

Analysis of Small Cell Partitioning in Urban Two-Tier Heterogeneous Cellular Networks

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Abstract—This paper presents a system model that enables the analysis of *indoor* downlink performance in urban two-tier heterogeneous cellular networks. The urban building topology is modeled as a process of randomly distributed circles. Each building is deployed with an indoor small cell with a certain *occupation probability*. Macro base stations are sited outdoors. Their signals experience *distance-dependent shadowing* due to the blockage of the buildings. Given a typical building at the origin, expressions for the coverage probability with- and without small cell occupation are derived. The analysis of the asymmetric interference field as observed by a typical indoor user is simplified by approximations. Their accuracy is verified by Monte Carlo simulations. Our results show that isolation by wall partitioning can enhance indoor rate, and that the improvement is more sensitive to building density, rather than penetration loss per building.

Index Terms—Small Cells; Heterogeneous Cellular Network; Urban Topology; Building Density; Boolean Model; Stochastic Geometry; Virtual Building Approximation;

I. INTRODUCTION

There is a broad consensus among communication engineers that two of the key characteristics of future wireless cellular networks are *spatial randomness* and *heterogeneity* [1–14]. Yet, numerous studies have met the challenge of finding representative, analytically tractable models for the emerging systems, most of which are based on techniques from stochastic geometry [7, 11, 15]. However, when it comes to convenient expressions, this mathematical framework imposes its own particular limitations [1, 14]. Firstly, in the analysis on stochastic geometry, *shadowing* is typically incorporated by log-normally distributed Random Variates (RVs) [8, 12, 13] or neglected at all [1–7, 9, 10]. A recent study on blockage effects in urban environments indicates a dependency on the link length [16]. It follows the intuition that a longer link increases the likelihood of buildings to intersect with it. Secondly, scenarios comprising *both indoor- and outdoor environments* have not received much attention in analytical studies due to the imposed inhomogeneities on signal propagation. The designated area of operation for small cells is indoors. Existing approaches either neglect the wall partitioning [8, 17], oversimplify the macro-tier topology [4–6] or they omit cross-tier interference [12]. Considering a two-tier cellular network with outdoor macro Base Stations (BSs) and indoor-deployed small cells, our contributions are:

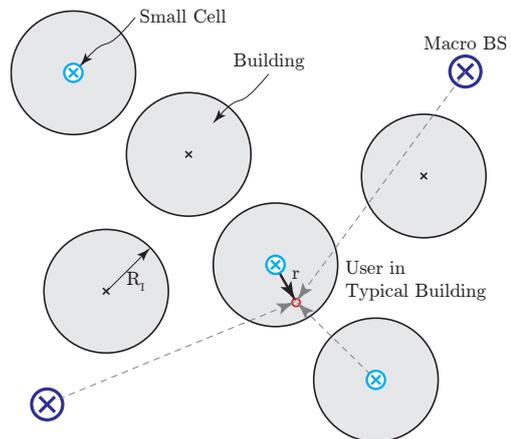


Fig. 1: Urban two-tier heterogeneous cellular network. Macro BSs are deployed in an outdoor environment. Buildings are modelled as a random process of circles and are assumed to have a fixed radius R_I . Only a fraction of buildings is occupied by small cells. The figure depicts a typical indoor user with macro BSs and neighboring small cells (dashed lines).

- A tractable model for urban environment topologies. It comprises an outdoor environment, which is partly covered by circular building objects with a certain density. Based on concepts from random shape theory, the model is applied to characterize both *signal propagation* and *network deployment*.
- A novel *virtual building approximation* to simplify aggregate-interference analysis. The key idea is to establish a user-centric interference environment by shifting the centers of the typical building and its exclusion regions to the user location.
- Analytical expressions for the *coverage probability* of indoor users with small cell- and macro BS association, assuming that a building is served by a small cell with a certain *occupation probability*.
- Evaluation of the spectral efficiency of *typical* indoor users with respect to building density, wall penetration loss and small cell occupation probability.

II. SYSTEM MODEL

A. Topology Model for Urban Environments

Consider a two-tier cellular network comprising outdoor BSs and indoor small cells, as shown in Figure 1. Buildings are modeled by a Boolean scheme of circles on the \mathbb{R}^2 plane. Therefore, the centers of the circles form a Poisson Point Process (PPP) Φ_B of intensity λ_B [18]. For simplicity, we assume that all circles have a fixed radius R_I . A point on the plane is said to be *indoors*, if it is covered by a building, and *outdoors* otherwise. Indoor- and outdoor environment are partitioned by wall penetration loss, which is hereafter denoted as L_W and assumed constant for all buildings.

B. Network Deployment

Macro BSs are distributed according to a PPP Φ_M of intensity μ_M . Note that we require these BSs to be located outdoors. Thus, the macro BS process can equivalently be constructed by independently thinning an initial PPP of density $\mu'_M = \mu_M/p_O$, where p_O equals the probability that a point on \mathbb{R}^2 is not covered by a building. According to [16, Corollary 1.2], the thinning probability is $p_O = \exp(-\lambda_B R_I^2 \pi)$.

A building will deploy an indoor small cell with a certain *occupation probability* η . Assume the indoor small cells to be located at the center points of the occupied buildings. Then, their spatial distribution can be modeled by a PPP Φ_S of intensity $\lambda_B \eta$, which results from independently thinning the object center PPP Φ_B [18].

C. User Association

In this paper, we aim to characterize the coverage and rate performance of indoor users. Noting that the buildings are assumed to form a Boolean scheme, the centers of the buildings form a PPP on the plane [16]. Therefore, by Slivnyak's theorem [18], when fixing a typical building at the origin, the centers of the other buildings still form a PPP. We will investigate the performance of users inside the typical building. We define separate association rules, depending on whether or not the typical building is occupied by a small cell.

Case 1 [Typical Building with Small Cell]: Consider a typical building at the origin, which is equipped with an indoor small cell. For simplicity, we assume that all users inside this building are associated with the small cell at the origin. We omit the cases in which indoor users at the edge of the typical building may receive stronger signals from a close-by outdoor macro BS, thus underestimating the coverage probability. Similar to the analysis in [11], *exclusion guard regions* are imposed on both macro- and small cell tier, where no BSs from the corresponding tier are allowed to distribute. For simplicity, we assume that the exclusion region for macro BSs is a ball of radius R_I centered at the origin, ensuring that no macro BSs are located inside the typical building. The exclusion region of the small cell tier is defined as a ball of radius $2R_I$ in order to prevent overlapping association regions of two small cells.

Case 2 [Typical Building without Small Cell]: When the typical building is not occupied by a small cell, the user is

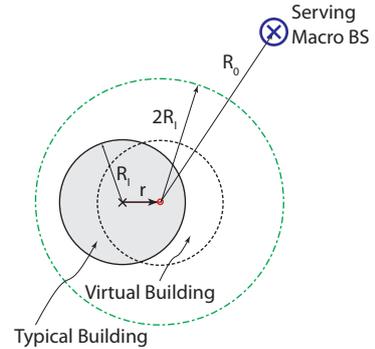


Fig. 2: Target area without small cell (gray shaded) and user-centric *virtual building* (dashed). Dashed-dotted circles denotes the shifted small cell exclusion region. The indoor user is assumed to be served by the nearest macro BS at distance R_0 .

either associated to the dominant macro BS or a small cell in the immediate vicinity. We regard the former as being of greater relevance and omit the latter, which leads to a lower bound on coverage probability. In this case, the indoor user will be served by the nearest BS of the macro tier. The same exclusion regions as defined in Case 1 are employed for macro BSs and small cells.

D. Virtual Building Approximation

Without loss of generality, a *typical indoor user* is assumed to be located at $(r, 0)$. Note that the exclusion regions as defined in Section II-C are centered at the origin rather than at the user. Consequently, the interference field as observed by the user is asymmetric and renders analysis difficult in general. Thus, we propose the following approximation.

Let (R, θ) denote the position of an interference. Its distance to a user located at $(r, 0)$ is determined as

$$d(r) = \sqrt{R^2 + r^2 - 2Rr \cos \theta}. \quad (1)$$

Since typically $R \gg r$, we approximate $d(r)$ as

$$d(r) \approx R, \quad (2)$$

which is independent of the angle θ . As shown in Figure 2, the approximation in (2) is equivalent to shifting all the BSs along with the exclusion regions by a vector $(r, 0)$, as if the typical building was centered at the user location. Thus, this approach is referred to as *virtual building approximation*, and is applied to simplify further analysis.

E. Signal Propagation

1) *Macro BS to Indoor User:* A signal originating from a macro BS experiences small scale fading, log-distance dependent path loss with path loss exponent α_O , attenuation due to building blockage and wall penetration, L_W . Small scale fading is modeled by a Rayleigh RV g_i , with $\mathbb{E}[g_i] = 1$. Along

the lines of [16, Theorem 1], the number of obstructing blockages along a link of length R is a Poisson RV with parameter $\beta_B R$, where $\beta_B = 2\lambda_B R_I$ in the introduced topology model. For analytical tractability we employ the *expected blockage attenuation*, as referred from [16, Theorem 6]. Combining blockage- and log-distance path loss along a link of length R yields

$$\ell(R) = e^{-\beta_B R(1-\gamma_B)} R^{-\alpha_O}, \quad (3)$$

where γ_B refers to the attenuation of a single blockage, also denoted as *building penetration loss*. Note that (3) characterizes *shadowing* entirely by the parameters of the underlying environment topology and accounts for the condition that the macro BS is deployed outdoors.

2) *Small Cell to Indoor User*: When user and small cell are situated in the same building, the signal experiences small scale fading and log-distance path loss with exponent α_I .

The signals from all other small cells are subject to small scale fading, log-distance path loss with exponent α_O and attenuation by a factor L_W^2 , as caused by the indoor-to-outdoor and outdoor-to-indoor wall penetration. Since small cell transmit power is typically low, only small cell interferers from *neighboring* buildings are taken into account. We define two building as being *neighbors* to each other, if the segment connecting their centers is not intersected by any other building.

III. PERFORMANCE ANALYSIS

In this section we derive analytical expressions for the coverage probability of an indoor users at position $(r, 0)$, regarding both buildings with- and without small cell deployment. We assume the network to be interference limited, as is typically the case in urban areas [19]. Thus, thermal noise is neglected in the analysis.

A. Typical Building with Small Cell

Assume the typical building to be occupied by a small cell. Then, the Signal-to-Interference-Ratio (SIR) at distance r , $0 < r \leq R_I$, is determined as

$$\text{SIR}^{(S)}(r) = (P_S g_0 r^{-\alpha_I}) \left(\sum_{i:R_i>R_I} P_M g_i L_W \ell(R_i) + \sum_{i:R_i>2R_I} S_i P_S g_i L_W^2 R_i^{-\alpha_O} \right)^{-1}, \quad (4)$$

where the terms P_M and P_S denote macro BS- and small cell transmit powers, $\ell(\cdot)$ corresponds to the combined blockage- and path loss attenuation, as defined in (3) and R_i refers to the length of link i . The RVs S_i are Bernoulli distributed and, by [16, Theorem 1], have parameters $\exp(-\beta_B R_i - p_B)$, where $p_B = \lambda_B R_I^2 \pi$. They indicate whether or not an interfering small cell is in a neighboring building of the typical user.

Theorem 1. *Consider a user at distance r , $0 < r \leq R_I$, away from the center of a small cell-occupied building. Then, its coverage probability is determined as*

$$P_c^{(S)}(\delta|r) = \mathbb{P}[\text{SIR}^{(S)}(r) > \delta|r] = e^{-2\pi(\mu_M I_M + \mu_S I_S)}, \quad (5)$$

where

$$I_M = \int_{R_I}^{\infty} \left(1 - \frac{\frac{P_S}{P_M}}{\frac{P_S}{P_M} + \delta \ell(t) L_W r^{\alpha_I}} \right) t dt \quad (6)$$

$$I_S = \int_{2R_I}^{\infty} \left(\frac{\delta L_W^2 r^{\alpha_I} e^{-(\beta_B t + p_B)}}{t^{\alpha_O} + \delta L_W^2 r^{\alpha_I}} \right) t dt \quad (7)$$

Proof: Applying (4), we exploit that g_i are i.i.d. exponential RVs and S_i are Bernoulli RVs with parameters $\exp(-\beta_B R_i - p_B)$. Then, it follows from Campbell's theorem (see e.g., [18]) that

$$\begin{aligned} P_c^{(S)}(\delta|r) &= \mathbb{P}[\text{SIR}^{(S)}(r) > \delta|r] \\ &= \mathbb{E}_{\Phi_M} \left[\prod_{i:R_i>R_I} \frac{\frac{P_S}{P_M}}{\frac{P_S}{P_M} + \delta \ell(R_i) L_W r^{\alpha_I}} \right] \\ &\quad \mathbb{E}_{\Phi_S} \left[\prod_{i:R_i>R_S} \left(1 - \frac{\delta L_W^2 r^{\alpha_I} e^{-(\beta_B R_i + p_B)}}{R_i^{\alpha_O} + \delta L_W^2 r^{\alpha_I}} \right) \right] \end{aligned} \quad (8)$$

Finally, (5) is obtained by computing the Laplace functional [18]. \blacksquare

B. Typical Building without Small Cell

Assume a dominant macro BS to be located at distance R_0 , $R_0 > R_I$ away from the center of the typical building and consider that this building is not occupied by a small cell. Then, the SIR at distance r , $0 < r \leq R_I$, calculates as

$$\text{SIR}^{(M)}(R_0) \approx (P_M g_0 \ell(R_0)) \left(\sum_{i:R_i>R_0} P_M g_i \ell(R_i) + \sum_{i:R_i>2R_I} S_i P_S g_i L_W R_i^{-\alpha_O} \right)^{-1} \quad (9)$$

Note that (i) the expression is independent of r and (ii) the factor L_W is omitted, since attenuation due to wall penetration is experienced by all signals and therefore cancels out in the SIR term.

Theorem 2. *Consider a user at distance r , $0 < r \leq R_I$, away from the center of a typical building without small cell and assume that it is associated with its dominant macro BS. Then, its coverage probability is determined as*

$$\begin{aligned} P_c^{(M)}(\delta) &= \mathbb{P}[\mathbb{E}_{R_0}[\text{SIR}^{(M)}(R_0) > \delta]] \\ &= \int_{R_I}^{\infty} P_c^{(M)}(\delta|R) f_{R_0}(R) dR, \end{aligned} \quad (10)$$

where

$$P_c^{(M)}(\delta|R_0) = e^{-2\pi(\mu_M I_M + \mu_S I_S)}, \quad (11)$$

with

$$I_M = \int_{R_0}^{\infty} \left(1 - \frac{\ell(R_0)}{\ell(R_0) + \delta \ell(t)} \right) t dt, \quad (12)$$

$$I_S = \int_{2R_I}^{\infty} \left(\frac{\delta L_W \frac{P_S}{P_M} e^{-(\beta_B t + p_B)}}{t^{\alpha_O} \ell(R_0) + \delta L_W \frac{P_S}{P_M}} \right) t dt, \quad (13)$$

TABLE I: Simulation Parameters

Macro-to-small cell power ratio	$\frac{P_S}{P_M}$	10^{-2}
Macro BS density	μ_M	$4.61 \times 10^{-6} \text{ m}^{-2}$
Outdoor path loss exponent	α_O	4
Indoor path loss exponent	α_I	2
Radius of building area	R_I	25 m

and

$$f_{R_O}(R) = \begin{cases} 2\pi\mu_M R e^{-\pi\mu_M(R^2 - R_I^2)} & , R \geq R_I \\ 0 & , \text{otherwise} \end{cases} \quad (14)$$

Proof: The conditional coverage probability $P_c^{(M)}(\delta|R)$ in (11) is derived along the lines of (5). Conditioning on the dominant macro BS distance leads to (10), where $f_{R_O}(R)$ in (14) is the nearest neighbor distance distribution of a homogeneous PPP outside a ball of radius R_I [18].

C. Typical Indoor User

The coverage probability of a *typical* indoor user at distance r , $0 < r \leq R_I$, is obtained by linearly combining $P_c^{(S)}(\delta|r)$ from (5) and $P_c^{(M)}(\delta)$ from (10) according to the small cell occupation probability η . Then,

$$P_c(\delta|r) = \eta P_c^{(S)}(\delta|r) + (1 - \eta) P_c^{(M)}(\delta). \quad (15)$$

IV. NUMERICAL EVALUATION

In this section, we numerically evaluate the performance of a typical user at the edge of a building, i.e., $r = R_I$. At this location, the proposed *virtual building approximation* is expected to perform worst.

Spectral efficiency is employed as a metric. It is defined as $\tau = \mathbb{E}_{\text{SIR}}[\log_2(1 + \text{SIR})]$ and can be reformulated in terms of coverage probability as

$$\tau(r) = \frac{1}{\log(2)} \int_0^{\delta_{\max}} \frac{P_c(\delta|r)}{\delta + 1} d\delta, \quad (16)$$

with $P_c(\delta|r)$ from (15) and $\delta_{\max} = 2^6 - 1$, referring to 64-Quadrature Amplitude Modulation (QAM), which is the highest modulation order in the current Long Term Evolution Advanced (LTE-A) standard [20].

Parameters for evaluation are listed in Table I. To verify the accuracy of the *virtual building approximation*, Monte Carlo simulations are carried out, using the system model as introduced in sections II-A and II-B. Macro BS density is chosen such that the inscribing ball of the typical cell has $R_c = 250$ m and the BSs are distributed over a field of $15 R_c \times 15 R_c$. The results are estimated from averaging over 500 fading- and 500 spatial realizations.

Figure 3 depicts spectral efficiency over indoor coverage ratio, which is defined as $1 - p_O$ in Section II-B. Note that when fixing the average building *size*, the indoor coverage ratio scales with the *density* of the buildings. Solid- and dashed lines correspond to analysis and simulations, respectively. Results

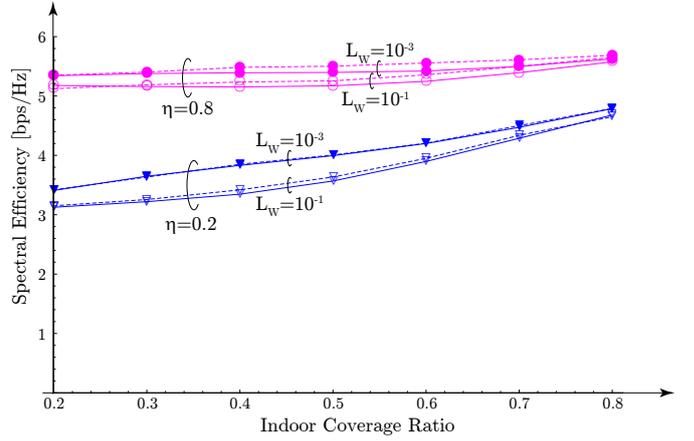


Fig. 3: Spectral efficiency [bps/Hz] over area ratio, which is covered by buildings. Solid- and dashed lines denote results from analysis and simulations, respectively. Curves are shown for varying small cell occupation probability (plot markers "▽" refer to $\eta = 0.2$ and "○" refer to $\eta = 0.8$, respectively) and wall penetration loss, L_W .

are shown for a sparse- and a dense small cell deployment, as quantified by the occupation probability η . For both scenarios, weak- and strong wall partitioning are investigated. The wall penetration loss is correlated to the building penetration loss γ_B , as introduced in (3). In this paper, we conservatively set $\gamma_B = L_W$. This assumption can be replaced by more elaborated models in further work.

It is observed that

- The achievable spectral efficiency is improved by increasing building density. This result follows the intuition that obstructions due to large objects establish a safe-guard against interference [16]. Note that for constant occupation probability, the small cell density grows in proportion to the building density. Therefore, the results render the existence of a *hotspot limited regime* in urban environments questionable and support simulation results in [5, 6, 21, 22].
- Low isolation by wall penetration deteriorates performance in both deployment scenarios. Intuitively, the isolation of the interfering small cells is decreased when the wall penetrations become weaker. The impact of penetration loss on coverage probability, however, becomes minor especially when the building density is high. Intuitively, this indicates that *the number of penetrations rather than the loss per penetration dominates the effect of partitioning between indoor and outdoor environment*.
- Even though we evaluate a user at the edge of a typical building, the analytical results closely resemble the simulations. This confirms the accuracy of the *virtual building approximation* as well as the inclusion of macro interferers in the immediate vicinity of the typical building, as claimed in Section II-C.

V. CONCLUSION

In this paper, we introduced a novel system model for two-tier heterogeneous cellular networks in urban environments. We focused on indoor users and derived analytical expressions for the coverage probability in buildings with- and without small cell deployment. Our proposed *virtual building approximation* considerably improved the tractability of the analysis and its accuracy was confirmed by simulation results. Numerical evaluations were carried out to investigate the performance of a *typical* indoor user in terms of spectral efficiency. The results revealed subtle but crucial effects of an urban environment. Observations such as the blockage safeguard and the vanishing impact of wall isolation with increasing building density have been missed by overly simplistic models. Further work includes physical aspects such as intra-building interference, transmitter-receiver height aspects and a distinction between line-of-sight- and non-line-of-sight dominant interferers.

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