Entropy-dissipative discretizations of nonlinear diffusive equations

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We present numerical discretizations which preserve the entropy structure of the analyzed nonlinear diffusive equations. More precisely, we develop numerical schemes for which the discrete entropy is stable or even dissipating. The key idea is to "translate" entropy-dissipation methods to the discrete case. We consider two situations.

First, an implicit Euler finite-volume approximation of porous-medium or fast-diffusion equations is investigated [2]. The scheme dissipates all zeroth-order entropies which are dissipated by the continuous equation. The proof is based on novel discrete generalized Beckner inequalities. Furthermore, the exponential decay of some first-order entropies is proved using systematic integration by parts and a convexity property with respect to the time step parameter.

Second, new one-leg multistep time approximations of general nonlinear evolution equations are investigated [1, 3]. These schemes preserve both the nonnegativity and the entropy-dissipation structure of the equations. The key idea is to combine Dahlquist's G-stability theory with entropy-dissipation methods. The optimal second-order convergence rate is proved under a certain monotonicity assumption on the operator. The discretization is applied to a cross-diffusion system from population dynamics and a fourth-order quantum diffusion equation.

References

[1] M. Bukal, E. Emmrich, and A. Jüngel. Entropy-stable and entropy-dissipative approximations of a fourth-order quantum diffusion equation. Preprint, 2012. `arXiv:1208.5474`.

[2] C. Chainais-Hillairet, A. Jüngel, and S. Schuchnigg. Entropy-dissipative discretization of nonlinear diffusion equations and discrete Beckner inequalities. Preprint, 2013. `arXiv:1303.3791`.

[3] A, Jüngel and J. P. Milišić. Entropy dissipative one-leg multistep time approximations of nonlinear diffusive equations. In preparation, 2013.