VALPARAISO NUMERICO IV
Séptimo Encuentro de Análisis Numérico de Ecuaciones Diferenciales Parciales
Facultad de Ingeniería, Pontificia Universidad Católica de Valparaíso, Diciembre 11–13, 2013

Rate optimality of adaptive algorithms: An axiomatic approach*

Carsten Carstensen†, Michael Feischl‡, Dirk Praetorius§

Abstract

The impact of adaptive mesh-refinement in computational partial differential equations cannot be overestimated, and convergence with optimal rates has mathematically been proved for certain model problems. We aim at a simultaneous axiomatic presentation of the proof of optimal convergence rates for adaptive finite element methods as well as boundary element methods in the spirit of [1]. For this purpose, an overall set of four axioms is sufficient and (partially even) necessary.

Compared to the state of the art in the temporary literature [2, 4, 5], the improvements can be summarized as follows: First, a general framework is presented which covers the existing literature on rate optimality of adaptive schemes for both, linear as well as nonlinear problems, which is fairly independent of the underlying finite element or boundary element method. Second, efficiency of the error estimator is neither needed to prove convergence nor quasi-optimal convergence behavior of the error estimator. Instead, efficiency exclusively characterizes the approximation classes involved in terms of the bestapproximation error plus data resolution. In particular, the constraint on optimal marking parameters does not depend on the efficiency constant. Third, some general quasi-Galerkin orthogonality is not only sufficient, but also necessary for the R-linear convergence of the error estimator, which is a fundamental ingredient in the current quasi-optimality analysis [1, 2, 3, 4, 5]. Finally, the general analysis allows for equivalent error estimators and inexact solvers as well as different non-homogeneous and mixed boundary conditions.

Key words: finite element method, boundary element method, a posteriori error estimate, adaptive algorithm, convergence, optimality

Mathematics subject classifications (1991): 65N30, 65N15, 65N38

*This research was supported by the Austrian Science Fund (FWF) under grant P21732 and W1245.

†Humboldt-University of Berlin, Department of Mathematics, 10099 Berlin, Germany, e-mail: `cc@math.hu-berlin.de`

‡Vienna University of Technology, Institute for Analysis and Scientific Computing, 1040 Vienna, Austria, e-mail: `Michael.Feischl@tuwien.ac.at`

§Vienna University of Technology, Institute for Analysis and Scientific Computing, 1040 Vienna, Austria, e-mail: `Dirk.Praetorius@tuwien.ac.at`

References

[1] Stevenson, R., *Optimality of a standard adaptive finite element method.* Found. Comput. Math., vol. 7, pp. 245–269, (2007).

[2] Cascon, J.M., Kreuzer, C., Nochetto, R.H., and Siebert, K., *Quasi-optimal convergence rate for an adaptive finite element method.* SIAM J. Numer. Anal., vol. 46, pp. 2524–2550, (2008).

[3] Feischl, M., Führer, and Praetorius, D., *Adaptive FEM with optimal convergence rates for a certain class of non-symmetric and possibly non-linear problems.* Preprint 2012, `arXiv:1210.8369`.

[4] Feischl, M., Karkulik, M., Melenk, J.M., and Praetorius, D., *Quasi-optimal convergence rate for an adaptive boundary element method.* SIAM J. Numer. Anal., vol. 51, pp. 1327–1348, (2013).

[5] Gantumur, T., *Adaptive boundary element methods with convergence rates.* Numer. Math., vol. 124, pp. 471–516, (2013).

[6] Carstensen, C., Feischl, M., Page, M., and Praetorius, D., *Axioms of adaptivity.* Preprint 2013.