energy densities fields



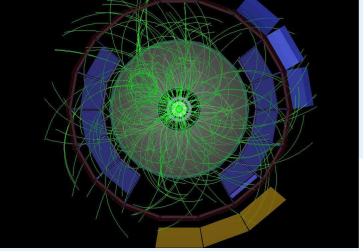
50 averaged runs $N_\perp * N_\eta * N_u * N_\phi = 40^2 * 128 * 128 * 32$: after onset one sees **rapid growth of B_l and E_L fields**, followed by non-Abelian interactions kick in.



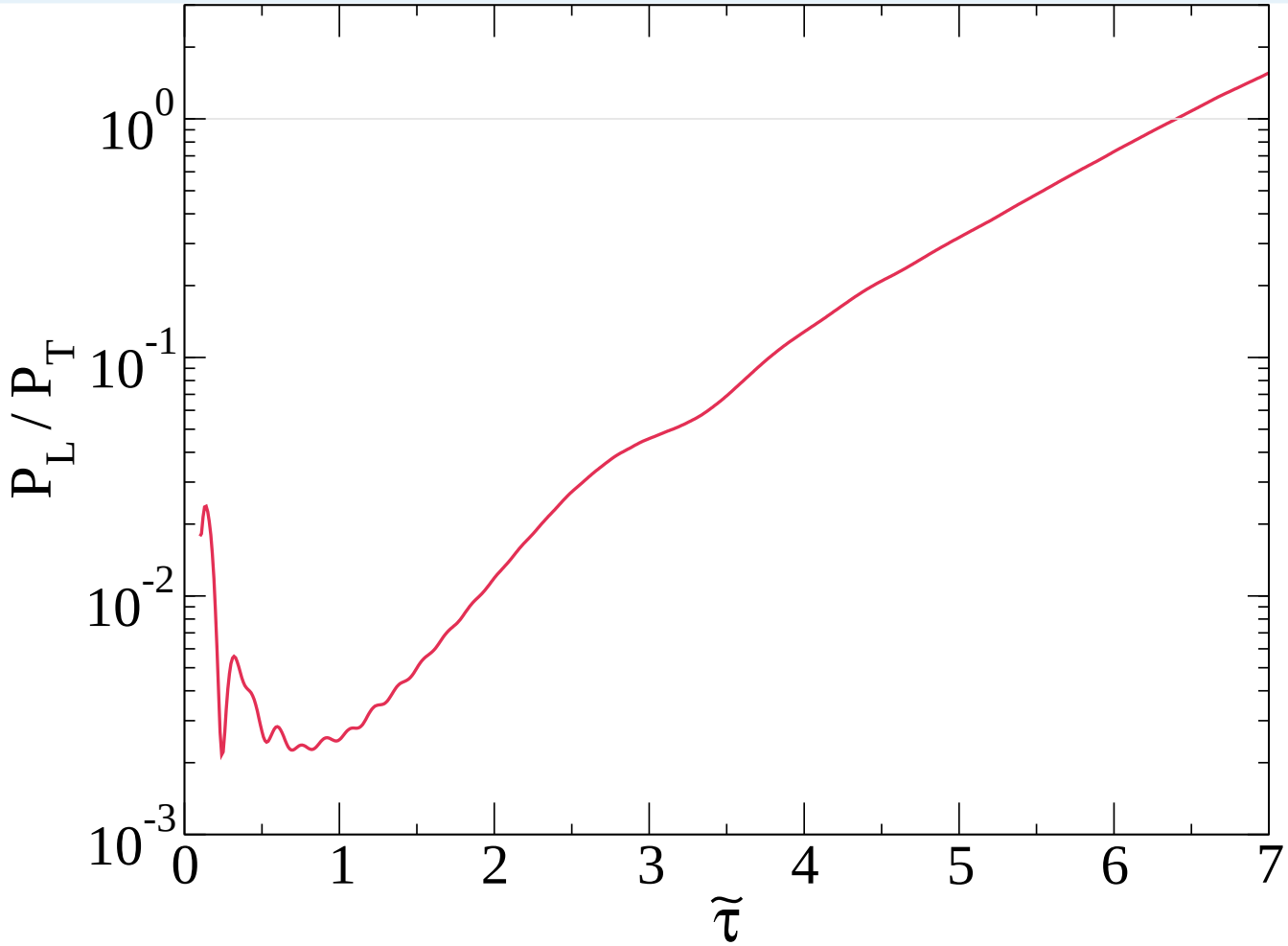
Pressures



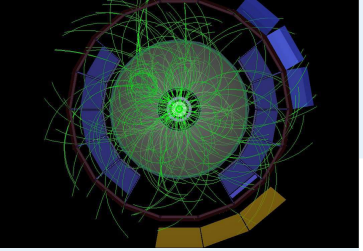
Initially highly anisotropic, note $P_{L,\mathrm{field}}(\tau = 0.3) < 0$, **growing field pressures**, $P_{L,\mathrm{field}}$ dominates at late times, $\tilde{\tau}$ scaled P_L drops $\propto 1/\tilde{\tau}^2$



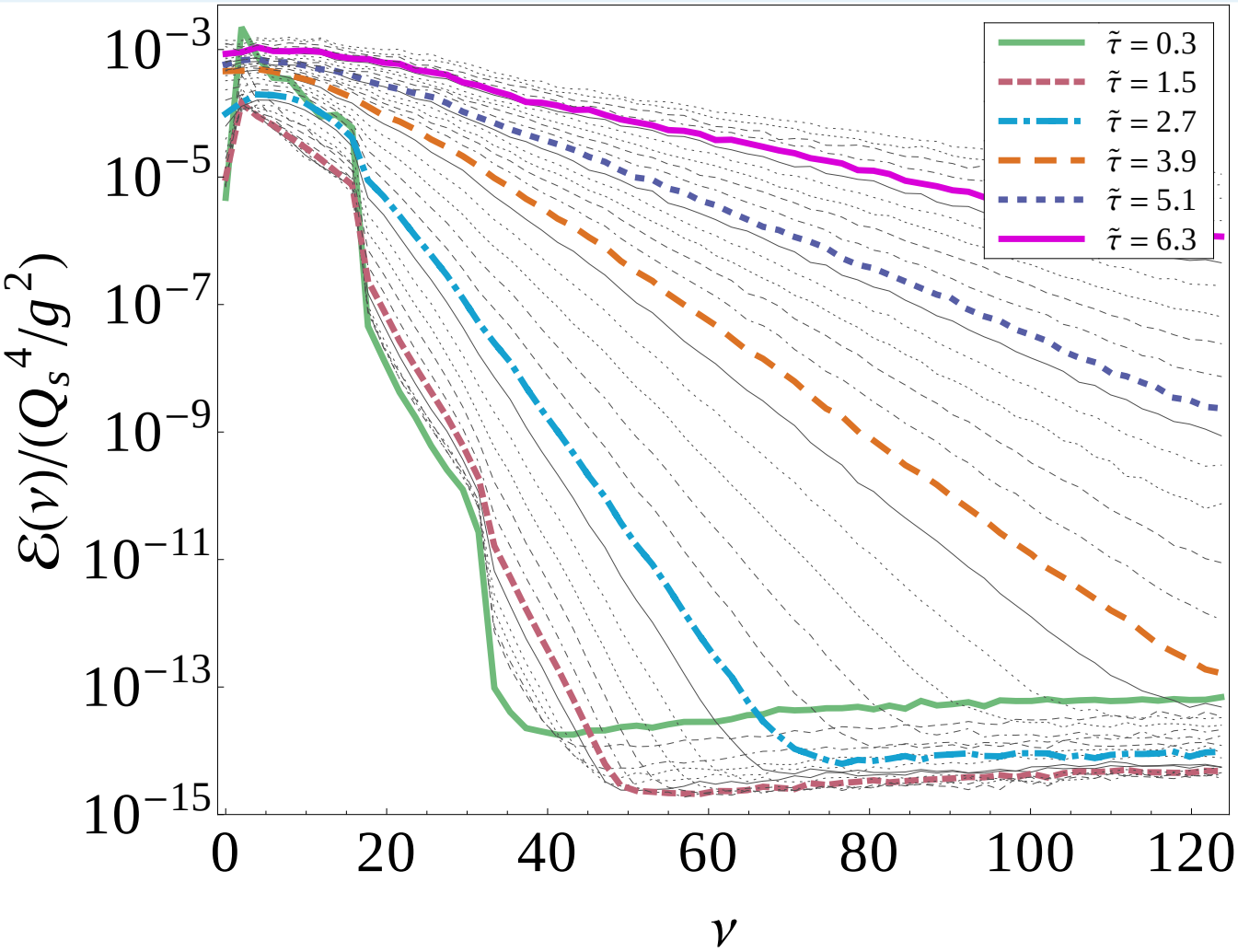
Pressure ratio



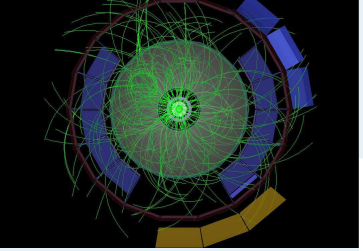
Chromo-Weibel instability restores isotropy on fm/c scale, approximately at $\tilde{\tau} = 6$.



Spectra



Fourier transform each E and B chromofields and sum all the components: **rapid emergence emergence of an exponential distribution of longitudinal energy**



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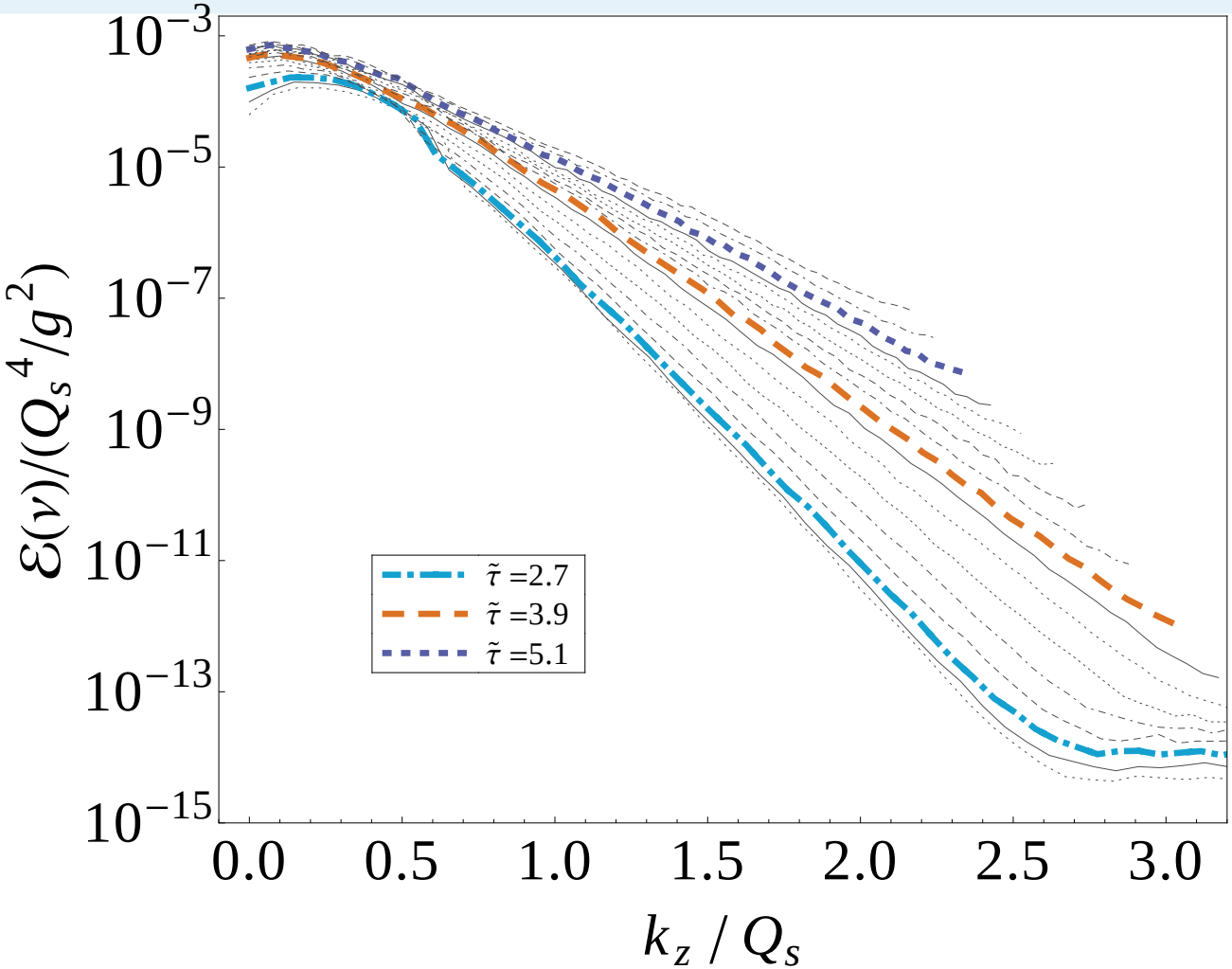
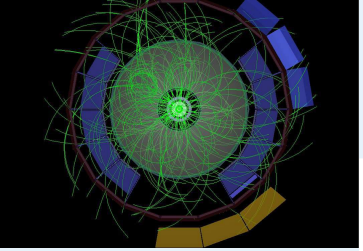
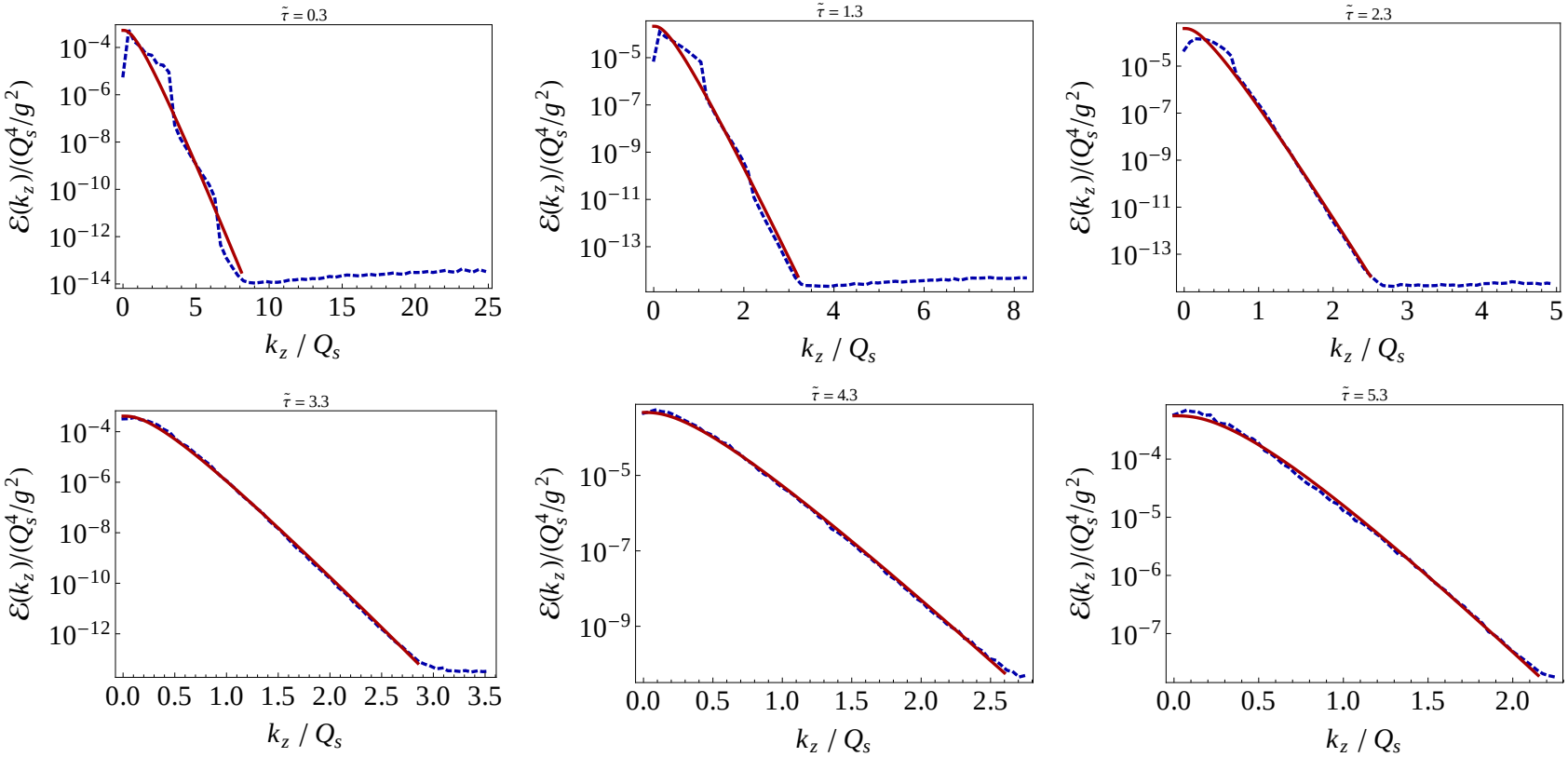


The **red-shifting** is even more visible in the k_z plot. Nonlinear mode-mode coupling is vital in order to populate high momentum modes.



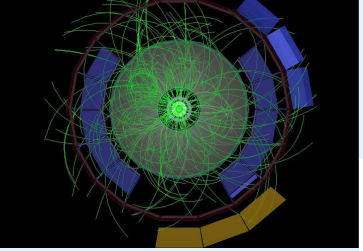
Spectra fits



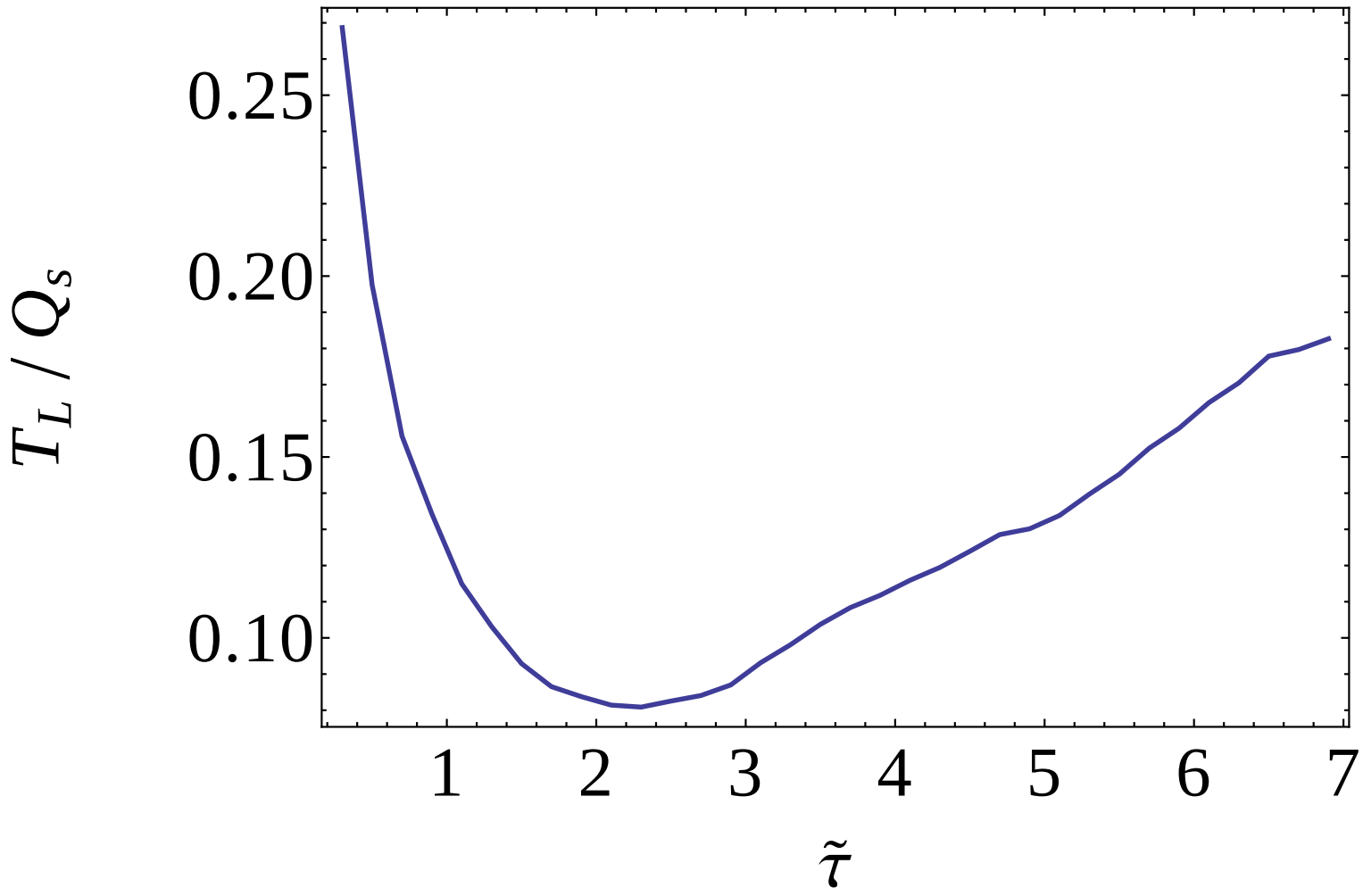
Massless Boltzmann distribution fits the longitudinal spectra:

$$\mathcal{E}_{\text{fit}}(k_z) = A\left(k_z^2 + 2|k_z|T + 2T^2\right)\exp\left(-|k_z|/T\right) \qquad (7)$$

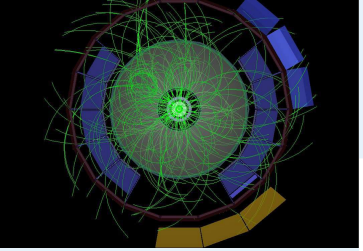
Comparison of data and fit function at six different $\tilde{\tau}$.



Longitudinal thermalization



First the soft sector cools down. Due to the instability longitudinal soft fields reheats.



Conclusions

- We performed the **first real-time 3d numerical** study of non-Abelian plasma in a longitudinally expanding system within hard expanding loops HEL.
- The **momentum space anisotropy** can persist for quite some time.
- There doesn't seem to be a "soft scale" saturation of the instability as was seen in static boxes.
- The longitudinal spectra seem to be well described by a Boltzmann distribution indicating **rapid longitudinal thermalization of the gauge fields**.
- We are now studying even larger lattices in order to better understand the infrared dynamics.

Real-time lattice parameters of the hamiltonian evolution in temporal axial gauge:

longitudinal lattice spacing	a_η	0.025
transverse lattice spacing	a	Q_s^{-1}
temporal time step	ϵ	$10^{-2}\tau_0$
first time step	τ_0	$1/Q_s$
longitudinal lattice points	N_η	128
transverse lattice points	$N_\perp$	40
lattice size in velocity space	$N_u \times N_\phi$	128×32
coupling constant	g	$(3.77)^{0.5}$

Assuming for LHC collisions

$$Q_s \sim 2\text{GeV} = (0.1\text{fm})^{-1} \ . \tag{8}$$

We match to CGC values

$$n(\tau_0) = c\frac{N_g Q_s^3}{4\pi^2 N_c \alpha_s(Q_s\tau_0)} \tag{9}$$

with the gluon liberation factor $c = 2\ln 2$. From this one can determine the isotropic Debye mass

$$m_D^2(\tau_{\text{iso}}) = 1.285/(\tau_0\tau_{\text{iso}}). \tag{10}$$