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On the Two-Architecture Connected Facility Location Problem

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Abstract

We introduce a new variant of the connected facility location problem that allows for modeling mixed deployment strategies (FTTC/FTTB/FTTH) in the design of local access telecommunication networks. Several mixed integer programming models and valid inequalities are presented. Computational studies on realistic instances from three towns in Germany are provided.

Keywords: Connected Facility Location, Branch-and-Cut, FTTx Deployment

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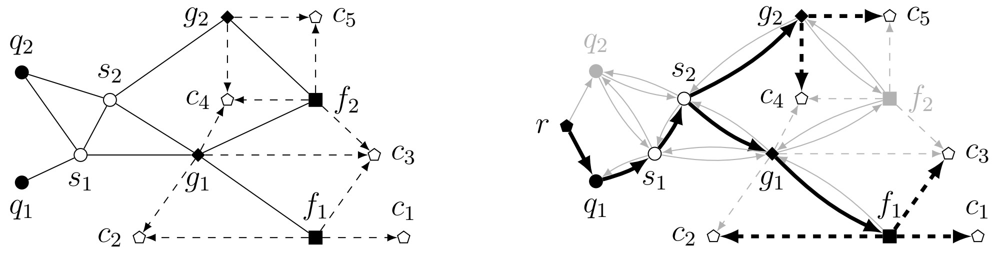
1 Introduction and Problem Definition

In the design of local access networks three main scenarios (deployment *architectures*) are considered: (i) "fiber-to-the-home" (FTTH), (ii) "fiber-to-the-building" (FTTB), and (iii) "fiber-to-the-curb" (FTTC). From an optimization point of view – abstracting from the more technical details and considering mainly topology decisions – FTTH deployment is modeled using variants of the Steiner tree problem [2,5], and FTTB or FTTC deployments are modeled as connected facility location (ConFL) [1,3,4]. In this paper we consider a new modeling and optimization approach for the *mixed deployment* which is motivated by the fact that in urban areas the lowest investment costs and the best bandwidth rates are achieved with a deployment that mixes FTTH and FTTC/FTTB. The main drawback of existing approaches is that they do not allow for the design of such a combined deployment. To overcome this, we propose to model the mixed deployment as *ConFL with two architectures*, which will be denoted by 2-ArchConFL. We consider two different architectures 1 and 2 (these could be FTTB and FTTC, or two FTTC quality-of-service levels) with associated minimum coverage rates, p_1 and p_2. The presented model can be easily generalized to more than two architectures, thus incorporating more deployment strategies, such as "fiber-to-the-air" (FTTA), if necessary.

More precisely, we are given a bipartite *assignment graph* between potential *facilities*, representing locations where equipment can be installed, and *customers*. Two types of facilities – one for each architecture – exist and give rise to two types of *assignment arcs* directed from facilities to customers. Each customer can be supplied by at most one facility and each supplying facility has to be opened in order to serve customers. In addition, each open facility must be connected to one of the *central offices*, via a path in the *core graph*. The (undirected) core graph consists of facilities, central offices and potential *Steiner nodes*, and its edges correspond to segments along which fibers can be laid out. See Figure 1(a) for an example.

The goal is to serve certain fractions of customers (determined according to *minimum coverage rates*) by each architecture while minimizing total cost.

Formally, the problem is described by a directed graph $G = (V, A)$ where the node set V is the disjoint union of (i) potential central offices (COs) Q with opening costs $c_q \geq 0, \forall q \in Q$, (ii) customer nodes C with demands $d_c \in \mathbb{N}, \forall c \in C$, (iii) potential facility locations $F = F^1 \cup F^2$ with opening costs $c_i^l \geq 0, \forall i \in F^l, l = 1, 2$, and (iv) potential Steiner nodes S. Hereby, potential facilities in F^l represent locations where equipment can be installed to connect customers using architecture l; note that F^1 and F^2 need not be



(a) Exemplary Instance.

(b) Exemplary Solution.

Fig. 1. (a) An exemplary instance with potential central offices q_1, q_2, type-1 facilities f_1, f_2, type-2 facilities g_1, g_2, potential Steiner nodes s_1, s_2, and customers $c_1, \dots, c_5$; assignment arcs are dashed. (b) A solution, including the root node, with selected CO q_1 supplying c_1, c_2, and c_3 using technology 1 via facility f_1, and c_4 and c_5 using technology 2 via facility g_2; note that g_1 is used as a Steiner node.

disjoint. The arc set A consists of (i) the core arcs $A_c = \{(i,j) \in A \mid i, j \notin C\}$, corresponding to forward and backward arcs for each edge of the core graph, with trenching costs $c_a \geq 0, \forall a \in A_c$, and (ii) assignment arcs $A^l = \{(i,j) \in A \mid i \in F^l, j \in C\}$, for each architecture $l = 1, 2$. Each potential assignment $(i,j) \in A^l$ is associated with costs $c_{ij}^l \geq 0$ for connecting customer j to facility i using architecture l. Finally, minimum coverage rates p_1 and p_2 are given with $0 \leq p_1 \leq p_2 \leq 1$, specifying the minimal fraction of total demand $D := \sum_{j \in C} d_j$ that must be satisfied by each architecture. Hereby we assume architecture 1 to be preferable to architecture 2, so that a coverage rate of p_2 means that $p_2 \cdot 100\%$ of the total demand needs to be satisfied by either architecture 1 or 2.

The total cost of a solution is the sum of all opening costs of used COs, trenching costs for used core arcs, assignment costs for the realized customer assignments, and opening costs of selected facilities. Note that CO nodes and facility locations can be used as Steiner nodes, in which case no opening costs are paid for passing through them. Furthermore, due to non-negative edge costs, there always exists an optimal solution which is a forest, or even – in case only a single CO is open – a tree. For an example of a feasible solution, see Figure 1(b).

2 Integer Linear Programming Models

For the above stated 2-ArchConFL problem integer linear programs (ILP) can be formulated; we explicitly present cut formulations here, but note that also other models, comprising flow or subtour elimination constraints, can be devised, as for the classical ConFL problem (cf. [3]).

For modeling purposes, we extend the graph G with an artificial root node $r \notin V$ connected via artificial arcs $A_r = \{(r,q) \mid q \in Q\}$ to all central offices (cf. Figure 1(b)). Their purpose is to select one or more COs to open and to incorporate their costs into the model: for each artificial arc (r,q), $q \in Q$, we set $c_{rq} := c_q$. Obviously, if $|Q| = 1$, i.e., there is only one potential CO node, creation of the root and artificial arcs can be skipped and the CO itself can act as the root. For abbreviation we use $A_{rc} := A_r \cup A_c$.

In the following subsections, we present ILP models based on various directed cutset constraints. We denote by $F^l_j = \{i \in F^l \mid (i,j) \in A^l\}$ the set of eligible facilities for a customer $j \in C$ for $l = 1,2$. Then the set of common decision variables for all the models is as follows: (i) core arc variables $x_{ij} \in \{0,1\}, \forall (i,j) \in A_{rc}$ indicate whether or not core/artificial arc (i,j) is used, (ii) assignment arc variables $x^l_{ij} \in \{0,1\}, \forall (i,j) \in A^l, l = 1,2$ indicate if customer j is supplied by facility i using architecture l, (iii) facility variables $y^l_i \in \{0,1\}$, $\forall i \in F^l, l = 1,2$ indicate whether or not facility i is open providing connections using architecture l, and (iv) customer variables $z^l_j \in \{0,1\}$, $\forall j \in C, l = 1,2$ indicate if customer j is connected using architecture l. For a given node set $W \subset V$, let $\delta^-(W) = \{(i,j) \in A \cup A_r \mid i \notin W, j \in W\}$ be the set of ingoing arcs in G. For an arc set $\hat{A} \subseteq A \cup A_r$ we use $x(\hat{A}) := \sum_{(i,j) \in \hat{A} \cap A_{rc}} x_{ij}$, as well as $x^l(\hat{A}) := \sum_{(i,j) \in \hat{A} \cap A^l} x^l_{ij}$ and $(x + x^l)(\hat{A}) := x(\hat{A}) + x^l(\hat{A})$ for $l = 1,2$.

Basic model

Using the previously described variables, we can formulate 2-ArchConFL as model (yC) given by (1)–(7).

$$\min \quad \sum_{(i,j) \in A_{rc}} c_{ij} x_{ij} + \sum_{l=1}^{2} \sum_{(i,j) \in A^l} c^l_{ij} x^l_{ij} + \sum_{l=1}^{2} \sum_{i \in F^l} c^l_i y^l_i \tag{1}$$

$$\text{s.t.} \quad \sum_{l=1}^{2} z^l_j \leq 1 \qquad \forall j \in C \tag{2}$$

$$\sum_{i \in F^l_j} x^l_{ij} = z^l_j \qquad \forall j \in C,\ l = 1,2 \tag{3}$$

$$x^l_{ij} \leq y^l_i \qquad \forall j \in C,\ i \in F^l_j,\ l = 1,2 \tag{4}$$

$$\sum_{\lambda=1}^{l} \sum_{j \in C} d_j z^\lambda_j \geq \lceil p_l D \rceil \qquad l = 1,2 \tag{5}$$

$$x(\delta^-(W)) \geq y_i^l \qquad \forall W \subseteq V \setminus C,\ i \in F^l \cap W,\ l = 1,2 \qquad (6)$$

$$(\boldsymbol{x}, \boldsymbol{x}^1, \boldsymbol{x}^2, \boldsymbol{y}^1, \boldsymbol{y}^2, \boldsymbol{z}^1, \boldsymbol{z}^2) \in \{0,1\}^{|A|+|A^1|+|A^2|+|F^1|+|F^2|+2|C|} \qquad (7)$$

Constraints (2) and (3) ensure that each connected customer uses a unique architecture and assignment arc; if $p_2 = 1$, Inequality (2) can be replaced by equality. Constraints (4) force a facility to be opened whenever an assignment arc issuing from it is chosen. Demanded coverage rates are satisfied due to Constraints (5). Finally, the connectivity constraints given by (6) (*y-cuts*) ensure that each opened facility is connected to the root node via opened core arcs. Since the root node is adjacent only to the CO nodes, at least one CO is opened in the solution. Hence (yC) is a valid model for 2-ArchConFL.

Note that the left-hand side matrix $M = (a_{ij})_{1 \leq i \leq 2|C|+|A_1 \cup A_2|, 1 \leq j \leq |A_1 \cup A_2|}$ defined by (3) and (4) has the following structure:

$$\left(\begin{array}{ccc} 1\;1\;\dots\;1 & & \\ & \ddots & \\ & & 1\;\dots\;1 \\ \hline \multicolumn{3}{|c|}{\mathbf{I}} \\ \hline \end{array}\right) \quad \begin{array}{l} \left.\vphantom{\begin{array}{c}1\\1\\1\end{array}}\right\} 2|C| \\ \left.\vphantom{\begin{array}{c}1\\1\end{array}}\right\} |A^1 \cup A^2| \end{array}$$

Here $\mathbf{I}$ denotes the unit matrix of size $|A^1 \cup A^2|$. Observe that each column of this 0/1-matrix contains exactly two nonzero entries; consider the partition (M_1, M_2) of its rows where M_1 contains the first $2|C|$ rows. Then for each column j we have $\sum_{i \in M_1} a_{ij} - \sum_{i \in M_2} a_{ij} = 0$. Hence M is totally unimodular and the integrality of the assignment variables can be relaxed to $x_{ij}^l \in [0,1]$.

zl-cuts

If some customer j is connected using architecture l, any cut between j and the root node must contain either a core arc or an assignment arc for l. Thus, the model can be strengthened by replacing the y-cuts (6) by *zl-cuts*:

$$(x + x^l)(\delta^-(W)) \geq z_j^l \qquad \forall W \subseteq V,\ j \in C \cap W,\ l = 1,2 \qquad (8)$$

If $|W \cap C| = 1$, we can reformulate (8) using (3) to obtain the following inequalities, which dominate (8):

$$x(\delta^-(W)) \geq \sum_{i \in W \cap F_j^l} x_{ij}^l \qquad \forall W \subseteq V,\ C \cap W = \{j\},\ l = 1,2 \qquad (9)$$

z-cuts

Similarly, if customer j is connected with any architecture then some core or assignment arc must be selected, which gives the *z-cuts*:

$$(x + x^1 + x^2)(\delta^-(W)) \geq z_j^1 + z_j^2 \qquad \forall W \subseteq V,\ j \in C \cap W \tag{10}$$

As for the zl-cuts, if $|W \cap C| = 1$, we obtain the dominating inequalities

$$x(\delta^-(W)) \geq \sum_{l=1}^{2} \sum_{i \in W \cap F_j^l} x_{ij}^l \qquad \forall W \subseteq V,\ C \cap W = \{j\} \tag{11}$$

In the following, we refer by (zlC) and (zC), to model (yC) with (6) replaced by (9) and (11), respectively. We denote by $v_{\mathrm{LP}}(\mathrm{X})$ the optimum objective value of the LP relaxation of MIP model (X). Then the following can be shown (in a similar way as in [3]):

Proposition 2.1 $v_{\mathrm{LP}}(\mathrm{zC}) \geq v_{\mathrm{LP}}(\mathrm{zlC}) \geq v_{\mathrm{LP}}(\mathrm{yC})$*, and there exist instances for which strict inequality holds for both inequalities. Furthermore, the integrality gap of (yC) is in* $\Omega(|V|)$*.*

3 Computational Results

To assess our models, branch-and-cut approaches have been implemented in C++ using IBM CPLEX 12.4 and tested on instances based on realistic networks representing deployment areas of three German towns. Table 1 gives further details on the instances. For each of the three given network topologies, 20 and 40 different instances are generated by varying the allowed sets of facilities and assignment arcs. We applied an absolute time limit of 7 200 CPU-seconds to all experiments which have been performed on a single core of an Intel Xeon processor with 2.53 GHz using at most 3GB RAM. We compared the computational performance of (yC), (zlC), and (zC) models, and also considered variant (yzC) where z-cuts are separated if no further violated y-cuts exist. The underlying branch-and-cut implementations follow the main ideas given in [3]. For each instance and cut strategy, nine combinations of (percentage) coverage rates are considered: $(p_1, p_2) \in \{(20, 60), (40, 60), (20, 80), (40, 80), (60, 80), (20, 100), (40, 100), (60, 100), (80, 100)\}$.

Figure 2 shows box plots of the runtimes of all computations for each network w.r.t. the different cut strategies. Each column corresponds to 9 coverage settings for each instance, i.e., we have 180 (for `berlin-tu` and `atlantis`) and

Network	**# Instances**	$\lvert V \setminus C\rvert$	$\lvert C\rvert$	$\lvert F\rvert$	$\lvert A_{rc}\rvert$	$\lvert A^1 \cup A^2\rvert$
`berlin-tu`	20	384	39	55–109	1124	84–269
`atlantis`	20	1001	345	361–447	2062	851–2952
`vehlefanz`	40	895	238	273–407	2197	544–3749

Table 1
Overview of test instances.

360 computations for `vehlefanz`. The numbers on top of each column indicate in how many computations the time limit was hit. In general, the y-cuts appear to be preferable over z- and zl-cuts. For the smaller network the z-cuts show a slightly better performance – this might be due to the fact that these instances significantly differ from the others with respect to the ratio of the number of customer nodes to the total number of nodes.

Figure 3 shows the influence of different coverage rates on the computational performance. Each column contains results of 20+20+40 computations over all instances, for a fixed coverage pair. Here the (yzC) cut strategy is considered, since this seems to be the best compromise between (yC) and (zC), considering all instance types. As can be seen from the three sections of the plot, increasing p_1 while keeping the values of p_2 fixed, yields a significant reduction in CPU-time. The picture is not that clear if p_1 is kept fixed and p_2 is increased (different greyscale levels): While for $p_1 = 20\%$ CPU-time decreases with higher p_2, no clear trend can be derived for $p_1 = 40\%$ and $p_1 = 60\%$.

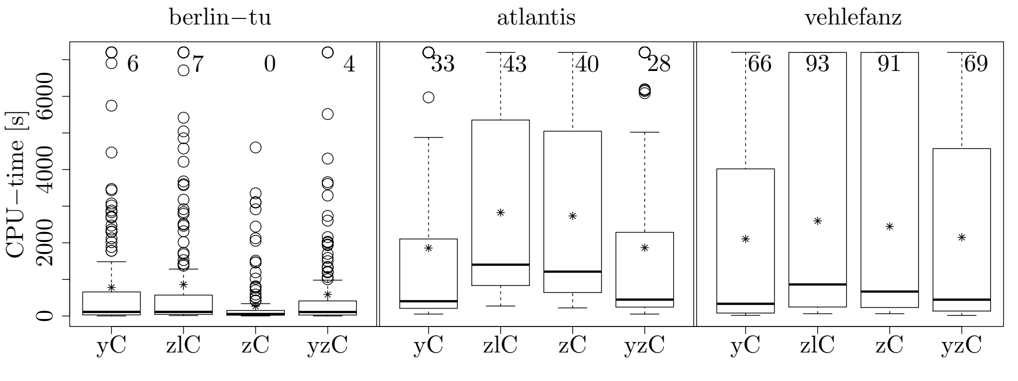


Fig. 2. CPU-time per instance for different cut strategies.

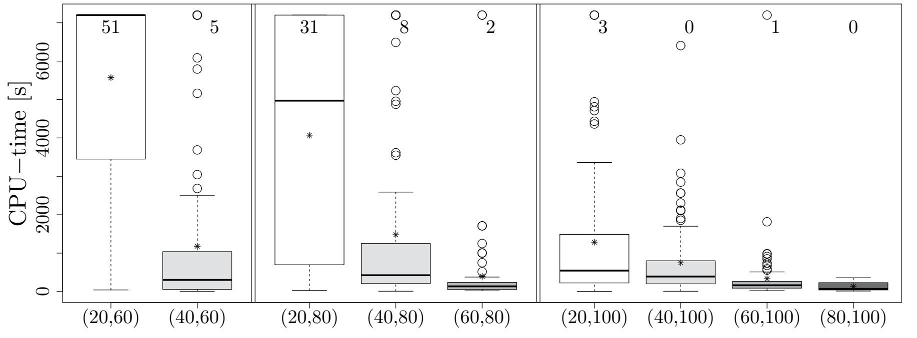


Fig. 3. CPU-time for different coverage rates.

4 Conclusions and Outlook

A new variant of the connected facility location problem has been introduced and a MIP model with cut inequalities has been presented and computationally tested on a set of realistic instances. For future studies, other valid inequalities and formulations for the problem are conceivable, such as variants of cover cuts, and Miller-Tucker-Zemlin or common flow formulations.

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