



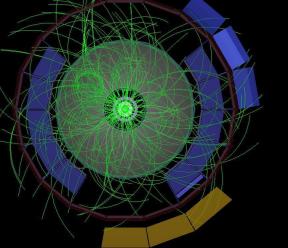
Longitudinal thermalization via the chromo-Weibel instability

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Frankfurt Institute of Advanced Studies

1207.5795

Collaborators: Anton Rebhan, Michael Strickland

December 14, 2012



Motivation

Hard Expanding
Loops (HEL)

Physical
Observables

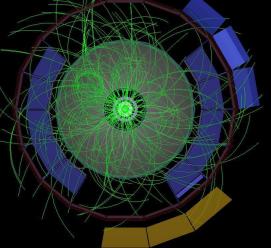
Weakly coupled inspired by Hard Thermal Loops (HTL)

Real-time physical quantities of non-equilibrium processes

Plasma turbulence affects parton transport
(isotropization, jet energy loss, viscosity,...)

Contrast predictions for early time dynamics of the quark gluon plasma

Derivation of time scales for the isotropization,
thermalization



Hard Expanding Loops (HEL)

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Assumptions

Stages of the Little Big Bang

Scales QGP

Weibel instabilities

Hard (Thermal) Loops - Vlasov

Bjorken expansion

Unstable modes growth rate

Physical Observables

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HEL checks

Energy densities fields

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Pressure ratio

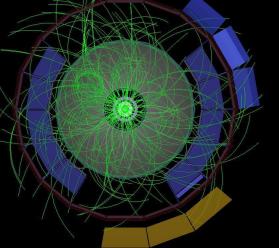
Non-Abelian spectra

Abelian spectra

Spectra fits

Longitudinal thermalization

Outlook



Hard-Expanding Loops Assumptions

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Free streaming background

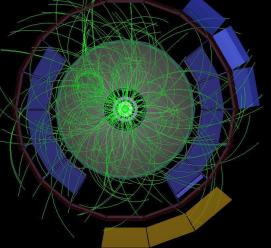
Anisotropy in momentum space

SU(2) particle content

Fixed transverse size

Extrapolate to $\alpha_s \sim 0.3$

Match CGC $n(\tau_0) \propto Q_s^3 \alpha_s^{-1}$



Stages of the Little Big Bang

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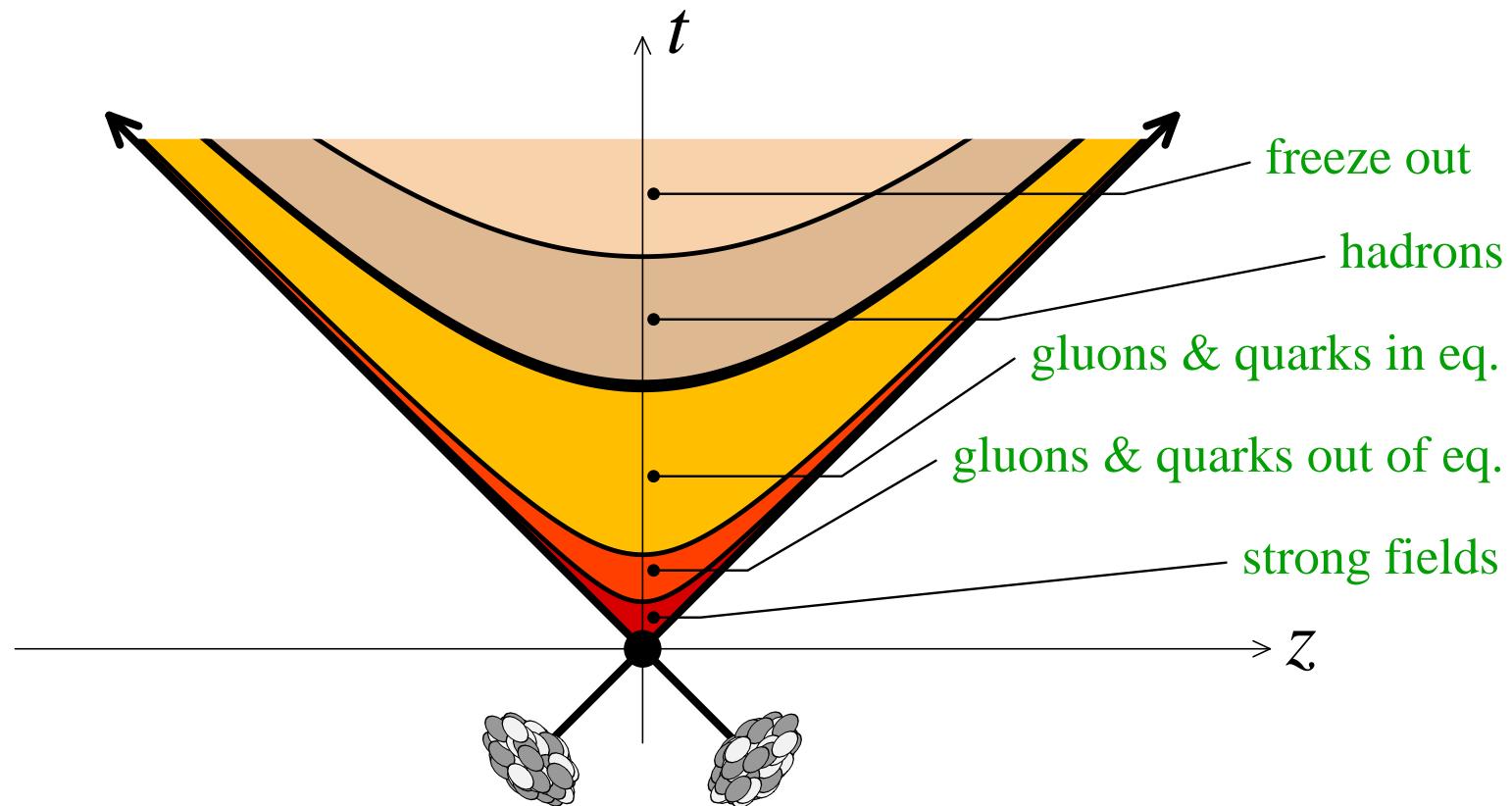
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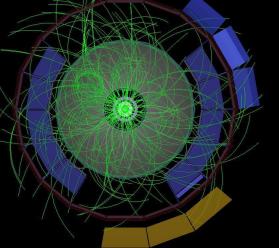
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Physical Observables



[Gelis 2010] Illustration of the stages of a heavy ion collision. This work focuses on the early phase with strong fields in an out of equilibrium situation.



Scales of weakly coupled QGP

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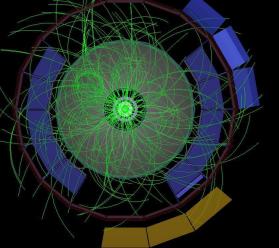
Weibel
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- T : energy of hard particles
- gT : thermal masses, Debye screening mass, Landau damping
- g^2T : magnetic confinement, color relaxation, rate for small angle scattering
- g^4T : rate for large angle scattering, $\eta^{-1}T^4$



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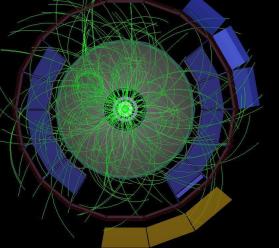
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Physical
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- T : energy of hard particles
- gT : thermal masses, Debye screening mass,
Landau damping, **plasma instabilities** [Mrowczynski 1988,
1993, ...]
- g^2T : magnetic confinement, color relaxation, rate for
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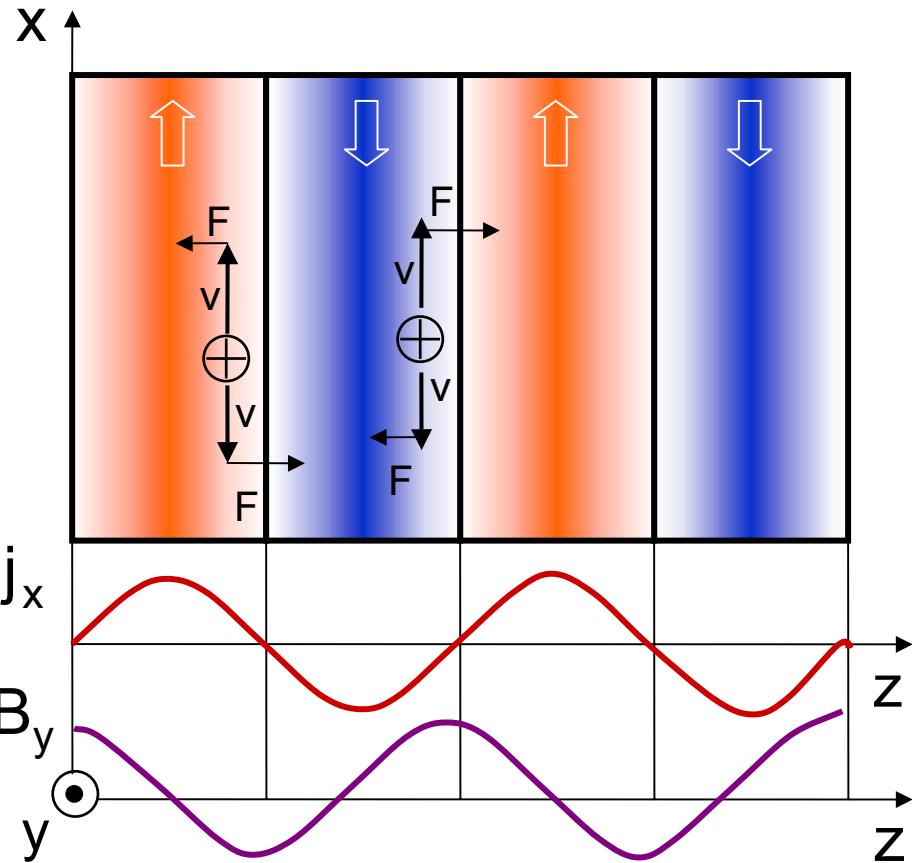
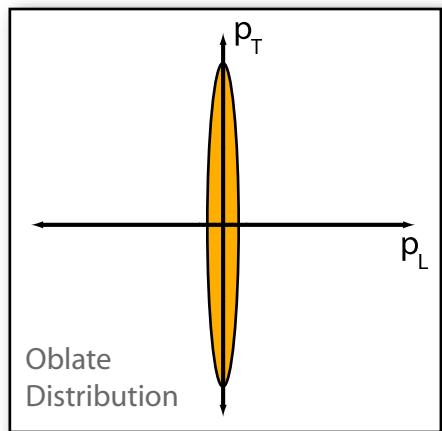
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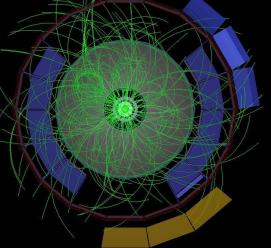
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[Strickland 2006]: Illustration of the mechanism of filamentation instabilities.



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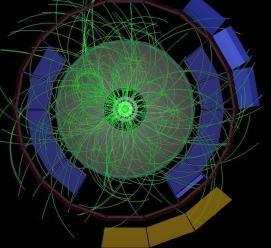
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Physical Observables

Hard (Thermal) Loops - Vlasov

Assuming free streaming, one solves the gauge covariant Vlasov equation

$$v \cdot D \partial f_a(\mathbf{p}, \mathbf{x}, t) = g v_\mu F_a^{\mu\nu} \partial_\nu^{(p)} f_0(\mathbf{p}, \mathbf{x}, t) \quad (1)$$



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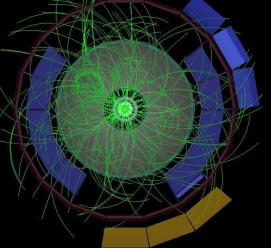
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coupled to Yang-Mills equation

$$D_\mu F_a^{\mu\nu} = j_a^\nu = g \int \frac{d^3 p}{(2\pi)^3} \frac{p^\mu}{2p^0} \delta f_a(\mathbf{p}, \mathbf{x}, t) \quad (2)$$



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in the HTL approximation

$$g A_\mu \ll |\mathbf{p}_{hard}|, \quad (3)$$

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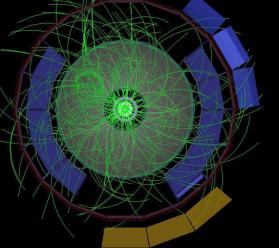
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the Romatschke, Strickland background distribution function

$$f_0(p_\perp, \tilde{p}_\eta) = f_{\text{CGC}}([p^2 + \xi(\tau)(\mathbf{p} \cdot \hat{\mathbf{n}})^2]/p_{\text{hard}}^2(\tau))^{0.5}. \quad (4)$$

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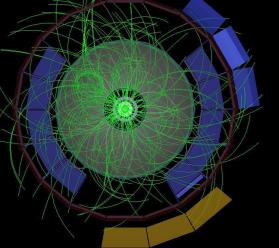
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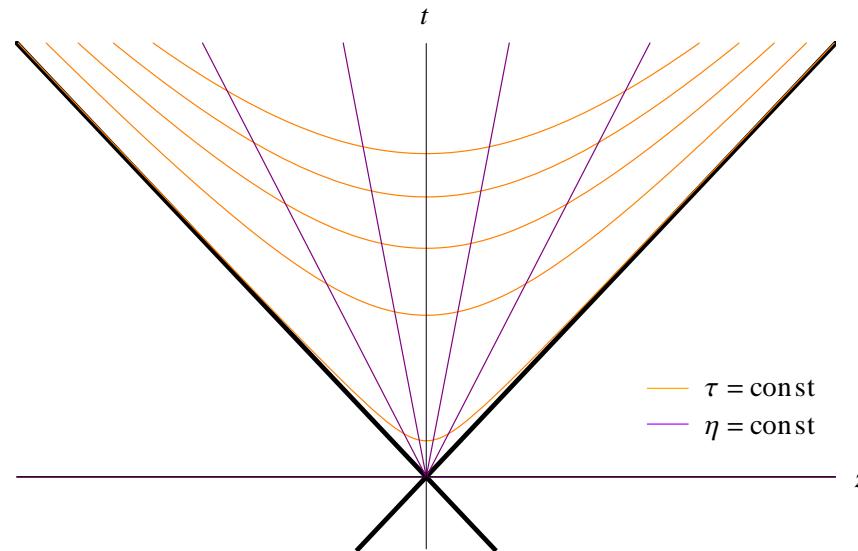
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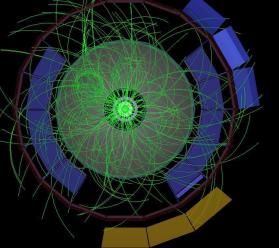
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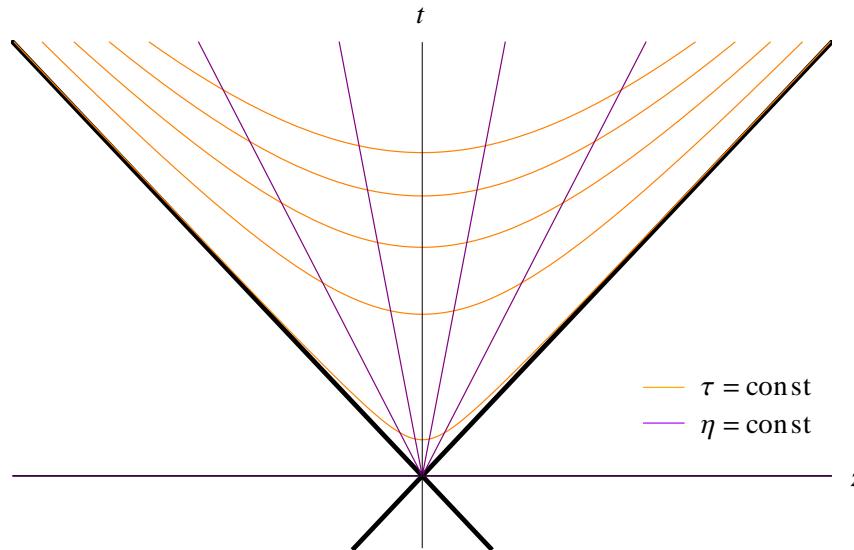
Unstable modes growth rate

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Bjorken expansion

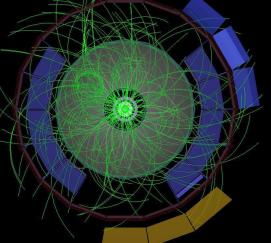


It is convenient to switch to comoving coordinates

$$\begin{aligned} t &= \tau \cosh \eta , & \tau &= \sqrt{t^2 - z^2} , \\ z &= \tau \sinh \eta , & \eta &= \operatorname{arctanh} \frac{z}{\tau} , \end{aligned} \tag{5}$$

with the corresponding metric

$$ds^2 = d\tau^2 - d\mathbf{x}_\perp^2 - \tau^2 d\eta^2 . \tag{6}$$



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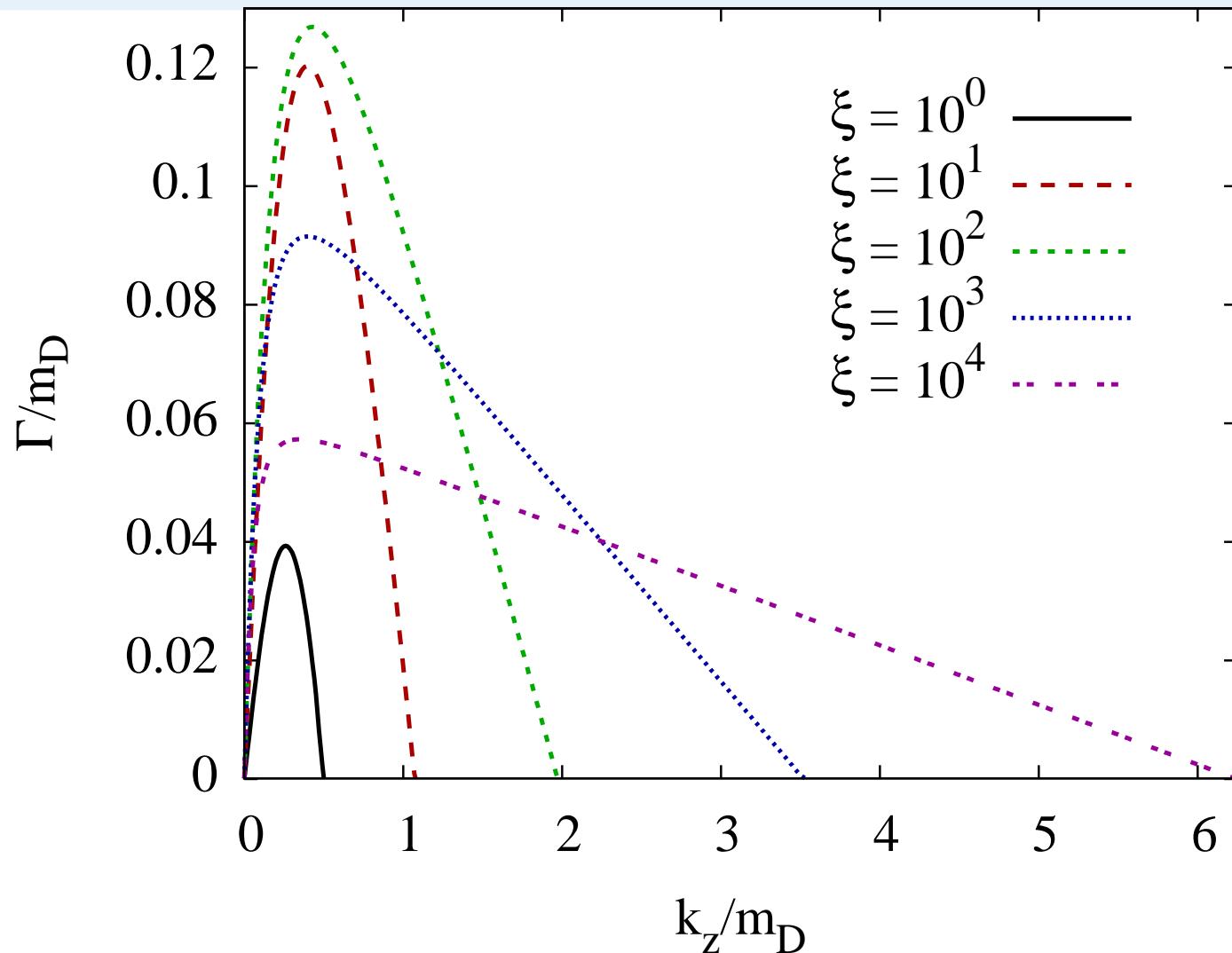
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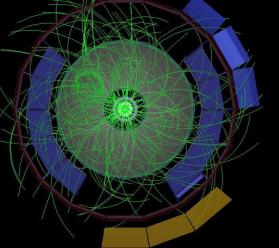
Unstable modes growth rate

Physical Observables

Unstable modes growth rate



Unstable mode spectra of purely longitudinal modes for specific anisotropies: $N(\tau) \approx \exp(2m_D \sqrt{\tau \tau_{CGC}})$



Expanding 3D+3V non-Abelian Plasma Instabilities

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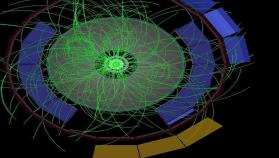
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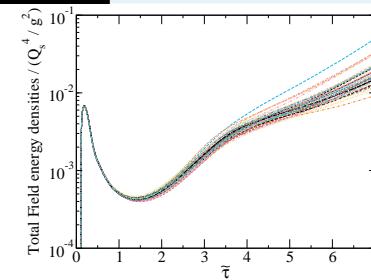
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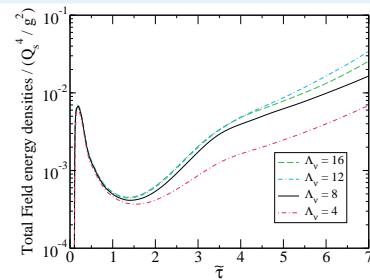
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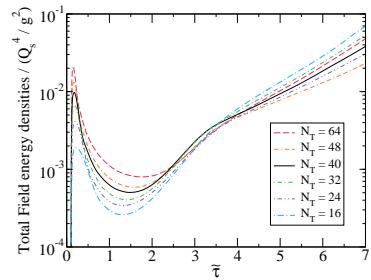
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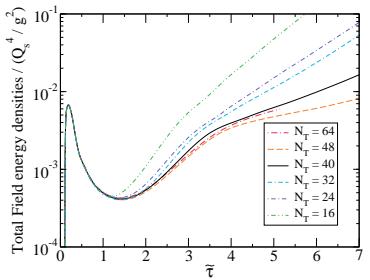
(a) Different seeds



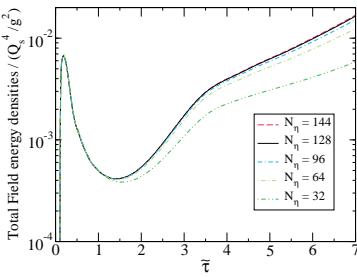
(b) Variation of Λ_ν



(c) Variation of a_\perp



(d) Variation of N_\perp



(e) Variation of a_η (f) Variation of N_η

Figure 1: Numerical check

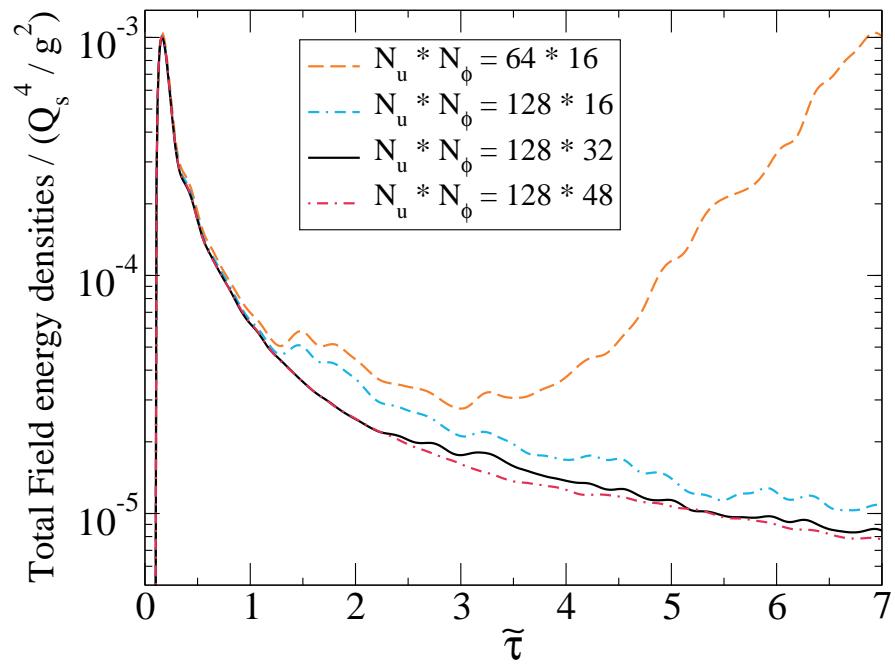
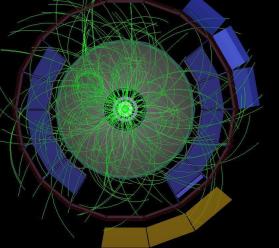


Figure 2: Evolution of stable modes



Energy densities fields

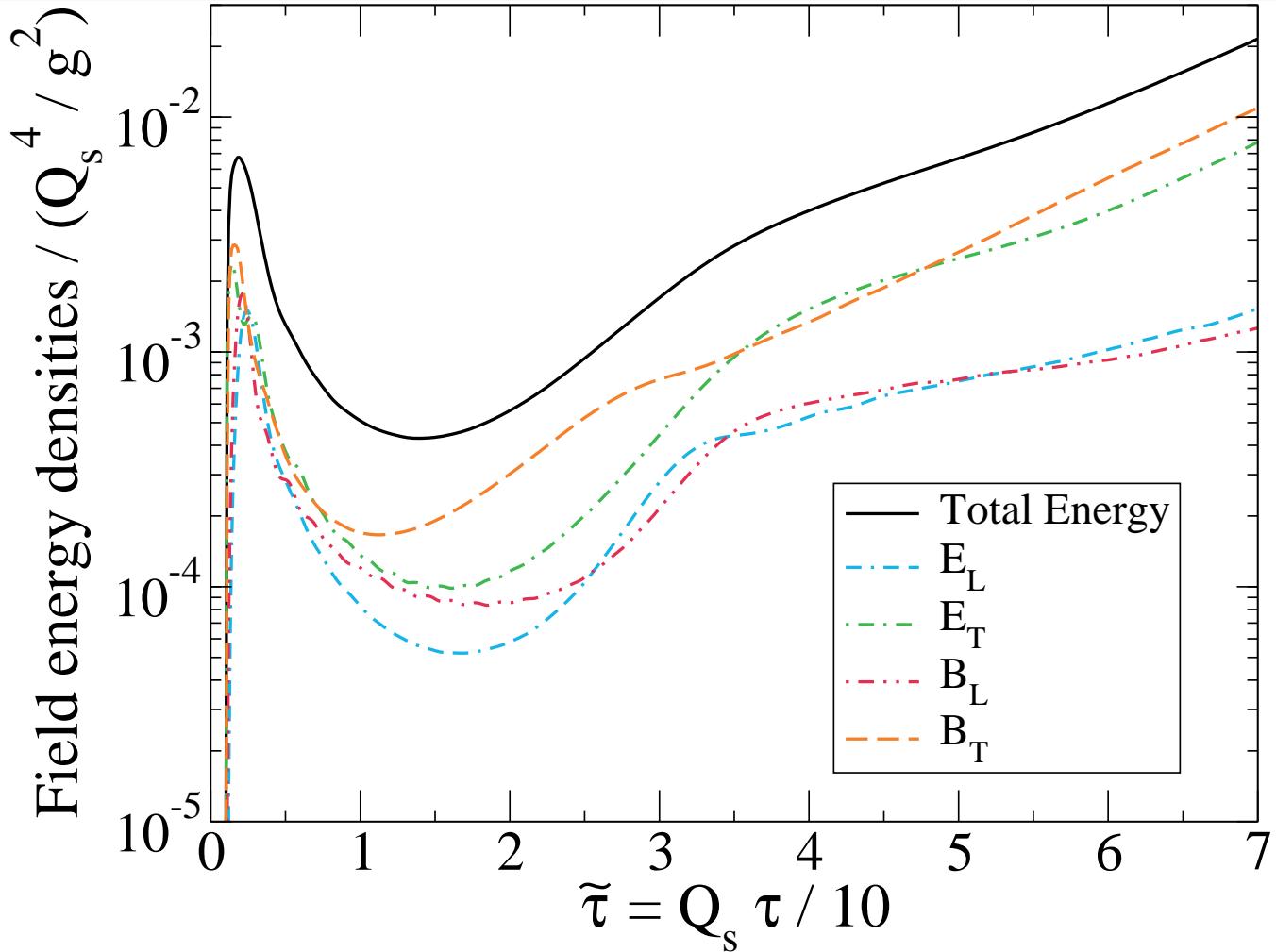
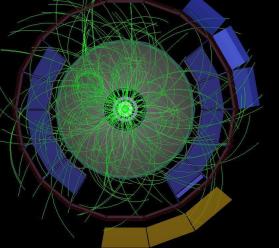


Figure 3: 50 averaged runs $N_\perp * N_\eta * N_u * N_\phi = 40^2 * 128 * 128 * 32$: after onset one sees **rapid growth of B_l and E_L fields**, followed by non-Abelian interactions kick in.



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Preliminary: Energy densities

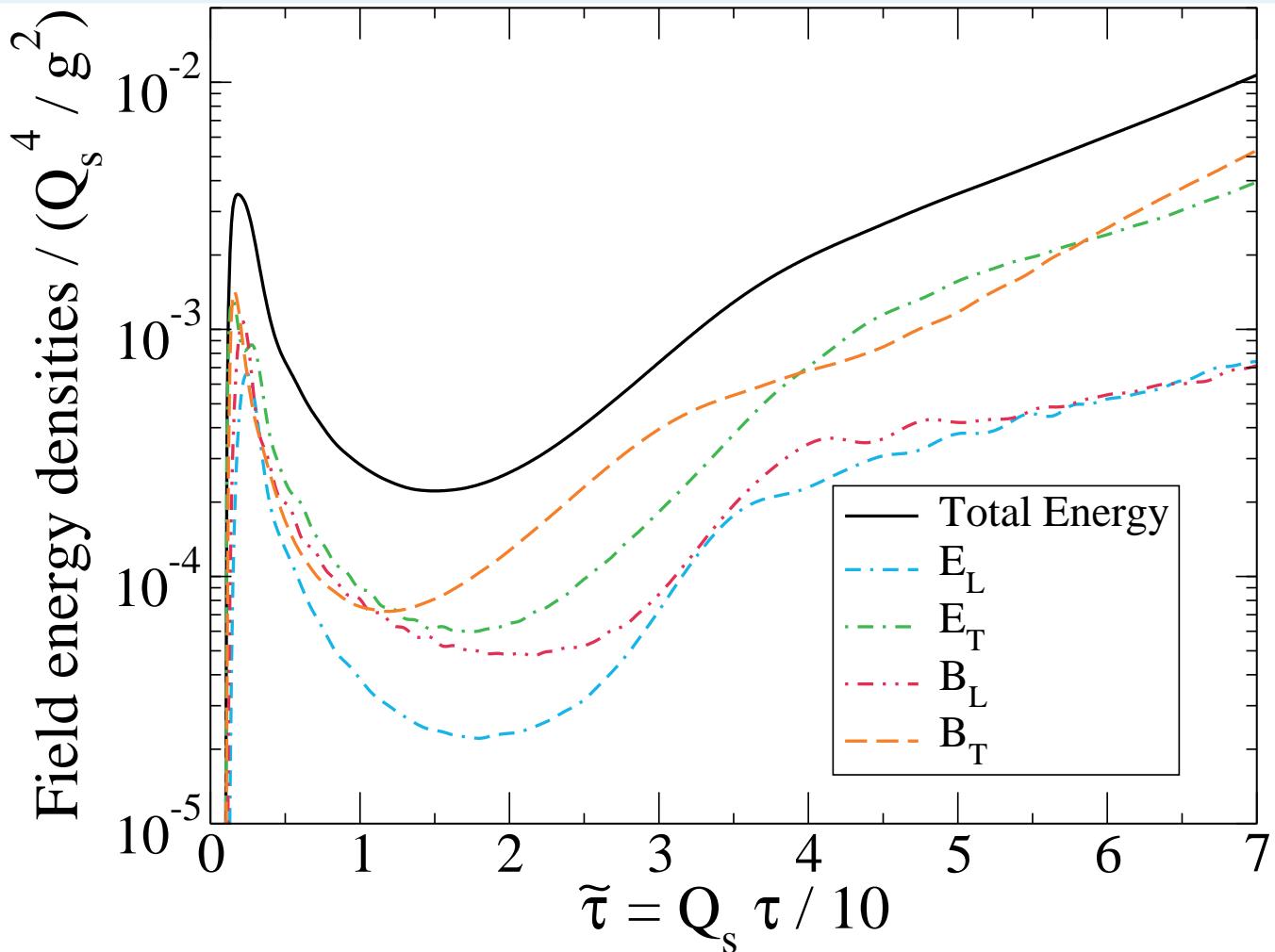
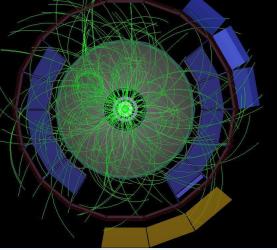


Figure 4: Single run $N_\perp * N_\eta * N_u * N_\phi = 40^2 * 1024 * 128 * 32$: Large longitudinal lattice runs confirm fields evolution.



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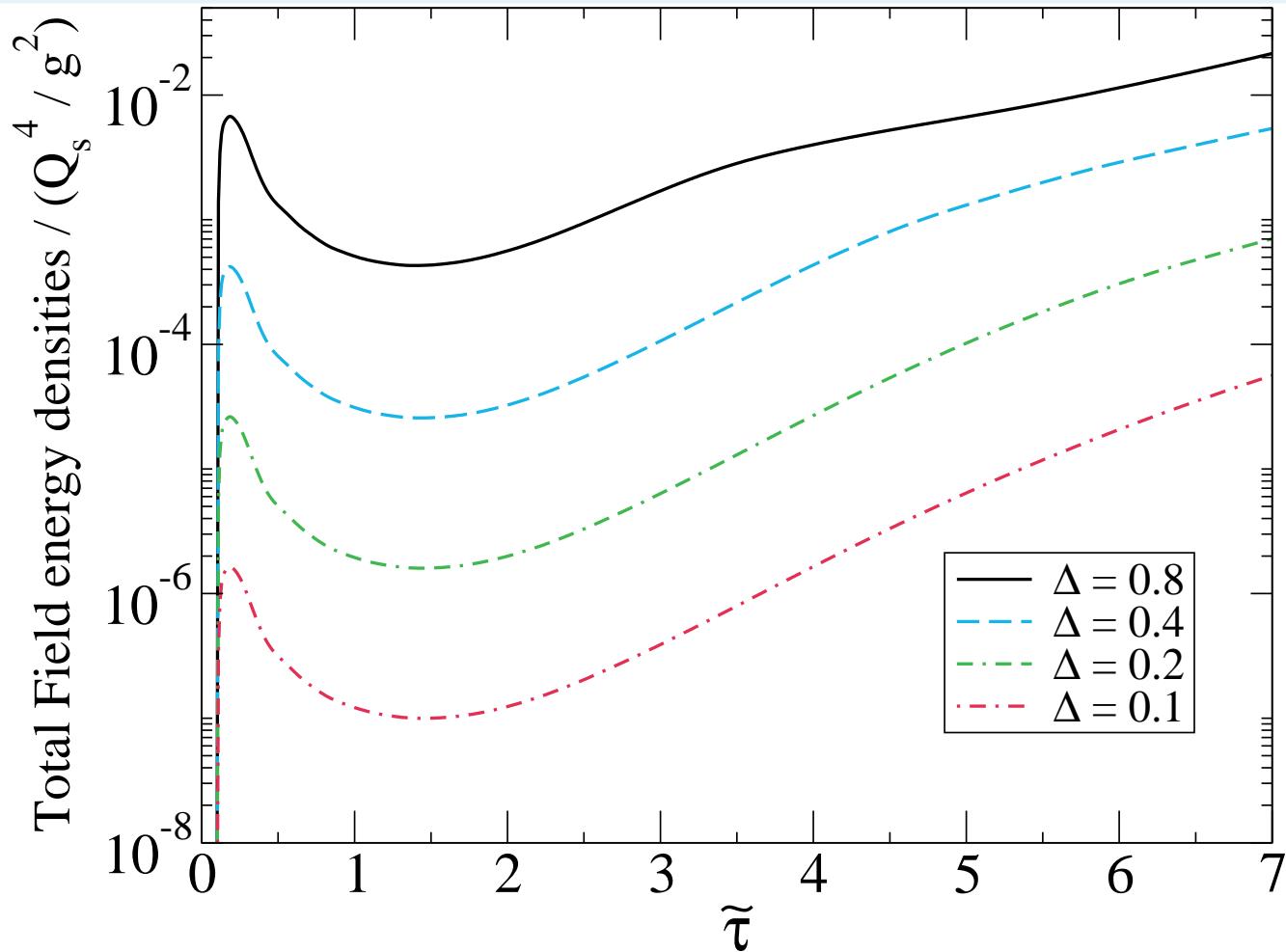
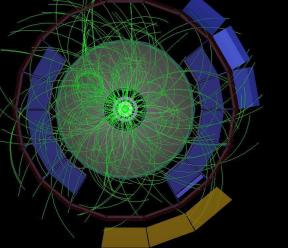


Figure 5: Total field energy density for different initial current fluctuation magnitudes showing similar behavior (apart from non-Abelian point).



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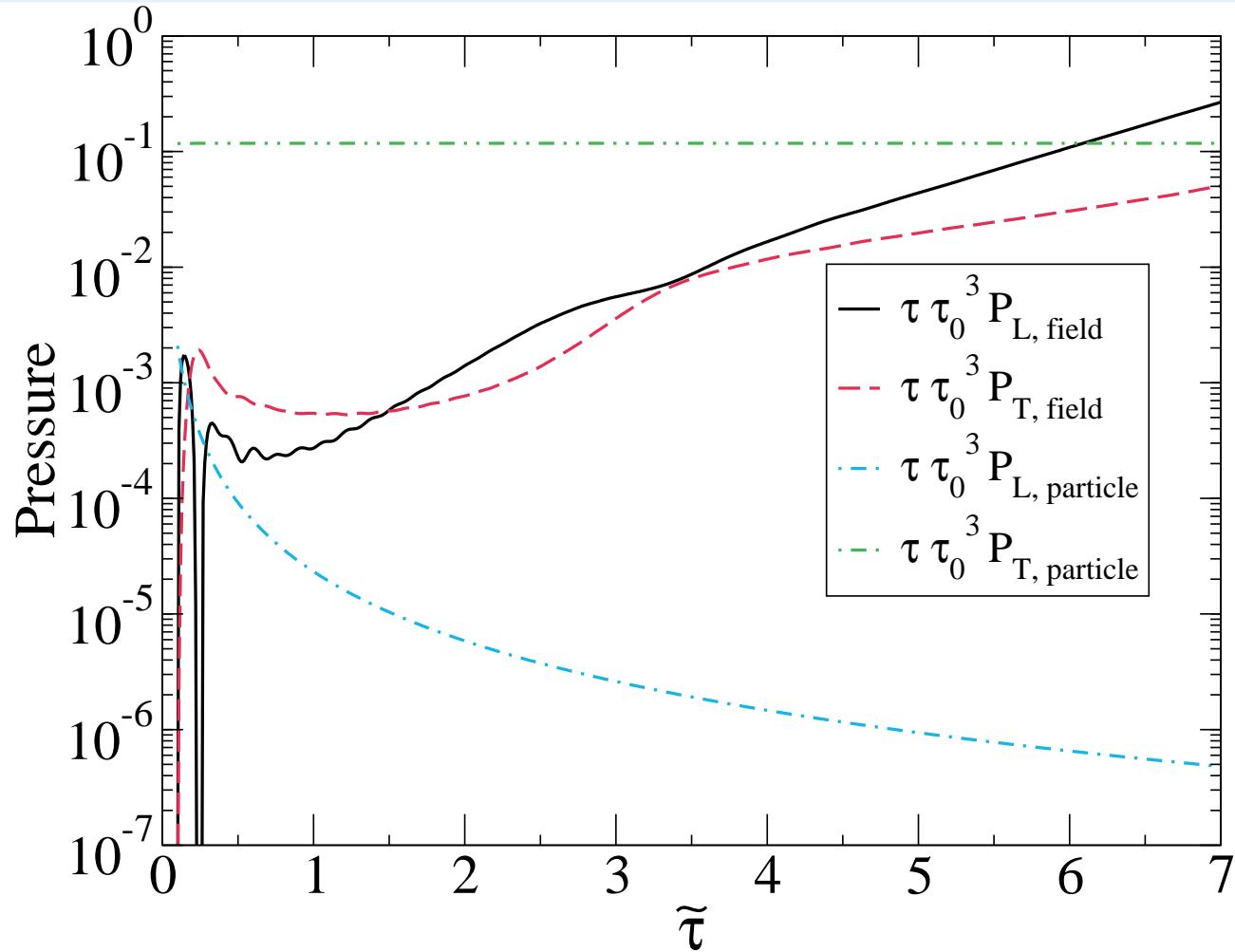
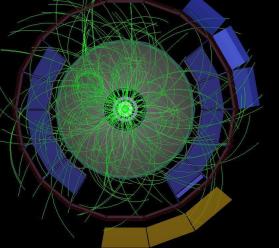


Figure 6: Initially highly anisotropic, note $P_{L,\text{field}}(\tau = 0.3) < 0$, **growing field pressures**, $P_{L,\text{field}}$ dominates at late times, $\tilde{\tau}$ scaled P_L drops $\propto 1/\tilde{\tau}^2$.



Pressure ratio

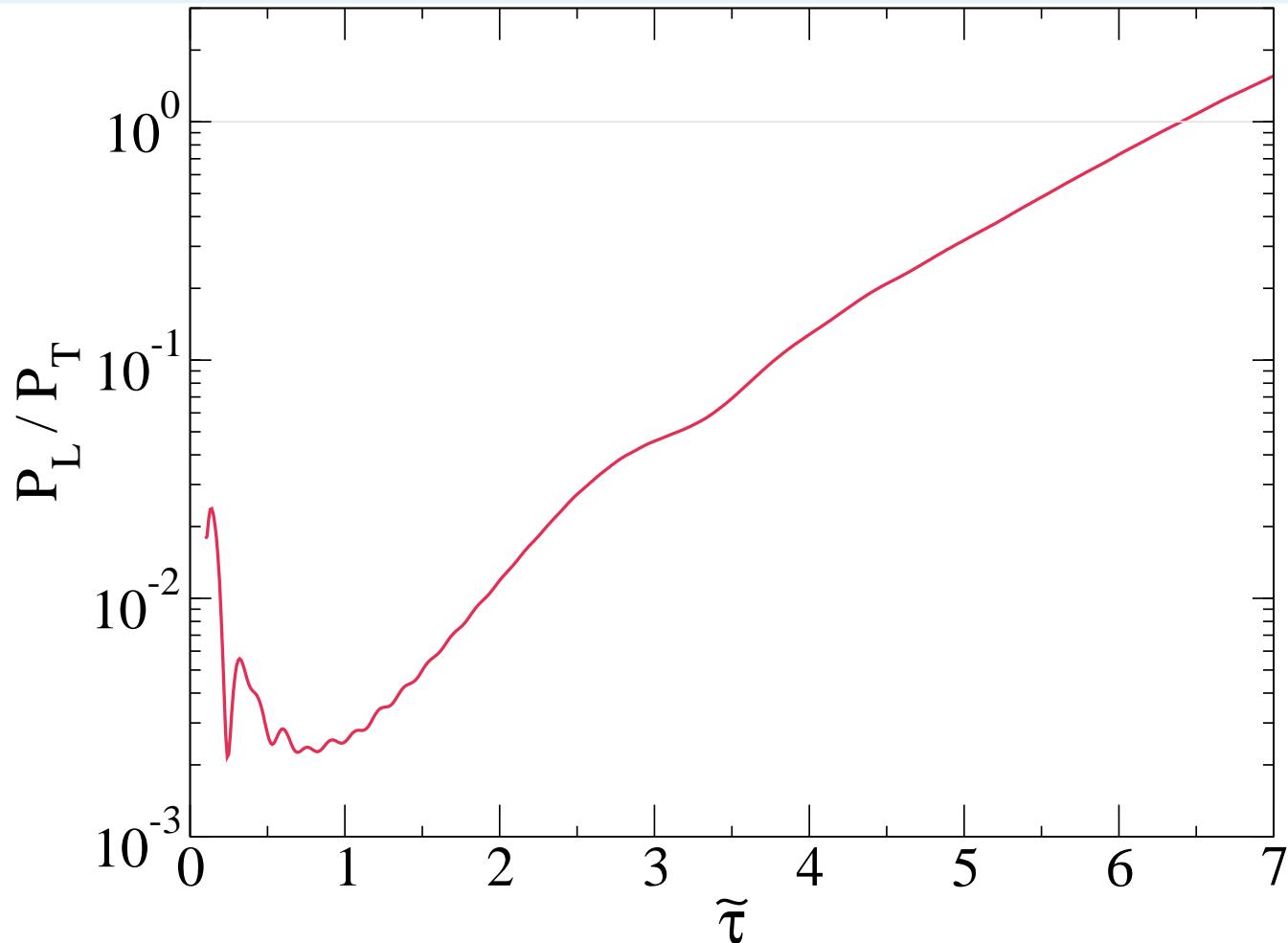
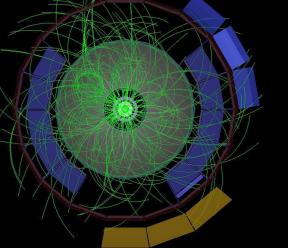


Figure 7: Chromo-Weibel instability restores isotropy on fm/c scale, at $\tilde{\tau} \approx 6$.



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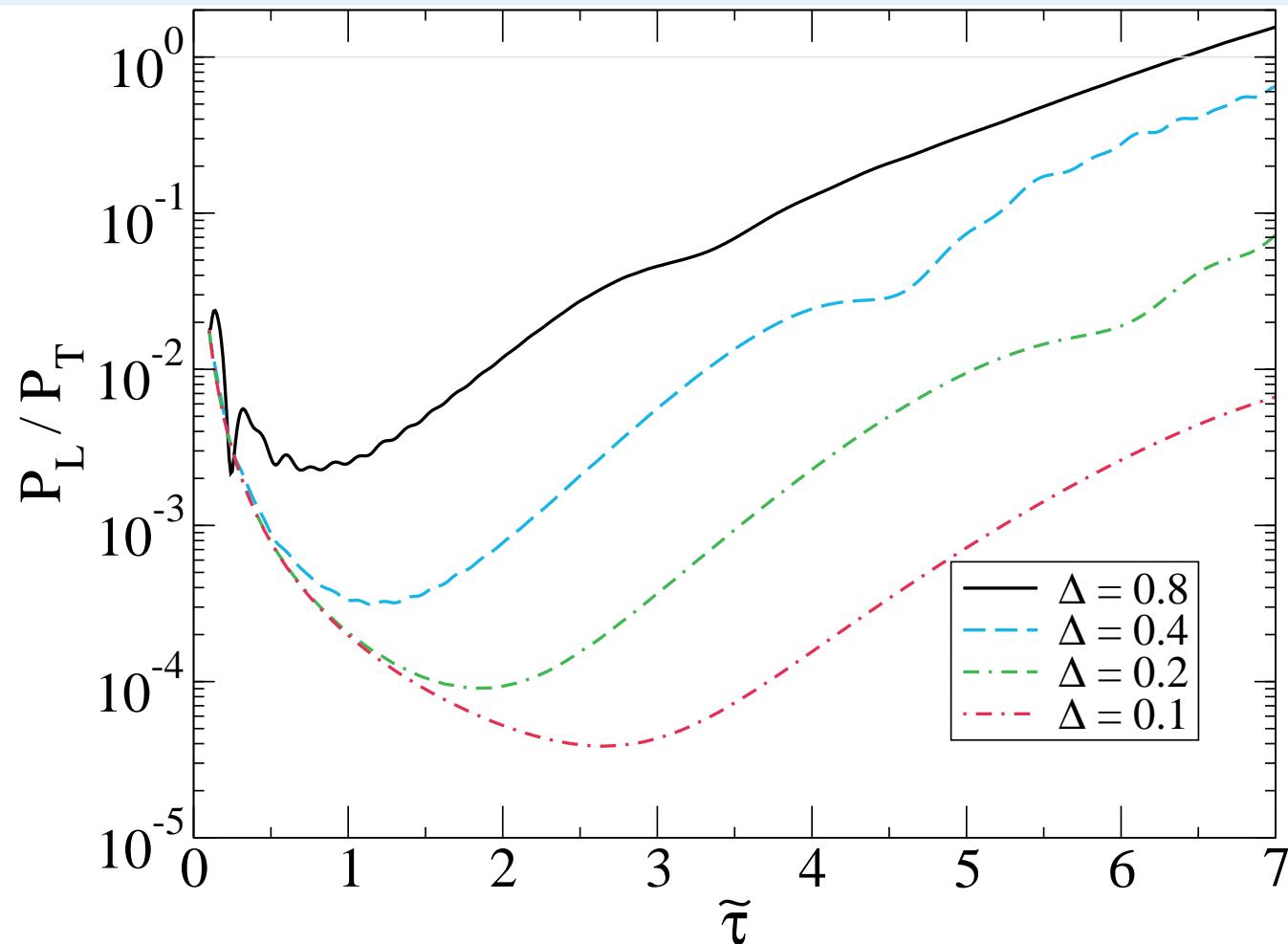
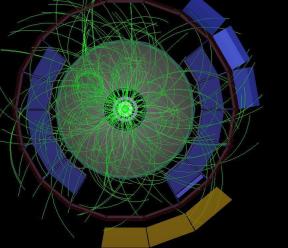


Figure 8: Different initial current fluctuation magnitudes Δ effecting the isotropization time.



Non-Abelian spectra

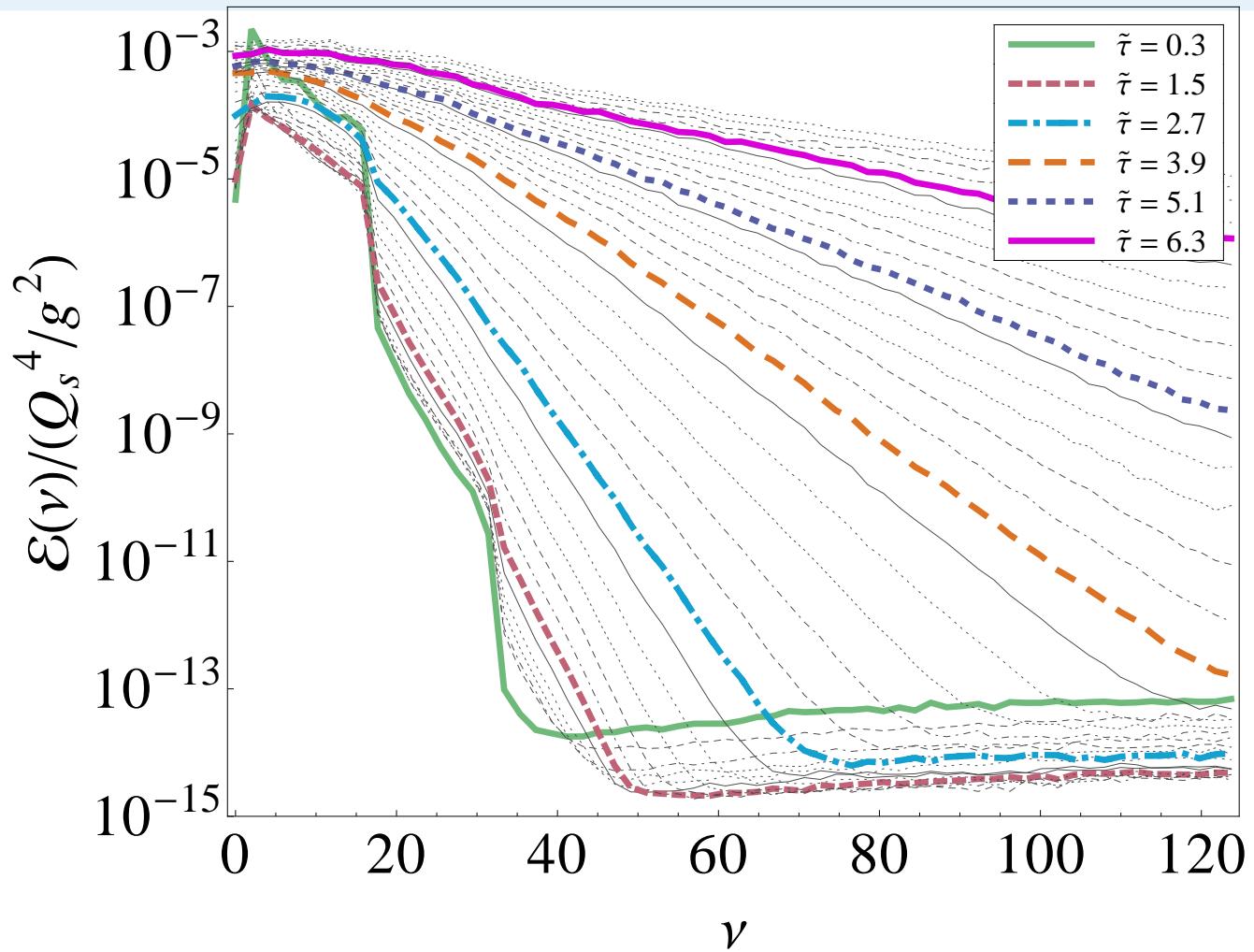
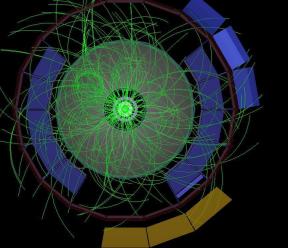


Figure 9: Fourier transform each E and B chromofields and sum all the components: **rapid emergence of an exponential distribution of longitudinal energy.**



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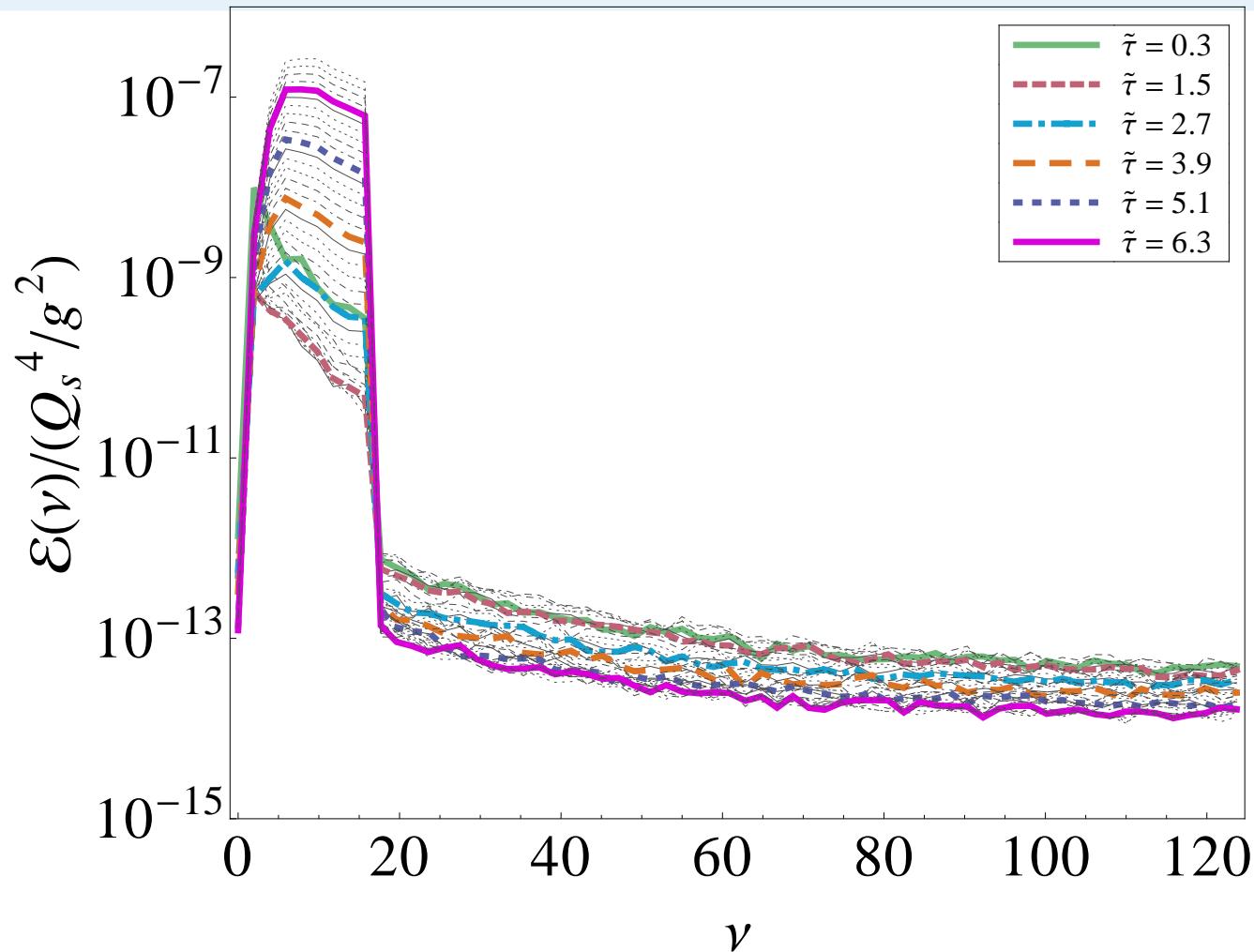
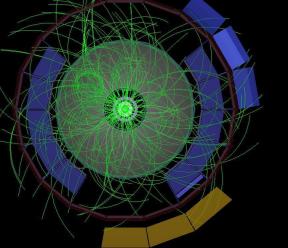


Figure 10: Longitudinal spectra for **abelian** runs shows amplification of the initial seeded modes.



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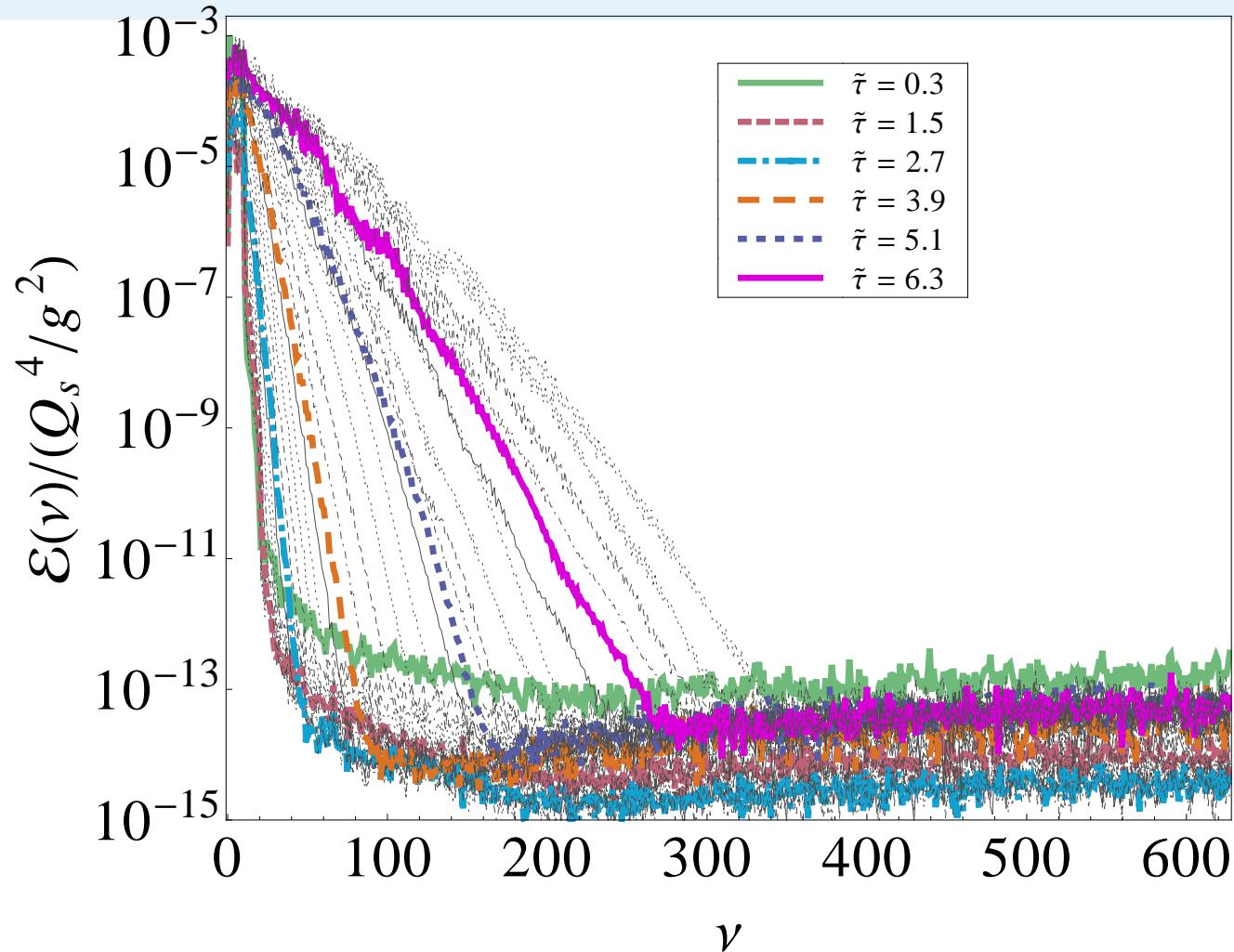
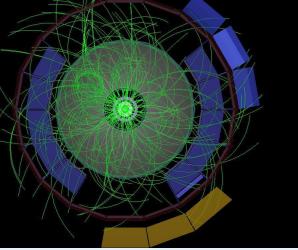


Figure 11: Exponential spectrum visible too for single big longitudinal lattice runs: Notice **high UV cutoff** ν_{\max} .



Spectra

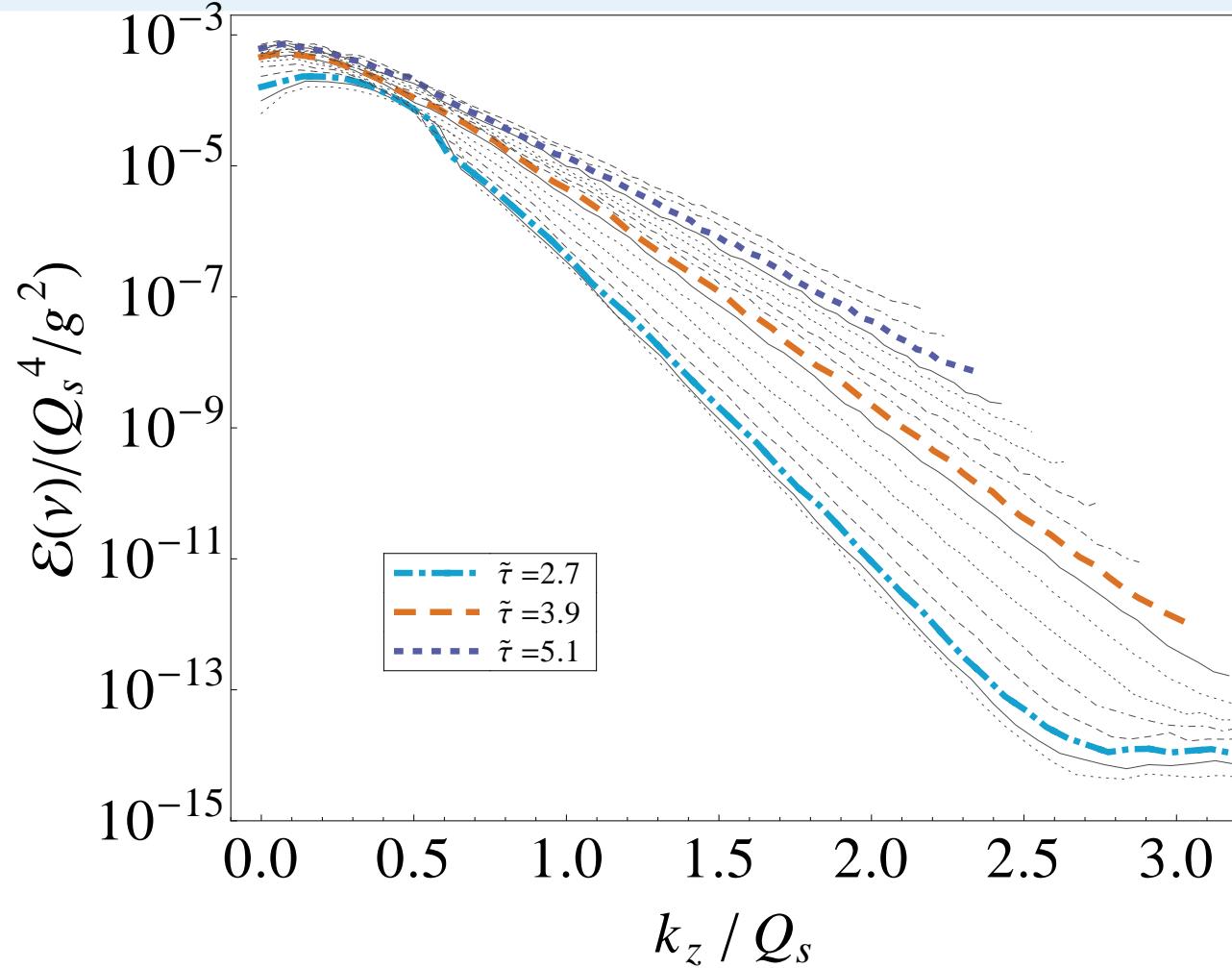
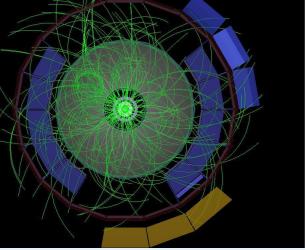


Figure 12: The **red-shifting** is even more visible in the k_z plot. Nonlinear mode-mode coupling is vital in order to populate high momentum modes.



Spectra fits

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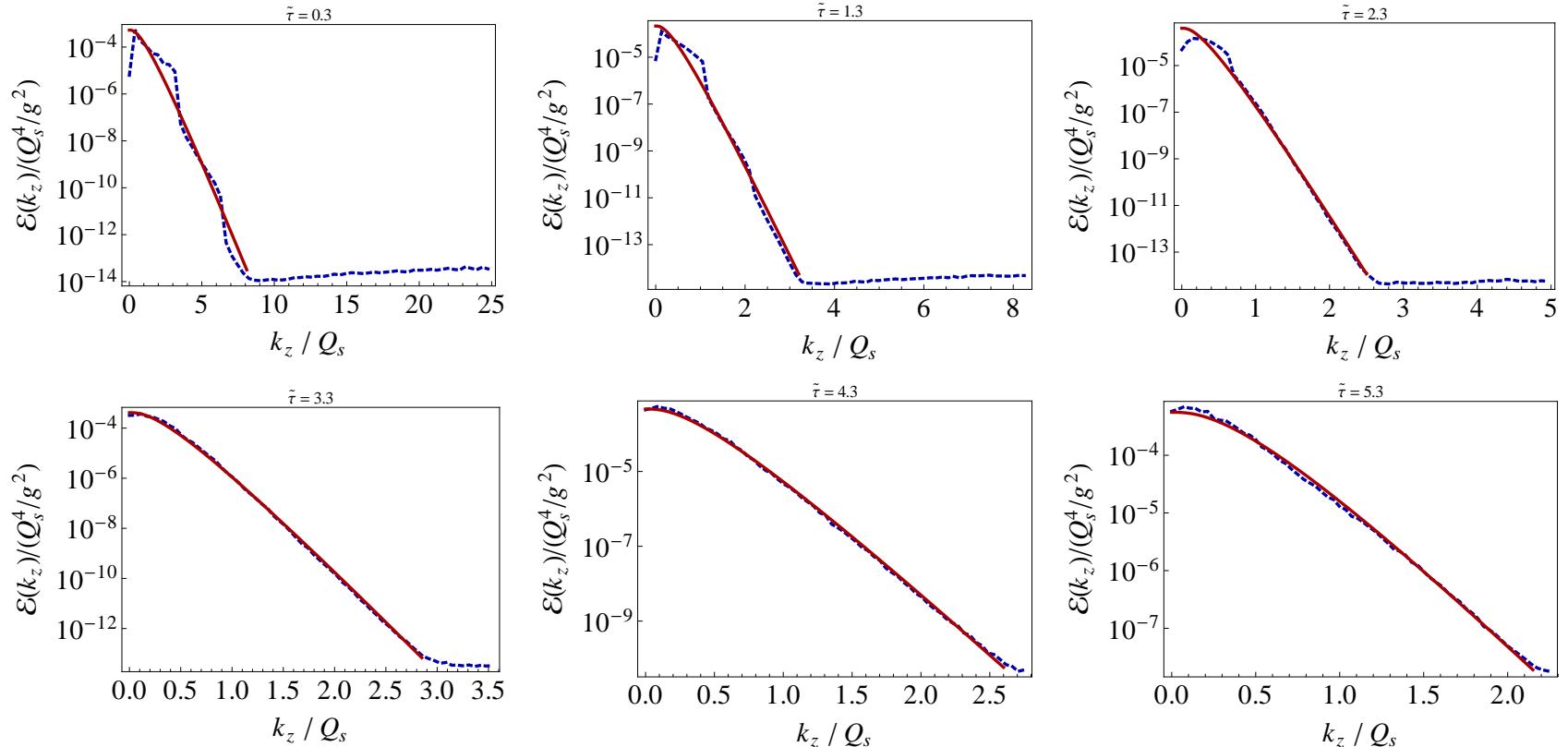
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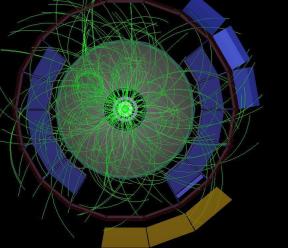
Longitudinal thermalization
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Massless Boltzmann distribution fits the longitudinal spectra:

$$\mathcal{E}_{\text{fit}}(k_z) = A \left(k_z^2 + 2|k_z|T + 2T^2 \right) \exp(-|k_z|/T) \quad (7)$$

Comparison of data and fit function at six different $\tilde{\tau}$.



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Longitudinal thermalization

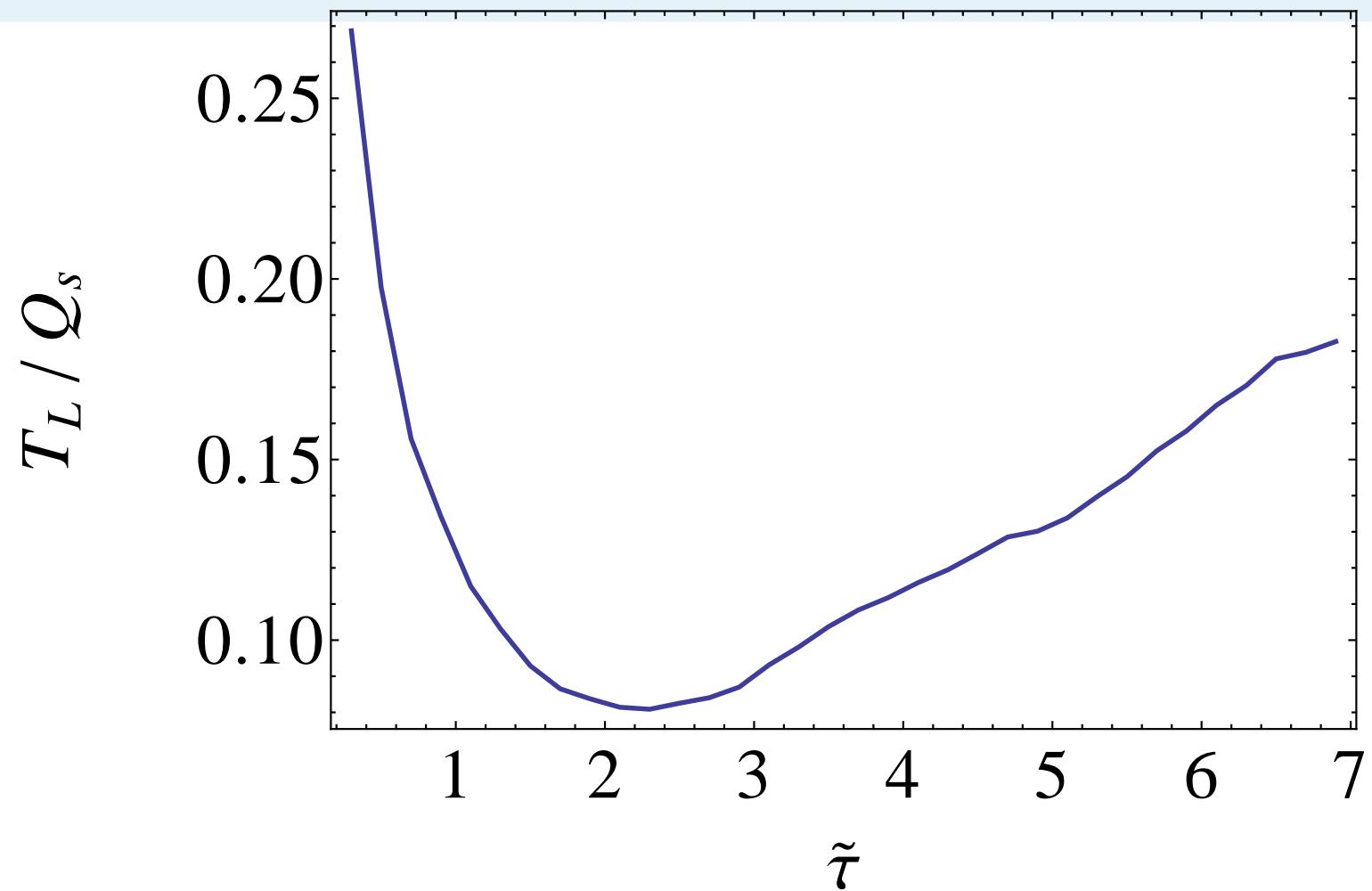
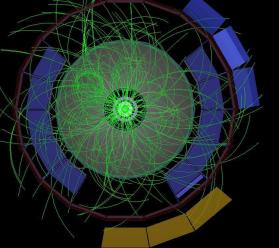


Figure 13: First the soft sector cools down. Due to the instability longitudinal soft fields **reheats**.



Outlook

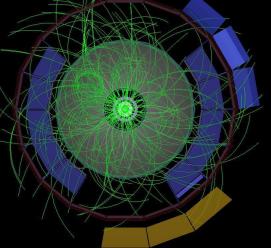
Hard Expanding
Loops (HEL)

Physical
Observables

HEL checks
Energy densities
fields

Pressures
Pressure ratio
Non-Abelian
spectra
Abelian spectra
Spectra fits
Longitudinal
thermalization
Outlook

- Experimental signatures
- Larger longitudinal N_η
- Improved IC conditions: k_\perp cutoff
- Probe diagramm with modified f_0
- Incorporate backreaction



Conclusions

Hard Expanding Loops (HEL)

Physical Observables

HEL checks
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fields

Pressures

Pressure ratio
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Outlook

- We performed the **first real-time 3d numerical study** of non-Abelian plasma in a longitudinally expanding system within hard expanding loops **HEL**.
- The **momentum space anisotropy** can persist for quite some time.
- There doesn't seem to be a “soft scale” saturation of the instability as was seen in static boxes.
- The longitudinal spectra seem to be well described by a Boltzmann distribution indicating **rapid longitudinal thermalization of the gauge fields**.
- We are now studying even larger lattices in order to better understand the infrared dynamics.

Real-time lattice parameters of the hamiltonian evolution in temporal axial gauge:

longitudinal lattice spacing	a_η	0.025
transverse lattice spacing	a	Q_s^{-1}
temporal time step	ϵ	$10^{-2}\tau_0$
first time step	τ_0	$1/Q_s$
longitudinal lattice points	N_η	128
transverse lattice points	N_\perp	40
lattice size in velocity space	$N_u \times N_\phi$	128×32
coupling constant	g	$(3.77)^{0.5}$

Assuming for LHC collisions

$$Q_s \sim 2\text{GeV} = (0.1\text{fm})^{-1}. \quad (8)$$

We match to CGC values

$$n(\tau_0) = c \frac{N_g Q_s^3}{4\pi^2 N_c \alpha_s(Q_s \tau_0)} \quad (9)$$

with the gluon liberation factor $c = 2 \ln 2$. From this one can determine the CGC Debye mass

$$m_D^2(\tau_{\text{CGC}}) = 1.285 / (\tau_0 \tau_{\text{CGC}}). \quad (10)$$