# A Circular Interference Model for Wireless Cellular Networks

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***Abstract*—In this work, we investigate downlink co-channel interference in wireless cellular networks. Our main target is to facilitate statistical analysis in networks with regular grid layout. In particular, we focus on the interference statistics outside the center of a grid scenario. First, a novel circular interference model is introduced. The key idea is to spread the power of the interferers uniformly along the circumcircle of the grid-shaping polygon. We then propose to model the aggregate interference statistics by a single Gamma random variate. The corresponding shape- and scale parameters are determined in closed form by employing this circular model. The analysis yields key insights on the distribution's formative components. We verify the accuracy of the Gamma approximation by qualitative- and quantitative measures. A basic guideline for applying and extending our approach indicates that it considerably improves analytic accessibility of regular grid models.**

***Index Terms*—Interference Modeling, Hexagonal Cellular Network, Hexagon System Model, aggregate Co-Channel Interference (CCI), Interference Statistics, Interference Distribution**

## I. Introduction and Contributions

The proposal of a cellular structure for mobile networks dates back to 1947. Two Bell Labs engineers, Douglas H. Ring and W. Rae Young were the first to mention the idea in an internal memorandum. Almost two decades later, in 1966, Richard H. Frenkiel and Philip T. Porter, shaped a "hexagonal cellular array of areas" to propose the first mobile phone system [1]. Although never published officially, the hexagon model gained high popularity within the research community and is still extensively utilized nowadays [2–7]. It serves either as the system model itself, or as a reference system for more involved simulation scenarios.

On the other hand, its geometric structure renders closed-form analysis of *aggregate interference statistics* difficult [8]. Hence, simulation results often lack a mathematical back up.

Recently, closed-form results have been reported with system models based on stochastic geometry [9–11]. However, these results are obtained only for particular parameter choices, typically assuming spatial stationarity and isotropy of the interference scenario. Thus, its potential to consider *non-symmetric interferer impact*, e.g., when the receiver is located outside the scenario center, is limited. Moreover, the stochastic approach is based on an ensemble of network realizations and is therefore not applicable when a fixed structure of the network is given.

Recently published work on hexagonal grid models has mainly focused on link-distance statistics [12, 13]. The authors also account for fading and provide closed-form approximations for the co-channel interference of a *single link*. However, closed-form expressions for the moments and the distribution of *aggregate co-channel interference* are not available yet.

The main contributions of this work are:

1) We introduce a *circular interference model* to facilitate interference analysis in cellular networks with regular grid layout. In particular, we focus on the hexagonal grid due to its ubiquity in wireless communication engineering [2–7]. The key idea is to consider the power of the interfering transmitters as being uniformly spread along the circumcircle of the hexagon.
2) We propose to model interference statistics in a hexagonal scenario by a single Gamma Random Variate (RV). Its shape- and scale parameters are determined in closed form by employing the *circular model*. The analysis yields key insights on the formative components of the interference distribution.
3) We provide a basic guideline for applying and extending our approach. Next to its Multiple Input Multiple Output (MIMO) capabilities, we also demonstrate the generalization to multiple tiers and arbitrary regular polygon models.

The remainder of this work is structured as follows: Section II specifies the hexagonal reference-system model. In Section III the circular interference model and its dual pendant are introduced. Section IV investigates Gamma-distributed interference and its parametrization by the proposed circular interference model. In Section V, the accuracy of the Gamma approximation is verified. This section also provides applications and extensions of the model. Section VI concludes the work.

## II. System Model

The reference hexagonal setup is composed of a central cell and six interfering transmitters, as shown in Figure 1. The interferers are equipped with omnidirectional antennas and are located at the edges of a hexagon with radius $R$ (marked as '+' in Figure 1). The signal from the $i$-th interfering transmitter with polar coordinates $(R, \Phi_i)$ to a receiver with polar coordinates $(r, \phi)$ experiences:

- Path loss

$$\ell\left(d^{(M)}_{\rho,\Delta_i}\right) = \begin{cases} c \cdot d_0^{\alpha} & , \quad d^{(M)}_{\rho,\Delta_i} < d_0 \\ c \cdot \left(d^{(M)}_{\rho,\Delta_i}\right)^{\alpha} & , \quad d^{(M)}_{\rho,\Delta_i} \geq d_0 \end{cases} \tag{1}$$

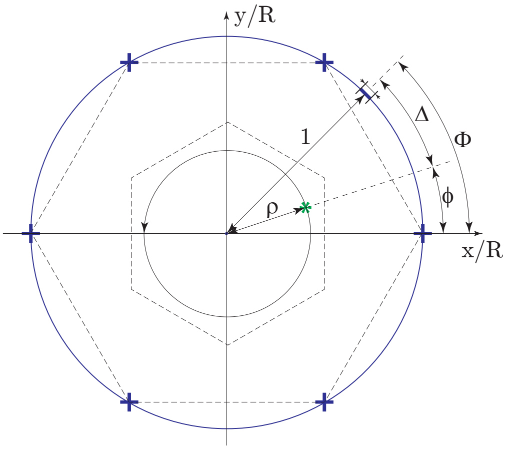


Fig. 1: System model: Center cell with receiver and interfering transmitters at normalized distances $\rho$ and 1, respectively. The receiver is marked by '$*$', the interferers of the hexagonal reference system are denoted by '$+$', respectively.

where $c$ is a constant, $\alpha$ is the path loss exponent and

$$d^{(M)}_{\rho,\Delta_i} = \sqrt{R^2 + r^2 - 2Rr\cos(\phi - \Phi_i)} \tag{2}$$
$$= R\sqrt{(1 + \rho^2 - 2\rho\cos(\Delta_i))},$$
$$\text{with } \Phi_i = \frac{2\pi}{M} i, \; i = 1, \ldots, M,$$
$$\text{and } \rho = \frac{r}{R}, \; \Delta_i = \phi - \Phi_i. \tag{3}$$

In the hexagonal scenario, $M = 6$. The terms $\rho$ and $\Delta_i$ denote the receiver's normalized distance to the center and its angle-difference to the $i$-th interfering transmitter, respectively.

- Fading, which is modeled by independent and identically distributed (i.i.d.) Gamma RVs $G_i \sim \Gamma[k_0, \theta_0]$, with shape- and scale parameters $k_0$ and $\theta_0$, respectively. Its Probability Density Function (PDF) is defined as

$$\Gamma[k_0, \theta_0] = \frac{1}{\Gamma(k_0)\,\theta_0^{k_0}} x^{k_0 - 1} e^{-x/\theta_0}. \tag{4}$$

Gamma-fading includes Rayleigh and Nakagami-m as special cases. It allows to model multiuser-MIMO systems and can accurately approximate composite fading distributions (e.g., Rayleigh-Lognormal). Therefore, it covers a wide range of scenarios.

## III. Circular Interference Model

In a one-tier hexagonal grid scenario, as presented in Section II, the received aggregate interference power at position $(r, \phi)$ can be expressed as

$$I_6(\rho, \phi) = \sum_{i=1}^{6} \frac{P\,G_i}{\ell\left(d^{(6)}_{\rho,\Delta_i}\right)}, \tag{5}$$

where $P$ denotes the transmit power, $G_i$ is the fading and $\ell(d^{(6)}_{\rho,\Delta_i})$ refers to the path loss at distance $d^{(6)}_{\rho,\Delta_i}$, with $d^{(6)}_{\rho,\Delta_i}$ from (2).

Outside the cell-center, i.e., $\rho > 0$, distribution and moments of $I_6(\rho, \phi)$ can in general not be evaluated in closed-form. In this section, we propose a circular interference model to facilitate and enable statistical analysis.

### A. Proposed Model

In the circular interference model, the power of the six reference transmitters is spread uniformly along a circle of radius $R$.

This is achieved by equally distributing the total transmit power $6\,P$ among $M$ equally spaced transmitters and considering the limiting case $M \to \infty$. With (5), we obtain

$$\lim_{M\to\infty} \frac{6\,P}{M} \sum_{i=1}^{M} \frac{G_i}{\ell\left(d^{(M)}_{\rho,\Delta_i}\right)} = \frac{6\,P\,\mathbb{E}[G_i]}{2\pi} \int_{-\pi}^{\pi} \frac{1}{\ell\left(d_{\rho,\Delta}\right)} d\Delta, \tag{6}$$

with $\ell(\cdot)$ from (1) and $d_{\rho,\Delta_i}$ from (2). The terms $d_{\rho,\Delta}$ and $\Delta$ denote distance and angle-difference between the receiver and an infinitesimal interfering circular segment, as indicated in Figure 1.

Assuming a path loss exponent $\alpha = 2$, i.e., free space propagation, and $R > d_0$, (6) can be evaluated as

$$I_C(\rho) = 6\,P\,\mathbb{E}[G_i]\,\frac{1}{cR^2}\,\frac{1}{1 - \rho^2}. \tag{7}$$

An intuitive interpretation of this result by the model's pendant is provided in the next subsection.

*Note*: In the remainder of this work, we stick to a path loss exponent $\alpha = 2$. It represents the worst case of low interference attenuation. However, previously- as well as all subsequently presented analysis can be carried out in closed-form for $\alpha = 2n$ with $n \in \mathbb{N}$. Values $\alpha$ other than these require the evaluation of elliptic integrals [14].

### B. The Dual Model

Consider a receiver in a hexagonal scenario, which is moved along a circle of radius $\rho$ from $-\pi$ to $\pi$, as depicted in Figure 1. The *average expected* interference along the circle can be calulated as

$$I'_C(\rho) = \frac{1}{2\pi} \int_{-\pi}^{\pi} \mathbb{E}[I_6(\rho, \phi')]\, d\phi' \tag{8}$$
$$= \sum_{i=1}^{6} P\,\mathbb{E}[G_i]\,\frac{1}{2\pi} \int_{-\pi}^{\pi} \frac{1}{\ell(d_{\rho,\Delta})} d\phi'. \tag{9}$$

The result is obtained by plugging (5) into (8), exchanging sum and integral and exploiting the linearity of the expectation.

The term $I'_C(\rho)$ in (6) is equivalent to $I_C(\rho)$ in (9) and, consequently, also yields (7). Thus, the result is *independent of the receiver's angle-position*. It can be interpreted as the *average expected* interference, i.e., the interference experienced by a typical receiver in a hexagonal scenario at distance $\rho$.

Note that the circular interference model is not restricted to hexagons. By replacing '6' by '$N$' in (5)–(9), it can generally be applied for substituting any convex regular $N$-polygonal model, as validated in Section V-A.

## IV. Statistics of Aggregate Interference

In this section, we investigate aggregate interference in a hexagonal scenario with i.i.d. Gamma fading. We propose to approximate its statistics by a *single Gamma RV*. Its distance-dependent shape- and scale parameters are determined by applying the previously presented circular model.

The rationale for this approximation are (i) the accurate fit, as verified in Section V-B and (ii) its applicability to evaluate average rate and Bit Error Ratio (BER) in closed form [11, 15].

### A. Interference Statistics at the Center

Assume i.i.d. Gamma fading with $G_i \sim \Gamma[k_0, \theta_0]$. Referring to (5), interference can be considered as a sum of RVs, which are weighted by the received power without fading; $P/\ell(d^{(6)}_{\rho,\Delta_i})$).

At the center of a hexagonal scenario ($\rho = 0$), all weighting factors are equal, i.e., $P/\ell(d^{(6)}_{\rho,\Delta_i}) = P/cR^2$. By virtue of the scaling- and summation property of a Gamma RV, the resulting interference is distributed as

$$I_6(0,\phi) \sim \Gamma\left[6\,k_0, \theta_0 \frac{P}{cR^2}\right]. \tag{10}$$

### B. Interference Statistics outside the Center

Outside the center ($\rho > 0$), a non-uniform impact of the interferers is observed: The distances $d^{(6)}_{\rho,\Delta_i}$ and, thus, also the weighting factors $P/\ell(d^{(6)}_{\rho,\Delta_i})$ generally differ from each other. The resulting interference distribution can be evaluated by recursive methods [16] or hypergeometric functions [8], which hamper closed-form analysis of further performance metrics such as average rate and BER.

Therefore, we propose to approximate the typically experienced interference distribution at distance $\rho$ by

$$\hat{I}(\rho) \sim \Gamma[\hat{k}(\rho), \hat{\theta}(\rho)]. \tag{11}$$

The rationale for this model are findings in prior work, where out-of-cell interference in stochastic networks is accurately approximated by a Gamma distribution [11].

The distribution is fully determined by the distance-dependent shape- and scale parameters $\hat{k}(\rho)$ and $\hat{\theta}(\rho)$, respectively. In order to evaluate the two parameters, we first employ the proposed circular interference model to determine expectation and variance of $\hat{I}(\rho)$. Then, we exploit the fact that $\mathbb{E}[\hat{I}(\rho)] = \hat{k}(\rho)\,\hat{\theta}(\rho)$ and $\mathrm{Var}[\hat{I}(\rho)] = \hat{k}(\rho)\,\hat{\theta}^2(\rho)$:

*a) Expectation:* As discussed in Section III-A, the unequal received powers from the interfering transmitters can be *averaged*. The *average* impact of each interferer is calculated as

$$P\frac{1}{2\pi}\int_{-\pi}^{\pi} \frac{1}{\ell\left(d_{\rho,\Delta}\right)} d\Delta = \frac{P}{cR^2}\frac{1}{1-\rho^2}, \tag{12}$$

and yields the *typically expected* aggregate interference at distance $\rho$ as

$$\mathbb{E}\left[\hat{I}(\rho)\right] = 6\,k_0\theta_0 \frac{P}{cR^2}\frac{1}{1-\rho^2}. \tag{13}$$

| Transmit power | P | 1 |
|---|---|---|
| Circle radius | R | 1 |
| Path loss exponent | $\alpha$ | 2 |

TABLE I: System parameters

*b) Variance:* The variance of the aggregate interference comprises of two components:

1) Variance of the fading:

$$\mathrm{Var}_f\left[\hat{I}(\rho)\right] = 6\,k_0 \left(\theta_0 \frac{P}{cR^2}\frac{1}{1-\rho^2}\right)^2. \tag{14}$$

2) Variance of the received power without fading, which is caused by the unequal distances $d_{\rho,\Delta_i}$. With

$$\frac{1}{2\pi}\int_{-\pi}^{\pi}\left(\frac{P}{\ell\left(d_{\rho,\Delta}\right)} - \frac{P}{cR^2}\right)^2 d\Delta = \left(\frac{P^2}{cR^2}\right)^2 \frac{2\rho^2+\rho^4-\rho^6}{(1-\rho^2)^3}, \tag{15}$$

the second variance component is obtained as

$$\mathrm{Var}_d\left[\hat{I}(\rho)\right] = 6k_0\left(\theta_0\frac{P}{cR^2}\right)^2 \frac{2\rho^2+\rho^4-\rho^6}{(1-\rho^2)^3} \tag{16}$$

Since the two components are statistically independent, the overall variance is calculated as

$$\begin{aligned}\mathrm{Var}\left[\hat{I}(\rho)\right] &= \mathrm{Var}_f\left[\hat{I}(\rho)\right] + \mathrm{Var}_d\left[\hat{I}(\rho)\right] \\ &= 6\,k_0\left(\theta_0\frac{P}{cR^2}\frac{1}{1-\rho^2}\right)^2\left(1+\frac{2\rho^2+\rho^4-\rho^6}{1-\rho^2}\right)\end{aligned} \tag{17}$$

where $\mathrm{Var}_f[\hat{I}(\rho)]$ and $\mathrm{Var}_d[\hat{I}(\rho)]$ refer to (14) and (16), respectively.

Finally, the distance-dependent shape- and scale parameter are derived from (13) and (17) as

$$\hat{k}(\rho) = 6\,k_0\,\frac{1-\rho^2}{1+\rho^2+\rho^4-\rho^6} \tag{18}$$

$$\hat{\theta}(\rho) = \theta_0\frac{P}{cR^2}\,\frac{1}{1-\rho^2}\left(1+\frac{2\rho^2+\rho^4-\rho^6}{1-\rho^2}\right) \tag{19}$$

## V. Numerical Results and Discussion

In this section, the accuracy of the circular model and the proposed Gamma approximation are verified. Ideal system parameters are chosen in order to facilitate traceability.

### A. Validation of Expected Aggregate Interference

First, we compare the expected interference powers in the hexagonal reference scenario and the proposed circular interference setup. The employed system parameters are summarized in Table I and fading is assumed to be distributed as $G_i \sim \Gamma[1, 1]$.

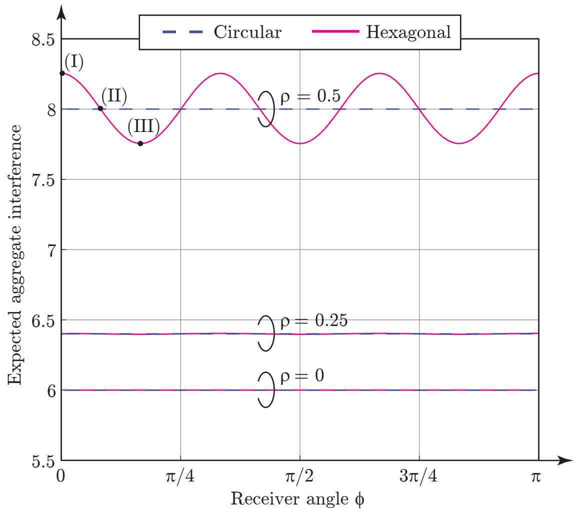


Fig. 2: Expected aggregate interference experienced at position $(\rho, \phi)$ in circular- ($I_C(\rho)$) and hexagonal model ($\mathbb{E}[I_6(\rho, \phi)]$), respectively. Receiver distances $\rho = \{0, 0.25, 0.5\}$ refer to cell-center, middle of cell and cell edge, respectively.

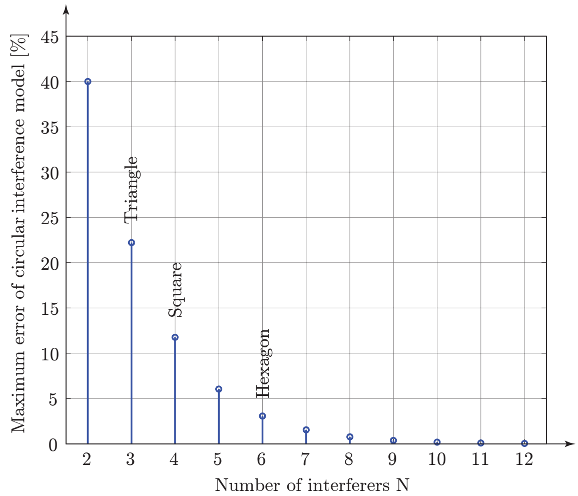


Fig. 3: Maximum error of circular interference model from expected interference in *convex regular N-polygonal* models. The labeled cell-shapes can be arranged in a grid without overlapping areas.

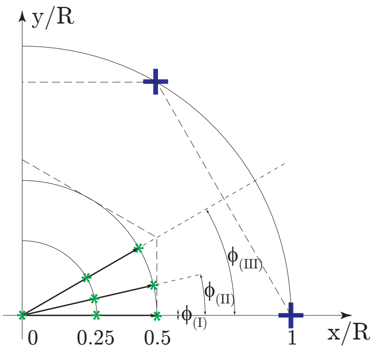


Fig. 4: Setup for evaluation. Cutout of Figure 1 (upper right quadrant).

Consider a receiver which is moved along a semi circle $\{(\rho, \phi)| \phi \in [0, \pi]\}$, as indicated in Figure 1. The expected interference in the hexagonal scenario is calculated as

$$\mathbb{E}\left[I_6(\rho, \phi)\right] = 6 \sum_{i=1}^{6} \frac{1}{\ell\left(d_{\rho,\Delta_i}^{(6)}\right)}, \tag{20}$$

with $I_6(\rho, \phi)$ from (5) and $\mathbb{E}[G_i] = 1$. For the circular model, we obtain

$$\mathbb{E}\left[I_C(\rho)\right] = I_C(\rho) = \frac{6}{1 - \rho^2}, \tag{21}$$

with $I_C(\rho)$ from (7). Figure 2 depicts the evaluated results of (20) and (21) for various distances $\rho$:

- At cell-center, i.e., $\rho = 0$, the expected interference powers in the hexagonal- and circular scenario ($\mathbb{E}[I_6(0, \phi)]$ and $I_C(0)$) are equal.
- Outside the center, i.e., $\rho > 0$, $\mathbb{E}[I_6(\rho, \phi)]$ fluctuates around $I_C(\rho)$. The deviation is weak in the middle of the cell ($\rho = 0.25$), and strong at cell edge ($\rho = 0.5$). Note that $\mathbb{E}[I_6(\rho, \phi)]$ is not symmetric about $I_C(\rho)$ due to the concavity of the path loss model.

The relative error of the circular interference model is calculated as

$$\epsilon\left(\rho, \phi\right) = \left|\frac{\mathbb{E}\left[I_6(\rho, \phi)\right] - I_C(\rho)}{\mathbb{E}\left[I_6(\rho, \phi)\right]}\right|, \tag{22}$$

with $\mathbb{E}[I_6(0, \phi)]$ and $I_C(0)$ from (20) and (21), respectively. The largest error occurs at cell edge, i.e.,

$$\max_{\rho,\phi} \epsilon\left(\rho, \phi\right) = \max_{\phi} \epsilon\left(0.5, \phi\right). \tag{23}$$

However, the expected interference powers of the circular- and the hexagonal model deviate by no more than 3.07 %, as shown in Figure 3 (corresponding to marker (I) in Figure 2).

### *B. Validation of Gamma Approximation*

In this subsection, we verify the accuracy of the Gamma approximation in (11). The exact position-dependent distributions of $I_6(\rho, \phi)$ (cf. (5)) are obtained by numerically evaluating the approach in [16].

In order to obtain a representative profile of distributions, a receiver is moved along a straight line from $\rho = 0$ to $\rho = 0.5$. The procedure is carried out for three receiver angle-positions $\Phi_{(I)}, \Phi_{(II)}$ and $\Phi_{(III)}$, as shown in Figure 4. The angles correspond to the markers (I), (II) and (III) in Figure 2:

- $\phi_{\text{(I)}} = 0$ represents a receiver, which is moved directly towards its strongest interferer.
- $\phi_{\text{(II)}}$ is implicitely given by $\mathbb{E}\left[I_6(0.5, \phi)\right] = I_C(0.5)$, i.e., the angle where the expected interference in the hexagonal grid equals interference in the circular scenario.
- $\phi_{\text{(III)}} = \frac{\pi}{6}$ represents the case, where a receiver is moved centrally between its two strongest interferers.

Fading is chosen as $G_i \sim \Gamma(2, 1)$. This corresponds to a $1 \times 2$ MIMO system with Rayleigh-fading and Maximum Ratio

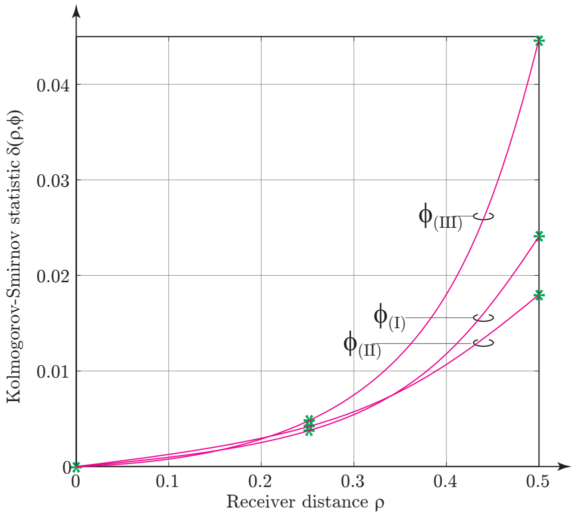


Fig. 5: Kolmogorov-Smirnov statistic at position $(\rho, \phi_m)$: Comparison of Gamma CDF and exact distributions. Receiver distances $\rho = 0$ and $\rho = 0.5$ refer to cell-center and cell-edge, respectively.

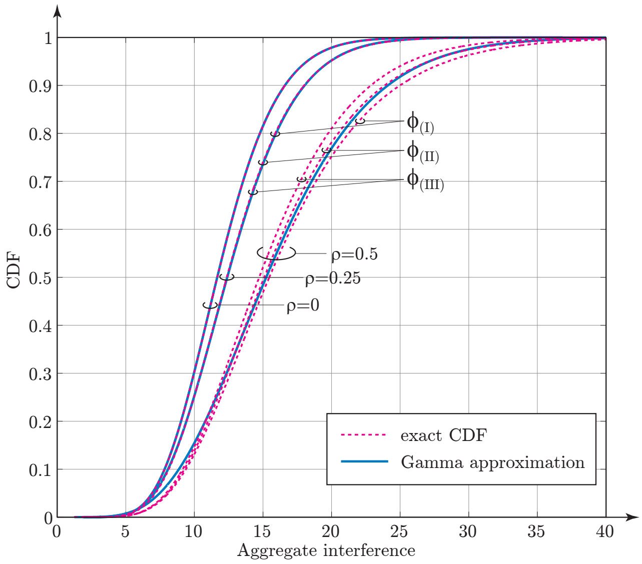


Fig. 6: Aggregate interference at particular receiver positions (see '∗' in Figure 4): Exact CDFs, as obtained by numerically evaluating [16] for a hexagon scenario (dashed lines) and corresponding Gamma approximations (solid lines).

Combining (MRC) at the receiver, or, equivalently, a $2 \times 1$ MIMO system with MRC at the transmitter.

Then, for each distance $\rho$ and angle $\phi_m \in \{\phi_{(I)}, \phi_{(II)}, \phi_{(III)}\}$ we evaluate

- the Cumulative Distribution Functions (CDFs) of the Gamma approximation, $F_{\hat{I}}(x; \hat{k}(\rho), \hat{\theta}(\rho))$, where $\hat{k}(\rho)$ and $\hat{\theta}(\rho)$ refer to (18) and (19), respectively.
- the CDFs $F_6(x; \rho, \phi_m)$ of $I_6(\rho, \theta_m)$, using the approach in [16].

In order to *quantify* the accuracy of the Gamma approximation, the Kolmogorov-Smirnov statistic is employed. It formulates as

$$\delta(\rho, \phi_m) = \sup_x \left| F_{\hat{I}}\left(x; \hat{k}(\rho), \hat{\theta}(\rho)\right) - F_6\left(x; \rho, \phi_m\right) \right|. \tag{24}$$

The results are depicted in Figure 5: The Gamma approximation most closely resembles the experienced interference distributions at $\phi_{(II)}$, i.e., the typical receiver (lower curve). In this case, the difference between exact CDFs and Gamma approximation is less than 1 % for $\rho < 0.39$ and 1.8 % at cell edge ($\rho = 0.5$). The largest deviation occurs at $\phi_{(III)}$, when the receiver is moved directly towards its strongest interferer (upper curve). Then, the distributions differ by less than 1 % for $\rho < 0.34$ and by 4.4 % at cell edge.

For *qualitative* evaluation, the exact CDFs and the corresponding Gamma approximations at representative receiver positions are depicted in Figure 6. These positions are shown in Figures 4 and 5 and denoted as '*'. The Gamma CDFs perfectly fit at cell center ($\rho = 0$) and in the middle of the cell ($\rho = 0.25$). At cell edge ($\rho = 0.5$), the Gamma approximation closely resembles the experienced interference of a receiver at $\phi_{(II)}$. The probability of high interference values at $\phi_{(III)}$ is slightly underestimated by at most 4.4 % (cf. Figure 5).

### *C. Applications and Extensions*

This subsection provides a basic guideline for applying and extending the circular interference model.

*1) Average Rate and BER:* The proposed Gamma approximation yields an input for quotient distributions, as typically appearing in Signal-to-Interference Ratio (SIR) metrics. In particular cases, these distributions are well known and allow to evaluate BER- and achievable rate statistics [11, 15].

*2) Heterogeneous Networks:* By fully characterizing the cellular contribution, the Gamma approximation considerably facilitates interference analysis in heterogeneous networks.

*3) Multiple Tiers:* The second tier of interferers in a hexagonal model consists of 12 transmitters, which are located on two hexagons with radius $\sqrt{3}R$ and $2R$, respectively. For these interferers, the cell edge of the center cell is located at $\rho = 1/(2\sqrt{3})$ and $\rho = 1/4$, respectively. According to (22), the corresponding interference powers deviate at most 0.12 % and 0.049 % from the exact solution.

Interference statistics in a multi-tier network can be calculated by applying the Gamma approximation for each tier separately. This yields a sum of Gamma RVs [16], where the amount of sum terms is determined by the *number of tiers* and *not* by the *total number of interfering transmitters*, as, e.g., in [17].

*4) Regular N-polygonal Models:* As stated in Section III, the circular model can substitute any *convex regular N-polygonal* system model, also denoted as *N-gon*[1]. This is verified by determining the error between circular model and various *N-gon* models, i.e., $\max_\phi \epsilon(0.5, \phi)$, as depicted in

[1]The term $N$ denotes the number of edges of the polygon and corresponds to the number of interfering transmitters.

Figure 3. The results are obtained by replacing '6' by '$N$' in Equations (5) to (9) and (22). It is observed that the error decreases for a higher number of interferers $N$, since the $N$-*gons* converge towards a circle.

*5) Non-uniform Power Spreading:* Non-symmetric impact of the interferers can be modeled by employing non-uniform power distributions along the circle. This also allows to take heterogeneous scenarios into account.

*6) Uplink:* Similar to [18], our approach is applicable as a framework for modeling interference in the uplink.

## VI. Conclusion

In this work, we introduce a novel circular interference model. We determine its distance-dependent aggregate interference and show that the results deviate by no more than 3 % from the position-dependent interference powers in a hexagonal grid. Then, we approximate the interference statistics outside the center of a hexagonal scenario by a single Gamma random variate. We determine its distance-dependent shape- and scale parameters in closed form and identify the two key formative components of the distribution: (i) the variance of the fading and (ii) the variance due to the eccentric receiver position. A qualitative- and quantitative comparison with the exact distributions confirms the accuracy of the approximation. Gamma distributed interference enables statistical analysis of further metrics such as average rate and bit error ratio in closed form. Therefore, our approach considerably widens the scope of performance evaluation in regular grid models.


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## References


[1] J. Gertner, *The Idea Factory: Bell Labs and the Great Age of American Innovation*. Penguin Group, 2012.

[2] P. Marsch and G. Fettweis, "Static Clustering for Cooperative Multi-Point (CoMP) in Mobile Communications," in *IEEE International Conference on Communications (ICC)*, Jun. 2011, pp. 1–6.

[3] C. Ball, R. Mullner, J. Lienhart, and H. Winkler, "Performance Analysis of Closed and Open Loop MIMO in LTE," in *European Wireless Conference (EW)*, May 2009, pp. 260 –265.

[4] A. Farajidana, W. Chen, A. Damnjanovic, T. Yoo, D. Malladi, and C. Lott, "3GPP LTE Downlink System Performance," in *IEEE Global Telecommunications Conference (GLOBECOM)*, Dec. 2009, pp. 1–6.

[5] J. Giese, M. Amin, and S. Brueck, "Application of Coordinated Beam Selection in Heterogeneous LTE-Advanced Networks," in *8th International Symposium on Wireless Communication Systems (ISWCS)*, Nov. 2011, pp. 730–734.

[6] Y. Liang, A. Goldsmith, G. Foschini, R. Valenzuela, and D. Chizhik, "Evolution of Base Stations in Cellular Networks: Denser Deployment versus Coordination," in *IEEE International Conference on Communications (ICC)*, May 2008, pp. 4128–4132.

[7] T. Novlan, R. Ganti, and J. Andrews, "Coverage in Two-Tier Cellular Networks with Fractional Frequency Reuse," in *IEEE Global Telecommunications Conference (GLOBECOM)*, Dec. 2011, pp. 1–5.

[8] F. Di Salvo, "A Characterization of the Distribution of a Weighted Sum of Gamma Variables through Multiple Hypergeometric Functions," *Integral Transforms and Special Functions*, vol. 19, no. 8, pp. 563–575, 2008.

[9] M. Haenggi, J. Andrews, F. Baccelli, O. Dousse, and M. Franceschetti, "Stochastic Geometry and Random Graphs for the Analysis and Design of Wireless Networks," *IEEE Journal on Selected Areas in Communications*, vol. 27, no. 7, pp. 1029–1046, Sept. 2009.

[10] M. Haenggi, *Stochastic Geometry for Wireless Networks*. Cambridge University Press, 2012.

[11] R. W. Heath, M. Kountouris, and T. Bai, "Modeling heterogeneous network interference using poisson point processes," *IEEE Transactions on Signal Processing*, vol. 61, no. 16, pp. 4114–4126, 2013.

[12] Y. Zhuang, Y. Luo, L. Cai, and J. Pan, "A Geometric Probability Model for Capacity Analysis and Interference Estimation in Wireless Mobile Cellular Systems," in *IEEE Global Telecommunications Conference (GLOBECOM)*, Dec. 2011, pp. 1–6.

[13] K. B. Baltzis, *Cellular Networks - Positioning, Performance Analysis, Reliability*. InTech, 2011, ch. Hexagonal vs Circular Cell Shape: A Comparative Analysis and Evaluation of the Two Popular Modeling Approximations.

[14] I. S. Gradshteyn and I. M. Ryzhik, *Table of Integrals, Series, and Products*, 7th ed. Elsevier/Academic Press, Amsterdam, 2007.

[15] R. Heath, T. Wu, Y. H. Kwon, and A. Soong, "Multiuser MIMO in distributed antenna systems with out-of-cell interference," *IEEE Transactions on Signal Processing*, vol. 59, no. 10, pp. 4885–4899, 2011.

[16] P. Moschopoulos, "The distribution of the sum of independent gamma random variables." *Annals of the Institute of Statistical Mathematics*, vol. 37, p. 541544, 1985.

[17] M.-S. Alouini, A. Abdi, and M. Kaveh, "Sum of gamma variates and performance of wireless communication systems over Nakagami-fading channels," *IEEE Transactions on Vehicular Technology*, vol. 50, no. 6, pp. 1471–1480, 2001.

[18] H. Tabassum, F. Yilmaz, Z. Dawy, and M.-S. Alouini, "A framework for uplink intercell interference modeling with channel-based scheduling," *IEEE Transactions on Wireless Communications*, vol. 12, no. 1, pp. 206–217, 2013.