Now Or Never
End-game effects in R&D project investment

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This paper investigates the valuation of an option to invest in a project of uncertain cost on a finite time horizon. The common body of knowledge[5][3] attributes a positive value to flexibility and the possibility to postpone an investment in order to wait for the arrival of new information. From this point of view an investor will assume that as time passes and the R&D projects deadline [1] approaches the value of the option to invest in the project will gradually vanish. Hence it appears logical that the willingness to invest in a project declines as time is running out. In my work I show that under certain conditions the opposite is the case i.e. as the deadline for completion approaches, investors increase their willingness to invest in the progress of the R&D project as a last effort to claim the reward and avoid failure. What appears as non-rational behavior of an investor who seeks to avoid sunk costs, is actually a fully rational investment strategy.

I define the investor's choice of options as follows. At any point in time the investor has to decide whether to keep the option to invest in the project or abandon it. If he decides to shut down, the control variable I is set to zero and the option value immediately becomes zero as well. If, on the other hand, the investor wants to keep the opportunity to successfully complete the project an investment of at least I_κ is required. I denote this the periodic *keep-alive-cost* of the project and the associated investor behavior *acting passive*. This minimal funding, however, does not contribute to the projects progress, yet it allows the investor to observe the next random-shock to estimated cost to completion C_t and thereby face a new decision of the same kind. This is economically comparable to the periodic maintenance cost mentioned in [2]. Any allocation that exceeds I_κ (up to a limit of $I_{\max}$) directly reduces the projects remaining cost i.e. $E(C_{t+1}) = C_t - (I - I_\kappa)dt = C_t - I_\pi dt$. I denote this *active investment* or I_π in equations respectively. In Equation 1 the choice of values for the investment variable I is indicated in a clear mathematical form.

$$I = \begin{cases} 0 & \text{, abandon project} \\ I_\kappa & \text{, keep alive and wait (passive project)} \\ (I_\kappa, I_{\max}] & \text{, invest in the project (active project)} \end{cases} \tag{1}$$

From this definition I expect to find three distinct regions within the vector space spanned by the time and cost axis. These regions, one for shutdown, passive and active project each, are separated by two curves. I denote these curves the *activation boundary*, i.e. the curve where an investor is indifferent whether to wait or push the project for another increment on the time-axis, and the *shutdown boundary*, i.e. the curve where an

[1] Consider milestones, contracts, venture capitalist's requirements et cetera as possible reasons for the terminal date.

investor is indifferent whether to wait for another random-shock along the time-axis or immediately abandon the R&D project. Figure 2 illustrates these boundaries by coloring discrete cost nodes according to investor's actions.

I characterize an arbitrary project by the reward for successful completion R, a finite time horizon i.e. deadline T, and a parameter σ that specifies the size of the random shocks that affect the projects cost to completion $C(t)$. The project is considered a success if $C(t) = 0$ and a failure if $C(T) > 0$. For all other points in time the R&D project's cost to completion evolves as controlled[2] CIR (Cox-Ross-Ingersoll) diffusion process (Eq.2) where the active project's investment $-I_\pi dt$ is the drift in remaining cost imposed by the investor, and $\sigma C_t^{\frac{1}{2}} dW_t$ are the random shocks that affect the project's development as information unfolds.[3]

$$dC_t = -I_\pi dt + \sigma C_t^{\frac{1}{2}} dW_t \tag{2}$$

I model the possible values of $C(t)$ as nodes on a discrete grid, with indexes n and t, and solve the optimization problem by application of Bellman's dynamic programming method (eq.3). From the optimization results, I find that the value function's form shown in fig.1 accords with the existing literature and that the smooth-pasting condition [3] is fulfilled. The eight-pointed star indicates the first node on the cost axis where the Tobin's marginal q equals 1.

$$V\{C[n,t]\} = \begin{cases} 0 & ,t = T \wedge C[n,t] > 0 \\ R & ,t \leq T \wedge C[n,t] = 0 \\ \max\limits_{I_\pi \in \mathbb{R}^+} [0; -I_\kappa \Delta t - I_\pi \Delta t + \frac{1}{(1+r)^{\Delta t}} \times [\pi(I_\pi) \times \dots & \\ V\{C[n+j+1, t+1]\} + (1 - \pi(I_\pi)) \times \dots & \\ V\{C[n+j-1, t+1]\}]] & ,t < T \wedge C[n,t] > 0 \end{cases} \tag{3}$$

If the remaining time span to the dead-line is large, the estimated cost to completion at which an investor is willing to fund the project strives toward a fixed point i.e. the stars will concentrate on a single spot. On a short horizon however, connecting these points does, in contrast to the common body of knowledge, not yield a monotonic curve but exhibits a distinct turning point in the investor's willingness to fund the R&D project instead. At the activation boundary, the investor considers the trade-off between investing a marginal monetary unit and just keeping the option alive. Since the value of waiting deteriorates faster than the value of the ongoing project does, this trade-off moves in favor of an *active project* before the investor gradually gives up due to the time constraint. Another explanation is that the further away from the terminal date the more potential call options[1] to begin the R&D project are available to the investor. As

[2]The description refers to the replacement of the CIR process' auto-regressive drift by a control variable.

[3]Note that as Pindyck points out $E[\int_0^T I(C_t, t) * e^{-rt} dt] = C_0$ [5]

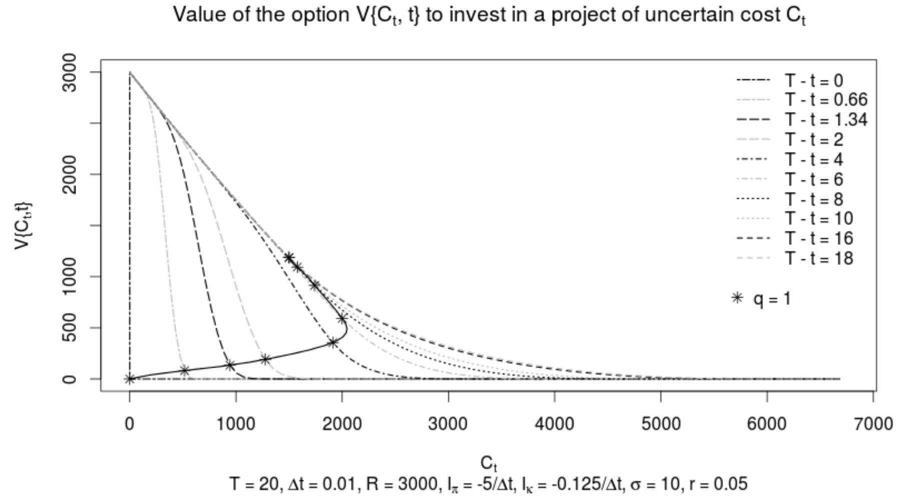

Figure 1: Value function curves with eight-pointed stars indicating the maximum remaining cost where an investor is willing to push the project, at these nodes the Tobin's marginal q is equal or slightly larger than 1.

these options deplete the incentive to invest grows and thereby the willingness to accept higher costs rises. Figure 2 focuses on the investor's decisions in discrete time and adds a line for the cost to completion where the willingness to fund the project comes to rest if the time horizon is sufficiently large. Additionally the plot visualizes the maximum drift that an investor may impose, be it because of technical or financial restrictions. Thereby it uncovers, that investors are willing to take gambles in order to keep their chances to claim the reward if the deadline is approaching.

With this article I show that a discrete evaluation of investments under uncertainty, e.g. large R&D projects, can shed new light on the optimum investor's behavior for projects of uncertain cost with a specified terminal date. I observe an investment strategy that would not be expected from a fully rational investor but from an individual that wishes to avoid sunk costs. Furthermore I relate the dynamic programming results with existing q-theory [1] and find investors willing to push the project as soon as $q \geq 1$. Also I find that the curve that connects the points where the Tobin's marginal $q = 1$ exhibits a distinct hump which I denote the *endgame effect in a rational investor's behavior*.

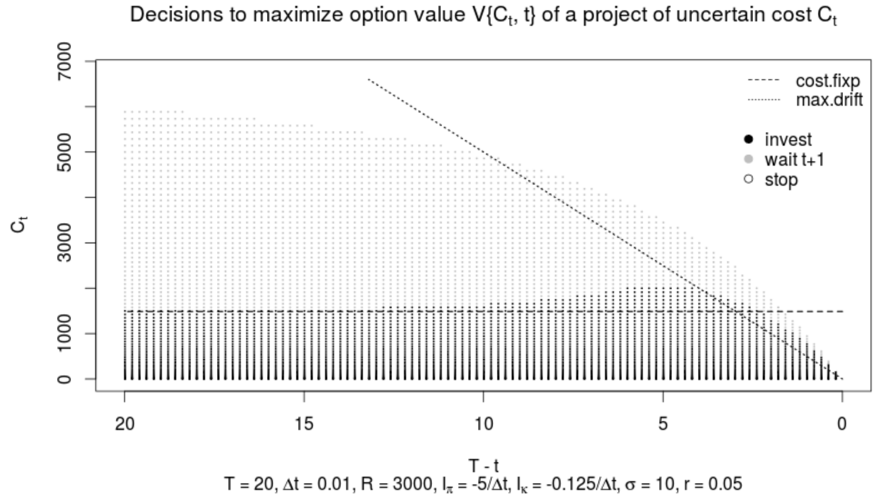

Figure 2: Investor behavior regions indicating the activation and exit boundary. Note how the investor is pushing the project way after the point where he could successfully finish by his own powers is passed.

References

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