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Anisotropic model for the numerical computation of magnetostriction in grain-oriented electrical steel sheets

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Abstract

Purpose – The modeling of magnetostrictive effects is a topic of intensive research. The authors' goal is the precise modeling and numerical simulation of the magnetic field and resulting mechanical vibrations caused by magnetostriction along the joint regions of electric transformers.

Design/methodology/approach – The authors apply the finite element (FE) method to efficiently solve the arising coupled system of partial differential equations describing magnetostriction. Hereby, they fully take the anisotropic behavior of the material into account, both in the computation of the nonlinear electromagnetic field as well as the induced magnetostrictive strains. To support their material models, the authors measure the magnetic as well as the mechanical hysteresis curves of the grain-oriented electrical steel sheets with different orientations (w.r.t the rolling direction). From these curves they then extract for each orientation the corresponding commutation curve, so that the hysteretic behavior is simplified to a nonlinear one.

Findings – The numerical simulations show strong differences both in the magnetic field as well as mechanical vibrations when comparing this newly developed anisotropic model to an isotropic one, which just uses measured curves in rolling direction of the steel sheets. Therefore, a realistic modeling of the magnetostrictive behavior, especially for grain-oriented electrical steel as used in transformers, needs to take into account the anisotropic material behavior.

Originality/value – The authors have developed an enhanced material model for describing magnetostrictive effects along the joint regions of electric transformers, which fully considers the anisotropic material behavior. This model has been integrated into a FE scheme to numerically simulate the mechanical vibrations in transformer cores caused by magnetostriction.

Keywords Magnetostriction, Finite element method, Anisotropic material behavior, Nonlinearity, Finite element analysis, Steel

Paper type Research paper

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1. Introduction

Magnetostrictive materials are widely used for actuator and sensor applications. However, often the magnetostrictive behavior of these alloys is an undesirable effect, as, e.g. in electric machines and transformers, where it is one of the main sources for noise generation. Unfortunately, these materials exhibit nonlinear behavior for the magnetic properties as well as the mechanical characteristics leading to the well-known magnetic hysteresis loop and the magnetostrictive hysteresis loop (so-called butterfly curve), respectively, (Vandevelde and Melkebeek, 2002; Linnemann *et al.*, 2009; Kaltenbacher *et al.*, 2009). A quite important aspect – especially for grain-oriented electrical steel as used in transformers – is the anisotropic material behavior both concerning the magnetic properties as well as the induced mechanical strains (Weiser *et al.*, 2000).

The modeling of magnetostrictive effects is a topic of intensive research. Among the huge amount of publications one can find three main approaches. The first one, which is widely used, is based on introducing a magnetostrictive strain tensor, where the entries depend on the magnetic induction (Delaere *et al.*, 2000; Vandevelde and Melkebeek, 2002). Thereby, these additional strains result in mechanical forces modeled as a right hand side term in the partial differential equation (PDE) for mechanics. In a second approach, a free energy as a tensor function depending on the mechanical strain and magnetic induction is used (Dorfmann and Ogden, 2003; Fonteyn *et al.*, 2010). Thereby, a fully coupled constitutive relation between mechanical and magnetic quantities is achieved. The last approach is based on a thermodynamic consistent model, where the mechanical strain and magnetic induction is decomposed in a reversible and an irreversible part (Linnemann *et al.*, 2009; Kaltenbacher *et al.*, 2009). Furthermore, the full constitutive model is based on a free energy function. Whereas in Linnemann *et al.* (2009) the irreversible part is modeled by a switching criterion using inner variables (Kaltenbacher *et al.*, 2009) uses hysteresis operators. Common to all models is the current restriction to isotropic and/or uni-axial behavior.

Our goal is the precise investigation of the magnetic field and resulting mechanical vibrations caused by magnetostriction along the joint regions of electric transformers. Therefore, we cannot apply any homogenization technique and fully resolve each individual steel sheet. This is clearly not possible for a whole transformer core, and so we restrict our investigation to some few steel sheets. To reduce the complexity, we choose an ansatz, in which we neglect the reaction of the mechanical stresses and strains on the magnetic properties and therefore decouple the computation of the magnetic and mechanical field. By help of an Epstein frame and a single sheet tester (SST), we measure the magnetic as well as the mechanical hysteresis curves of the grain-oriented electrical steel sheets with different orientations (w.r.t the rolling direction). From these curves we then extract for each orientation the corresponding commutation curve, so that the hysteretic behavior is simplified to a nonlinear one. This approach is then applied to a stack of six electrical steel sheets with a 90° joint region, excited by two current loaded coils. We compare this anisotropic model to an isotropic one, where the nonlinear magnetic and mechanical material parameter are just used from the rolling direction.

The rest of the paper is organized as follows. In Section 2 we describe our physical model and its integration into the magnetic and mechanical PDE as well as their finite element (FE) formulation. The measurement setups, which provide us the nonlinear curves, are discussed in Section 3. In Section 4 the numerical results are presented, demonstrating the importance of taking anisotropy for grain-oriented electrical steel sheets as used in transformers into account. Finally, Section 5 summarizes our achievements.

2. Physical modeling and FE discretization

Magnetostrictive materials are characterized by the magnetic hysteresis between the magnetic induction $\boldsymbol{B}$ and magnetic field intensity $\boldsymbol{H}$ as well as the mechanical hysteresis between the mechanical strain $\boldsymbol{S}$ and magnetic induction $\boldsymbol{B}$ (Figure 1).

According to a thermodynamically consistent model, we decompose the physical quantities, magnetic induction and mechanical strain, into a reversible and an irreversible part[1] (indicated by the superscripts r and i, respectively):

$$\boldsymbol{S} = \boldsymbol{S}^{\mathrm{r}} + \boldsymbol{S}^{\mathrm{i}}, \quad \boldsymbol{B} = \boldsymbol{B}^{\mathrm{r}} + \boldsymbol{B}^{\mathrm{i}}. \tag{1}$$

To allow for the history of the driving magnetic field intensity, the irreversible magnetic induction $\boldsymbol{B}^{\mathrm{i}}$ is set to be equal to the magnetization $\boldsymbol{M}$, which is modeled, e.g. by a Preisach hysteresis operator (Kaltenbacher *et al.*, 2009):

$$\boldsymbol{B}^{\mathrm{i}} = \boldsymbol{M} = \mathcal{H}[\boldsymbol{H}]\boldsymbol{e}_M. \tag{2}$$

The irreversible strain can be, e.g. expressed by the following polynomial ansatz (Kaltenbacher *et al.*, 2009):

$$[\boldsymbol{S}^{\mathrm{i}}] = \frac{3}{2}(\beta_1 \cdot \mathcal{H}[\boldsymbol{H}] + \beta_2 \cdot (\mathcal{H}[\boldsymbol{H}])^2 + \cdots + \beta_n \cdot (\mathcal{H}[\boldsymbol{H}])^n)\left(\boldsymbol{e}_M \otimes \boldsymbol{e}_M^{\mathrm{t}} - \frac{1}{3}[\boldsymbol{I}]\right), \tag{3}$$

while the parameters $\beta_1, \ldots, \beta_n$ need to be fitted to measurement data and $[\boldsymbol{I}]$ denotes the identity tensor.

Now, magnetostriction is a property of ferromagnetic materials and can be described as a coupling between the mechanical and the magnetic field. This relation is described by the well-known magnetostrictive constitutive equations modeling the linear coupling of the magnetic and the mechanical deformation (Linnemann *et al.*, 2009):

$$\boldsymbol{\sigma} = [\boldsymbol{c}^H]\boldsymbol{S}^{\mathrm{r}} - [\boldsymbol{e}]^{\mathrm{t}}\boldsymbol{H} \tag{4}$$

$$\boldsymbol{B}^{\mathrm{r}} = [\boldsymbol{e}]\boldsymbol{S}^{\mathrm{r}} + [\boldsymbol{\mu}^S]\boldsymbol{H}. \tag{5}$$

In equations (4) and (5) $\boldsymbol{\sigma}$ denotes the Cauchy stress tensor in Voigt notation, $[\boldsymbol{c}^H]$ the tensor of mechanical moduli (at constant magnetic field intensity), $[\boldsymbol{e}]$ the piezomagnetic coupling tensor and $[\boldsymbol{\mu}^S]$ the tensor of magnetic permeability (at constant mechanical strain).

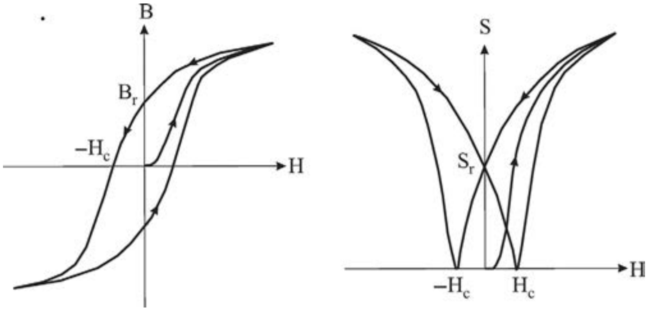


Figure 1.
Magnetic and mechanical hysteresis (butterfly curve)

By using these constitutive relations, we have presented in Kaltenbacher *et al.* (2009) a formulation based on the magnetic scalar potential, and in Volk *et al.* (2011) have even extended it for the magnetic vector potential to also take eddy current effects into account. However, both models are currently restricted to scalar hysteresis operators, and do not take into account the anisotropic behavior, which is a crucial point for grain-oriented electrical steel used in transformer cores (Weiser *et al.*, 2000). Furthermore, both models are quite expensive concerning computational time. Therefore, we have developed a dedicated physical model for grain-oriented electrical steel. In doing so, we first assume that the entries of the piezomagnetic coupling tensor $[e]$ are small, and we are allowed to neglect this coupling in equations (4) and (5). Next the anisotropic and nonlinear magnetic behavior of the steel sheets is modeled by its vector relation between the magnetic induction $\boldsymbol{B}$ and field intensity $\boldsymbol{H}$:

$$\boldsymbol{B} = \boldsymbol{B}(\boldsymbol{H}) = B_\varphi(H)\boldsymbol{e}_B; \quad \boldsymbol{e}_B = \frac{\boldsymbol{B}}{B}. \tag{6}$$

Here, we compute the unit vector $\boldsymbol{e}_B$ and evaluate the magnetic commutation curve B_φ for which the orientation fits best with $\boldsymbol{e}_B$. Therefore, the defining PDE for the magnetic field reads as:

$$\gamma\frac{\partial \boldsymbol{A}}{\partial t} - \nabla \times \nu(B_\varphi)\nabla \times \boldsymbol{A} = \boldsymbol{J}_{\mathrm{i}} \tag{7}$$

with $\boldsymbol{A}$ the magnetic vector potential, $\boldsymbol{J}_{\mathrm{i}}$ the impressed current density, ν the magnetic reluctivity depending on $B\varphi$ (equation (6)) and γ the electric conductivity.

The PDE for mechanics is given by:

$$\rho\frac{\partial^2 \boldsymbol{u}}{\partial t^2} - \mathcal{B}^{\mathrm{t}}\boldsymbol{\sigma} = 0 \tag{8}$$

with $\boldsymbol{u}$ the mechanical displacement, ρ the density, and $\mathcal{B} = \nabla^{\mathrm{s}}$ the differential operator. As in equation (3), we assume the conservation of volume for the irreversible strain. However, we now model instead of the hysteretic behavior a nonlinear, anisotropic behavior, and denote it by $[\boldsymbol{S}^{\mathrm{m}}]$ (magnetostrictive induced strain tensor), which is computed as follows:

$$[\boldsymbol{S}^{\mathrm{m}}] = \frac{3}{2}\left(\boldsymbol{e}_B \otimes \boldsymbol{e}_B^{\mathrm{t}} - \frac{1}{3}\boldsymbol{I}\right)S_\varphi^{\mathrm{m}}(B). \tag{9}$$

Here, we compute the direction of $\boldsymbol{B}$ and evaluate the magnetostrictive commutation curve $S_\varphi^{\mathrm{m}}(B)$ for which the orientation fits best with $\boldsymbol{e}_B$. Now, we can express the reversible mechanical strain $\boldsymbol{S}^{\mathrm{r}}$ by the difference of the total strain $\boldsymbol{S} = \mathcal{B}\boldsymbol{u}$ and the irreversible (magnetostrictive) strain $\boldsymbol{S}^{\mathrm{i}} = \boldsymbol{S}^{\mathrm{m}}$ via:

$$\boldsymbol{S}^{\mathrm{r}} = \mathcal{B}\boldsymbol{u} - \boldsymbol{S}^{\mathrm{m}}. \tag{10}$$

This relation in combination with equation (4) by neglecting $[\boldsymbol{e}]$ results for equation (8) into:

$$\rho\frac{\partial^2 \boldsymbol{u}}{\partial t^2} - \mathcal{B}^{\mathrm{t}}[\boldsymbol{c}^H]\mathcal{B}\boldsymbol{u} = -\mathcal{B}^{\mathrm{t}}[\boldsymbol{c}^H]\boldsymbol{S}^{\mathrm{m}}. \tag{11}$$

The FE formulation of equations (7) and (11) is straight forward. For equation (7) we use edge finite elements and solve the arising algebraic system of equations by an efficient Newton scheme utilizing a two level solver (Hauck *et al.*, 2013). For equation (11) we apply nodal finite elements (Kaltenbacher, 2007).

Summarizing, the developed magnetostrictive model has the following features:

- Decoupling of magnetic and mechanical PDEs; so both PDEs can be solved separately with optimal conditions.
- Anisotropy and eddy currents are taken into account.
- No hysteresis considered; instead it uses commutation curves computed from measured hysteresis curves.
- Change of magnetic properties due to the mechanical field within a working point is neglected (working point can be determined by pre-stressing of measured samples).

3. Measurement setups

First of all, to obtain reliable measurement data for the magnetic behavior, we have constructed an Epstein frame according to IEC 60404-2 (Figure 2). The 25 cm Epstein apparatus consists of four coils with primary windings, secondary windings, a compensation coil and the material sample as core. The sheets are stratified in stripes. In this way, the measurement setup represents a transducer, whose characteristics are specified. The primary outer windings are used to magnetize the material and the secondary inner windings are needed for magnetic flux density determination over the induced voltage. We have performed measurements for steel sheets, which have been cut out at different angles according to the rolling direction. Thereby, for each stack of steel sheets, we have measured the outer and all inner hysteresis loops, as demonstrated in Figure 3. Out of all the hysteresis loops, we compute for each angle a commutation curve (Figure 4), which we then use in our numerical computation for the magnetic field.

To measure the mechanical hysteresis of the electrical steel sheets a second measurement setup was constructed on the basis of a SST as shown in Figure 5.

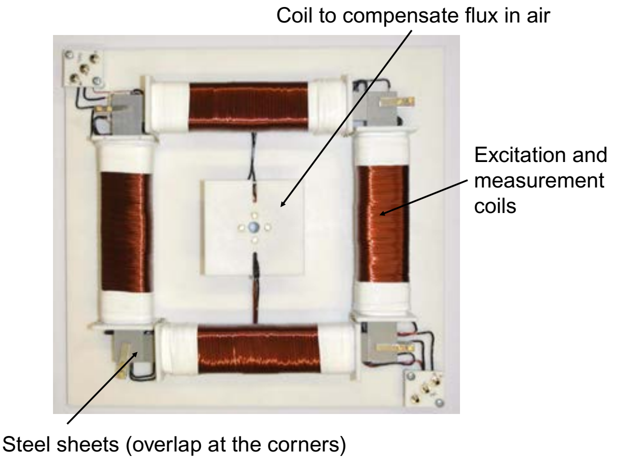


Figure 2.
25 cm Epstein frame for measuring the magnetic properties of grain-oriented electrical steel sheets

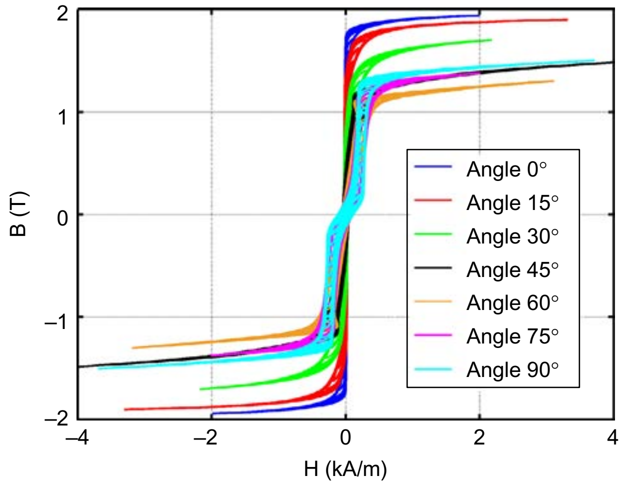


Figure 3.
Magnetic hysteresis curves for grain-oriented electrical steel sheets being cut out at different angles according to the rolling direction (0° corresponds to the rolling direction)

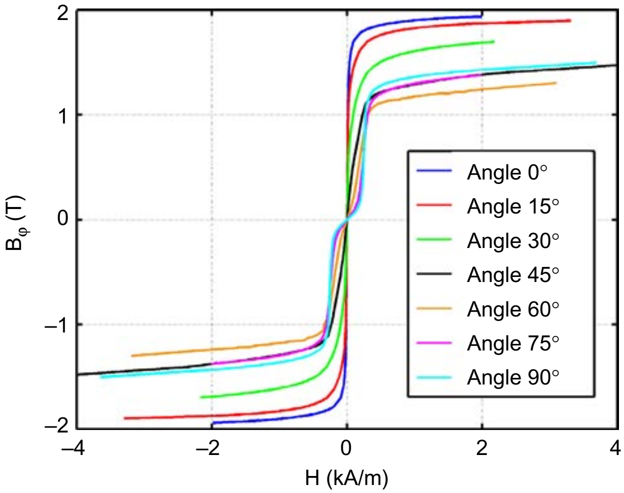


Figure 4.
Nonlinear BH curves for different angles (0° corresponds to the rolling direction)

This extended setup also captures the magnetic induction as well as the magnetic field intensity. However, we did not use the SST to determine magnetic hysteresis since it has to be calibrated to a certified setup to obtain reliable measurement results. To capture the mechanical hysteresis the SST was extended by a lifting mechanism to unload the sample sheet to ensure its stress-less vibration. The mechanical vibration due to magnetic excitation of the SST is measured by a laser vibrometer, which compared to strain gauges provides high accuracy without electromagnetic cross-sensitivity and is contact-free. The measurement of the mechanical strain as a function of the magnetic field results in the magnetostrictive hysteresis loop (so-called butterfly curve). Additionally the extended SST permits pre-stressing of the steel sheets in order to capture the reaction on the magnetic properties which corresponds to

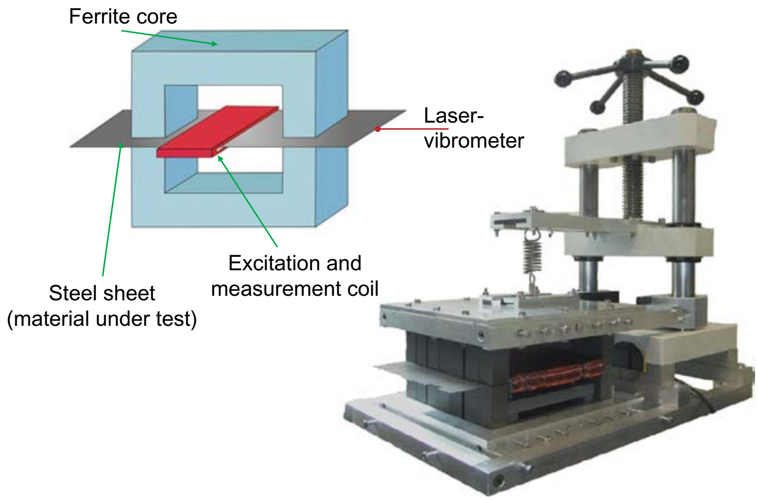


Figure 5.
SST as used to obtain the mechanical hysteresis (principle and manufactured setup)

a working point that is used in the simulation. To consider anisotropy, again a series of measurement is performed with different electrical steel sheets which have been cut out with varying cutting angles with respect to the grain orientation of the steel. As for the magnetic hysteresis, we also convert the mechanical hysteresis in a single commutation curve, which leads to angle-dependent nonlinear magnetostriction curves, as shown in Figure 6.

4. Numerical results

For the numerical investigation, we choose a setup of six stacked electrical steel sheets with a 90° joint and an excitation coil along each yoke as shown in Figure 7. We model just

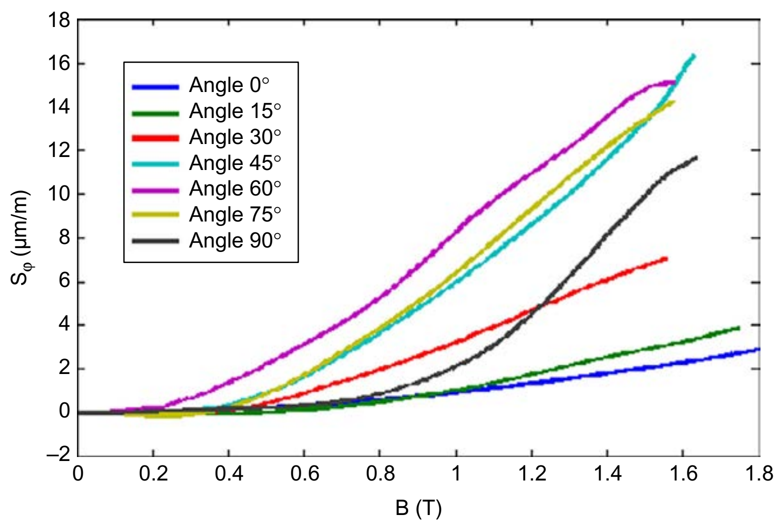


Figure 6.
Nonlinear SB curves for different angles (0° corresponds to the rolling direction)

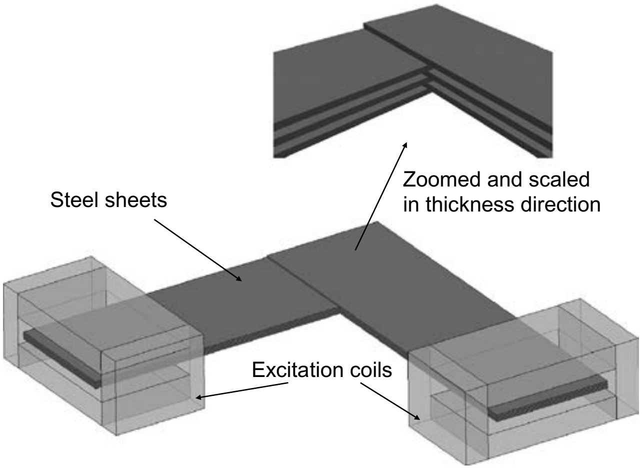
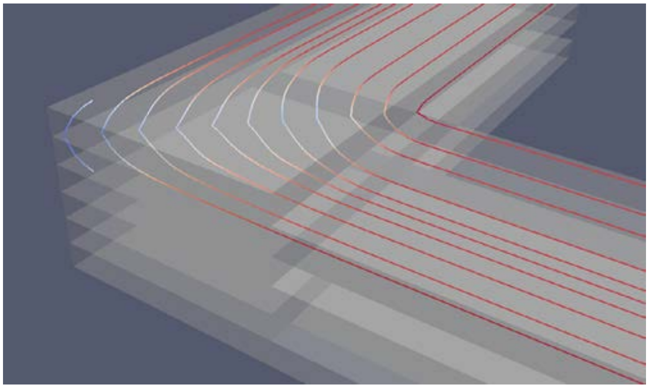


Figure 7.
Computational model: quarter symmetry is considered

a quarter symmetry by applying appropriate boundary conditions at the symmetry planes. Our main goal is to study the difference between an isotropic and anisotropic magnetostrictive computation. Thereby, we choose for the isotropic computation the measured material curves along the rolling direction (angle of 0°), whereas for the anisotropic computation we use all measured material curves (Figures 4 and 6).

In a first step, we compute the magnetic field and compare the flux lines at the joints. Figure 8 shows the flux lines for the isotropic and Figure 9 for the anisotropic case at the time step of maximal magnetic induction (about 1.7T). We display the flux lines just for the two last layers and zoom into the joint region. Comparing the results, one can clearly see the difference. For the isotropic case, the amplitude of the magnetic induction immediately drops to a low one at the beginning of the joint due to the increased effective cross-section when turning the flux direction in the rectangular joint region. Since the magnetic material properties are homogeneous and independent of direction, the change of the magnetic flux direction itself is continuously across the joint region. The transition



Note: For better visualization we have scaled the thickness direction by a factor of ten

Figure 8.
Magnetic flux lines for the two upper steel sheets in case of isotropic computation

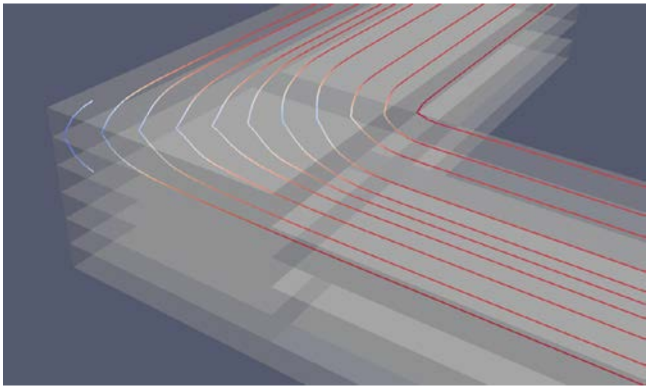


Figure 9.
Magnetic flux lines for the two upper steel sheets in case of anisotropic computation

Note: For better visualization we have scaled the thickness direction by a factor of ten

of the magnetic flux between the two vertical stacked steel sheets is mainly limited when entering and leaving the joint region. Accordingly the magnetic flux density reaches its full value just at the end of the joint when entering the opposite yoke.

In the anisotropic case, the guiding effect of the preferred magnetic direction in the grain orientation of the electrical sheet keeps the amplitude and direction of the magnetic flux for some distance in the joint region. In the area of the central diagonal of the joint region (at 45°), the reduced magnetic permeability perpendicular to the grain orientation forces the magnetic flux to a vertical transition into neighboring steel sheets. This behavior of the magnetic field has a strong impact on the mechanical vibrations. In a second step we use the computed magnetic induction and calculate the mechanical deformation according to the additional magnetostrictive strain. In Figures 10 and 11 we show all three components of the mechanical displacement

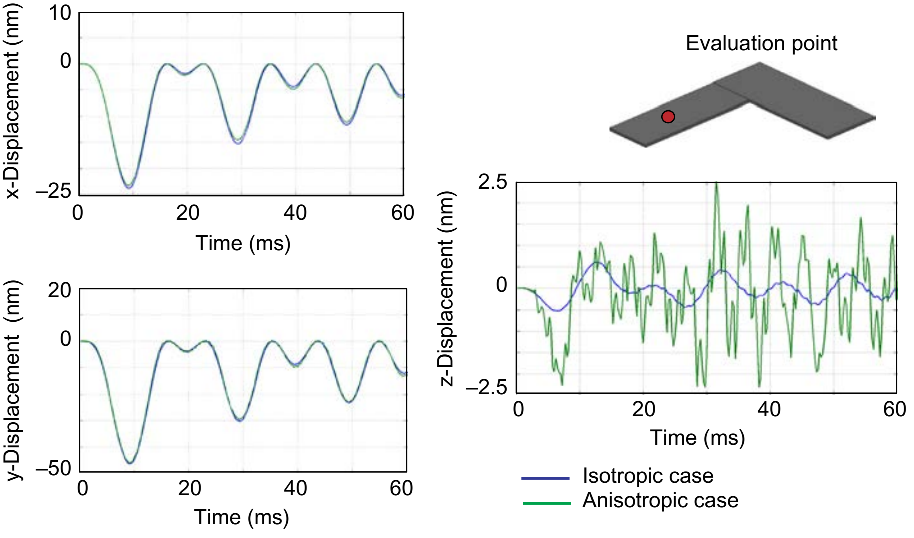
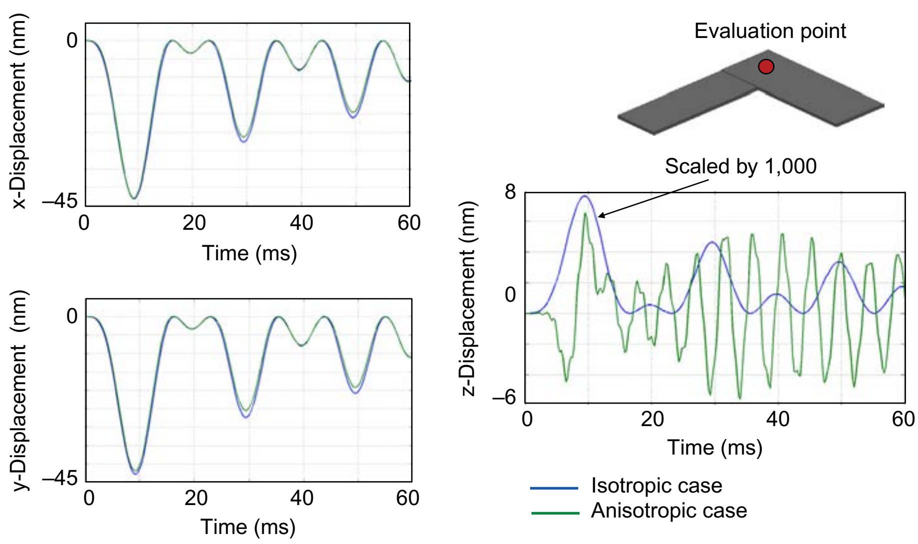


Figure 10.
Mechanical displacement at an observation point along the yoke



Note: For display reasons, we have multiplied the z-displacement of the isotropic case by a factor of 10^3

Figure 11. Mechanical displacement at an observation point at the joint region

over time at two different observation points. In general, we observe that the displacement in plane direction (x- and y-displacement) show almost no difference. However, the displacement in thickness direction (z-displacement) is quite different both concerning amplitude and frequency content. Especially at the joint region the amplitude of the mechanical vibration is a factor of about 1,000 larger in the anisotropic case than in the isotropic case. Furthermore, we can state that the computation for the isotropic material model exhibits mainly the 100 Hz component (current excitation is at 50 Hz). In the anisotropic case higher harmonics are predominant. The related frequency spectrum in vibration and noise is typical of what can be measured at real transformers.

5. Conclusion and outlook

We have presented a magnetostrictive constitutive model which fully takes the anisotropy of grain-oriented electrical steel sheets as used in electrical transformers into account. The model itself is simplified in this sense that the magnetic as well as mechanical hysteretic behavior is reduced to a nonlinear one by computing commutation curves out of the corresponding hysteresis measurements. Furthermore, we neglect the impact of the mechanical field on the magnetic properties within a working point, which can be determined by pre-stressing the measured sample sheets. However, the model needs measurements provided by an Epstein frame and a SST. The computations show strong differences both in the magnetic field as well as mechanical vibrations when comparing this anisotropic model to an isotropic one, which just uses measured curves in the rolling direction of the steel sheets.

Currently we are working on an experimental validation setup, where we can study different joint techniques, especially step-lap joints.

Note

1. With [$\boldsymbol{S}$] we denote the tensor of mechanical strain and with $\boldsymbol{S}$ the algebraic vector containing the three normal and three shear strains according to Voigt notation.

References

Delaere, K., Heylen, W., Hameyer, K. and Belmans, R. (2000), "Local magnetostriction forces for finite element analysis", *IEEE Trans. on Magnetics*, Vol. 36 No. 5.

Dorfmann, A. and Ogden, R.W. (2003), "Magneto-elastic modeling of elastomers", *Eur. J. Mechanics and Solids*, Vol. 22.

Fonteyn, K., Belahcen, A., Kouhia, R., Rasilo, P. and Arkkio, A. (2010), "FEM for directly coupled magneto-mechanical phenomena in electrical machines", *IEEE Trans. on Magnetics*, Vol. 46 No. 8.

Hauck, A., Ertl, M., Schöberl, J. and Kaltenbacher, M. (2013), "Accurate simulation of transformer step-lap joints using anisotropic higher order FEM", *COMPEL*, Vol. 32 No. 5.

Kaltenbacher, M. (2007), *Numerical Simulation of Mechatronic Sensors and Actuators*, 2nd ed., Springer, Berlin.

Kaltenbacher, M., Meiler, M. and Ertl, M. (2009), "Physical modeling and numerical computation of magnetostriction", *COMPEL*, Vol. 28 No. 4.

Linnemann, K., Klinkel, S. and Wagner, W. (2009), "A constitutive model for magnetostrictive and piezoelectric materials", *International Journal of Solids and Structures*, Vol. 46.

Vandevelde, L. and Melkebeek, J.A. (2002), "Modeling of magnetoelastic material", *IEEE Trans. on Magnetics*, Vol. 38 No. 2.

Volk, A., Kaltenbacher, M., Hauck, A., Ertl, M. and Lerch, R. (2011), "Finite element scheme based on magnetic vector potential and mechanical displacement for modeling magnetostriction", *Proceedings of the 8th International Conference on Computation in Electromagnetics CEM*.

Weiser, B., Pfützner, H. and Anger, J. (2000), "Relevance of magnetostriction and forces for the generation of audible noise of transformer cores", *IEEE Trans. on Magnetics*, Vol. 36 No. 5.

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