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Overlapping Generations Models with Immigration

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Abstract

In this work we develop overlapping generations models (OLG) which explicitly include immigration. In the following, we propose two general equilibrium models. In a first approach, we investigate the effect of a sudden variation in the number of immigrants on the host country to shed light on the welfare effects of immigration for the various generations of the host country's population. Subsequently, in a second model, we vary the age structure of the inflowing migrants and determine the impact of immigration on the pension system and capital accumulation. The impact of age-specific immigration on the social security rate and the pension expenditure rate in a benefit-defined pay-as-you-go pension scheme are presented. Moreover, scaled pension expenditures and tax payments for the two groups, natives and immigrants, are given. For the presented numerical experiment the social security rate decreases with the age of the arriving immigrants although the old-age dependency ratio increases substantially. This is because of the fact that immigrants qualify for fewer pensions in the host country. Moreover, across all age groups immigrants are net payers of the pension system. Hence, they are at least to a small extend able to close the financial gap caused by the aging of the native population.

Keywords: Immigration, continuous time overlapping generations models, pension systems

1. Modeling an immigration shock with a continuous time OLG model

1.1. Introduction

Immigration is a complex process. People who immigrate to a country differ, among other characteristics, in ethnicity, religious believes, age, skill level and their economic situation. In this work we will focus only on the last three of these features. The number of annual immigrants as well as their skill level and age distribution impact the population structure and the productivity of the work force. The economic situation of immigrants, in here, reflected by their capital endowment when entering the country, changes the capital labor ratio in the presumably closed economy. As a consequence, all these characteristics impact the economic activities in the country and henceforth the welfare of its inhabitants.

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OLG models are typically used to investigate how different generations interact with each other in an economy. Here, we extend this model structure by adding a new heterogeneity, namely by explicitly modeling natives and immigrants.

Here, we aim to determine the welfare consequences of an exogenous immigration shock to a closed economy. This means a rapid change in the number of immigrants for a small period of time due to policy changes or possibly also unstable conditions such as wars or economic crises in other countries.

In order to identify the welfare effects for different cohorts of the native population, the model must replicate the age structure of the population and the life cycle choices for the agents of the different vintages.

In [2, 3] the population is modeled in form of overlapping dynasties where arriving immigrants are the founders of new dynasties. This provides a first step to a realistic description of the population structure. However, immigrants have all the same age when entering the host country. Moreover, these dynasties live infinitely long. In contrast to these articles, here we consider an age pattern of inflowing immigrants and finite but uncertain life times.

In [4], like here, it is investigated how an immigration shock affects the welfare of different cohorts. There, they use the framework of a three period discrete time overlapping generations model. The age structure of the population is determined by assigning the individuals to these three periods. As a result there are only three coexisting generations at each point of time. In order to depict a realistic finite life time, the length of these periods is approximately 25-30 years.

Unlike the second model treated in this report, where the focus is on the long term effects, i.e. steady state changes, here we investigate the intertemporal changes in the macroeconomic variables and therefore also consider a temporal shock which lasts only for a couple of years.

In the following, we assume that immigrants enter without any assets. The life cycle of both, natives and immigrants, is divided into a schooling, a working and a retirement period. Cohorts living in different periods are linked by intergenerational transfers, i.e. labor taxes are redistributed to the old in form of pension payments. While receiving their education, native agents accumulate debts, due to own consumption and the lack of labor income. They pay back and start saving during their working period.

It is assumed that immigrants enter the country after finishing their education. This is in accordance with Austrian data, where more than 85 % of all immigrants between ages 16-24 enter after finishing schooling. This holds for even 95 % of all immigrants over the age of 35. We assume that human capital is solely accumulated by education. There are also no intergenerational knowledge spillovers.

We distinguish two different cases. First, we assume that the number of schooling years, this means the length of education, is exogenously given and may vary between natives and immigrants. Since education determines the efficiency of labor, we aim to investigate how differences in education between natives and immigrants impact the welfare of the native population. This impact on the welfare varies for different generations. As a consequence, some generations win and others lose in terms of life cycle utility.

Later, we endogenize the education decision of natives and let them decide over their optimal number of schooling years.

While natives accumulate capital through saving, immigrants consume immediately what they earn during working life and receive pensions after retirement. Therefore, immigrants do not hold any assets in course of their life cycle. Here, we follow [16] where it is argued that if it is assumed that immigrants are close to the bottom of the income structure, they have little incentives to save and invest in the host country, because saving incentives are correlated with income. Moreover, most immigrants remit much of their savings to their country of origin and some of them even intend to go back after some time.

It is assumed that immigrants and natives feature the same fertility and mortality rates. Immigrants' offspring is considered as native and therefore also acts as saver and capital owner.

1.2. Model

Age structured populations are studied in economics through overlapping generations models. These models allow for a realistic determination of life-cycle behaviors. Here, an age-structured population with immigration is considered. The household side of the closed economy is modeled by an overlapping generations framework. The firm sector is assumed to consist of one representative firm that uses aggregate capital and labor for the production of a single good.

1.2.1. Population Structure

The economy is populated by different age cohorts whose lifespan is uncertain but bounded. In the following, time is denoted by $t \geq 0$, where $t = 0$ is the starting time of the consideration of the economy, and a cohort's birth date is τ. The age of death is a random variable over $[0, \omega]$, where $\omega < \infty$ is the maximal reachable age. The probability of surviving of an individual born at time τ until age $a = t - \tau \in [0, \omega]$ is again denoted by $l(a) \in C^1[0, \omega]$ and does not change with time. Therefore, $-l'(a)$ is again the unconditional probability of dying at age a. Accordingly,

$$\mu(a) = -\frac{l'(a)}{l(a)},$$

is equal to the conditional probability of dying at age a, given that the individual survives until this age. Therefore, $\mu(a)$ is the density function of the random variable describing the age of death. For the probabilistic density function $\mu(a)$ it holds $\int_0^\omega \mu(a)\, \mathrm{d}a = l(0) = 1$.

Let $N(\tau, t)$ denote the number of natives[1] and $M(\tau, t)$ the number of individuals born outside the host country at time τ and still being alive at time $t > \tau$. Then $N(\tau, 0)$ and

[1]The term "number of people"' is strictly speaking not correct. To be more correct, one would have to speak of $N(\cdot, t)$ as a density representing the distribution of individuals along cohorts.

$M(\tau, 0)$ for $\tau \in [-\omega, 0]$ represent the age structure of natives and immigrants at the starting point of the economy. The native cohort with birth date τ changes over time according to

$$\frac{dN(\tau, t)}{dt} = -N(\tau, t)\mu(t - \tau). \tag{1}$$

The births are given as a boundary condition for this equation at $\tau = t$ as

$$N(\tau, \tau) = \int_{\tau-\omega}^{\tau} f(\tau - s) \left(N(s, \tau) + M(s, \tau)\right) \mathrm{d}s, \tag{2}$$

where $f(\cdot) \in C[0, \omega]$ denotes the age-specific fertility rate. Again we assume that

$$NRR < 1.$$

Additionally, we assume that the children of immigrants are part of the native population and that immigrants and natives have the same age-specific fertility and mortality rates. The dynamics of the population $M(\tau, t)$ reads as,

$$\frac{dM(\tau, t)}{dt} = -M(\tau, t)\mu(t - \tau) + m(\tau, t), \quad M(\tau, \tau) = 0. \tag{3}$$

Here, $m(\tau, t)$ denotes the age-specific and possibly time-varying immigration profile. Then, the number of natives in the population at time t is given by

$$N(t) = \int_{t-\omega}^{\omega} N(t, \tau)\ d\tau,$$

and the number of immigrants is

$$M(t) = \int_{t-\omega}^{\omega} M(t, \tau)\ d\tau.$$

1.2.2. Individual Optimal Behavior

Let the utility from consumption $c > 0$ of any individual be denoted by $u(c)$ and consider a CRRA-utility function

$$u(c) = \begin{cases} \frac{c^{1-\sigma}}{1-\sigma} & \text{if } \sigma \in (0, 1) \cup (1, +\infty), \\ \ln(c) & \text{if } \sigma = 1, \end{cases}$$

where σ is the risk aversion coefficient and $1/\sigma$ is the intertemporal elasticity of substitution between consumption over time. The higher $1/\sigma$, i.e. the lower the risk aversion coefficient, the more willing is the household to substitute consumption over time. Function $u(c)$ belongs to the family of constant relative risk aversion utilities (CRRA).

Since in our considerations the age of dying $a \in [0, \omega]$ is not a fixed number but a random variable, we adopt the expected utility hypothesis. An individual of cohort τ chooses a consumption profile $c(\tau, \cdot)$ such that her expected life-time discounted utility $\mathrm{E}[u]$ is maximized.

The subjective discount rate is denoted by ρ. It defines how the preference for consumption decreases over the life time and is assumed to be constant.

Agents have perfect foresight meaning that agents perfectly forecast the rates of return on capital, $r(t)$, and labor, $w(t)$. Consequently, they supply labor such that its actual return, in form of wage, meets their expectations and the same holds for the saving decision determining the supplied capital and the expected return on capital. A typical life cycle consists of a schooling period, h, a work period and retirement after the fixed age R. We assume that people born at time τ are identical from the economic point of view.

An agent of the cohort born at time τ takes the real return on assets $r(t)$ and the wage rate $w(t)$ as given and chooses her consumption in order to maximize her expected utility

$$\int_{\tau}^{\tau+\omega} e^{-\int_{\tau}^{t}(\rho+\mu(\eta-\tau))d\eta} \, \frac{c^{\sigma}(\tau,t)}{\sigma} \, \mathrm{d}t \tag{4}$$

subject to the flow dynamics

$$\begin{aligned} \frac{da(\tau,t)}{dt} &= (r(t)+\mu(t-\tau))a(\tau,t) + (1-\theta)w(t)e(h,t-\tau) - c(\tau,t) \\ &\quad + \mathbb{I}_{[R,\omega]}(t-\tau)p(t), \ \ t \in (\tau, \tau+\omega). \end{aligned} \tag{5}$$

Here, $a(\tau,t)$ denote the financial assets of an agent born in τ at time t. We assume that each agent depending on her age $t-\tau$ is endowed with *efficient units of labor*, $e(h,t-\tau)$: $[0,R]\times[0,\omega] \to [0,\infty)$, i.e. for a given age $t-\tau$ function $e(h,\cdot)$ determines her productivity in the production process. Consequently, labor income equals $w(t)e(h,t-\tau)$.

For a fixed number of schooling years h we define

$$e(h,t-\tau) = \begin{cases} e^{g(h,t-\tau)} & \text{if } h \le t-\tau \le R, \\ 0 & \text{otherwise,} \end{cases}$$

where $g(h,t-\tau) := \delta_1 h + \delta_2(t-\tau-h) + \delta_3(t-\tau-h)^2$ and $\delta_1, \delta_2 > 0$ and $\delta_3 < 0$.

Therefore, the productivity, and consequently, the wage of an agent is a concave function in work experience measured in working years $t-\tau-h$. This representation of $e(h,t-\tau)$ follows [13], where the logarithm of wages is modeled as the sum of a linear function of years of education h and a quadratic function of years of experience. During schooling and after retirement there is no supply of labor. A share θ of the labor income must be paid into a Pay-As-You-Go (PAYG) pension system. They benefit from the payments of the working cohorts when they retire in form of pension payments $p(t)$, $t \in [\tau+R, \tau+\omega]$.

Agents hold all their assets in form of annuities, cf. [27]. Since the life-insurance company redistributes the wealth of the agents who died to those who survived in the same age cohort, the real rate of return $r(t)$ is augmented by the age-specific mortality rate $\mu(t-\tau)$.

Agents have no assets when they enter the economy except of those who are alive at time $t=0$:

$$a(\tau,0) \text{ given, if } \tau \in (-\omega, 0), \tag{6}$$

$$a(\tau,\tau) = 0, \text{ if } \tau \ge 0. \tag{7}$$

Moreover, one cannot die indebted[2]:

$$a(\tau, \tau + \omega) = 0. \tag{8}$$

Since the individual utility maximizing problem (4)–(8) constitutes a dynamic optimization problem, we apply Pontryagin's maximum principle to obtain the optimal consumption profile. The corresponding present value Hamiltonian reads as

$$\begin{aligned}\bar{H}(t, \tau, c, a, \lambda) = {} & e^{-\rho(t-\tau)} l(t-\tau) \frac{c^{1-\sigma}(\tau, t)}{1-\sigma} \\ & + \bar{\lambda}(\tau, t)((r(t) + \mu(t-\tau))a(\tau, t) + (1-\theta)w(t)e(h, t-\tau) \\ & - c(\tau, t) + \mathbb{I}_{[R,\omega]}(t-\tau)p(t)).\end{aligned}$$

The first order necessary optimality conditions are:

$$\frac{\partial \bar{\lambda}(\tau, t)}{\partial t} = -\frac{\partial \bar{H}(t, \tau, c, a, \lambda)}{\partial c} = -\frac{\delta H}{\delta a} = -\bar{\lambda}(\tau, t)(r(t) + \mu(t-\tau)), \tag{9}$$

$$\frac{\partial \bar{H}(t, \tau, c, a, \lambda)}{\partial c} = e^{-\rho(t-\tau)} l(t-\tau) c^{-\sigma}(\tau, t) - \bar{\lambda}(\tau, t) = 0. \tag{10}$$

From (10) we obtain the following expression for the optimal consumption profile

$$c(\tau, t) = e^{\frac{-\rho(t-\tau)}{\sigma}} \left(\frac{\bar{\lambda}(\tau, t)}{l(t-\tau)} \right)^{\frac{-1}{\sigma}}. \tag{11}$$

By introducing

$$\lambda(\tau, t) := \bar{\lambda}(\tau, t) e^{\rho(t-\tau)} \frac{1}{l(t-\tau)} \tag{12}$$

we derive a differential equation for the shadow price that is independent of the mortality function:

$$\frac{\partial \lambda(\tau, t)}{\partial t} = \frac{\partial \bar{\lambda}(\tau, t)}{\partial t} \frac{e^{\rho(t-\tau)}}{l(t-\tau)} + \rho e^{\rho(t-\tau)} \frac{\bar{\lambda}(\tau, t)}{l(t-\tau)} - e^{\rho(t-\tau)} \bar{\lambda}(\tau, t) \frac{l'(t-\tau)}{l^2(t-\tau)}.$$

And by using the optimality condition (9) and relation (12) we find that

$$\frac{\partial \lambda(\tau, t)}{\partial t} = (-r(t) + \rho)\lambda(\tau, t)$$

holds. Therefore, the equation

$$\lambda(\tau, t) = e^{\int_\tau^t (-r(\eta)+\rho)\, \mathrm{d}\eta} \lambda_0(\tau) \tag{13}$$

[2]In fact we should require that, $a \geq 0$, but at the optimum equality holds.

holds. Inserting (13) into expression (11) and defining

$$\tilde{\lambda}(\tau) := \lambda_0^{\frac{-1}{\sigma}}(\tau),$$

yields an expression for $c(\tau, t)$ that is solely represented by exogenous variables except for the initial value $\lambda_0(\tau)$ which yet has to be determined

$$c(\tau, t) = e^{\frac{-1}{\sigma}\int_\tau^t (r(\eta)-\rho)\,\mathrm{d}\eta}\tilde{\lambda}(\tau). \tag{14}$$

We substitute consumption (14) into the budget constraint (5) and determine $\tilde{\lambda}(\tau)$ in such a way that the boundary conditions (6), (8) or alternatively (7)–(8) are fulfilled.

More precisely, for $(\tau, t) \in \{(\tau, t) : \tau \in (0, \infty), t \in [\tau, \tau + \omega)\}$ the optimal consumption is given by

$$c(\tau, t) = \tilde{\lambda}(\tau)e^{\frac{1}{\sigma}\int_\tau^t (r(\eta)-\rho)\,\mathrm{d}\eta},$$

with

$$\tilde{\lambda}(\tau) = \frac{\int_\tau^{\tau+\omega} e^{-\int_\tau^t (r(\eta)+\mu(\eta-\tau))\,\mathrm{d}\eta}\big((1-\theta)w(t)e(h, t-\tau) + \mathbb{I}_{[R,\omega]}(t-\tau)p(t)\big)\,\mathrm{d}t}{\int_\tau^{\tau+\omega} e^{\int_\tau^t (\frac{1}{\sigma}((1-\sigma)r(\eta)-\rho)+\mu(\eta-\tau))\,\mathrm{d}\eta}\,\mathrm{d}t},$$

and for $(\tau, t) \in \{(\tau, t) : \tau \in (-\omega, 0), t \in (0, \tau + \omega)\}$

$$c(\tau, t) = \tilde{\lambda}(\tau)e^{\frac{1}{\sigma}\int_\tau^t (r(\eta)-\rho)\,\mathrm{d}\eta},$$

$$\tilde{\lambda}(\tau) = \frac{a(\tau, 0)}{\int_0^{\tau+\omega} e^{\int_\tau^t (\frac{1}{\sigma}((1-\sigma)r(\eta)-\rho)+\mu(\eta-\tau))\,\mathrm{d}\eta}\,\mathrm{d}t} + \frac{\int_0^{\tau+\omega} e^{-\int_\tau^t (r(\eta)+\mu(\eta-\tau))d\eta}\big((1-\theta)w(t)e(h, t-\tau) + \mathbb{I}_{[R,\omega]}(t-\tau)p(t)\big)\,\mathrm{d}t}{\int_0^{\tau+\omega} e^{\int_\tau^t (\frac{1}{\sigma}((1-\sigma)r(\eta)-\rho)+\mu(\eta-\tau))\,\mathrm{d}\eta}\,\mathrm{d}t}.$$

The case of $c(\tau, t) < 0$ for some t can be ruled out because it could only happen for $\tilde{\lambda}(\tau) < 0$ which would imply, see Equation (14), that the shadow price would be negative for all $t \in [\tau, \tau + \omega]$, thus $c(\tau, t)$ would be negative for all t, which contradicts the optimality of $c(\tau, \cdot)$.

1.2.3. Endogenous education decision

It is well-known that education plays an important role when it comes to economic performance of a country in general, and hence also when one aims to determine economic effects of immigration because immigrants, among other things, change the skill composition of the labor force. Whereas many models, cf. [11, 15], consider different skill groups to account of the educational heterogeneity in the population, here we explicitly model the accumulation of human capital of the agent which determines her efficiency in the production process and therefore is related to her skill level. While it is assumed that immigrants have an exogenous, fixed education level when they enter the country, a native agent endogenously chooses her optimal period of education. Children of immigrants can become higher educated than their parents when they choose to be so. In general, human capital can be accumulated through education and/or learning-by-doing. Here, we only allow for an education period at the beginning of the life time. In the beginning of the life-cycle agents dedicate their time to education and during that time they do not work. An increase of education leads to an increase in the efficiency units of labor.

Subsequently, we model the agent's decision on her optimal length of schooling. Doing so, a rational agent compares the future income stream of an additional schooling year with the potential income of quitting schooling now.

We assume that the decision of quitting school is once and for all. We obtain a necessary condition for the optimal number of schooling years by using the Lagrange method. Since the only heterogeneity here is the vintage of the cohort represented by τ, all members of a cohort receive the same education. Therefore, the schooling period is a function of the vintage τ, $h(\tau)$. Wherever we consider a fixed cohort τ we suppress the dependence on τ and simply write h.

The schooling problem of those belonging to the cohort τ reads as

$$\max_{h\in[0,R]} \int_0^\omega e^{-\int_0^s(\rho+\mu(\eta))d\eta} u(c(\tau,\tau+s))\,\mathrm{d}s,$$

subject to the budget constraint

$$\begin{aligned}&\int_0^\omega e^{\int_s^\omega(r(\tau+\eta)+\mu(\eta))d\eta}((1-\theta)w(\tau+s)e(h,s)+\mathbb{I}_{[R,\omega]}(s)p(\tau+s))\,\mathrm{d}s\\ &=\int_0^\omega e^{\int_s^\omega(r(\tau+\eta)+\mu(\eta))d\eta}c(\tau,\tau+s)\,\mathrm{d}s. \qquad (15)\end{aligned}$$

Equation (15) is obtained by using the Cauchy formula for the linear differential equation in (5).

The corresponding Lagrangian $\mathcal{L}$ reads as

$$\begin{aligned}\mathcal{L}(h,\mu)=\int_0^\omega &(e^{-\int_0^s(\rho+\mu(\eta))\,\mathrm{d}\eta}u(c(\tau,\tau+s))\\ &+\mu(e^{\int_s^\omega(r(\tau+\eta)+\mu(\eta))\,\mathrm{d}\eta}((1-\theta)w(\tau+s)e(h,s)\\ &+\mathbb{I}_{[R,\omega]}(s)p(\tau+s)-c(\tau,\tau+s))))\,\mathrm{d}s.\end{aligned}$$

Hence by using the necessary condition $\frac{\partial\mathcal{L}}{\partial h}=0$, we obtain the optimal number of schooling years h,

$$\begin{aligned}&\int_h^R e^{\int_s^\omega(r(\tau+\eta)+\mu(\tau+\eta))\,\mathrm{d}\eta}(1-\theta)w(\tau+s)\frac{\partial e(h,s)}{\partial h}\,\mathrm{d}s\\ &=e^{\int_h^\omega(r(\tau+\eta)+\mu(\eta))\,\mathrm{d}\eta}(1-\theta)w(\tau+h)e(h,h). \qquad (16)\end{aligned}$$

Observe that

$$\frac{\partial e(h,s)}{\partial h} = \begin{cases} 0 & \text{if } s < h, s > R \\ e^{g(h,s)}(\delta_1 - \delta_2 - 2\delta_3 s + 2\delta_3 h) & \text{if } s \in [h, R]. \end{cases}$$

The right hand side of Equation (16) is the expected forgone income when not realizing h as the number of years to be spent at school and the left hand side determines the expected gain during the remaining working years from postponing the working entry age.

The term $\frac{\partial e(h,s)}{\partial h}$ in the left hand side of Equation (16) determines the resulting marginal increase in productivity for age s.

The optimal schooling time is not easy to determine explicitly for non-constant $r(t)$ and $w(t)$ and general, non rectangular survival laws. In our model $w(t)$ and $r(t)$ are determined endogenously through profit maximizing of the representative firm at every instant of time. In general, (16) can only be solved numerically and only provides a necessary condition for the optimal $h(\tau)$. Moreover, existence of an optimal number of school years is not granted.

To simplify the implicit relation (16) for h we make the assumption of $\delta_3 = 0$, which reflects a linear increase of efficiency in experience. Then we obtain

$$\begin{aligned}(\delta_1 - \delta_2) \int_h^R & e^{-\int_0^s r(\tau+\eta)\,\mathrm{d}\eta} e^{\delta_2(s-h)} l(s)(1-\theta)w(\tau+s)\,\mathrm{d}s \\ &= e^{-\int_0^h r(\tau+\eta)\,\mathrm{d}\eta} l(h)(1-\theta)w(\tau+h). \end{aligned} \tag{17}$$

For constant r and w and a rectangular survival function, (17) reduces to

$$(\delta_1 - \delta_2) \int_h^R e^{-rs} e^{\delta_2(s-h)}\,\mathrm{d}s = e^{-rh}.$$

Hence,

$$(\delta_1 - \delta_2) e^{-\delta_2 h} \frac{-1}{r - \delta_2} \left(e^{R(\delta_2 - r)} - e^{(\delta_2 - r)h} \right) = e^{-rh}.$$

Then, the explicit expression for the optimal solution h reads as

$$h = R + \frac{1}{r - \delta_2} \ln\left(1 - \frac{r - \delta_2}{\delta_1 - \delta_2}\right).$$

We see that h is independent of the constant wage rate w.

1.2.4. Government

Each time t the government collects taxes θ on labor to finance the implemented PAYG pension system. It is required that at any time t the government must have a balanced budget:

$$\begin{aligned}\theta w(t) & \int_{t-R}^{t} \left(e(h(\tau), t-\tau) N(\tau,t) + e(h^M, t-\tau) M(\tau,t) \right) \mathrm{d}\tau \\ &= p(t) \int_{t-\omega}^{t-R} (N(\tau,t) + M(\tau,t))\,\mathrm{d}\tau.\end{aligned}$$

1.2.5. Firms

In our model economy agents interact with firms. We apply the representative firm hypothesis. The firm produces output $Y(t)$ with labor $L(t)$ and capital $K(t)$ as input factors. The firm pays wages for labor input and borrows the services of capital from households and also pays for these services. The production function is of neoclassical type,

$$Y(t) = F(K(t), L(t)) = K^{\alpha}(t)L^{1-\alpha}(t),$$

where

$$\begin{aligned} K(t) &= \int_{t-\omega}^{t} N(\tau,t)a(\tau,t)\,\mathrm{d}\tau, \\ L(t) &= \int_{t-R}^{t} \big(e(h(\tau), t-\tau)N(\tau,t) + e(h^M, t-\tau)M(\tau,t)\big)\,\mathrm{d}\tau. \end{aligned}$$

We assume here that immigrants and natives are perfect substitutes. Output can either be used for consumption or for increasing the capital stock. Firms maximize their profits by choosing capital $K(t)$ and labor $L(t)$ in an optimal way. The firm's problem reads as

$$\max_{K,L}\{Y(t) - R(t)K(t) - w(t)L(t)\}.$$

Factors receive their marginal products,

$$\begin{aligned} R(t) &= F_K(K(t), L(t)), \\ w(t) &= F_L(K(t), L(t)). \end{aligned}$$

Let us denote by $k(t) = \frac{K(t)}{L(t)}$ the capital-(effective) labor ratio and let $f(k) := k^{\alpha}$. Therefore the factor returns can be obtained by

$$\begin{aligned} R(t) &= f'(k(t)), \\ w(t) &= f(k(t)) - f'(k(t))k(t). \end{aligned}$$

1.3. Numerical Experiments

Subsequently, we consider a benchmark case where at the moment of shock the economy as well as the population are in a steady state.

Demography

In [1] it was shown that any population with below-replacement fertility and a constant number of annual immigrants with a fixed age distribution as well as constant age-specific mortality rates, eventually converge to a stationary population.

Here, for each $a = t_0 - \tau$ we calibrate $N(a, t_0)$ with the number of members of cohort τ in the native female population of Austria in 2001 and $M(a, t_0)$ is the number of individuals born outside the country of the corresponding cohort τ [3]. We simulate equations (1) – (3)

[3] No later data could be found for $M(a, t_0)$.

with constant age-specific fertility rates $f(a)$, where again $a = t - \tau$, and constant age-specific mortality rates $\mu(a)$ and a constant inflow of immigrants $m(a)$ until a stationary population is reached, see Figure 1. In the following, time is measured in years.

For the fertility rates $f(a)$ and the immigration rates $m(a)$ we took linearly interpolated Austrian data of 2008. For the numerical examples below we follow [5] and consider a survival function of the form

$$l(a) = \frac{e^{-a\mu_0} - \epsilon}{1 - \epsilon},$$

with $\epsilon > 1, \mu_0 < 0$. This survival law fulfills $l(0) = 1$ and ω is determined such that $l(\omega) = 0$ holds,

$$\omega = -\frac{\ln(\epsilon)}{\mu_0}.$$

Therefore, $\lim_{a \to \omega} \mu(a) = +\infty$. We fully specify $l(a)$ by setting $\mu_0 = 0.068$ and $\omega = 80$. For these specifications, the net reproduction rate (NRR) is approximately 0.7, which is below replacement level.

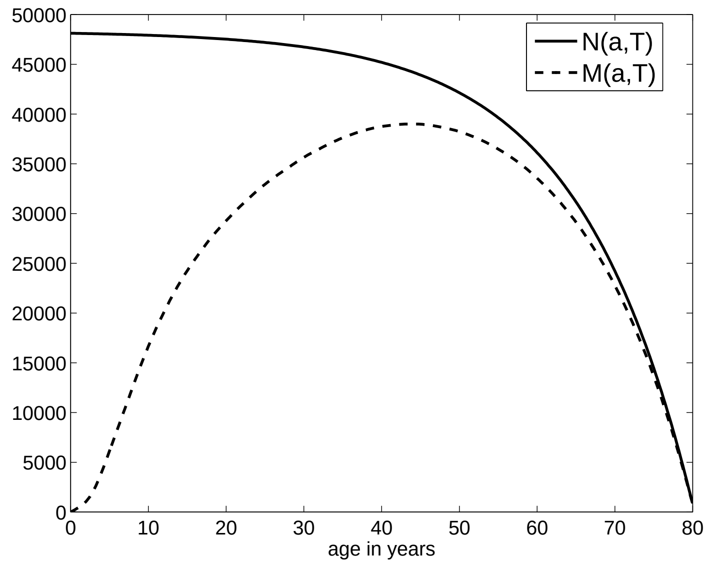

Figure 1: Steady state population age structure at time of the shock

Figure 1 shows the steady state age structure of the two sub-populations. Notice that in the stationary population the share of immigrants is about 35%.

Economic Parameters

We set the initial assets profile $a(\tau, 0)$ to the steady state solution,

$$a(\tau, 0) = a(t - \tau),\ \tau < t,$$

before the immigration shock. Table 1 summarizes the important parameters for the calculations we present in this section. In the economic model age 0 corresponds to the real

Parameters	*Core Model*
education migr. h_i^m	0,6,11
retirement age R	49 (65-16)
μ_0	0.068
life span ω	80 (96-16)
tax θ	0.12
CES σ	1
capital share α	1/3
δ_1	0.041
δ_2	0
δ_3	0
time pref. rate ρ	0
depreciation rate δ	0
shock period	$t \in [100, 105]$

Table 1: Parameter calibration

age of 16, because this is the age when compulsory schooling typically ends. The period of the life-cycle before age 16 is not modeled explicitly. The consumption of these agents is assumed to be part of the parents consumption.

During the additional education time agents accumulate debts due to their lack of labor income.

Immigration shock

We normalize time such that the time when the immigration shock happens is $t = 0$. The immigration shock is modeled as a doubling of the number of immigrants from a pre-shock value of 35000 annual immigrants and lasts for 5 years. During this time twice as many immigrants enter the country while the age structure is held constant.

Such a scenario could be compared to the years 1989 -1993, where due to the war in former Yugoslavia, the numbers of net migrants to Austria where in some years even three times as high. Figure 2 shows how the number of natives $N(t)$ and immigrants $M(t)$ change over time as a consequence of the shock. The immigration shock leads in later consequence to a higher number of natives, since the immigrants children are assumed to integrate themselves fully in the host country.

Numerical Results

We first analyze the case where also the education of the natives is exogenously given and consider three scenarios with respect to the educational achievements of immigrants. First, we assume that immigrants who enter have only the basic education, $h_1^m = 0$, then we consider that immigrants obtain the same number of schooling years as natives, $h_2^m = h = 6$, and the last case reflects an inflow of immigrants with a high education i.e. $h_3^m = 11$ [4]. We

[4]However, this last scenario $h_3^m = 11$ does not really suit our setting, because high educated immigrants, might as well (similar to high educated natives) accumulate savings and therefore would also contribute to

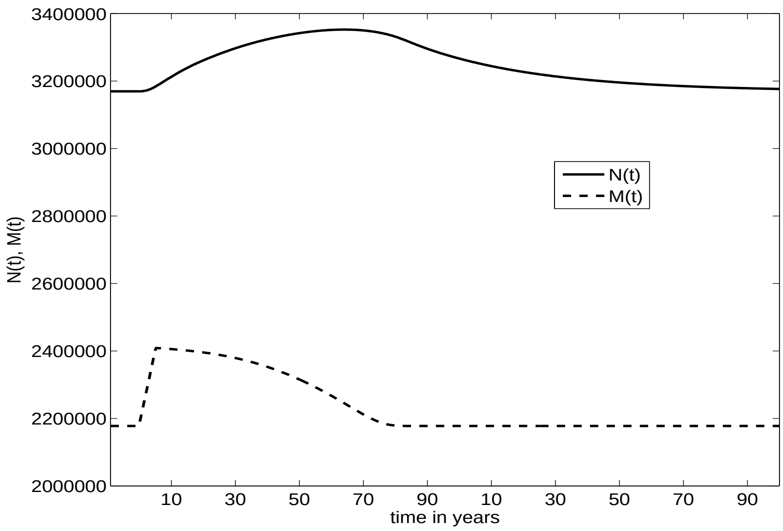

Figure 2: Number of natives and immigrants over time: natives (solid); immigrants (dashed)

then compare the utility changes of the different cohorts for all three scenarios.

Due to the increase in the number of immigrants at the beginning of the shock, i.e. at time $t = 0$, the capital (effective) labor ratio $k(t)$ decreases. Consequently, the interest rate goes up and the wage rate goes down as can be seen in Figure 3. This favors those who are owners of capital and affects adversely workers.

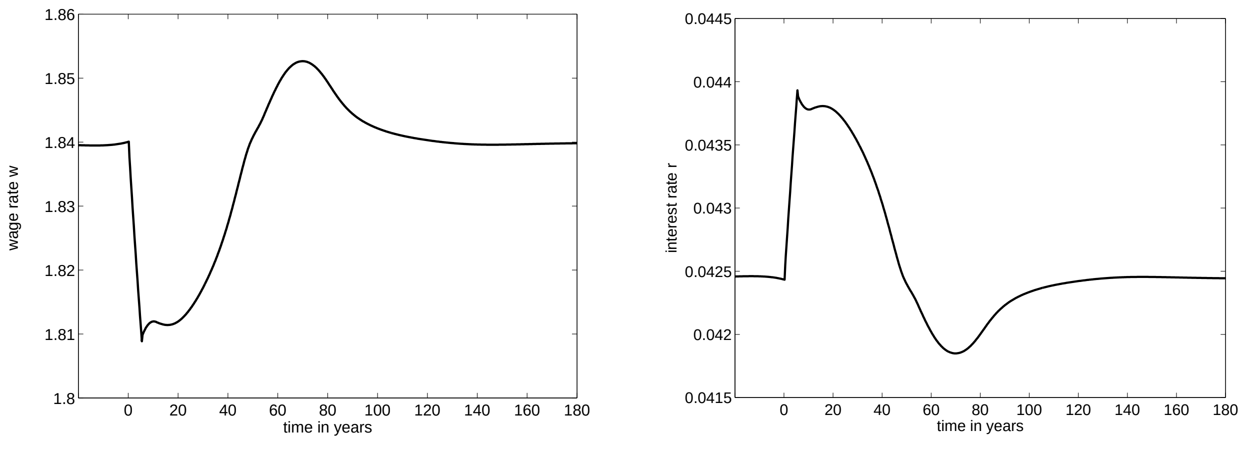

Figure 3: Wage rate (left) and interest rate (right) over time for h_1^m

Figure 4 represents the immigration shock effects for the welfare of different cohorts. It shows the relative change in life time utility over time for the various cohorts compared to the steady state value for the various h_i^m.

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the capital stock.

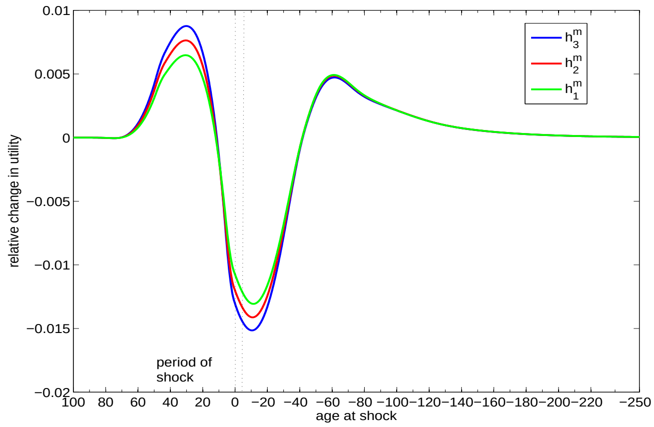

Figure 4: Relative change in welfare for different cohorts: $h_1^m = 0$ (green), $h_2^m = 6$ (blue), $h_3^m = 11$ (red) for $h = 6$

The figure shows that the cohorts who are middle aged and old at the time of the shock benefit. This is because they are the owners of capital at that time. The welfare is the highest for those who are currently 46 (30 + 16) years old at the time of the shock. The second peak in Figure 4 is due to the flattening out of the number of immigrants and the fact that equipped offspring of the immigrants enter the economy. This causes a peak in the capital labor ratio. As a consequence the interest rate r decreases and the wage rate w increases. However, this peak is already damped as compared to the initial one.

The cohorts which have the severest drawbacks of the immigration shock are those who enter the economy in the decade after the shock. This is because they face very high interest rates at the beginning of their lifetime, when they actually accumulate debts because they are still educating themselves. Moreover, they face a very low wage rate during their working life and decreasing interest rates. With respect to the different education of the immigrants, one can say that the higher the education of the immigrants, the more severe are the effects on the utility.

In a next step, we endogenize the education decision of the native individuals in order to see how the increase of labor effects the skill composition in the country. In the left graphics of Figure 5 the utility changes for various cohorts in case of an endogenous determination of the schooling period by the native agents is depicted. It shows that endogenous education slightly decreases the loss of future generations and the gain of old generations. Therefore, by choosing their education optimally young natives can damp the negative effect of the immigration shock on their life time utility.

In the right Figure 5 the change in the length of the schooling period of the various cohorts of vintages τ younger than the shock is depicted. One observes that at the shock the schooling period goes down. This fall is then followed by a period where cohorts go even longer to school then before the shock.

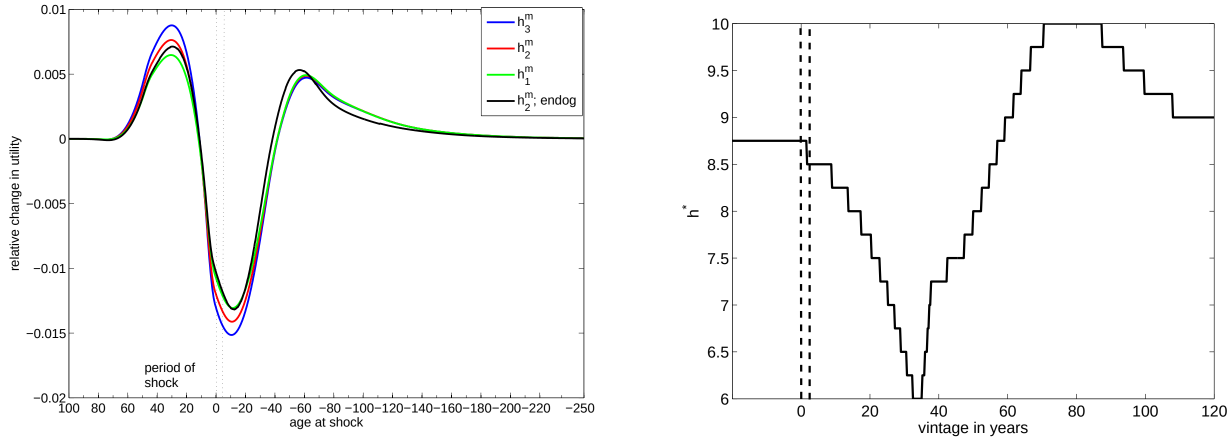

Figure 5: Relative change in welfare for different cohorts: $h_1^m = 0$ (green), $h_2^m = 6$ (blue), $h_3^m = 11$ (red), $h_2^m = 0$ with endogenous $h(\tau)$ (left). Optimal schooling time (right)

1.3.1. Conclusions

The model presented in this section focuses on the welfare effects induced by an immigration shock. The shock is modeled by an increase of the number of immigrants for a short period of time. By developing a continuous time overlapping generations model for a closed economy, we determine the resulting changes in life-time utility of different age cohorts. Numerical results for Austrian data are provided.

We conclude that the immigration shock is welfare improving for those cohorts being at the end of their working life or already retired. They benefit from the increased interest rate. Moreover, retirees may have an additional benefit from the increased tax payments of the higher number of workers due to the incoming immigrants. However, since the wage rate goes down, pension payments might also go down. So the effect of the immigration shock on pensions of retirees is not unambiguous.

The shock leads to the highest decrease in life cycle utility of those cohorts born during or after the shock. This result is interesting in terms of immigration policies. It implies that young cohorts would prefer closed borders whereas older cohorts would not. Moreover, we may conclude that the increased number of immigrants during the shock leads to an increase in the work force and therefore the wage rate goes down due to increased competition. This is accompanied by an increase in the interest rate because of the induced reduction in the capital-labor ratio.

1.3.2. Outlook

The present model can be extended in various ways. So far it only represents the first attempt to depict possible effects of an inflow of individuals who change the demographic and economic situation in the host country.

So far it is assumed that immigrants' offspring make the same life-cycle decisions as the natives of the same cohort do and are therefore considered as natives. In further investigations one could relax this assumption and investigate how the results change if a share of the children still behave as their parents did.

Moreover, so far we haven't accounted of emigration.

By considering a different pension system rather than the contribution defined PAYG system one may get more insight in what is driving the results and how they depend on this modeling assumption.

It would be of particular interest how the results depend on the steady-state age structure of the population. Since changes in the two factor returns, interest rate r and wage rate w, affect different cohorts, one may expect that the intermediate-term effects induced by the shock depend critically on the age structure.

2. Long-run impact of age-specific immigration

2.1. Introduction

The subsequent model investigates the long-run impact of immigration by explicitly modeling the life-cycle of the immigrants in the host country.

The aging of the populations of Western countries goes hand in hand with the aging of their labor forces. This has severe effects on the sustainability of the social security system and especially the pension schemes. In many countries the pension system is a so-called pay-as-you-go (PAYG) system, where the currently working population pays for those in retirement. Hence, a growing number of older persons in comparison to a shrinking labor force, caused by low mortality rates at older ages and additionally low fertility rates, implies a shrinking money flow into the system. In what follows, the Austrian PAYG pension system will be mimed.

One remedy for population aging may be to step up immigration. It is a common belief that since the age-structure of immigrants is younger than the one of the native population, immigration could help to reduce the fiscal imbalance caused by the aging process. However, clearly these immigrants would also grow older and hence many people argue that in the long-run there would be no positive effect of immigration with respect to the fiscal balance.

In [24] it was shown that for a calibration with US data immigration is slightly beneficial for the government finances. [7] investigated how an immigration policy which consists of the admission of unskilled immigrants, whose children incur assimilation costs in order to become skilled workers, positively influences the net pension benefits for native residents and immigrants under a defined-benefit pension system. They find that native residents do not always become net beneficiaries, even if the government admits an unlimited number of immigrants. [7] also shows that this result does not hold in a defined contribution system.

Empirical studies on the effect of immigration on the Austrian economy have already been made, for example, by [26] and [12]. In [26], it was investigated how an increase in immigration affects wages of young native blue collar workers in Austria. It is found that in regions, industries, or firms with a higher share of foreign workers, natives earn higher wages.

In an empirical paper, [12] used the general accounting method to study the intertemporal fiscal impact of immigration to Austria. It is concluded that under the assumption that future immigrants resemble those of the current immigration the total fiscal effect of immigration is positive. The reasons for the positive effect of immigration are (i) the young age structure, and (ii) lower per capita net transfer payments during retirement compensating for lower per capita net tax payments during working age. We try to replicate these empirical findings in a theoretical model where we explicitly take into account the age structure of immigrants and the fact that immigrants qualify for lower pensions.

Hence, in contrast to the aforementioned theoretical papers, where typically two- or three-period OLG models are considered and immigrants are assumed to arrive either in period one or period two, which roughly distinguishes between immigrants who arrive as children or in adulthood, we explicitly model how the age-structure of the immigrants affects the host country. With the subsequent model one may tackle the following questions:

- What is the long-term effect of immigration on the sustainability of the pension system measured in terms of the social secruity rate and the pensions-to-output ratio?
- Are immigrants net beneficiaries or net payers of the pension system?
- Since the age of the immigrants has a strong impact on the age structure and size of a population the obvious question is whether the age structure of the immigrants matters for the pension system, and how?

While in [7] a two-period OLG model is considered, we consider a continuous time OLG model, where a continuum of overlapping generations coexist at the same time. This allows a more accurate modeling of the demography.

Clearly, under the assumption of preserving below-replacement fertility, immigration is needed in order to avoid a major shrinking and aging of the population. Hence, we assume that fertility would remain on low levels, and investigate how age-specific immigration rates would be able to compensate for this. Moreover, we assume that immigrants have higher fertility than natives.

We explicitly model a pension system which realistically resembles current practice in many European countries and consider a pay-as-you-go pension system with defined benefits such as it is the case in Austria. In this work we focus on the steady state, and it is assumed that the government budget is balanced. Hence, there are no debts.

From an economic point of view, immigrants and natives differ significantly. Hence, in contrast to other macroeconomic models such as [6], where the focus was solely on the macroeconomic aspect of immigration, we explicitly distinguish between natives and immigrants in the model. As a matter of fact, they participate quite differently in the pension system. While natives spend their majority or even the whole working life in their home country and consequently earn high pensions, many immigrants arrive in the middle of their productive period and hence qualify for lower pensions in the host country. In what follows, we investigate how this difference is reflected in terms of the pension system. Moreover, immigrants and natives also differ during their productive period, which affects their contributions to the pension system. We model this difference by allowing that immigrants and natives do not act as perfect substitutes in the production process and they may have different productivity profiles. This leads to different wages for immigrants and natives.

In the numerical example we again focus on the Austrian case. We conclude that immigrants are net contributors to the pension system. In contrast to the native population the immigrant population pays more into the pension system then it earns and hence immigration contributes for the closing of the financial gap caused by the aging of the population.

In a stylized scenario, we find that although immigrants who enter in their mid-thirties spend a shorter time working in the host country and lead to a sharp increase of the aging ratio they still lead to a smaller social contribution rate and a lower pensions-to-output ratio in comparison to a scenario where all immigrants enter in their early twenties. However, we also find that with immigration alone it is not possible to keep the social contribution rate on current levels. Hence, additional measures have to be taken for a balanced pension system.

2.2. Population dynamics

In this section, we describe the demographic side of the model which is exogenous to the economic model. We consider a bench mark demographic scenario, where under the assumption of a constant annual inflow of immigrants, we denote by $M(a, a^*)$ the number of immigrants at age a who have arrived in the country at age $a^* \leq a$. Once immigrants have migrated, they stay in the host country for the rest of their lives.

The age-specific immigration density $m(\cdot)$ fulfills

$$\int_{a_{min}}^{a_{max}} m(a^*)\,\mathrm{d}a^* = 1, \quad m(a^*) \geq 0.$$

Moreover, we assume that age-specific immigration patterns $m(\cdot)$ as well as fertility $f(\cdot)$ and mortality $\mu(\cdot)$ are time-invariant. Natives' fertility and mortality are such that fertility is again under the replacement level, i.e.

$$\int_0^{\omega} f(a)l(a)\,\mathrm{d}a < 1,$$

holds. Moreover, we assume that age-specific mortality rates of natives and immigrants are the same.

The resulting population is stationary through immigration, cf. [19], and consists of natives $N(a)$ and immigrants $M(a) = \int_{a_{min}}^{a_{max}} M(a; a^*)\,\mathrm{d}a^*$. The parameters a_{min} and a_{max} are the minimal and maximal age of immigration, where $a \wedge b := \min\{a, b\}$ and $0 < a_{min} < a_{max} < R$ holds. Here, $\omega = 110$ denotes the maximal attainable age. The number of annual intakes is given by the exogenous parameter I which determines together with $m(\cdot)$ the size of the steady-state population. Hence, the changes in age structure of the immigrants follows the subsequent dynamic law

$$M'(a; a^*) = -\mu(a)M(a; a^*), \;\; a^* < a < \omega, \tag{18}$$

$$M(a^*; a^*) = m(a^*)I, \;\; a^* \in [a_{min}, a_{max}]. \tag{19}$$

Here, $M'(a; a^*)$ denotes the derivative with respect to age a. The age structure of the native population fulfills

$$N'(a) = -\mu(a)N(a), \;\; 0 < a < \omega, \tag{20}$$

$$N(0) = \int_0^{\omega} \left(f(a)N(a) + f_M(a) \int_{a_{min}}^{a_{max} \wedge a} M(a; a^*)\,\mathrm{d}a^* \right) \mathrm{d}a, \tag{21}$$

where $N(a)$ gives the number of natives of age a and $f_M(\cdot)$ is the age-specific fertility of immigrants. Equation (21) gives the number of births in the population, where it is assumed that the children of immigrants are considered as natives. This means that while immigrants of the first generation have a higher fertility than the average native, $f_M \geq f$, their children, i.e. immigrants of the second generation, show already no significant difference in their child bearing behavior compared with natives. According to [21], the fertility of immigrants converges to the fertility levels of the host country. We assume that this assimilation happens within one generation which is also indicated by some data as mentioned in [21].

According to the Cauchy formula, the solution of (18) – (19) reads as follows:

$$M(a,a^*) = Im(a^*)l(a), \ a^* < a < \omega, \tag{22}$$

and hence

$$\begin{aligned} M(a) &= \int_{a_{min}}^{a_{max}} M(a;a^*)\,\mathrm{d}a^*, \\ &= \int_{a_{min}}^{a_{max}\wedge a} \frac{l(a)}{l(a^*)} m(a^*) I \,\mathrm{d}a^*, \ a_{min} < a < \omega. \end{aligned}$$

As before $l(a) = e^{-\int_0^a \mu(s)\,\mathrm{d}s}$. For the solution of (20) it holds that

$$\begin{aligned} N(a) &= e^{-\int_0^a \mu(s)\,\mathrm{d}s} N(0) \\ &= l(a)N(0). \end{aligned}$$

Inserting this into (21) gives

$$\begin{aligned} N(0) &= N(0)\int_0^\omega f(a)l(a)\,\mathrm{d}a + \int_0^\omega f_M(a)M(a)\,\mathrm{d}a, \\ &= \frac{\int_0^\omega f_M(s)M(s)\,\mathrm{d}s}{1-\int_0^\omega f(s)l(s)\,\mathrm{d}s}, \end{aligned}$$

and hence

$$N(a) = \frac{\int_0^\omega f_M(s)M(s)\,\mathrm{d}s}{1-\int_0^\omega f(s)l(s)\,\mathrm{d}s} l(a), \ 0 < a < \omega. \tag{23}$$

2.3. The pension system

The literature distinguishes between two different prototypical social security systems: the pay-as-you-go (*PAYG*) system and the fully-funded system. In the fully-funded system, the contributions of the individuals earn the market interest. They accumulate over their working period and are paid out after retirement. In the PAYG system the currently working people finance the pensions of the retired. Due to this, the PAYG system leads to a crowding out of capital. There are two variants of the PAYG system: benefit defined (BD) and contribution defined (CD). In the BD version the pensions are fixed and the corresponding social security tax rate is determined by the general equilibrium mechanism. In the CD system the opposite holds true meaning that the pension benefits are calculated such that the government's financial goals are reached.

Subsequently, we aim to mime the Austrian pension system. The Austrian pension system consists of three pillars, where the first and dominant pillar is a PAYG. There were three major reforms: 2000, 2003, 2004, which led to changes in N_B and p_1, see [10]. The following notions are of importance:

- N_B is the *assessment period*, i.e. it equals the number of an individual's working years used for calculating the pension entitlements,
- the so-called *assessment base* is derived from the average earnings over the assessment period N_B,
- p_1 is the annual *accrual rate* of the pension, which is the percentage of the annual wage payed out as pension. The accrual rate annually adds up over the whole working period to a maximum of 80%. Currently, $p_1 = 1.78\%$

Hence, natives' pensions are given by

$$p_N = p_1 \frac{R}{N_B} \int_{R-N_B}^{R} e_N(a) w_N \, \mathrm{d}a,$$

where $e_N(a)$ are the efficiency units of labor and w_N is the wage rate. Immigrants' pension payments are determined as follows. In Austria, immigrants who arrive later than 15 years before the statutory retirement age R, i.e. $a^* \in (R-15, a_{max}]$ get a minimal pension. Otherwise public pension payments p depend on the age of arrival a^*:

$$p(a^*) := \begin{cases} p_1 \frac{R-a^*}{N_B} \int_{R-N_B}^{R} w_M e_M(a;a^*) \, \mathrm{d}a & \text{if } 0 \le a^* \le R-N_B, \\ p_1 \int_{a^*}^{R} w_M e_M(a;a^*) \, \mathrm{d}a & \text{if } R-N_B < a^* \le R-15, \\ p_1 \int_{R-15}^{R} w_M e_M(a;a^*) \, \mathrm{d}a & \text{if } R-15 < a^* < a_{max}, \end{cases} \tag{24}$$

where again $e_M(a;a^*)$ are the efficiency units of labor and w_M is the wage rate. This follows the set up of the third pillar of the Austrian pension system. It holds that $p_N = p(0)$. Here, for the sake of simplicity, we neglect pension portability from the home country to the host country. Pension portability would lead to a higher income of immigrants during their retirement but it would not affect the government budget since this part of the immigrant's pension would be financed by the sending country. Hence, we would have to deal with an open economy framework. For a discussion of pension portability see, for example, [8]. As a consequence of these assumptions, the pensions received by an average immigrant are considerably smaller and depend on her age of arrival in the host country.

2.4. Utility maximization of natives

Native households maximize their life-time utility by choosing the age-dependent consumption profile. Households are comprised of one adult and dependent children, and the number of households of a certain age is determined by the population structure. The number of new households entering the economy is determined by the country's fertility, mortality and immigration rates. It is assumed that children become independent, enter into the labor market and start a new household at age $a_0 = 18$. This is in accordance with empirical findings, cf. [17].

Let the utility from consumption $c_N > 0$ of any individual be denoted by $u(c_N)$. In the following we choose the specific utility function

$$u(c_N) = \begin{cases} \frac{c_N^{1-\sigma}}{1-\sigma} & \text{if } \sigma \in (0,1) \cup (1,+\infty), \\ \ln(c_N) & \text{if } \sigma = 1, \end{cases}$$

where σ is the risk aversion coefficient. For this particular utility function it is related to the intertemporal elasticity of substitution which is simply $1/\sigma$. The intertemporal elasticity of substitution gives the change in marginal consumption growth with respect to marginal utility growth. The higher $1/\sigma$, i.e. the lower the risk aversion coefficient, the more willing is the household to substitute consumption over time. When $\sigma \to \infty$, this is the case of infinite risk aversion. Function $u(c)$ belongs to the family of constant relative risk aversion utilities (CRRA). It is assumed that the dependence of the children is not directly reflected in the utility function as for example considered in [18].

Since in our considerations the age of dying $a \in [0, \omega]$ is uncertain, we adopt the expected utility hypothesis. Therefore, an individual chooses a consumption profile $c(\cdot)$ such that her expected life-time discounted utility $\mathrm{E}[u]$ is maximized. The subjective discount rate is denoted by ρ, and is assumed to be constant. It gives the impatience of households for consumption. A high impatience means that households weigh later time points in life less. This leads to a higher consumption at earlier ages compared to a scenario with a low value of ρ. Hence, ρ defines how the preference for consumption decreases over the life time.

Again we recall that $l'(a)$ is the unconditional probability of dying at age a. We denote by $U_N(c_N)$ the expected, discounted and aggregated utility from consumption over the whole life horizon:

$$U_N(c_N) = \int_0^\omega -l'(s) \int_0^s e^{-\rho a} u(c_N(a)) \,\mathrm{d}a \,\mathrm{d}s, \tag{25}$$

$$= -\int_0^\omega e^{-\rho a} u(c_N(a)) \int_a^\omega l'(s) \,\mathrm{d}s \,\mathrm{d}a, \tag{26}$$

$$= \int_0^\omega e^{-\rho a} u(c_N(a)) l(a) \,\mathrm{d}a, \tag{27}$$

$$= \int_0^\omega e^{-\int_0^a (\rho+\mu(\tau))d\tau} u(c_N(a)) \,\mathrm{d}a. \tag{28}$$

In Equation (26) we changed the order of integration. Moreover, it holds that $l(a) = -\int_a^\omega l'(s)\, ds$.

Households have perfect foresight and perfectly forecast the rates of return on capital, r, and labor, w. Consequently, they make their saving decisions in such a way that they meet their expectations on the return of capital. Individuals start working with 18 and retire at the fixed age R. They take the real return on assets r and the wage rate w as given and choose their consumption in order to maximize their expected utility:

$$\int_0^\omega e^{-\int_0^a (\rho+\mu(s))ds} \, \frac{c_N^{1-\sigma}(a)}{1-\sigma} \,\mathrm{d}a, \tag{29}$$

subject to the flow dynamics

$$k'_N((a) = ((r + \mu(a))k_N(a) + (1-\theta)w_N(a)e_N(a) - c_N(a) + \mathbb{I}_{[R,\omega]}(a)p_N,\ a \in (0,\omega) \tag{30}$$

Individuals have no assets when they enter the economy:

$$k_N(0) = 0. \tag{31}$$

Moreover, individuals cannot die indebted[5]:

$$k_N(\omega) = 0. \tag{32}$$

In (30), r denotes the rate of return of capital. We assume that each individual depending on her age a is endowed with $e_N(a)$ *efficient units of labor*, i.e. for a given age a function $e_N(\cdot)$ determines her productivity in the production process. Consequently, gross labor income equals $y_N(a) = w_N e_N(a)$. During schooling and after retirement the individual does not supply labor.

Individuals hold all their assets in form of annuities, cf. [27]. Due to these life-insurances, the wealth of the individuals who died are redistributed to those who survived in the same age cohort. Hence, the real rate of return r is augmented by the age-specific mortality rate $\mu(a)$.

During working life individuals pay a share θ of their labor income into a contributions defined PAYG pension system. They benefit from the payments of the working cohorts when they retire in form of pension payments p_N. Using the Cauchy formula for the life cycle profile of the financial assets in Equation (30), we obtain that

$$k(a) = \int_0^a e^{\int_s^a (r+\mu(\eta))\,\mathrm{d}\eta} \big((1-\theta)w_N e_N(s) - c_N(s) + p_N \mathbb{I}_{[R,\omega]}(s)\big)\,\mathrm{d}s. \tag{33}$$

holds.

2.4.1. Optimal consumption profile

The corresponding present-value Hamiltonian of problem (29)–(32) reads as

$$\begin{aligned} H_N = {} & e^{-\rho a}u(c_N(a)) + \lambda_N(a)((r+\mu(a))k_N(a) \\ & + (1-\theta)w_N e_N(a) - c_N(a) + \mathbb{I}_{[R,\omega]}(a)p_N). \end{aligned}$$

We apply Pontryagin's maximum principle and obtain the first order necessary optimality conditions:

$$\begin{aligned} \lambda'_N(a) &= -\lambda_N(a)(r+\mu(a)), \\ \frac{\partial H_N}{\partial c_N} &= e^{-\rho a} l(a) c_N^{-\sigma}(a) - \lambda_N(a) = 0. \end{aligned} \tag{34}$$

[5]In fact we should require that $k_N \geq 0$ but at the optimum equality holds.

Hence, we obtain

$$\lambda_N(a) = l(a)e^{-ra}\lambda_0.$$

From (34) we obtain the following expression for the optimal consumption profile

$$c_N(a) = \left(e^{\rho a}\frac{\lambda_N(a)}{l(a)}\right)^{-\frac{1}{\sigma}}. \tag{35}$$

Hence, it holds that

$$c_N(a) = e^{\frac{(r-\rho)a}{\sigma}}c_0, \tag{36}$$

where $c_0 := \lambda_0^{-\frac{1}{\sigma}}$. In order to determine c_0 we substitute consumption (35) into the dynamic budget constraint (30). Then c_0 should be determined in such a way that boundary conditions (31)–(32) are fulfilled. To this end we use (33) with $a = \omega$ and express

$$c_0 = \frac{\int_0^\omega e^{-ra}l(a)((1-\theta)w_N e_N(a) - p_N \mathbb{I}_{[R,\omega]}(a))\,\mathrm{d}a}{\int_0^\omega l(a)e^{(r(1+\frac{1}{\sigma})-\frac{\rho}{\sigma})a}\,\mathrm{d}a}.$$

With the so determined c_0, formula (36) gives an explicit representation of the optimal consumption of the native population.

2.5. Remaining Life Time Utility Maximization of Immigrants

Now let us turn to the immigrant's perspective. We assume that immigrants, once they have migrated to a new country, remain their for the rest of their lives. It is assumed that they arrive without any assets[6] and maximize their rest of life time utility out of consumption. This means, that we do not model the immigrants' life time in the home country. This is consistent because of our assumption of a closed economy and therefore do not know the economic characteristics of the rest of the world. In contrast to many other models, where it is assumed that immigrants only enter at the beginning of the life-cylce, see e.g. [6], we assume that immigrants enter the country at ages $a^* \in [a_{min}, a_{max}]$. They arrive without any assets and after their arrival they choose optimally their consumption level $c_M(\cdot; a^*)$ over the remaining life cycle. Similarly as for natives, the life-time utility of consumption $c_M(\cdot)$ is

$$\begin{aligned} U_M(c_M) &= \int_{a^*}^{\omega} -l'(s)\int_{a^*}^{s} e^{-\rho a}u(c_M(a))\,\mathrm{d}a\,\mathrm{d}s, \\ &= -\int_{a^*}^{\omega} e^{-\rho a}u(c_M(a))\int_a^{\omega} l'(s)\,\mathrm{d}s\,\mathrm{d}a, \\ &= \int_{a^*}^{\omega} e^{-\rho a}u(c_M(a))l(a)\,\mathrm{d}a. \end{aligned}$$

[6]This corresponds to the fact that immigrants use their assets for the journey to the host country or leave the assets for their dependents in the home country

Hence, the utility maximizing problem reads as,

$$\max_{c_M} \int_{a^*}^{\omega} e^{-\int_0^a (\rho+\mu(\eta))\,\mathrm{d}\eta} \frac{c_M^{1-\sigma}(a)}{1-\sigma}\,\mathrm{d}a, \tag{37}$$

subject to

$$\begin{aligned} k_M'(a;a^*) &= (r+\mu(a))k_M(a;a^*) + (1-\theta)w_M e_M(a;a^*) \\ &\quad - c_M(a;a^*) + \mathbb{I}_{[R,\omega]}(a)p(a^*), \end{aligned} \tag{38}$$

$$k_M(a^*;a^*) = 0 \quad k_M(\omega;a^*) = 0. \tag{39}$$

We assume that each individual depending on her age a is endowed with $e_M(a;a^*)$ *efficient units of labor*, i.e. for a given age a function $e_M(\cdot;a^*)$ determines her productivity in the production process. Consequently, gross labor income equals $y_M(a;a^*) = w_M e_M(a;a^*)$. In general, we allow for dependence of productivity on a^* as empirically found in [24] for the US. The constant θ denotes the wage tax and p are the pensions as explained above. Using again the Cauchy formula for the life cycle profile of the financial assets of immigrants in Equation (38) we obtain that

$$k_M(a^*;a^*) = \int_{a^*}^{a} e^{\int_s^a (r+\mu(\eta))\,\mathrm{d}\eta} \big((1-\theta)w_M e_M(s;a^*) - c_M(s;a^*) + p_M(a^*)\mathbb{I}_{[R,\omega]}(s)\big)\,\mathrm{d}s. \tag{40}$$

2.5.1. Optimal consumption profile

The corresponding present-value Hamiltonian reads as

$$\begin{aligned} H_M &= e^{-\rho a}u(c_M(a;a^*)) + \lambda_M(a;a^*)((r+\mu(a))k_M(a;a^*) \\ &\quad + (1-\theta)w_M e_M(a;a^*) - c_M(a;a^*) + \mathbb{I}_{[R,\omega]}(a)p_M(a^*)). \end{aligned}$$

The first order necessary optimality conditions are:

$$\begin{aligned} &\lambda_M'(a;a^*) = -\lambda_M(a;a^*)(r+\mu(a)), \\ &\frac{\partial H_M}{\partial c_M} = e^{-\rho a} l(a) c_M^{-\sigma}(a;a^*) - \lambda_M(a;a^*) = 0. \end{aligned} \tag{41}$$

Hence, we obtain

$$\lambda_M(a;a^*) = l(a)e^{-ra}\lambda_{a^*}.$$

From (41) we obtain the following expression for the optimal consumption profile

$$c_M(a;a^*) = \left(e^{\rho a}\frac{\lambda_M(a;a^*)}{l(a)}\right)^{-\frac{1}{\sigma}}.$$

Then

$$c_M(a;a^*) = e^{\frac{(r-\rho)}{\sigma}a} c_{a^*}, \tag{42}$$

where $c_{a^*} := \lambda_{a^*}^{-\frac{1}{\sigma}}$. We determine c_{a^*} by inserting the above expression in (40) :

$$c_{a^*} = \frac{\int_0^\omega e^{-ra} l(a)((1-\theta)w_N e_N(a) - p_M(a^*)\mathbb{I}_{[R,\omega]}(a))\,\mathrm{d}a}{\int_0^\omega l(a) e^{(r(1+\frac{1}{\sigma})-\frac{\rho}{\sigma})a}\,\mathrm{d}a}.$$

With the so determined c_{a^*} formula (42) gives an explicit representation of the optimal consumption of the immigrant population.

2.6. The government budget

In the following we give formulas for the aggregate values of the pension expenditures and the tax payments of the two sub-populations.

The pension expenditures for the immigrant population are, see (24),

$$PE_M := \int_R^{\omega} \int_{a_{min}}^{a_{max} \wedge a} p(a^*) M(a; a^*) \, \mathrm{d}a^* \, \mathrm{d}a.$$

The pension expenditures for the native population are

$$PE_N := p(0) \int_R^{\omega} N(a) \, \mathrm{d}a.$$

Hence, total pension expenditures depend on the age structure of the population, the parameters of the pension system, N_B and p_1, as-well as the wage rates of natives and immigrants w_M and w_N, respectively.

The tax payments of the immigrant population are

$$tax_M := \theta \int_0^{\omega} \int_{a_{min}}^{a_{max} \wedge a} w_M e_M(a; a^*) M(a; a^*) \, \mathrm{d}a^* \, \mathrm{d}a,$$

and finally, tax payments of the native population are

$$tax_N := \theta \int_0^{\omega} w_N e_N(a) N(a) \, \mathrm{d}a.$$

Hence, the aggregate values are given by $PE_{tot} = PE_M + PE_N$ and accordingly $tax_{tot} = tax_M + tax_N$.

Then, the *social security system* is balanced if

$$tax_{tot} = PE_{tot}. \tag{43}$$

The Austrian pension system is a PAYG defined benefits. Therefore, θ has to be adjusted such that (43) holds. In this work we focus on the steady state. It is assumed that the government budget is always balanced and there are no debts. Hence, the sustainability of the pension system is reflected by changes in the contribution rate θ. Higher benefits are counteracted by an increase in the contribution rate. The contribution rate can be viewed as a generalization of the demographic old-age dependency ratio because it relates the aggregate expenses for the pensions in a population to the total contributions of the working people. Similarly, the old- age dependency ratio relates the number of non-working people in a population to those who are working. A lower contribution rate means that less taxes have to be used to close the gap of the pension system caused by the demographic change. Hence, the additional contributions could be used for other pillars of the social security system such as health insurance.

2.7. Firm's problem

The production sector of the economy is modeled by a representative firm which uses capital and labor to produce a single consumption good. The consumption good can either be saved or consumed. To which extend the product is consumed or saved is decided by the individuals who inhabit the economy.

The production function is given by

$$Y = K^{\alpha}(AL)^{1-\alpha},$$

where Y is the output, L is the effective aggregate labor input and K is the capital stock. The constant α is the capital share and A is the labor-augmenting technological level. It is assumed that immigrants and natives are imperfect substitutes. Therefore, the aggregate effective labor L is taken to be a so-called CES (constant elasticity of substitution) aggregator which combines the two different kinds of labor:

$$L = \left(\gamma L_M^{\frac{\beta-1}{\beta}} + (1-\gamma) L_N^{\frac{\beta-1}{\beta}}\right)^{\frac{\beta}{\beta-1}},$$

where L_M and L_N are the effective labor input of immigrants and natives, respectively,

$$L_M = \int_0^{\omega} \int_{a_{min}}^{a_{max} \wedge a} e_M(a; a^*) M(a; a^*) \, \mathrm{d}a^* \, \mathrm{d}a, \tag{44}$$

$$L_N = \int_0^{\omega} e_N(a) N(a) \, \mathrm{d}a. \tag{45}$$

The weights γ and $1-\gamma$ are associated with the two different forms of labor in the labor force. The constant β, $0 < \beta < \infty$, is the *elasticity of substitution* between native labor and immigrant labor. If $\beta > 1$, then the two types of labor are substitutes, meaning that a reduction in the supply of one type increases the demand for the other. For $\beta < 1$, the two types of labor are compliments and therefore a reduction of the supply of one does not increase the demand for the other. If $\beta \to 1$, the CES aggregator reduces to a Cobb-Douglas function. The limit of $\beta \to \infty$ describes the case of perfect substitutes and $\beta \to 0$ means that immigrant labor and native labor are perfect compliments. The aggregate capital stock is given by:

$$K = \int_0^{\omega} \int_{a_{min}}^{a_{max} \wedge a} k^M(a; a^*) M(a; a^*) \, \mathrm{d}a^* \, \mathrm{d}a + \int_0^{\omega} k^N(a) N(a) \, \mathrm{d}a. \tag{46}$$

The representative firm maximizes profit by hiring labor L and renting capital K from households. Therefore, prices for workers and capital equal the corresponding marginal product

$$R = A^{1-\alpha} \alpha \hat{k}^{\alpha-1}, \tag{47}$$

$$\log(w_M) = \log(A^{1-\alpha}(1-\alpha)\hat{k}^{\alpha}) + \frac{1}{\beta}\log(L) + \log(\gamma) - \frac{1}{\beta}\log(L_M), \tag{48}$$

$$\log(w_N) = \log(A^{1-\alpha}(1-\alpha)\hat{k}^{\alpha}) + \frac{1}{\beta}\log(L) + \log(1-\gamma) - \frac{1}{\beta}\log(L_N), \tag{49}$$

where $\hat{k} := K/L$ is the capita per effective labor.

2.8. Definition of steady-state equilibrium

A *steady-state competitive equilibrium* is defined as the policy functions of individuals ($c_N(\cdot)$ and $c_M(\cdot, a^*)$), labor and capital demand of firms (K and L), factor prices (w_M, w_N and r), social contribution rate (θ) and the value of pensions (p_N and $p(a^*)$), that fulfill the following conditions:

- The functions $c_N(a)$ and $c_M(a, a^*)$ are optimal in terms of the optimization problems given by (29)–(32) and (37)–(39).
- Factor prices are equal to marginal products given by (47)–(49).
- The goods market clears.
- The budget of the pension system is balanced, i.e. Equation (43) holds.

In Table 2 we summarized the equations that have to be fulfilled in equilibrium. The determination of a steady-state equilibrium turns out to be a fixed-point problem in $\hat{k}$

$$\hat{k} = \phi(\hat{k}), \tag{50}$$

where ϕ is a non-linear function in $\hat{k}$. For more details on the solution of (50) see the description of the numerical algorithm in section 2.9 below.

2.9. Numerical Experiments

2.9.1. Calibration

The above model is now calibrated with Austrian data.

Demography. For the computations we initialize the age structure of demographic variables, $f(a)$, $f_M(a)$, $\mu(a)$, $m(a)$, referring to Austrian data as of 2008 provided by, and interpolate these data piecewise linearly to obtain continuous representations of the vital rates. For the influx of migrants we take the mean value of net migration to Austria over the past 10 years, $I = 35000$. We assume a maximal attainable age of $\omega = 110$.

Households. To construct age-specific efficiency profiles for immigrants and natives, we used the 2008, 2009 and 2010 Income, Social Inclusion and Living Conditions (EU-SILC) survey data for Austria. Due to a lack of data, we assumed that $e_M(a; a^*) = e_M(a)$, i.e. we did not account of the differences in wages depending on the age of arrival of the immigrant. In Figure 6 we plotted the estimated efficiency profiles. Notice that while the efficiency of the natives is always increasing with age, that of the immigrants is slightly bending backwards in the ages before retirement.

We set the subjective discount factor $\rho = 0$, meaning that the only source of discounting future preferences is the survival probability and the relative risk aversion $\sigma = 1.6$ which is in line with [18].

$c_N(a) = e^{\frac{(r-\rho)}{\sigma}a} \frac{\int_0^\omega e^{-ra} l(a)((1-\theta)w_N e_N(a) - p_N \mathbb{I}_{[R,\omega]}(a))\,\mathrm{d}a}{\int_0^\omega l(a) e^{(r(1+\frac{1}{\sigma})-\frac{\rho}{\sigma})a}\,\mathrm{d}a}$, $c_M(a;a^*) = e^{\frac{(r-\rho)}{\sigma}a} \frac{\int_0^\omega e^{-ra} l(a)((1-\theta)w_N e_N(a) - p_M(a^*) \mathbb{I}_{[R,\omega]}(a))\,\mathrm{d}a}{\int_0^\omega l(a) e^{(r(1+\frac{1}{\sigma})-\frac{\rho}{\sigma})a}\,\mathrm{d}a}$, $k_N(a) = \int_0^a e^{\int_s^a (r+\mu(\eta))\,\mathrm{d}\eta} \left((1-\theta)w_N e_N(s) - c_N(s) + p_N \mathbb{I}_{[R,\omega]}(s)\right)\,\mathrm{d}s$, $k_M(a;a^*) = \int_{a^*}^a e^{\int_s^a (r+\mu(\eta))\,\mathrm{d}\eta} \left((1-\theta)w_M e_M(s;a^*) - c_M(s;a^*) + p_M(a^*) \mathbb{I}_{[R,\omega]}(s)\right)\,\mathrm{d}s$,
$L_N = \int_0^\omega e_N(a) N(a)\,\mathrm{d}a$, $L_M = \int_0^\omega \int_{a_{min}}^{a_{max}\wedge a} e_M(a;a^*) M(a;a^*)\,\mathrm{d}a^*\,\mathrm{d}a$, $K = \int_0^\omega \int_{a_{min}}^{a_{max}\wedge a} k^M(a;a^*) M(a;a^*)\,\mathrm{d}a^*\,\mathrm{d}a + \int_0^\omega k^N(a) N(a)\,\mathrm{d}a$, $R = A^{1-\alpha} \alpha \hat{k}^{\alpha-1}$, $\log(w_M) = \log(A^{1-\alpha}(1-\alpha)\hat{k}^\alpha) + \frac{1}{\beta}\log(L_M + L_N) + \log(\gamma) - \frac{1}{\beta}\log(L_M)$, $\log(w_N) = \log(A^{1-\alpha}(1-\alpha)\hat{k}^\alpha) + \frac{1}{\beta}\log(L_M + L_N) + \log(1-\gamma) - \frac{1}{\beta}\log(L_N)$, $\theta \int_0^\omega \left(\int_{a_{min}}^{a_{max}\wedge a} w_M e_M(a;a^*) M(a;a^*)\,\mathrm{d}a^* + w_N e_N(a) N(a)\,\mathrm{d}a \right)\,\mathrm{d}a$ $= p(0) \int_R^\omega \left(N(a) \int_{a_{min}}^{a_{max}\wedge a} p(a^*) M(a;a^*)\,\mathrm{d}a^* \right)\,\mathrm{d}a$,
$M(a,a^*) = Im(a^*)l(a)$, $M(a) = \int_{a_{min}}^{a_{max}\wedge a} \frac{l(a)}{l(a^*)} m(a^*) I\,\mathrm{d}a^*$, $N(a) = \frac{\int_0^\omega f_M(s) M(s)\,\mathrm{d}s}{1 - \int_0^\omega f(s) l(s)\,\mathrm{d}s} l(a)$.

Table 2: System of equations to determine the endogenous variables: microeconomic relations (first block); macroeconomic relations (second block); demography (third block);

Firm. To properly estimate the weight γ, we assume that the differences in wages of immigrants and natives is solely given by their efficiency of labor

$$\frac{y_M(a)}{y_N(a)} = \frac{e_M(a)}{e_N(a)}.$$

Hence, $\frac{w_M}{w_N} = 1$, holds and consequently we can estimate γ by

$$\gamma = \frac{\left(\frac{L_M}{L_N}\right)^{1/\beta}}{1 + \left(\frac{L_M}{L_N}\right)^{1/\beta}}.$$

Note the dependence of γ on β.

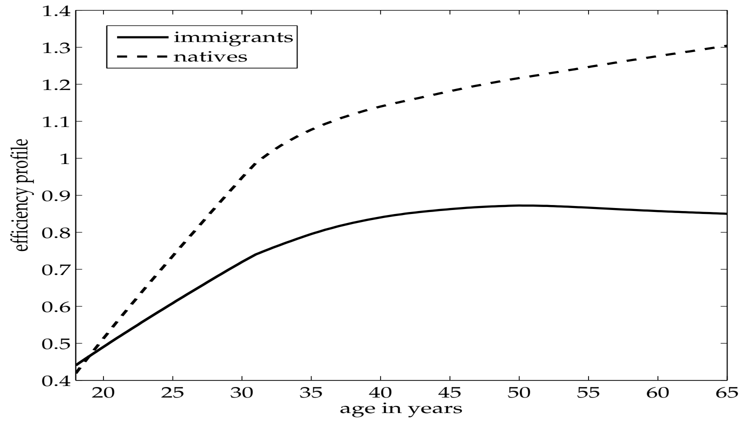

Figure 6: The efficiency profiles of immigrants and natives

We follow [18] where the author's also dealt with the Austrian economy and chose the capital share, $\alpha = 0.31$ and the rate of capital depreciation, $\delta = 0.04$. The labor-augmenting productivity factor $A = 4.2489 \cdot 10^4$ is chosen such that aggregate output Y approximates the value of Austria's GDP.

Pension system. We set the parameters of the pension system to those of the current Austrian pension system, where $N_B = 25$, $p_1 = 1.78\%$ and $R = 62.5$ holds.

2.9.2. Solution algorithm for the numerical solution

Below we numerically solve the system of equations of Table 2 determining the general equilibrium in the economy. Similar systems of equations have already been solved in other economic papers dealing with general equilibrium models. The general equilibrium mechanism is the most famous nonlinear equation problem in economics. A general solution algorithm is, for example, proposed in [9].

Since finding the equilibrium can be summarized to solving a non-linear fixed point equation in $\hat{k}$, in the following we apply a fixed-point iteration method. Subsequently, we assume that there exists a unique solution of equation (50). We take an initial guess $\hat{k}_0$ and insert it into the equations determining the marginal products R, w_M and w_N and with them we calculate the tax rate θ. Then, we compute per-capita consumption $c_N(a)$ and $c_M(a, a^*)$ and per-capita capital $k_N(a)$ and $k_M(a; a^*)$.

Here, in particular, we follow the below algorithm to find an equilibrium solution $\hat{k}^*$:

Step 1: First we choose an adjustment factor $\eta \geq 0$ and a tolerance $\epsilon > 0$ and small. The adjustment factor is chosen to guarantee stable convergence. The tolerance ϵ determines a stopping criterion for the solution algorithm. We initially compute the age densities of immigrants and natives, $M(a; a^*)$ and $N(a)$ according to (22), (23).Then, we make an initial guess $\hat{k}_0$.

Step 2: Given the initial guess $\hat{k}_0$ we compute the marginal products of capital and labor, R, w_M and w_N, according to equations (47)–(49).

Step 3: We subsequently determine the social security rate θ by solving (43).

Step 4: In the next step we compute the household problem for natives (29)–(32) and immigrants (37)–(39) meaning that we compute the age-specific consumption profiles $c_N(a)$ and $c_M(a, a^*)$ and subsequently also $k_N(a)$ and $k_M(a, a^*)$ with the previous determined values of R, w_M, w_N and θ. Everything else in the equations is exogenously given. Notice that for the immigrants the household problem depends on the arrival age a^* and hence has to be calculated for all $a_{min} \leq a^* \leq a_{max}$ separately.

Step 4: Subsequently, we compute the aggregate variables, K, L_M and L_N where K is determined as in (46) and L_M and L_N are given as in (44)–(45).

Step 5: Then, we compute a new guess $\hat{k}_{i+1} = K/(L_M + L_N)$.

Step 6: The procedure is stopped if $\|\eta\hat{k}_{i+1} + (1-\eta)\hat{k}_i\| < \epsilon$. Otherwise we go back to Step 2 and set $i = i + 1$.

In the numerical example below it was necessary to set $0 < \eta < 1$ because for $\eta = 0$ unstable iterations appeared.

2.9.3. Numerical Results

Demography. In Table 3 we summarize the various demographic scenarios, where we assumed that all immigrants arrive in a specific 5–year long sub-interval between $a_{min} = 18$ and $a_{max} = 40$.

In Figures 7 – 10, the resulting stationary through immigration populations are plotted. Notice that the younger the immigrants, the bigger is the resulting population. This is caused by a higher fertility of younger immigrants.

Since for the sustainability of the pension system not the total dependency of a population matters but instead the ratio of working to retired people, we calculated the resulting old-age ratios (OADs), see Table 3. For the calculation of the various OADs we used age groups $18 - 61$ and $62+$. We find that unlike in [20] where the optimal age to minimize the dependency was in the mid-thirties the OAD clearly rises with the age of immigration a^*. This is because for the dependency ratio, a high number of children, caused by a high fertility rate of young immigrants, is not beneficial since they increase the dependent population.

Demography				
$a^* \in$	[18,25]	[25,30]	[30,35]	[35,40]
$N_{tot} + M_{tot}$	9.1 m	7.2 m	4.2 m	2.2 m
$\frac{M_{tot}}{N_{tot}+M_{tot}}$	0.21	0.26	0.41	0.70
$\frac{M_{tot}}{N_{tot}}$	0.26	0.35	0.68	2.38
OAD	0.38	0.40	0.45	0.57

Table 3: Demographic results

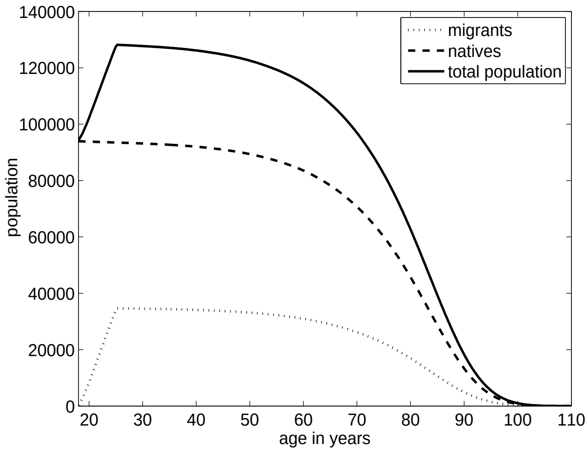

Figure 7: Long run population structure for immigration in ages $a^* \in [18, 25]$

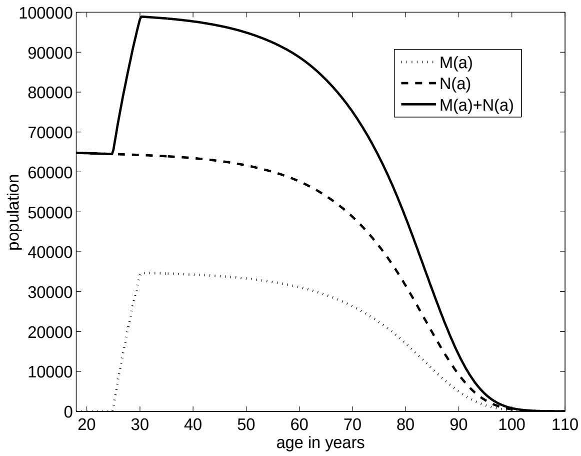

Figure 8: Long run population structure for immigration in ages $a^* \in [25, 30]$.

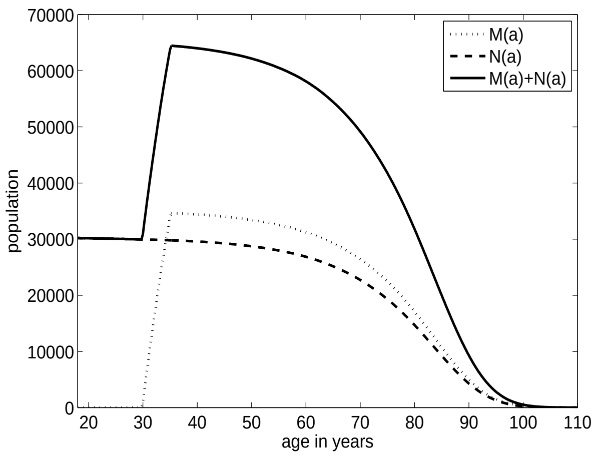

Figure 9: Long run population structure for immigration in ages $a^* \in [30, 35]$

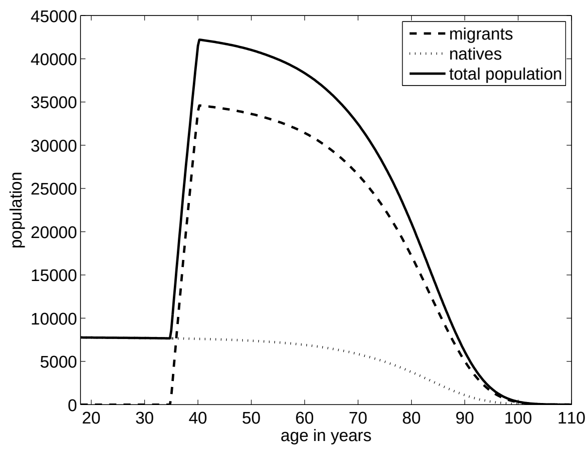

Figure 10: Long run population structure for immigration in ages $a^* \in [35, 40]$.

Effects on the Pension System. In Table 4 the impact of age-specific immigration on the social security rate θ and the pension expenditure rate PE_{tot}/Y are presented. Moreover, scaled pension expenditures and tax payments for the two groups, natives and immigrants,

$N_B = 25$, R=62.5, $e_N(a) \neq e_M(a)$, $\beta = 10000$				
$a^* \in$	[18,25]	[25,30]	[30,35]	[35,40]
θ	0.34	0.33	0.32	0.31
PE_M/w	$3.4 \cdot 10^5$	$3.0 \cdot 10^5$	$2.6 \cdot 10^5$	$2.2 \cdot 10^5$
PE_N/w	$14.5 \cdot 10^5$	$10.0 \cdot 10^5$	$4.6 \cdot 10^5$	$1.2 \cdot 10^5$
tax_N/w	$14.2 \cdot 10^5$	$9.7 \cdot 10^5$	$4.4 \cdot 10^5$	$1.2 \cdot 10^5$
tax_M/w	$3.7 \cdot 10^5$	$3.3 \cdot 10^5$	$2.8 \cdot 10^5$	$2.3 \cdot 10^5$
net transfers migrants/PE_{tot}	1.3 %	2.0 %	3.0 %	3.0%
PE_{tot}/Y in %	23.3 %	23.0 %	22.6%	21.4 %

Table 4: Pension expenditures and contribution rates

are given. One can see that for the given scenario the social security rate decreases with the age of the arriving immigrants although the OAD increases substantially. This is because of the fact that immigrants qualify for fewer pensions in the host country. Moreover, in Table 4 we see that across all age groups immigrants are net payers of the pension system. Hence, they are at least to a small extend able to close the financial gap caused by the aging of the native population. However, one also sees that immigration alone cannot solve the fiscal problems arising with the demographic change because an increase of the social security rate to $\theta \in [0.31, 0.34]$ would be necessary to guarantee a balanced budget. Also pension expenditure rates would have to rise from currently 12.8%, cf. [14] to values between 21% and 23%. Hence, also other measures such as an increase in the statutory retirement age and changes in the parameters of the pension system would additionally be necessary.

Impact of immigration on economic variables and life cycle behavior. Subsequently, we investigate the life cycle behavior of consumption and asset accumulation of natives and immigrants.

$N_B = 25$, R=62.5, $e_N(a) \neq e_M(a)$, $\beta = 10000$				
$a^* \in$	[18,25]	[25,30]	[30,35]	[35,40]
r	0.020	0.019	0.017	0.012
w	30840	31080	31560	32790

Table 5: Economic parameters

In Figures 11–12 the native's life-cycle profile of consumption and financial assets for the various entry scenarios of the immigrants are plotted. We note that unlike natives, immigrants even if they arrive at relatively young ages, they do not become net borrowers, see Figures 13 and 15. This is because they do not earn as high pensions as natives do and hence immediately start accumulating assets. There is also a clear dependence of the native's capital accumulation on the age of arrival of the immigrants. If immigrants arrive in early ages natives accumulate more capital. This is caused by a higher interest rate on capital, see Table 5, although there is a reverse effect caused by an increased θ. A higher θ is usually responsible for a crowding out of capital. Hence, the higher interest rate R

compensates the crowding out effect of an increased θ. In Figures 12, 14 and 16 the life-cycle consumption profiles of immigrants and natives are plotted. Figure 12 shows that the earlier the immigrants enter the country the lower is the initial consumption level and native individuals borrow more at the beginning of the life cycle.

Moreover, we find that if immigrants enter the host country later in life they accumulate even more assets than a native individual because they have to anticipate the missing pension payments at the end of their life.

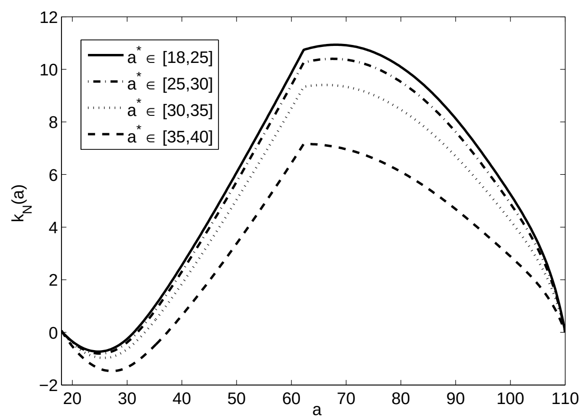

Figure 11: Scaled assets of natives over the life-cycle for $\beta = 10000$

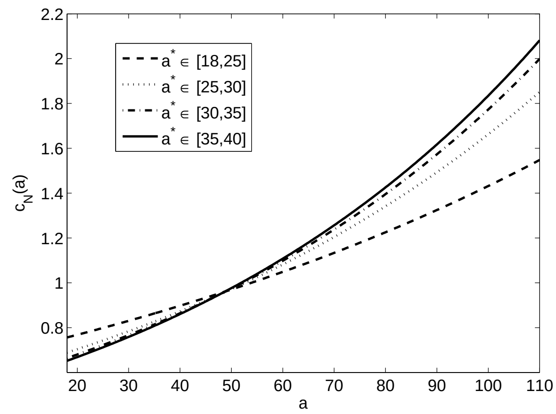

Figure 12: Scaled consumption of natives over the life-cycle for $\beta = 10000$

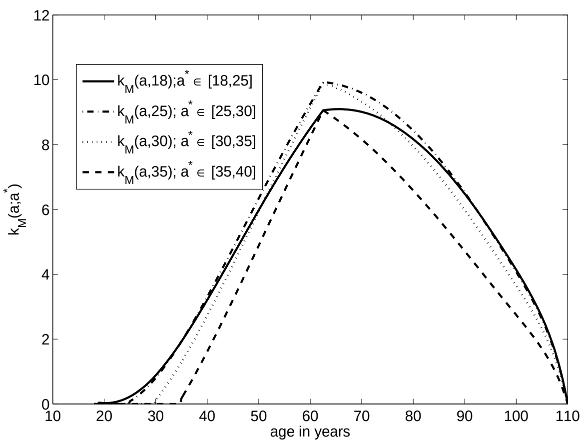

Figure 13: Scaled assets of immigrants over the life-cylce for $\beta = 10000$

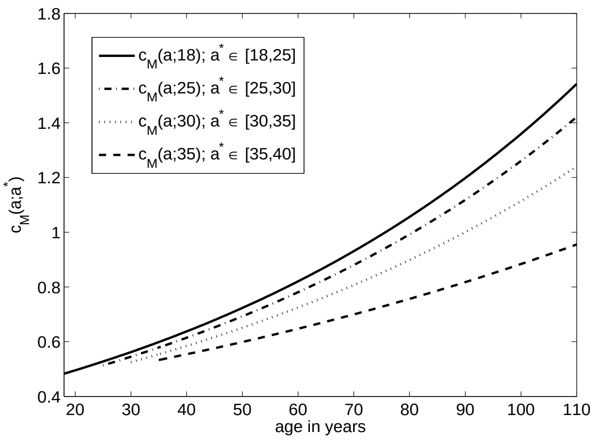

Figure 14: Scaled consumption of immigrants over the life-cylce for $\beta = 10000$

2.10. Outlook

In this work, the focus was on the steady state effects of immigrants' age structure regarding the sustainability of the pension system. Hence, in further investigations an

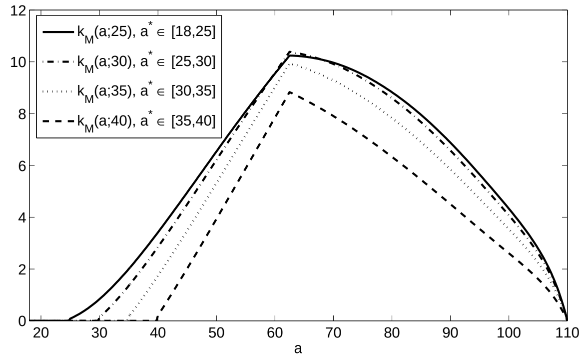

Figure 15: Scaled assets of immigrants over the life-cylce for $\beta = 10000$

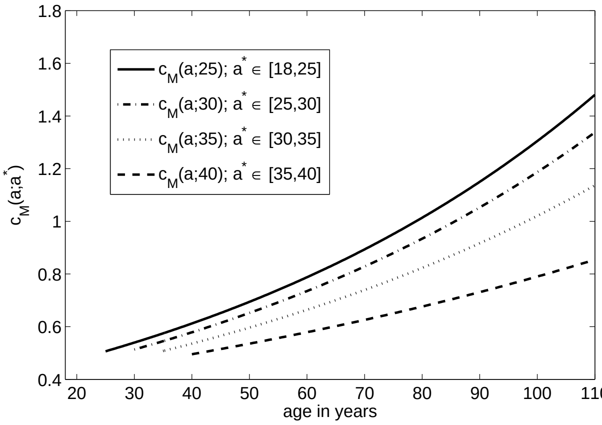

Figure 16: Scaled consumption of immigrants over the life-cylce for $\beta = 10000$

extension to the transitory dynamics would make it possible to study the short term effects which would shed light on more recent developments. We found that since immigrants are heterogeneous with respect to their age of arrival they also earn different pensions after retirement. This heterogeneity may lead to different insentives for retirement when the age of retirement is not exogenous anymore. Hence, an interesting extension of this model could be to investigate how the results would change in case of an endogenous retirement decision. Moreover, one could also drop the strong assumption of no capital movement and pass over to an open economy framework.

Acknowledgments

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